Supplementary Material

S1 Small N, large T

Parameters:

- N=20
- T=30

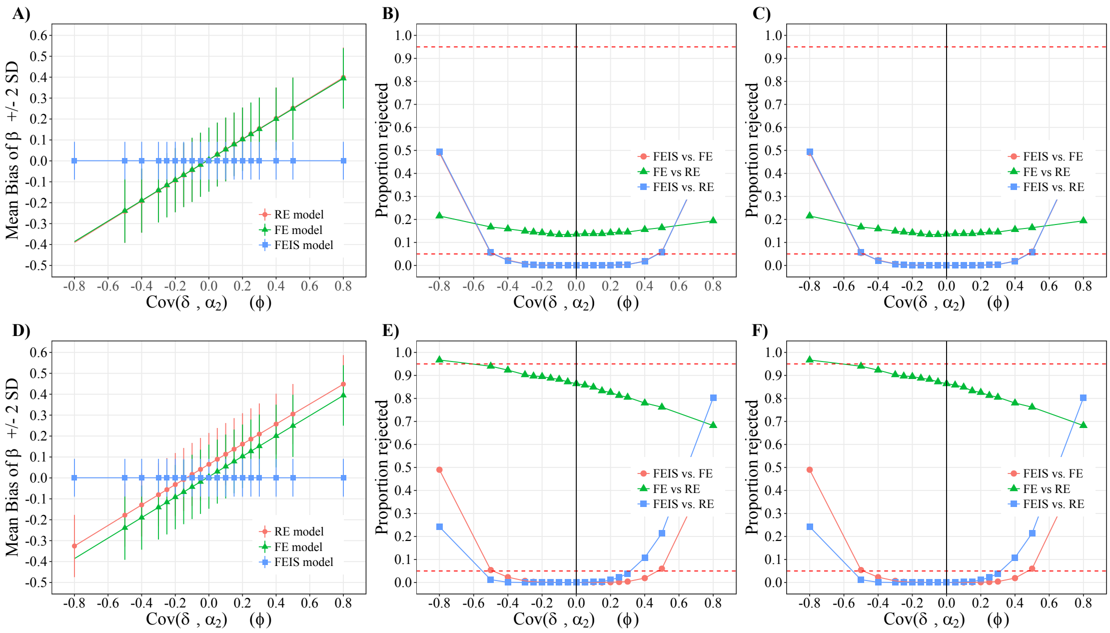

Figure S1: Simulated bias of RE, FE and FEIS estimators and rejection rates of ART (B,E) and BSHT (C, F) tests for $\theta = 0$ (A, B, C) and $\theta = 1$ (D, E, F). θ = effect of time-constant unobserved heterogeneity $\boldsymbol{\alpha}_1$ on covariate $\boldsymbol{x}$; $\phi = \text{Cov}(\boldsymbol{\delta}, \boldsymbol{\alpha}_2)$; $\boldsymbol{\delta}$ = effect of slope variable $\boldsymbol{w}$ on covariate $\boldsymbol{x}$; $\boldsymbol{\alpha}_2$ = effect of slope variable $\boldsymbol{w}$ on outcome $\boldsymbol{y}$; $N = 20$, $T = 30$, $R = 1000$, $R_b = 100$.

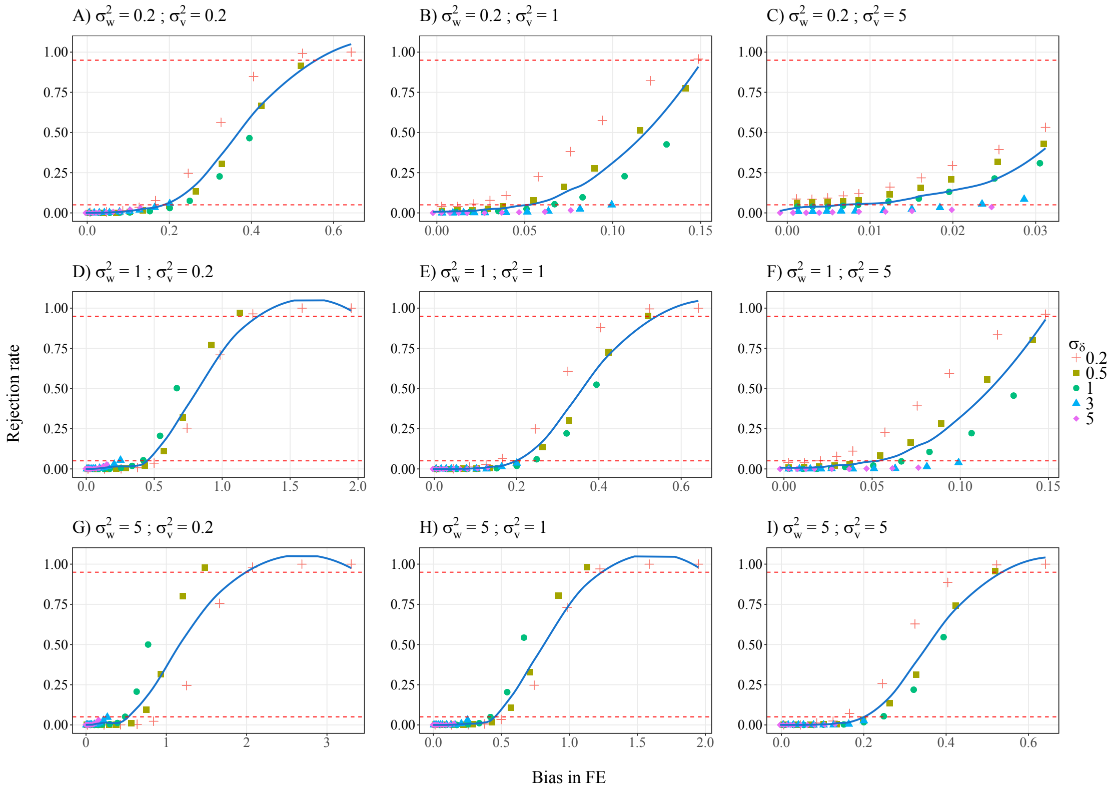

Figure S2: Rejection rates of Artificial Regression Test with robust standard errors. $\boldsymbol{\delta}$ = effect of slope variable $\boldsymbol{w}$ on covariate $\boldsymbol{x}$; $\boldsymbol{\alpha}_2$ = effect of slope variable $\boldsymbol{w}$ on outcome $\boldsymbol{y}$; $\boldsymbol{v}$ = independent random vector in covariate $\boldsymbol{x}$; σ^2 = variance of $\boldsymbol{\delta}$, $\boldsymbol{w}$, $\boldsymbol{v}$ respectively; $N = 20$, $T = 30$, $R = 1000$, $R_b = 100$.

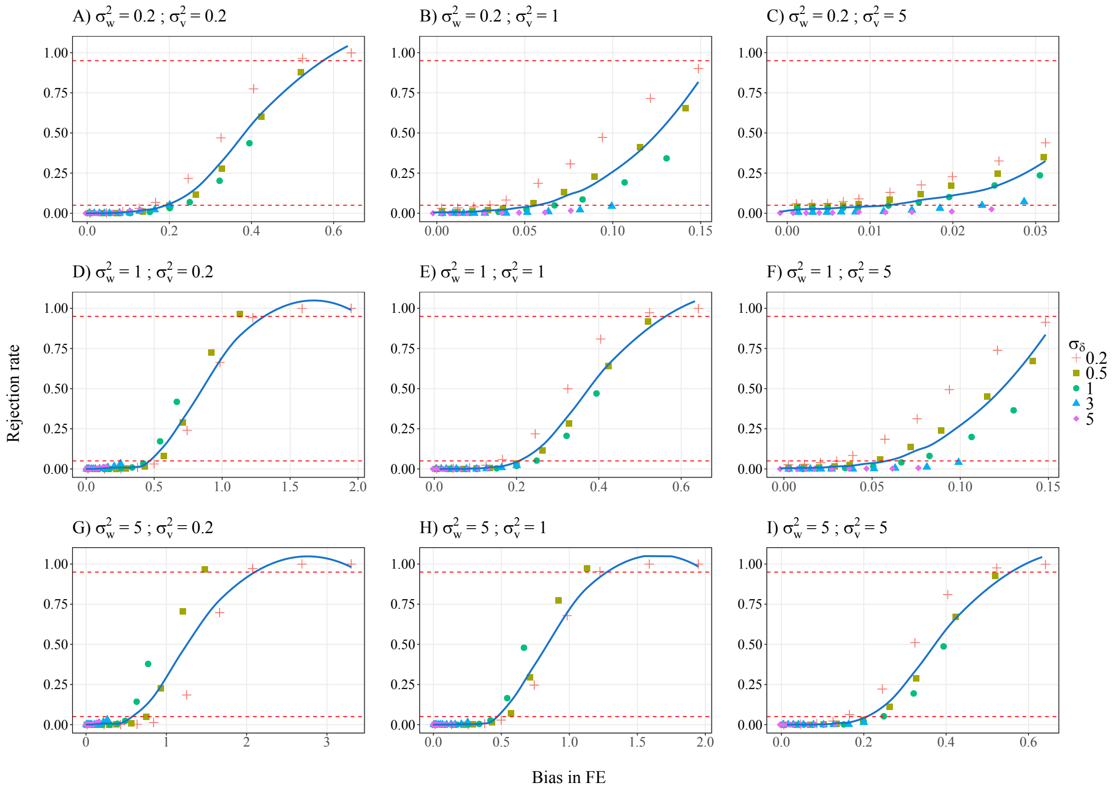

Figure S3: Rejection rates of Bootstrapped Hausman Test. $\boldsymbol{\delta}$ = effect of slope variable $\boldsymbol{w}$ on covariate $\boldsymbol{x}$; $\boldsymbol{\alpha}_2$ = effect of slope variable $\boldsymbol{w}$ on outcome $\boldsymbol{y}$; $\boldsymbol{v}$ = independent random vector in covariate $\boldsymbol{x}$; σ^2 = variance of $\boldsymbol{\delta}$, $\boldsymbol{w}$, $\boldsymbol{v}$ respectively; $N = 20$, $T = 30$, $R = 1000$, $R_b = 100$.

S2 Lagged treatment effects

- DGP: $y_{in} = x_{in}\beta + x_{in-1}\beta\zeta + x_{in-2}\beta\zeta + \alpha_{1i} + w_{in}\alpha_{2i} + \epsilon_n$, with data sorted ascending based on the slope variable w. Everything else equal to setting 1 in the main text.
- Estimation model includes only x_{in}.

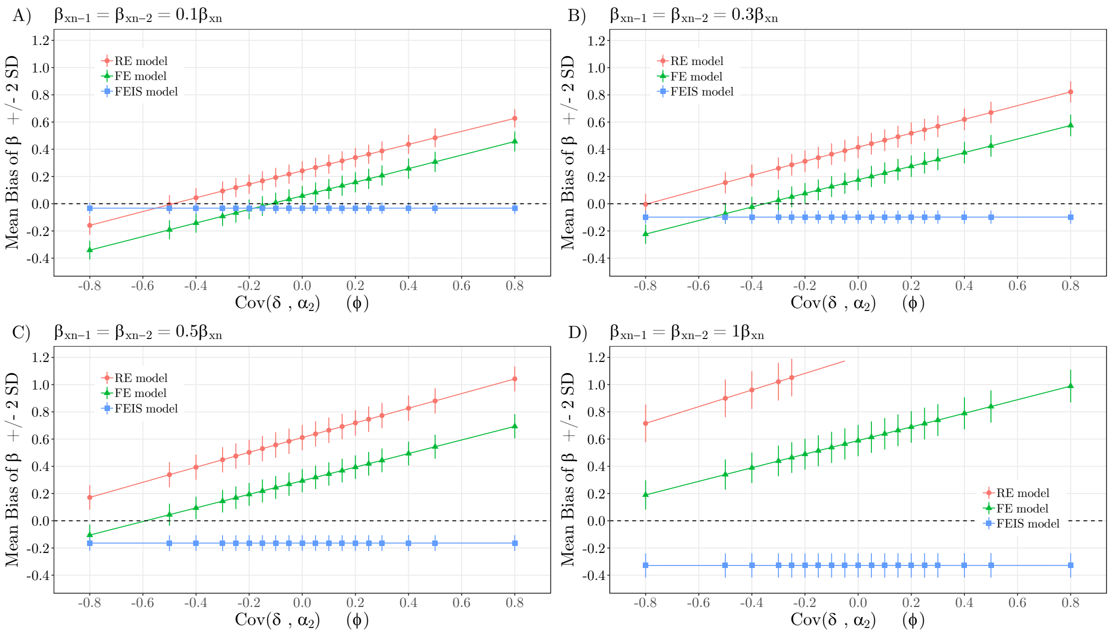

Figure S4: Simulated bias of RE, FE and FEIS estimators for x_n under different scenarios of lagged treatment effects ($x_{n-1}, x_{n-2} \neq 0$, where data is sorted according to slope $\boldsymbol{w}$). θ = effect of time-constant unobserved heterogeneity $\boldsymbol{\alpha}_1$ on covariate $\boldsymbol{x}$; $\phi = \text{Cov}(\boldsymbol{\delta}, \boldsymbol{\alpha}_2)$; $\boldsymbol{\delta}$ = effect of slope variable $\boldsymbol{w}$ on covariate $\boldsymbol{x}$; $\boldsymbol{\alpha}_2$ = effect of slope variable $\boldsymbol{w}$ on outcome $\boldsymbol{y}$; $\theta = 1$, $N = 300$, $T = 10$, $R = 1000$.

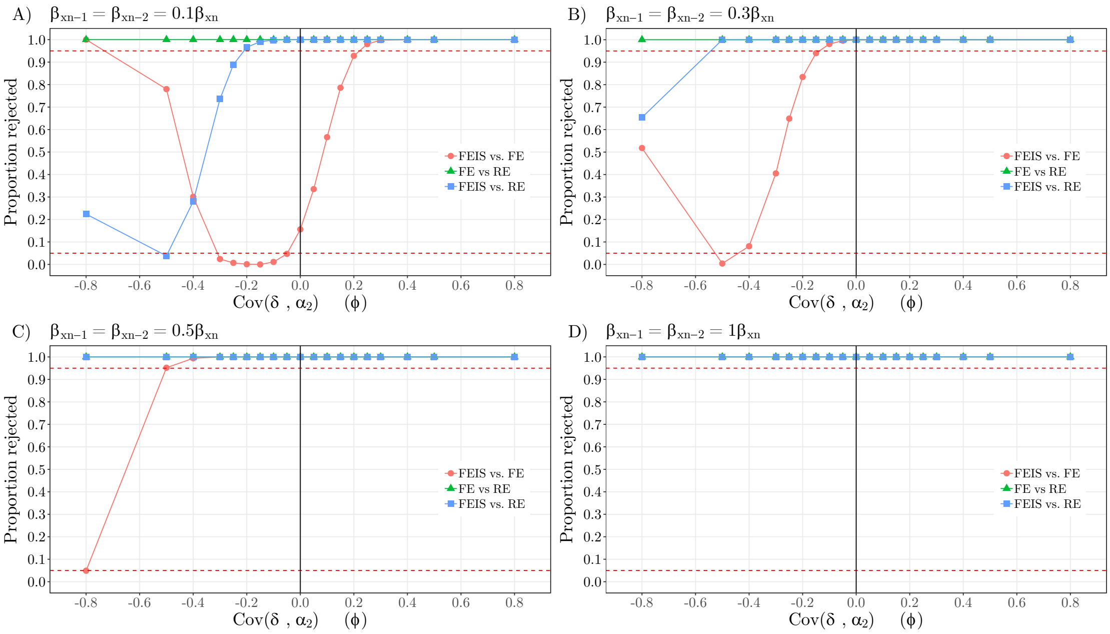

Figure S5: Simulated rejection rate of robust ART for x_n under different scenarios of lagged treatment effects ($x_{n-1}, x_{n-2} \neq 0$, where data is sorted according to slope $\boldsymbol{w}$). θ = effect of time-constant unobserved heterogeneity $\boldsymbol{\alpha}_1$ on covariate $\boldsymbol{x}$; $\phi = \text{Cov}(\boldsymbol{\delta}, \boldsymbol{\alpha}_2)$; $\boldsymbol{\delta}$ = effect of slope variable $\boldsymbol{w}$ on covariate $\boldsymbol{x}$; $\boldsymbol{\alpha}_2$ = effect of slope variable $\boldsymbol{w}$ on outcome $\boldsymbol{y}$; $\theta = 1$, $N = 300$, $T = 10$, $R = 1000$.

S3 Lagged treatment effects (specified in estimation model)

- DGP: $y_{in} = x_{in}\beta + x_{in-1}\beta\zeta + x_{in-2}\beta\zeta + \alpha_{1i} + w_{in}\alpha_{2i} + \epsilon_n$, with data sorted ascending based on the slope variable w. Everything else equal to setting 1 in the main text.
- Estimation model includes $x_{in} + x_{in-1} + x_{in-2}$.
- Note: Bias is shown in % of true $\beta\zeta$, as $\beta\zeta \neq 1$ for panels B, C in Figure S6.

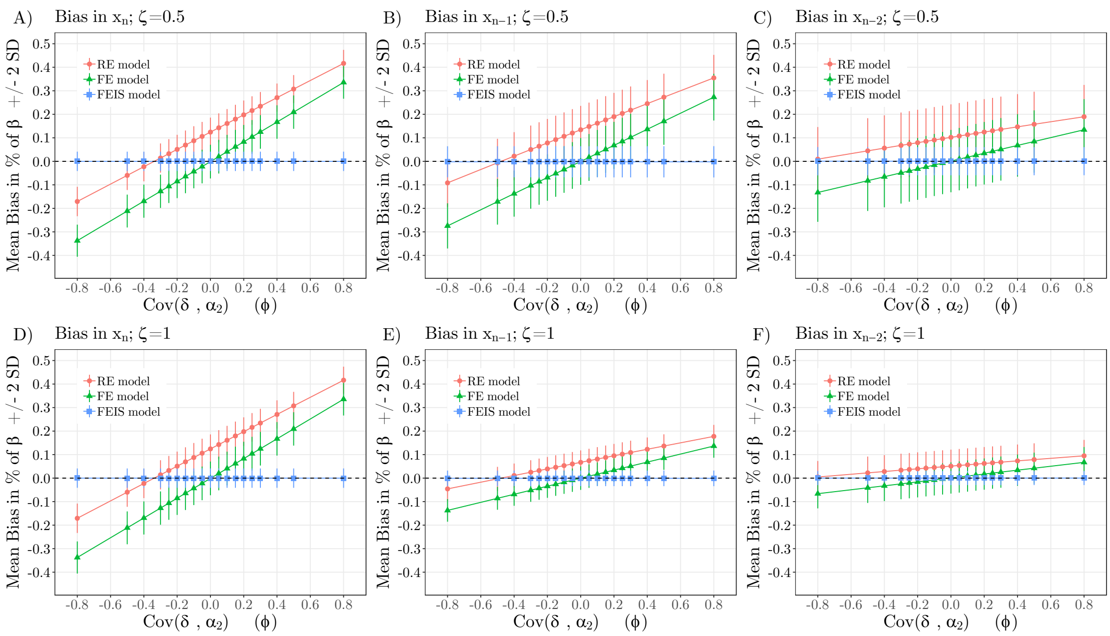

Figure S6: Simulated bias in % of RE, FE and FEIS estimators for x_n, x_{n-1}, x_{n-2} for different strength of ζ, where data is sorted according to slope $\boldsymbol{w}$. θ = effect of time-constant unobserved heterogeneity $\boldsymbol{\alpha}_1$ on covariate $\boldsymbol{x}$; $\phi = \text{Cov}(\boldsymbol{\delta}, \boldsymbol{\alpha}_2)$; $\boldsymbol{\delta}$ = effect of slope variable $\boldsymbol{w}$ on covariate $\boldsymbol{x}$; $\boldsymbol{\alpha}_2$ = effect of slope variable $\boldsymbol{w}$ on outcome $\boldsymbol{y}$; $\theta = 1$, $N = 300$, $T = 10$, $R = 1000$.

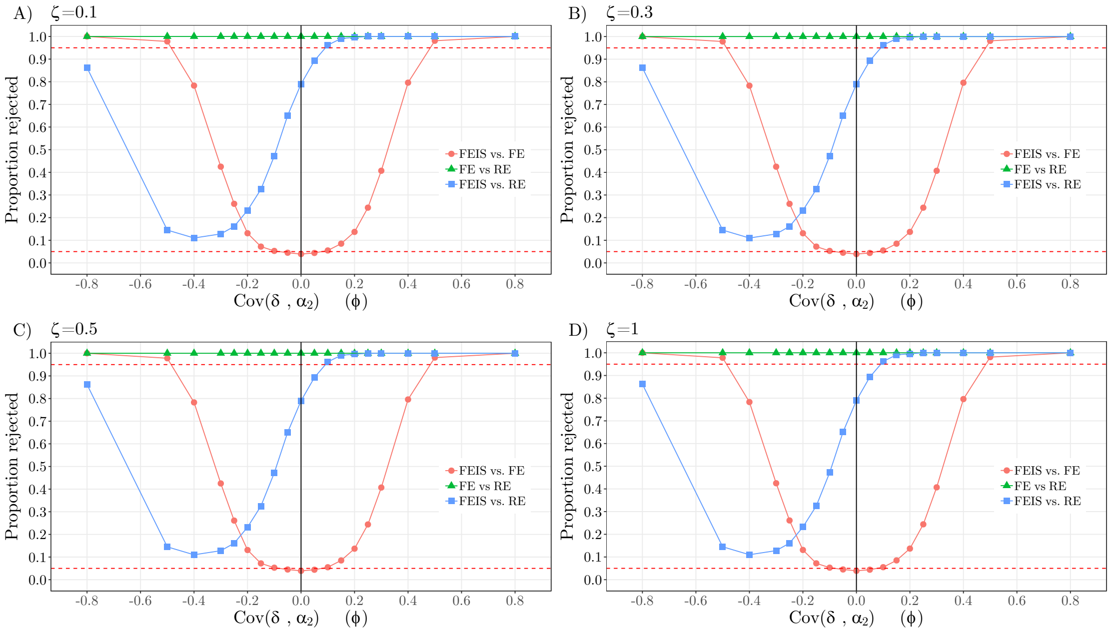

Figure S7: Simulated rejection rate of robust ART for x_n under different scenarios of lagged treatment effects ($x_{n-1}, x_{n-2} \neq 0$, where data is sorted according to slope $\boldsymbol{w}$). θ = effect of time-constant unobserved heterogeneity $\boldsymbol{\alpha}_1$ on covariate $\boldsymbol{x}$; $\phi = \text{Cov}(\boldsymbol{\delta}, \boldsymbol{\alpha}_2)$; $\boldsymbol{\delta}$ = effect of slope variable $\boldsymbol{w}$ on covariate $\boldsymbol{x}$; $\boldsymbol{\alpha}_2$ = effect of slope variable $\boldsymbol{w}$ on outcome $\boldsymbol{y}$; $\theta = 1$, $N = 300$, $T = 10$, $R = 1000$.

S4 Panel selection / attrition

- A) and B) Panel selection based on response: $\boldsymbol{s} = 1[p(\ddot{\boldsymbol{Y}}/\sigma_{\ddot{Y}}) + (1-p)\boldsymbol{\varepsilon} > 0]$ and $\boldsymbol{s} = 0$ otherwise, where $\sigma^2_{\ddot{Y}}$ is the variance of $\ddot{\boldsymbol{Y}}$ and $\boldsymbol{\varepsilon} \sim N(0,1)$. p indicates the proportion of the variance in the selection indicator $\boldsymbol{s}$ which comes from the response variable $\boldsymbol{Y}$.
- C) and D) Panel selection based on heterogeneous treatment groups: sample units are randomly allocated to group g_1 or g_2 with heterogeneous treatment effects $\beta_{g1} = 2\beta_{g2}$, while we still keep $(\beta_{g1} + \beta_{g2})/2 = 1$. Likewise we specify diverging parameters for the probability of contributing only two observations / time periods for each unit: $P(T_i = 2|g_1) = q$ and $P(T_i = 2|g_2) = (1-q)$, $T_i = 10$ otherwise. Subsequently, for each selected unit, 2 observations / time periods are randomly drawn to remain in the estimation sample.

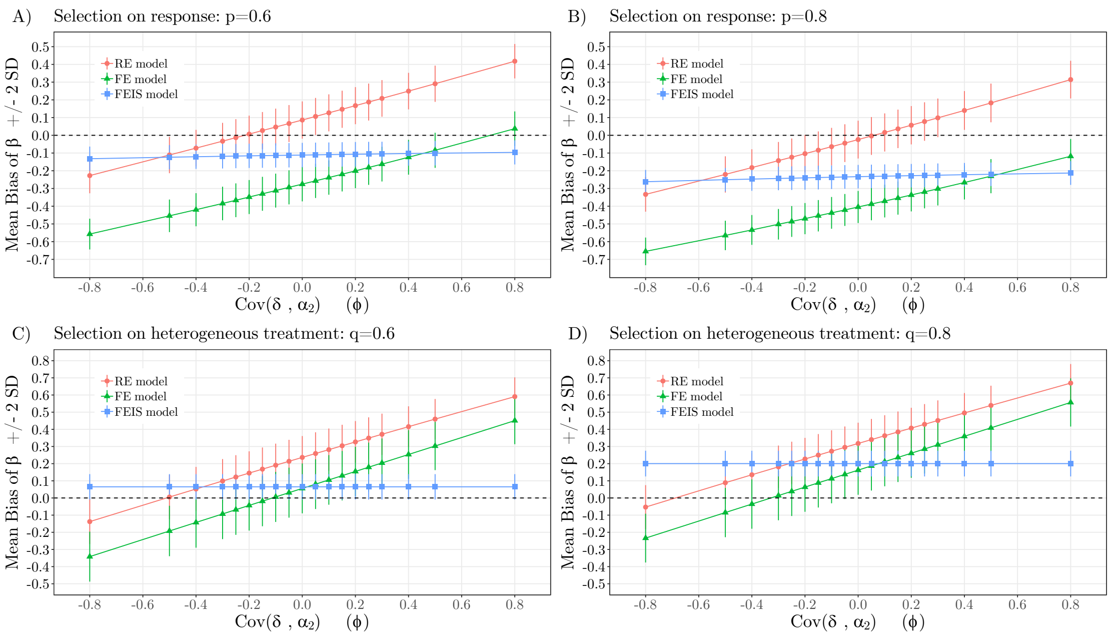

Figure S8: Simulated bias of RE, FE and FEIS estimators under different scenarios of panel selection as a function of $\ddot{Y}_t$ (A, B), and as a function of heterogeneous treatment groups, where $\beta_{g1} = 2\beta_{g2}$ and $s_{g1} > s_{g2}$ (C, D). θ = effect of time-constant unobserved heterogeneity $\boldsymbol{\alpha}_1$ on covariate $\boldsymbol{x}$; $\phi = \mathrm{Cov}(\boldsymbol{\delta}, \boldsymbol{\alpha}_2)$; $\boldsymbol{\delta}$ = effect of slope variable $\boldsymbol{w}$ on covariate $\boldsymbol{x}$; $\boldsymbol{\alpha}_2$ = effect of slope variable $\boldsymbol{w}$ on outcome $\boldsymbol{y}$; $\theta = 1$, $N_{total} = 300$, $T = 1 - 10$, $R = 1000$.

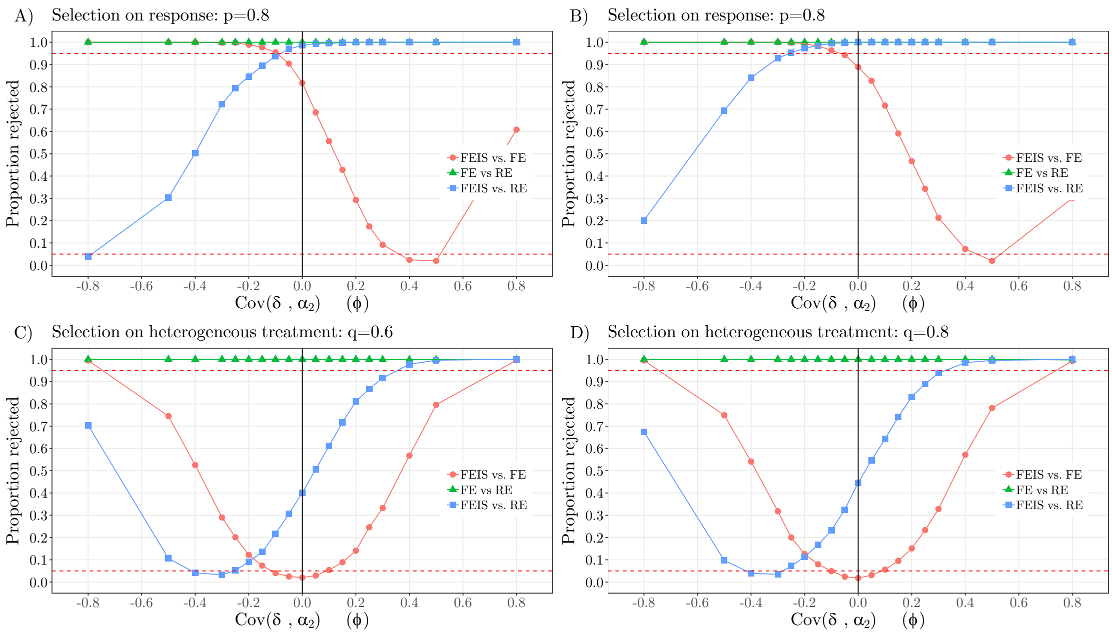

Figure S9: Simulated rejection rate of robust ART under different scenarios of panel selection as a function of $\ddot{Y}_t$ (A, B), and as a function of heterogeneous treatment groups, where $\beta_{g1} = 2\beta_{g2}$ and $s_{g1} > s_{g2}$ (C, D). θ = effect of time-constant unobserved heterogeneity $\boldsymbol{\alpha}_1$ on covariate $\boldsymbol{x}$; $\phi = \text{Cov}(\boldsymbol{\delta}, \boldsymbol{\alpha}_2)$; $\boldsymbol{\delta}$ = effect of slope variable $\boldsymbol{w}$ on covariate $\boldsymbol{x}$; $\boldsymbol{\alpha}_2$ = effect of slope variable $\boldsymbol{w}$ on outcome $\boldsymbol{y}$; $\theta = 1$, $N_{total} = 300$, $T = 1 - 10$, $R = 1000$.