exercise3 (Score: 6.0 / 12.0)

1. Task (Score: 6.0 / 12.0)
2. Comment

Lab 5

1. 提交作業之前，建議可以先點選上方工具列的**Kernel**，再選擇**Restart & Run All**，檢查一下是否程式跑起來都沒有問題，最後記得儲存。
2. 請先填上下方的姓名(name)及學號(stduent_id)再開始作答，例如：

```
name = "我的名字"
student_id= "B06201000"
```

3. 演算法的實作可以參考lab-5 (https://yuanyuyuan.github.io/itcm/lab-5.html), 有任何問題歡迎找助教詢問。
4. **Deadline: 12/11(Wed.)**

In [1]:

```
name = "鄭如芳"
student_id = "B05602020"
```

(Top)

Exercise 3

Analyse the convergence properties of the Jacobi and Gauss-Seidel methods for the solution of a linear system whose matrix is

$$\left[\begin{matrix}

```
\alpha &&0 &&1\\
0 &&\alpha &&0\\
1 &&0 &&\alpha
\end{matrix}\right],
\quad \quad
\alpha \in \mathbb{R}.$$
```

Comments:
What's the condition of alpha?

$$= (D - L - U) = \begin{bmatrix} \alpha & 0 & 0 \\ 0 & \alpha & 0 \\ 0 & 0 & \alpha \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$Solve\, Ax = b i.e\, solve\, (D - L - U)x = b and\, get\, x_{(k)} = \begin{bmatrix} 0 & 0 & \frac{-1}{\alpha} \\ 0 & 0 & 0 \\ \frac{-1}{\alpha} & 0 & 0 \end{bmatrix} x_{(k-1)} + \begin{bmatrix} \frac{1}{\alpha} & 0 & 0 \\ 0 & \frac{1}{\alpha} & 0 \\ 0 & 0 & \frac{1}{\alpha} \end{bmatrix} b$$

$$x_0 = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} x_1 = \begin{bmatrix} \frac{-x_3}{\alpha} \\ 0 \\ \frac{-x_1}{\alpha} \end{bmatrix} + \begin{bmatrix} \frac{b_1}{\alpha} \\ \frac{b_2}{\alpha} \\ \frac{b_3}{\alpha} \end{bmatrix}$$

$$x_2 = \begin{bmatrix} \frac{x_1}{\alpha^2} \\ 0 \\ \frac{x_3}{\alpha^2} \end{bmatrix} + \begin{bmatrix} \frac{-b_3}{\alpha^2} \\ 0 \\ \frac{-b_1}{\alpha^2} \end{bmatrix} + \begin{bmatrix} \frac{b_1}{\alpha} \\ \frac{b_2}{\alpha} \\ \frac{b_3}{\alpha} \end{bmatrix}$$

$$x_3 = \begin{bmatrix} \frac{-x_3}{\alpha^3} \\ 0 \\ \frac{-x_1}{\alpha^3} \end{bmatrix} + \begin{bmatrix} \frac{b_1}{\alpha^3} \\ 0 \\ \frac{b_3}{\alpha^3} \end{bmatrix} + \begin{bmatrix} \frac{-b_3}{\alpha^2} \\ 0 \\ \frac{-b_1}{\alpha^2} \end{bmatrix} + \begin{bmatrix} \frac{b_1}{\alpha} \\ \frac{b_2}{\alpha} \\ \frac{b_3}{\alpha} \end{bmatrix}$$

$$If\, n\, is\, even x_n = \begin{bmatrix} \frac{x_1}{\alpha^n} \\ 0 \\ \frac{x_3}{\alpha^n} \end{bmatrix} + \begin{bmatrix} \frac{-b_3}{\alpha^n} \\ 0 \\ \frac{-b_1}{\alpha^n} \end{bmatrix} + \begin{bmatrix} \frac{b_1}{\alpha^{(n-1)}} \\ 0 \\ \frac{b_3}{\alpha^{(n-1)}} \end{bmatrix} \ldots + + \begin{bmatrix} \frac{-b_3}{\alpha^2} \\ 0 \\ \frac{-b_1}{\alpha^2} \end{bmatrix} + \begin{bmatrix} \frac{b_1}{\alpha} \\ \frac{b_2}{\alpha} \\ \frac{b_3}{\alpha} \end{bmatrix}$$

$$If\, n\, is\, odd x_n = \begin{bmatrix} \frac{-x_3}{\alpha^n} \\ 0 \\ \frac{-x_1}{\alpha^n} \end{bmatrix} + \begin{bmatrix} \frac{b_1}{\alpha^n} \\ 0 \\ \frac{b_3}{\alpha^n} \end{bmatrix} + \begin{bmatrix} \frac{-b_3}{\alpha^{(n-1)}} \\ 0 \\ \frac{-b_1}{\alpha^{(n-1)}} \end{bmatrix} \ldots + + \begin{bmatrix} \frac{-b_3}{\alpha^2} \\ 0 \\ \frac{-b_1}{\alpha^2} \end{bmatrix} + \begin{bmatrix} \frac{b_1}{\alpha} \\ \frac{b_2}{\alpha} \\ \frac{b_3}{\alpha} \end{bmatrix}$$