Advanced Methods in Text Analytics

Word Representations

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What are representations? (1)

- Let’s **represent** a hypothetical country called Countryland!
- What **features** should we use?
  - Area size? E.g. 400.000 km^2
  - Population size? E.g. 20M
  - Continent? E.g. Europe
- Let’s group those features as a single 3D **feature vector**
  - Countryland: (400.000; 20M; ‘Central Europe’)
- Features can have different types
  - Real or integer numbers, e.g. population size
  - Categorical, e.g. the continents
- We refer to that vector as a **representation** of ‘Countryland’
- Is that useful for, say, inferring size of the economy?

What are representations? (2)

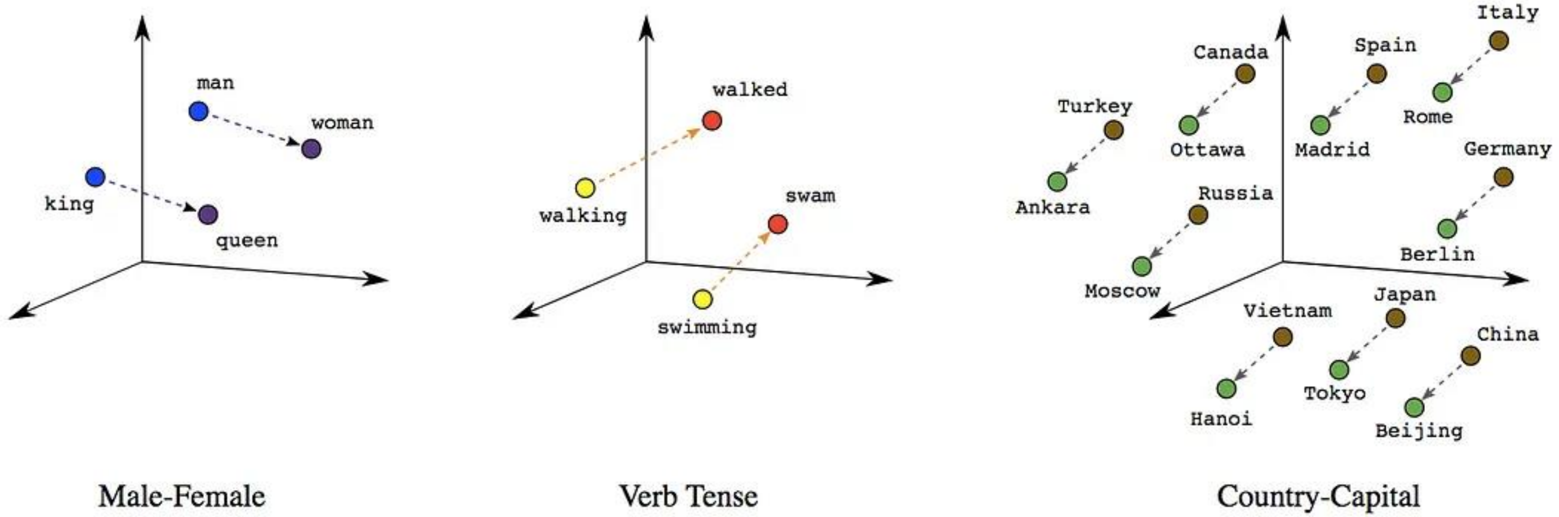
- How about the following features then?
  - GDP? E.g. 50 billion USD
  - Interest rate? E.g. 2.2%
  - Inflation rate? E.g. 7.2%
- This gives us a different representation of Countryland:
  - (50B; 2.2; 7.2)
- Manually constructing representations: **feature engineering**
  - Often requires expertise in the area
  - E.g. 'government debt to GDP' is a common metric in Economics
- **Feature vectors:** most typical form of input to machine learning (ML) methods

Can we *learn* useful representations?

- Can we **learn useful features**?
  - Then no more feature engineering
  - We may not even need an Economics degree to get useful features
- **Yes!** Deep Learning (DL) methods are useful for this.
  - But ML/DL is mostly engineering, so back to square one?
  - No! These methods are applicable to any domain, e.g. economics, applied mathematics, etc.
- The **learned representations** may be *generally* useful, e.g. word representations
  - Potentially useful for any application with text as input

Word Embeddings

- Learned vector representations of words/tokens
- Can be learned in several different ways
- Classic example: **word2vec**
  - Useful geometrical properties between related word vectors



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Challenges

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- Not straightforward as previous examples suggest!
  - Vector spaces with hundreds or thousands of dimensions
  - Variety in methods for learning word representations
- Thus, many challenges!
  - Here's some examples of those challenges
- **Interpretability:**
  - Learned features often not interpretable (real numbers)
  - Geometric properties of space not straightforward (high-dimensional space)
- **Cost:**
  - Train on terabytes of text
  - Latest LLMs have *trillions* of parameters
  - GPU memory limited
- This **fixed (static) word -> vector mapping seldom used** anymore
  - But still part of architectures, relevant to understand representations

Outline

1. Machine Learning Refresher

2. Deep Learning Basics

3. Word Embeddings

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Machine Learning Refresher

About this Section

- **Goal:** discuss basic concepts and terms that will be useful for the entire course
- **Outline:**
  1. Basic Machine Learning Concepts
  2. Probability Theory Refresher
  3. Maximum Likelihood Estimation
  4. Summary

What is machine learning? (1)

- Methods for developing algorithms by **learning from data**
  - In contrast to manually coding knowledge we already have
  - Example: flying a helicopter (how many situations must we consider?)
- Umbrella term encompassing **many families of methods**
  - E.g. Inductive Logic Programming, Deep Learning, etc.
- Related (umbrella) terms
  - **Artificial Intelligence:** solutions that show intelligent behavior
  - **Data Mining:** extract value from data (ML usually part of that pipeline)
- Machine learning **in this course**?
  - Mostly **deep learning and related methods**

What is machine learning? (2)

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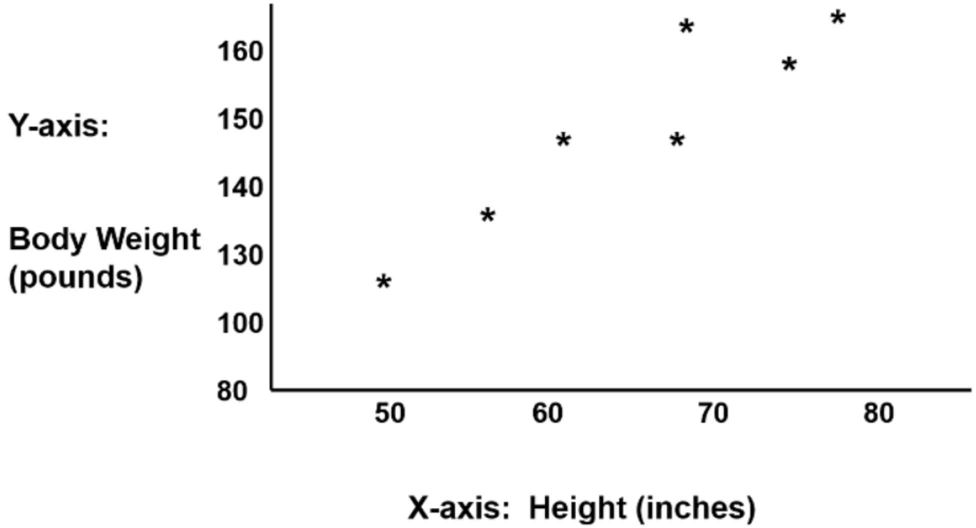
- Classic definition:
  - “A computer program is said to learn from **experience** *E* with respect to some class of **tasks** *T* and **performance measure** *P*, if its performance, as measured by *P*, improves with experience *E*.”

    Tom Mitchell, Machine Learning, 1997
- **Experience:** data in some format
  - In our case, text (that needs to be processed so models can "read" it)
- **Tasks:** depends on perspective
  - ML: classification, regression, etc.
  - NLP: machine translation, question answering, etc.
- **Performance measure:** depends on task
  - E.g. F1 score for classification tasks, ROUGE scores for summarization

What is a task?

- **Task:** predict weight of a person (regression, supervised)
- **Data:** How do we represent a person?
  - Let's go with a simple 2D vector: (height, weight)
- We might see a linear relation between input and target
  - **Input:** 1D feature vector (usually denoted by $\mathbf{x}_i$, here: height)
  - **Target:** weight (usually denoted by $\mathbf{y}$, here: real number)



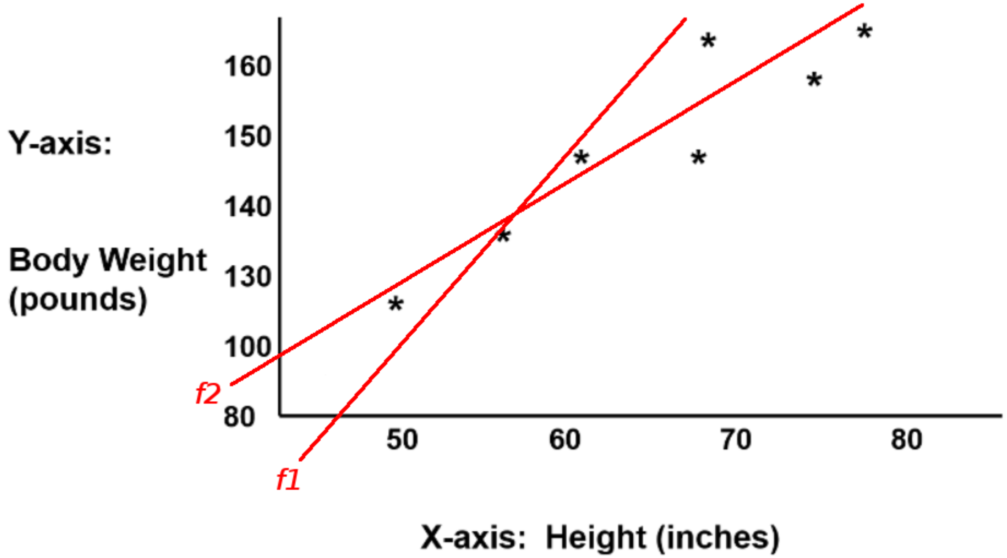
[Example source]

What is a model?

- Family of methods that **we choose**
  - Each comes with a set of **assumptions**
- E.g. based on observed data, we assume linear relation between height and weight
  - Thus, we choose a linear regression model
- **Linear regression:**
  - **Main assumption:** linear relation between inputs and output
  - Formally: learn function *f(x) = y*
  - Here: *f(height) = weight*
  - This model comes from statistics, studied more formally, e.g. more assumptions, ML is more "relaxed", focuses on learning (generalizing)
- How exactly do we **learn** a function?
  - To get there, we need to talk about parameters

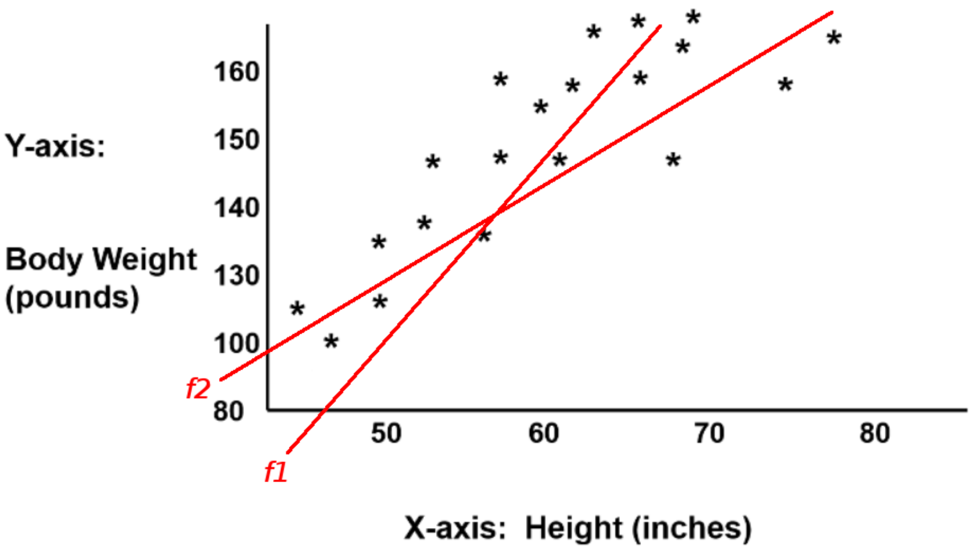
What are parameters?

- We can describe the assumption in linear regression with the **equation of a line**:
  - $y = mx + b$ where m is slope and b is y-intercept
  - In our case: *weight = m height + b*
  - Thus, we have **two parameters: *m, b***
- Each value assignment of m, b: different member of linear regression family
  - Which one **fits** the data better?
  - f_1 or f_2?
  - How do you know? Depends on the goal



What is generalization?

- **Goal of ML:** model should perform well at the task on new data
  - Not just data it was trained on
  - Thus, goal of training a model: **generalize to unseen data**
- For example, which function did you say fits our data best?
  - f_1 or f_2?
- What if we collect more data?
  - Before, f_2 seemed best
  - With more data, f_1 is best
- How do we train for generalization?
  - We **evaluate models on *held-out* evaluation data**, represents unseen data
  - Train and eval data assumed from same distribution

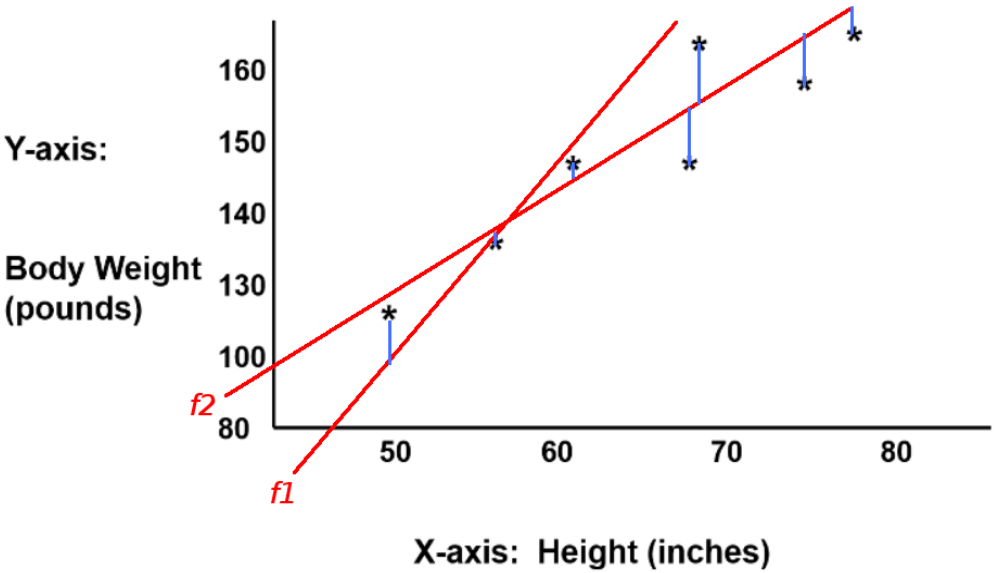


How do we evaluate a model?

- How can we tell which member of the family is more suitable for our task?
  - We need to **select a model** from the many options
- **Performance metric**
  - A **metric we choose to evaluate** how good a **model** is at a task
  - **For example:** mean squared error (MSE)

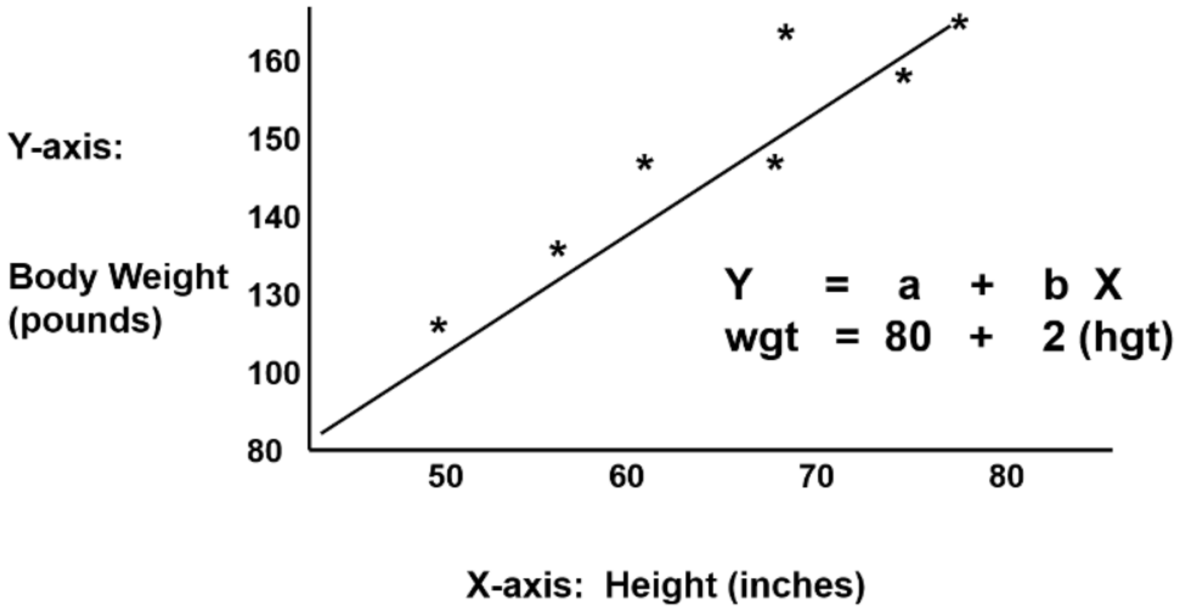
$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2$$

where:
- n: # of examples
- Y_i: target values
- Y'_i: predicted values



How do we use our selected model?

- **Model selection:** choosing a value assignment for our parameters
  - E.g. *m = 80, b = 2*
- To make a **prediction**, we make use of our selected model (function)



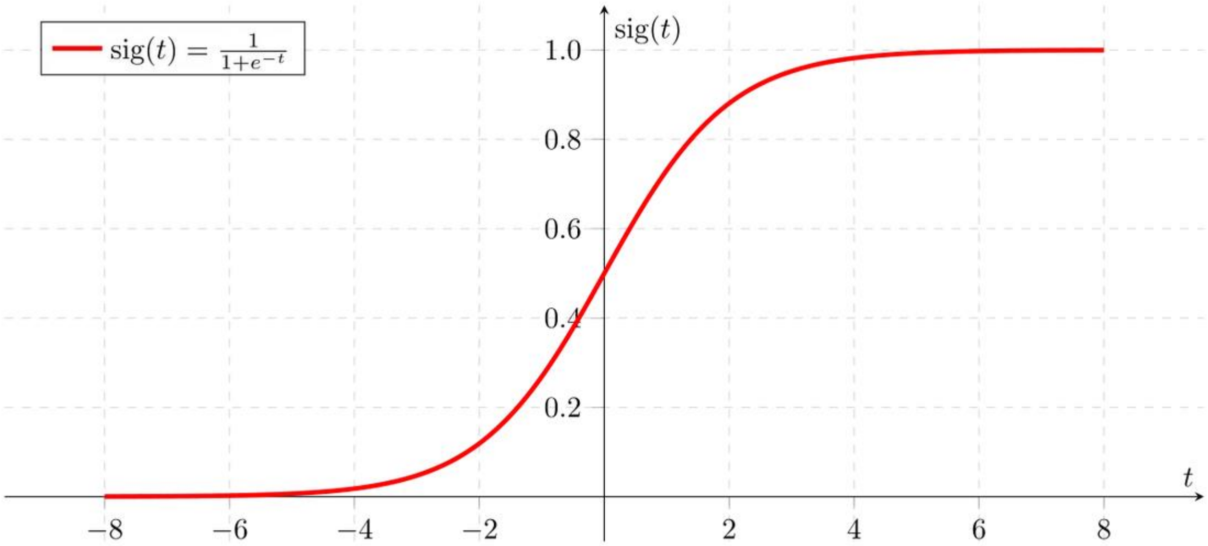
- Important distinction: inference vs prediction
  - **Inference:** use model to learn about data generation process
  - **Prediction:** predict specific outcomes for new input points

What are *hyper*parameters?

- Other parameters that are **typically not learned**
  - Instead, they are manually set by the user
  - Note that *meta-learning* does exist, e.g. AutoML
- Example in linear regression: using a bias or not
  - No bias: line passes by origin (assumes data is centered)
  - See scikit-learn's documentation on linear regression (fit_intercept)
- How do we choose values for hyperparameters?
  - **Hyperparameter optimization methods**
  - E.g. grid search, random search (known to be better than grid search)
  - Requires familiarity with known tricks/experience/expert knowledge
- Often **involves making assumptions**
  - E.g. this setting worked well in the past
  - E.g. the search space I designed benefits all models equally

What about classification?

- **Logistic regression:** popular model for *classification*
  - **Main assumption:** linear relation between inputs and output
- Essentially: linear regression with a logistic function
  - $y = \sigma(m\,x + b)$ where σ is the logistic (sigmoid) function



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- Logistic function maps inputs to real interval [0,1]
  - **Can be *interpreted* as a probability distribution**

How do we use logistic regression?

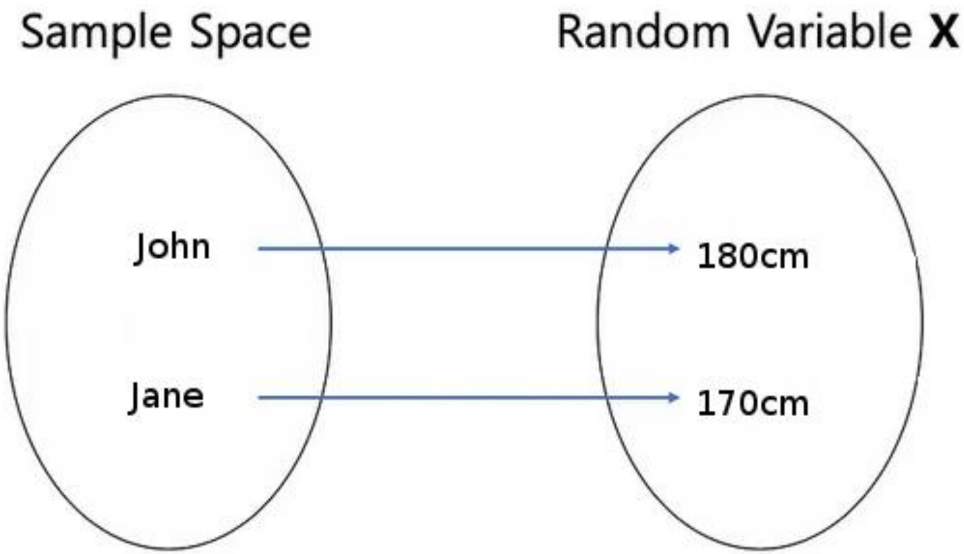
- **Binary** classification:
  - *$P(y = 1) = \sigma(m\ x + b)$ -->* interpreted as probability for positive class
  - *$P(y = 0) = 1 - \sigma(m\ x + b)$ -->* complementary probability for negative class
- That is, we get probabilities for both positive and negative class
  - We can do some *inference*, e.g. how much more likely is one class than the other?
  - But *prediction?*
- **Prediction:** we decide on a threshold
  - *$p(y = 1) >= 0.5$*, we predict positive class
  - *$p(y = 0) < 0.5$*, we predict negative class
- We get the following **decision boundary**
  - *$p(y = 1|x) = p(y = 0|x) = 0.5$*
- Describing tasks and models with **probability theory** is very common!
  - We'll do this often, so let's review the basic concepts

Probability Theory Basics

- This is a **quick refresher** about **probability theory**
- This is later **useful for many things**
  - Describing tasks
  - Describing models, e.g. what language models do
  - More concepts commonly used in NLP research
- Today, some basic intuitions/definitions
  - Random variables
  - Probability distributions
  - Independence of events
  - Sum rule / product rule
  - Joint distributions
  - Conditional probability
  - Conditional independence
  - Bayes rule

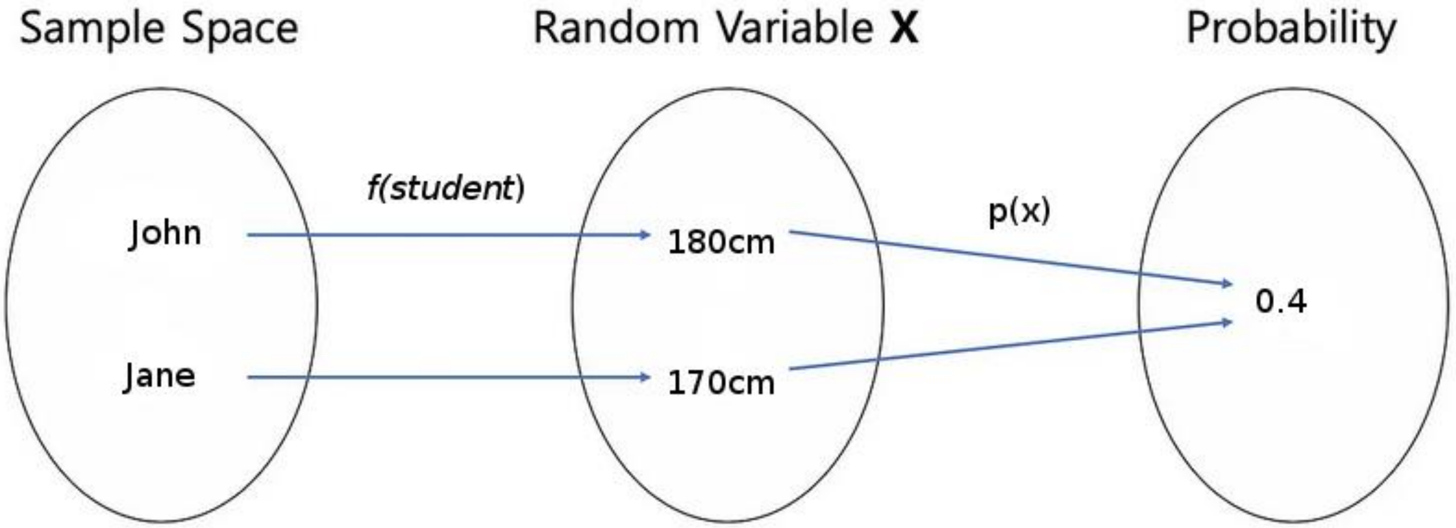
Random Variables

- **Function** between events and something relevant about the events
  - **Usually designed** according to something we want to model
- Example: say we are interested in average height of students
  - **Event:** randomly select a student
  - **Event space:** set of all students the course
- **Random variable X**: *f(student) = height of student*



Probability Distributions (1)

- Essentially a way to describe random variables
- **Random variables:** take values as a result of some random process
- **Distribution of random variable:** how often may each value be observed
- How does a distribution describe this?
  - A(nother) function *p(**X**):* maps set of possible values a random variable can take to interval [0,1]



Probability Distributions (2)

- **Output of *p* for given event *a*:** probability of observing event *a*
  - Written as $p(\mathbf{X}=a)$
  - Example: what is the probability of observing a height of 185cm?
  - $p(\mathbf{X} = 185cm) = 0.1$, i.e. probability is 10%
- **Discrete distributions:** random variable can take finite set of values
  - *p* usually referred to as *probability mass function*
- **Continuous distributions:** r.v. can take infinite set of values
  - *p* usually referred to as *probability density function*
- Examples:
  - Bernoulli (discrete): models output of single binary experiment
  - Gaussian (continuous): normal distribution
- Is **X** in our students' height example discrete or continuous?
  - Depends on how to measure height, as reals or integers

Independence of events

- Events **A** and **B** are **independent** when one event happening does not affect the probability of the other happening
- For example:
  - Event **A**: Germany wins the world cup in 2014
  - Event **B**: Germany loses world cup semifinal
  - Event **C**: The stock market crashed in 2008
  - **Which of these events are dependent/independent?**
- A and B are not independent
  - If Germany loses the semifinal, it cannot win the final
- A and C are independent, as well as B and C
  - Very unlikely the 2008 crash has any impact on the 2014 world cup
- Independence perhaps best understood with **sum rule**

Sum Rule

- Probability of *union* of *n* events happening, e.g. A **OR** B
  - Formally: $p(A \cup B) = p(A) + p(B) - p(A \cap B)$
- For example: r.v. **Y** shows number in die, relevant events are:
  - **A:** "We observe a multiple of 3", i.e. either 3 or 6
  - **B:** "We observe a multiple of 2", i.e. either 2, 4 or 6
- Are **A** and **B** independent? **No**, see overlap in event spaces
- For $p(A \cup B)$ we subtract probability of overlapping events: $p(\mathbf{Y}=6)$
  - $p(A) = 1/6 + 1/6 = 1/3$
  - $p(B) = 1/6 + 1/6 + 1/6 = ½$
  - $p(y = 6) = 1/6$
  - $p(A \cup B) = 1/3 + ½ - 1/6 = 0.66$
- With independent events, last term becomes zero:
  - Event **A**: "We observe an even number", i.e. either 2, 4 or 6
  - Event **B**: "We observe an uneven number", i.e. either 1, 3 or 5

Product Rule

- Probability of conjunction of n events happening, i.e. A **AND** B
- Formally: $p(A,B) = p(A|B)p(B)$
  - $p(A,B)$ = joint distribution of A and B
  - $p(A|B)$ = conditional distribution of A given B
- Side note:
  - All of these rules are *derived* from axioms of probability theory
  - For more, see probability theory literature
- Let's go over joint distributions and conditional distributions

Joint Distributions (1)

- Function $p(X_{1\ldots n})$ from *n* random variables to interval [0,1]
  - Distribution of all possible combinations of those *n* random variables
- Easier to see with joint probability table
- For example, joint distribution of flipping two coins **A** and **B**:

A / B	Heads	Tails
Heads	0.1	0.2
Tails	0.3	0.4

  - Note that all elements in table add up to 1, i.e. a distribution

Joint Distributions (2)

- We can derive many things from a joint distribution
- A very common operation: **marginalization**
  - Given joint distribution of A and B, derive distribution of A or B
- For example, we get distribution of coin A by summing over all possible values of B with the sum rule:

A/B	Heads	Tails	*p(A)*
Heads	0.1	0.2	= 0.3
Tails	0.3	0.4	= 0.7

- New column adds up to 1, i.e. a probability distribution
  - p(A) known as **marginal distribution** of A
- This process is referred to as **marginalizing out** B
  - Normally expressed with sum rule: $p(A) = \sum_b p(A,B = b)$

Conditional Probability

- Probability of **seeing event A knowing event B was already seen**
- Formally, $p(A|B) = p(A,B)/p(B)$
- Intuitively, **space of all possible events shrinks** (for dependent RVs)
  - Imagine we toss a die
  - **A:** "We observe an even number", i.e. either 2, 4 or 6
  - **B:** "We observe a multiple of 3", i.e. either 3 or 6
  - After observing A, event space for B is not {3, 6}, it is {6}
- Note that $p(A|B)$ can be derived from joint distribution $p(A,B)$
  - Marginalize out A to get $p(B)$
  - Then you have everything you need

Conditional Independence

- Two events are **independent given a third event**
  - Observing **A** tells me something about **B**
  - But when observing **C**, **A** no longer says anything about **B**
- For example:
  - Event **A:** knowing a person has good vocabulary
  - Event **B:** knowing a person's height
- Are **A** and **B** independent?
  - No, since children have limited vocabulary
- Now consider event **C**: knowing a person's age
  - Given **C**, **A** and **B** are independent, since height implies age
  - Once we have age, height gives us no new information
  - Thus, **A** and **B** are conditionally independent given **C**

Bayes' Rule

- Apply product rule on conditional probability
- That is, $p(A|B) = p(A,B) / p(B) = p(B|A)\ p(A) / p(B)$
  - Note that we want probability of A given B, i.e. $p(A|B)$
  - And to get it, we use marginal distribution of A, i.e. $p(A)$
  - This may seem **counterintuitive**
- This is typically interpreted like so:
  - $p(A)$ known as **prior probability** distribution of event A
    - Describes **what we know/believe** about event A
  - $p(B|A)$ known as **likelihood**
    - Given event A, what is the likelihood that we get our data
  - $p(A|B)$ known as **posterior probability** of A
    - Interpreted as **updated description of A** after considering what we originally know about A (prior), as well as our data (event B)
- This interpretation gives rise to the *frequentists vs bayesian* debate
- And with this, we are **done with our probability refresher!**

A Probabilistic Perspective of ML

- **Task:** predict outcome of a coin tossed by your (unreliable) friend
  - Many factors: mass distribution, position in fingers, friction, etc.
  - Difficult to *model* physically
- Let's use a probabilistic model
  - Abstracts away complications of physical model
- **Probability:** uncertainty of some *random* event, e.g. coin toss
  - In practice: ***relative frequency of observations*** about the event
  - **Event in this case:** *seeing heads* when tossing the coin
- **Simple Model:** Bernoulli distribution
  - **Intuition:** it models outcome of *single binary* experiment
  - ***Formally:*** $p(k|\Theta) = \Theta^k (1-\Theta)^{1-k}$ where k in {0,1} is possible outcomes
  - **Θ is the only parameter of model:** probability of seeing heads
- How do we get a value for this parameter so we can use our model?
  - We could **estimate this parameter**
  - We could blindly guess, or we could **use data for that estimation**!

Training Models: Parameter Estimation

- Say your friend allows you to observe ten tosses
  - You observe 3 out the 10 are heads
- You can estimate the value of Θ from this data
  - $\Theta = 3/10 = 0.3$ chance of seeing heads in a single toss
- We just counted **relative frequencies**
  - Training example: a single coin toss, i.e. y_i in *{heads, tails}*
  - Training data: 10 coin tosses
  - Relevant observations: we observe heads
- With a value estimated for parameter Θ, we can make predictions!
  - Our model has been trained and is ready to be used **on unseen data**
- Thus, **learning a function** means **estimating its parameters**
  - We don't know the parameters of a model
  - Instead, we have data and use it to estimate those parameters
- Let's look at a common framework for parameter estimation

Maximum Likelihood Estimation (1)

- Common approach for parameter estimation
  - Otherwise known as **MLE**
- Finds model that **maximizes likelihood** of observed data
  - But what does likelihood mean?
- Our model for the coin toss:
  - $p(k|\Theta) = \Theta^k (1-\Theta)^{1-k}$ is a function of k with Θ fixed (how we predict with it)
  - Let's ***instead see it as*** function of Θ with fixed k, i.e. $L(\Theta|k) = \Theta^k (1-\Theta)^{1-k}$
  - That is, a function of parameters given *fixed data* (assumed during training)
- We call $L(\Theta|k)$ the **likelihood function**
  - It is **not a probability distribution**
  - Instead, we use it to find distributions that fit observed data
- How do we use the likelihood function during training?
  - We want distribution with *highest* (max) probability to our data
  - **Assumption:** set of observed data is most likely sample among all data

Maximum Likelihood Estimation (2)

- Formally, we want $\Theta = max_{\Theta} L(\Theta|k)$
  - Equivalently, we want $\Theta = argmax_{\Theta}$ **log** $p(k|\Theta)$
- **First**, we make a common **assumption**: examples are **i.i.d.**
  - This stands for **independent and identically distributed**
  - Examples are independent, belong to same distribution
- **Then**, we apply this assumption to our coin toss setting
  - Our model for a single example: $p(k|\Theta) = \Theta^k (1-\Theta)^{1-k}$
  - The **joint probability of *N* observed examples** is then given by:

$$\begin{aligned} P(D) &= P(y_1, y_2, \ldots, y_N) \\ &= P(y_1)P(y_2)\ldots P(y_N) \\ &= P(head)^k P(tail)^{N-k} \\ &= \theta^k (1-\theta)^{N-k}. \end{aligned}$$

  - Here we used the i.i.d. assumption from line 1 to line 2

Maximum Likelihood Estimation (3)

- **Finally**, to find this maximum value, we:
  - Compute the first derivative
  - Set it to zero
  - Extract a value for Θ

$$\begin{aligned}
\frac{\partial \log P(D)}{\partial \theta} &= \frac{\partial\left(\log \theta^k (1-\theta)^{N-k}\right)}{\partial \theta} \\
&= \frac{\partial\left(k \log \theta + (N-k)\log(1-\theta)\right)}{\partial \theta} \\
&= \frac{k}{\theta} - \frac{N-k}{1-\theta} = 0 \\
&\Rightarrow \hat{\theta} = \frac{k}{N},
\end{aligned}$$

- We obtained relative frequencies, as we had done already!
  - In practice, **MLE often does not have a closed-form solution**
  - Instead, **we rely on numerical methods**
- MLE related to many aspects of ML: cross entropy, MAP, etc.

Empirical Risk Minimization

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- This is a general training approach
- **Main goal:** estimate how a model will work in practice by averaging a **loss function** $L(x_i, y_i)$ on a training set
  - Here x_i is a training example and y_i its corresponding label
- Formally:

$$R_{\mathrm{emp}}(h) = \frac{1}{n}\sum_{i=1}^{n} L(h(x_i), y_i).$$

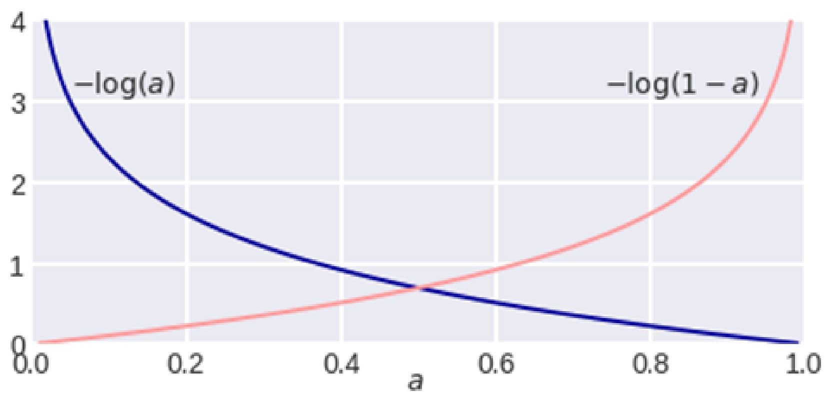
  - Note the training set is of size n here

- This is the most common training approach in ML/DL/NLP.
  - But what is a loss function, exactly?

What are loss functions?

- Functions that tell us **how a model performs during training**
  - Being a cost/loss, we want to **minimize it**
- Desiderata:
  - **Low loss:** when model prediction matches given label
    - The closer, the lower the loss/cost
  - **High loss:** when model prediction does not match label
    - The farther, the higher the loss/cost
- Example: **log loss**
  - Here *a* is model prediction
  - Note the desired behavior
  - Works with probabilistic models only

$$-\begin{cases}\log a_i, & y_i = 1,\\ \log(1-a_i), & y_i = 0.\end{cases}$$



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Let‘s design a training objective

- Combine empirical risk minimization with log loss
  - Assume binary classification
- We have:
  - $J = - 1/N \sum_n [\, y_i \log(p_i) + (1 - y_i) \log (1 - p_i) \,]$
    where N is size of training and p_i is prediction for example i
- Note the following:
  - Labels act as indicator factors
  - Log loss means bad model predictions get punished
  - We negate the entire expression so we can minimize it
- Thus, this is an optimization problem
  - This one can be shown to be a form of maximum likelihood estimation
- **Training objectives:** very important in NLP
  - E.g. masked vs autoregressive language have different, manually designed, training objectives

Summary: ML Refresher

- **Goal:** learn models that perform a task well on unseen data
- **Task:** from some input to some output
  - Regression: real-valued output
  - Classification: categorical output
- **Model:** abstract, often simplified version of a task
  - Comes with set of assumptions, e.g. linearity
- **Evaluation:** given predictions, compute relevant metric
- **Training:** parameter estimation
  - Objective functions (more not covered in *refresher*, e.g. regularization)
- All of these concepts are common in advanced NLP!
  - E.g. LLMs are **models** with trillions of **parameters**
  - Different LLMs **train** on different **objective functions** for different **tasks**
  - **Empirical risk minimization** and **MLE** used during training
  - They are **evaluated** on various **metrics** relevant to various tasks

Deep Learning Basics

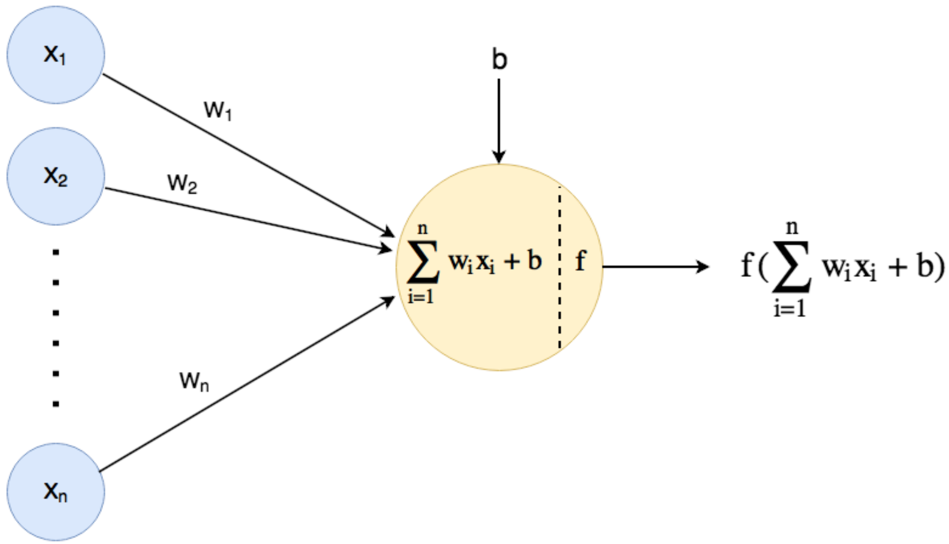
About this Section

- **Goal:** discuss basic concepts and terms that will be useful for the entire course
- **Outline:**
  1. Basic Deep Learning Concepts
  2. Backpropagation
  3. Language Models with Feed-forward Neural Networks

Mathematical Notation

- Scalars: a, b
- Vectors: **x**, **y** (by default column vectors, i.e. size is *n x 1*)
- Matrices: **X**, **Y**
- Scalar product: a**x**
- Dot product: $\mathbf{x}^T\mathbf{w}$ (transpose because **x** is column vector)
- Matrix-vector product: **Xy**
- Matrix-matrix product: **XY**

What is an artificial neuron?



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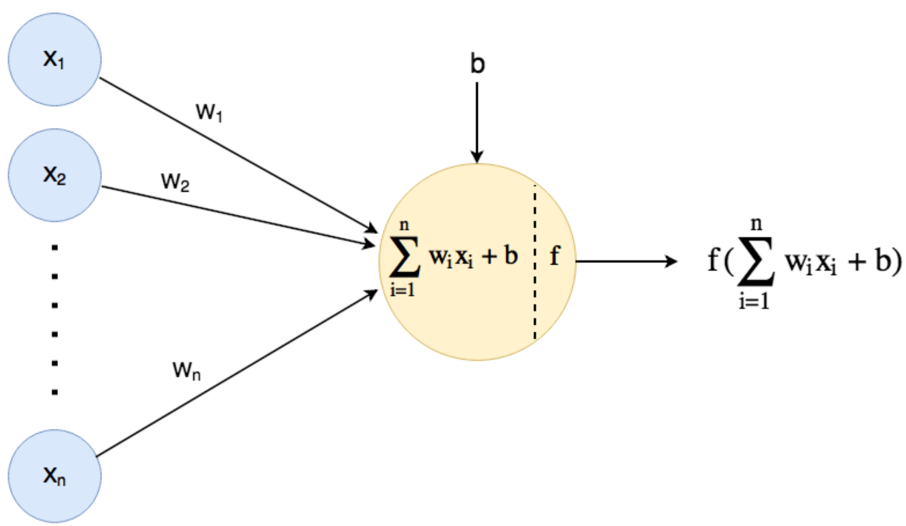
- Input is n-dimensional vector $\mathbf{x}$
- Features x_i multiplied with weights w_i, their products then added
  - What common operation is this?
  - A bias is optional, but almost always there, adds lots of flexibility to model
- Then an *activation function f* is applied to the result
- Different activation functions determine different artificial neurons

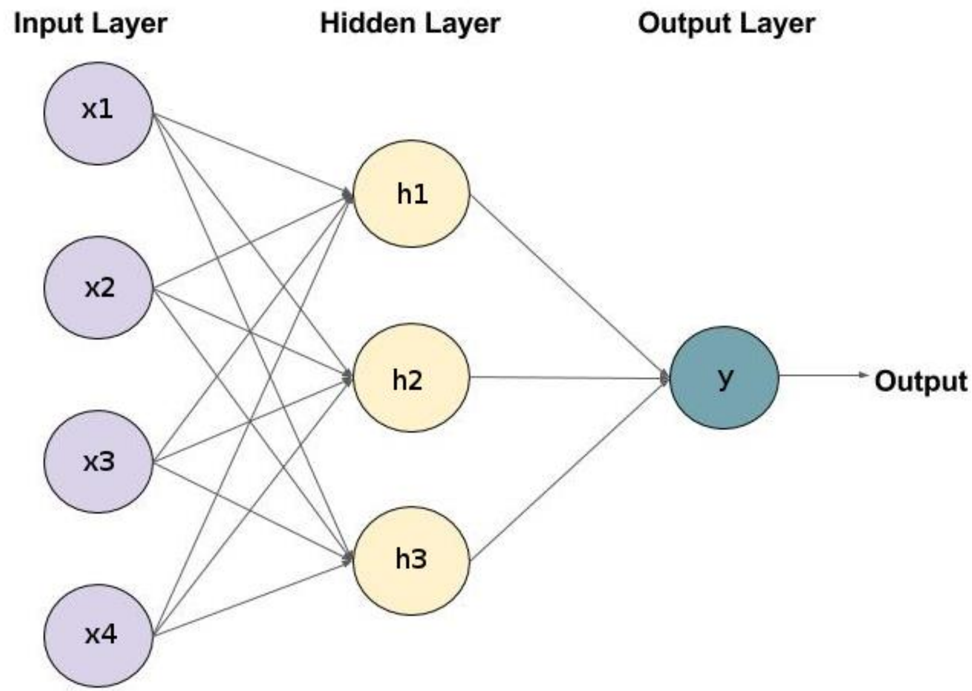
Types of Artificial Neurons

- They are mainly distinguished by their activation functions, e.g.
  - **Linear neuron:** $f(x) = x$
  - **Logistic neuron:** $f(x) = \sigma(x)$ (sigmoid function)
- What model do we obtain with a linear neuron?
  - Linear regression
- And with a logistic neuron?
  - Logistic regression
- More types:
  - **Tanh:** maps to [-1,1]
  - **ReLu:** $f(x) = max(0,x)$
  - Etc.
- By adjusting weights:
  - We learn functions!



Feed-forward Neural Networks

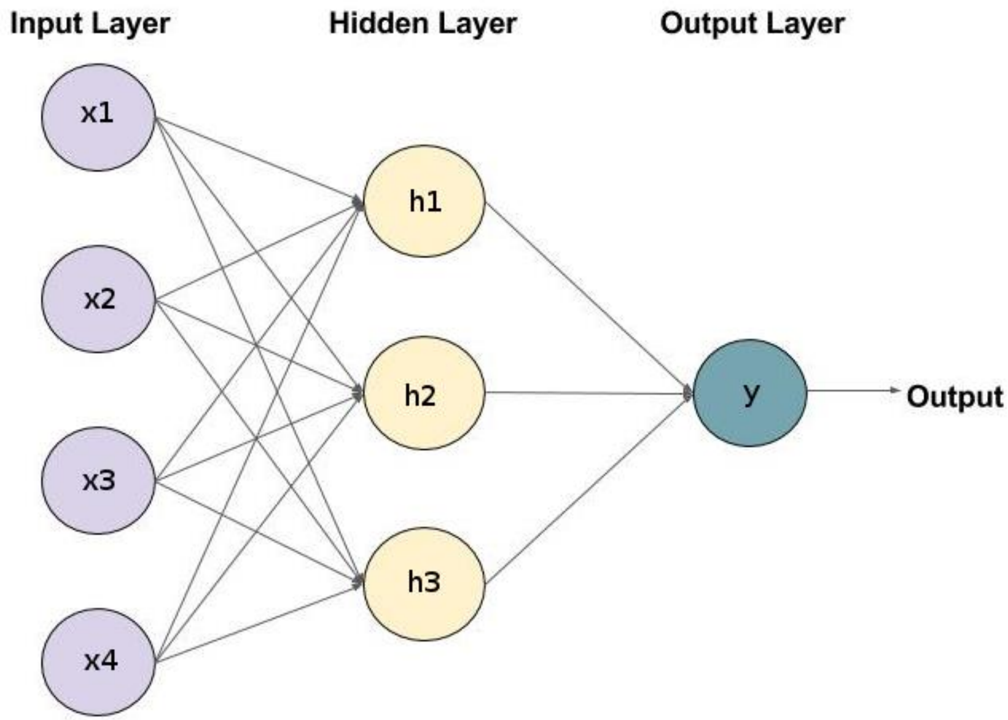
- Let's do some **feature learning**!
  - We use *hidden* layers
  - Input to h_i: ***x* (our data)**
  - h_i: learned values
  - Input to y: ***h* (learned!)**
- If y is linear neuron:
  - Linear regression with learned features
- If y is logistic neuron:
  - Logistic regression with learned features



- Thus, FNNs are a framework for *designing* functions (*algebraic circuits*)
  - Information flows forward (but also backward during training, more later)
  - Main restriction: **no loops! Computations** seen as **directed acyclic graphs**

Operations in FNNs

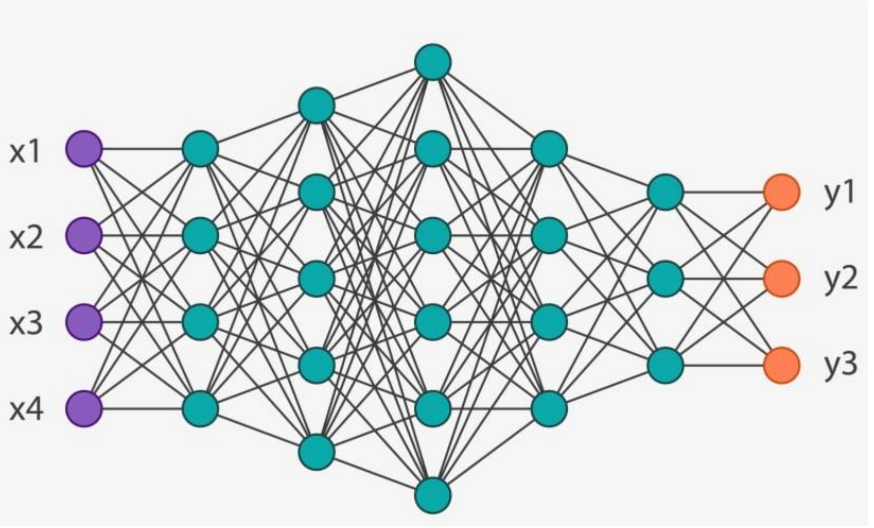
- Example so far: **fully-connected neural network** (useful intuition)
- What does it compute?
- $\boldsymbol{x}$: $n \times 1$ input vector
- $\boldsymbol{h}$: $m \times 1$ hidden representation
- h_k has n w_i, together: $\boldsymbol{w}_k$
- Let $\boldsymbol{W}_1$ be $n \times m$ matrix
  - m columns, each a $\boldsymbol{w}_i$
- Then $\boldsymbol{h} = \boldsymbol{W}_1^T\boldsymbol{x}$ if h_k linear
  - $\boldsymbol{h} = f(\boldsymbol{W}_1^T\boldsymbol{x})$ w. activ. f
  - $\boldsymbol{h} = f(\boldsymbol{W}_1^T\boldsymbol{x} + \boldsymbol{b})$ w. bias



- Similarly: $y = f(\boldsymbol{W}_2^T\boldsymbol{h} + b)$ where $\boldsymbol{W}_2$ is $m \times 1$, constructed as $\boldsymbol{W}_1$
  - Note network $y = \boldsymbol{W}_3^T\boldsymbol{x} + b$ exists with $\boldsymbol{W}_3 = \boldsymbol{W}_1\boldsymbol{W}_2$, i.e. no hidden layers
  - Why do we always use hidden layers then?

Why use Deep Networks?

- When is a network deep?
  - Not well defined
  - Some say >4 hidden layers
  - Others say >2
- More important: **why?**
- Universality theorem
  - Formalities about FNN limits to approximate functions
  - One takeaway: no depth *needed*



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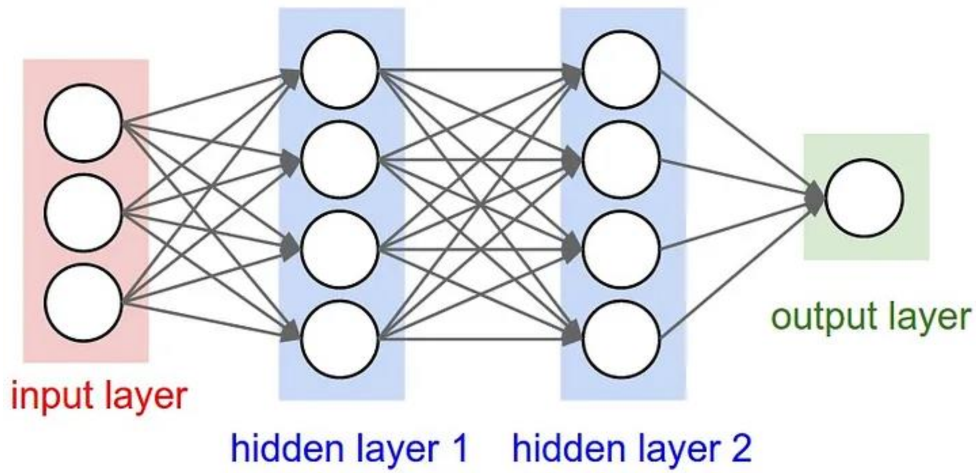
- Why use depth then? **Hidden representations!**
  - Each hidden layer is a learned feature vector, *hierarchical features*
  - Component for designing networks: number and size of hidden layers
    - Smaller layers force models to learn compact but useful representations
    - More/larger hidden layers: more freedom, more parameters, difficult to train
- More parameters -> stronger model, may overfit, requires lots of data

FNNs for Regression

- Let's design a fully-connected network for regression
- **Input:** n-dimensional $\boldsymbol{x}$
- **Output:** real number y
- So, how many input units?
  - What type of units?
- How many output units?
  - What type of units?
- How many hidden layers?
  - What size/type of units?



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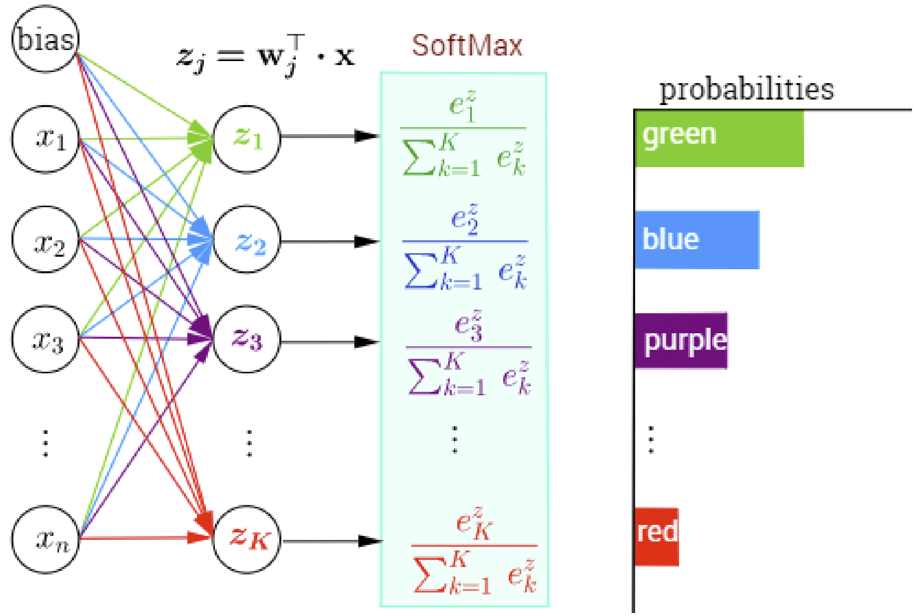
- How to ensure this number of output units? **Linear layer!**
  - *Project* down to the size that you need
  - Remember: $y = \boldsymbol{W}_k^T \boldsymbol{h} + b$ where $\boldsymbol{W}_k$ is m x n matrix
  - $\boldsymbol{W}_k^T$ projects n dimensional vector to m dimensional vector space
- **Linear transformation** are **everywhere** in Deep Learning!

FNNs for Classification

- Let's design a fully-connected network for classification
- **Input:** *n*-dimensional **x**
- **Output:** one of *K* classes
- So, how many input units?
  - What type of units?
- How many output units?
  - How do we ensure that?
  - What type of units?
- How to get predictions?
  - **Softmax layer**



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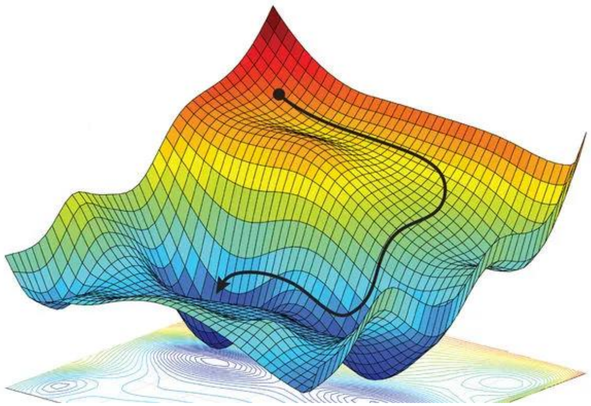
- Softmax *function*: turns given vector into *probability vector*
  - **x** is a probability vector if $x_i \in [0,1]$ for all i and $\Sigma_i x_i = 1$ (see first tutorial)
- Softmax *layer*: linear layer for projection + softmax function
  - **Input:** vector of logits; **Output:** probability vector for inference/prediction

Information Flow

- "Feed-forward neural networks: information flows forward"
  - Oversimplified! Important information flows backward *during training*
  - This influences design, inspired creation of important components
  - E.g. LSTMs, residual units, etc.
- Training often done with gradient-based optimizers
  - E.g. SGD, Adagrad, etc. (briefly discussed in first tutorial)
  - **Gradient information tells us how to update model parameters!**
- **Forward pass**
  - Compute function we are learning
- **Backward pass**
  - Compute gradient information to learn parameters
- How to compute gradients of functions computed by FNNs? **Backpropagation!**



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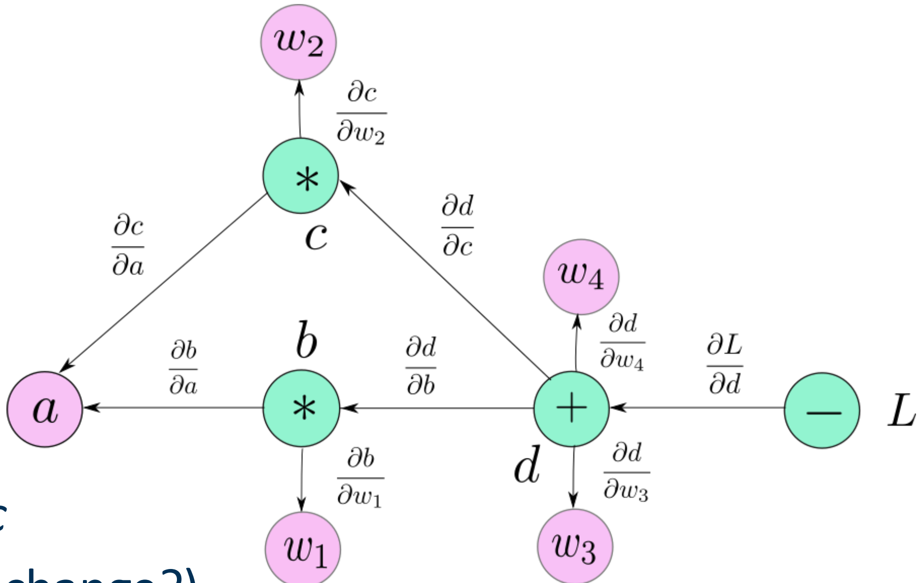
Backpropagation (1)

- What is a derivative?
  - Let $y = f(h)$
  - dy/dh asks: **how much does *y* change when *h* changes?**
- A gradient is a multidimensional derivative
  - Let $\boldsymbol{h}$ be n-dimensional vector and $y = f(\boldsymbol{h})$ a function from R^n to R
  - $\partial y/\partial \boldsymbol{h} = (\partial y/\partial h_1, \partial y/\partial h_2, \ldots, \partial y/\partial h_n)$ is gradient of y w.r.t. $\boldsymbol{x}$
  - I.e. the gradient of y w.r.t. $\boldsymbol{h}$ is also an n-dimensional (row) vector
  - Geometrically, **gradient vector is direction of the largest change of *y***
- **What rate of change are we interested in during training?**
  - How much does loss L change when model parameters $\boldsymbol{\Theta}$ change?
  - I.e. $\partial L/\partial \boldsymbol{\Theta}$
- What if L is output of a compound parameterized function (FNN)?
  - E.g. $L = f(y)$ where $\boldsymbol{y} = g(\boldsymbol{h}|\boldsymbol{\Theta}_2)$, $\boldsymbol{h} = z(\boldsymbol{x}|\boldsymbol{\Theta}_1)$ (z is 1st layer, g is 2nd, f is loss)
  - Here $\partial \boldsymbol{y}/\partial \boldsymbol{h}$ is a Jacobian ($m \times n$ matrix where m is size of $\boldsymbol{y}$, n size of $\boldsymbol{h}$)
  - Same for $\partial \boldsymbol{h}/\partial \boldsymbol{x}$

Backpropagation (2)

- We **propagate the change/error back** using the chain rule!
- Recall chain rule
  - Let $y = g(h)$ and $h = f(\Theta)$
  - $\partial y/\partial \Theta = \partial y/\partial h \cdot \partial h/\partial \Theta$
- Take this computation graph
  - **Internal nodes:** operations
  - **Leaves:** learnable weights w_i, input a
- Here $L = d - label$, $d = w_3 b + w_4 c$
- We want $\partial L/\partial w_4$ (which rate of change?)
- **Chain rule:** $\partial L/\partial w_4 = \partial L/\partial d \cdot \partial d/\partial w_4$
- Similarly, $\partial L/\partial w_2 = \partial L/\partial d \cdot \partial d/\partial c \cdot \partial c/\partial w_2$ where $c = w_2 a$
  - *Note that* $\partial L/\partial d$ is useful for both $\partial L/\partial w_4$ and $\partial L/\partial w_2$
  - **Reusing gradients is common during backpropagation**



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Backpropagation (3)

- Perhaps more important, take $\partial L/\partial w_2 = \partial L/\partial d \cdot \partial d/\partial c \cdot \partial c/\partial w_2$
  - Here, gradient $\partial c/\partial w_2$ *is* multiplied by factor $\partial L/\partial d \cdot \partial d/\partial c$
  - $\partial L/\partial d$ comes from *passing gradient* through minus operator in L
  - $\partial d/\partial c$ comes from *passing gradient* through plus operator in d
- Thus, **operations have impact on their "input" gradients!**
  - Important to consider when designing networks
  - E.g. *plus* operator has no effect on gradient
  - *Concat* operator splits gradient into two
- E.g. if either $\partial d/\partial c$ or $\partial L/\partial d$ are close to zero, $\partial L/\partial w_2$ will be small
  - In other words, we get little info about how to change w_2 to improve L
  - Generally, small gradients = slow/difficult to learn
- Many challenges in *training* deep networks, e.g. vanishing gradients
  - Inspired solutions like LSTMs, residual connections (used in transformers)
- **Let's look at a FNN for a practical NLP use case!**

Language Modeling Recap

- Predict the next word:
  - "Every Thursday there is a Schneckenhof …"
- This suggests:
  - **Some words are more likely to appear than others given some context**
- Probabilistically:
  - *p(meeting|"Every Thursday there is a Schneckenhof")*
- **Generally:** $p(w_{n+1}|w_1,w_2,\ldots,w_n)$
  - Conditional probability of w_{n+1} given joint distribution of $w_1,w_2,\ldots,w_n$
  - w_i are words/tokens in some fixed vocabulary V the model can process
- **Such models can predict entire sequences with probability chain rule**
  - $p(w_1,w_2,\ldots,w_n) = p(w_1)p(w_2|w_1)\, p(w_3|w_1,w_2)\ldots\, p(w_n|w_1,w_2,\ldots,w_{n-1})$
- Useful in many NLP settings, e.g. machine translation
  - A good language model should identify more common phrases
  - p(*This sentence makes sense*) < p(*Makes sense this sentence does*)

Language Modeling with FNNs (1)

- Bengio et al. (2003) proposed a FNN that simultaneously learns:
  1. "Distributed representations of words"
  2. "Probability function of word sequences expressed in terms of these representations"
- Size of word vectors (30, 60, 100) much smaller than vocabulary (17K)
- **Intuition: similar words have similar feature vectors**
- Thus, training sentence *The cat is walking in the bedroom* can be generalized to, e.g.:
  - *A dog* is walking in a *room*
  - The cat is *running* in a *room*
  - *A dog* is walking in a bedroom
  - The dog is walking in the *room*
- **How did they implement such a model?**
  - **A feed-forward neural network**
  - Not the first time it was done, but the first time at that scale

Language Modeling with FNNs (2)

- How did they model $p(w_m | w_1, w_2 \ldots, w_{m-1})$?
  1. A mapping C between every word $w_i \in V$ to a m-dimensional vector $\boldsymbol{w}_i$
     Today known as an **embedding layer**.
     Given w_i, get row i in embedding matrix $\boldsymbol{C}$ (V rows, each a vector $\mathbf{w}_\mathrm{i}$)
  1. Function g that maps input sequence of word vectors to a probability vector of size $|V|$, i-th component is probability of i-th word being next
- **Embeddings are parameters that are learned**
- Function g uses these embeddings plus additional parameters $\boldsymbol{\omega}$
  - $p(w_i | w_1, w_2 \ldots, w_{m-1}) = g(i, \boldsymbol{w}_1, \boldsymbol{w}_2 \ldots, \boldsymbol{w}_{m-1} | \boldsymbol{\omega})$ where i is index of word w_i
- Overall, they learned function f, a composition of C and g, with a FNN
  - $f(i, w_1, w_2 \ldots, w_{m-1}) = g(i, C(w_1), C(w_2) \ldots, C(w_{m-1}) | \boldsymbol{\Theta})$
  - All model parameters are $\boldsymbol{\Theta} = (\boldsymbol{C}, \boldsymbol{\omega})$
- It's easy to see what C might look like, but what does g look like?
  - In other words, what did their network architecture look like?

Language Modeling with FNNs (3)

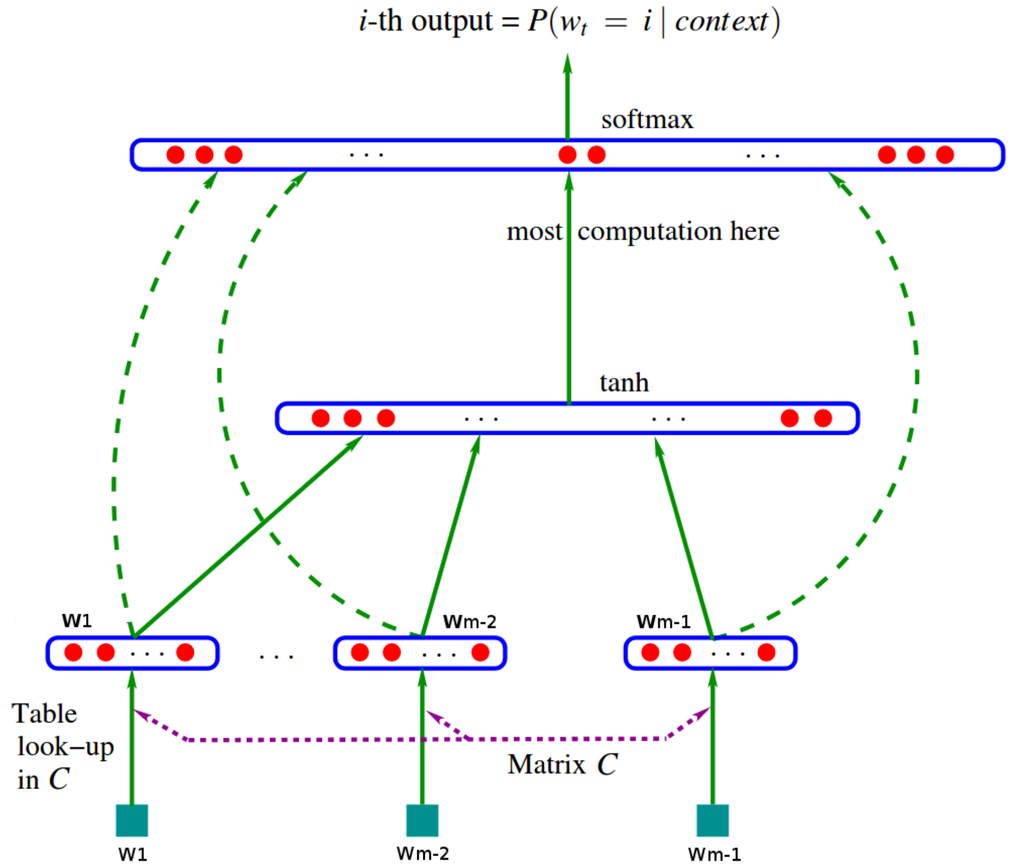
1st: Get word vectors $\boldsymbol{w_i}$

2nd: Concatenate vectors to form $\boldsymbol{x}$

3rd: Hidden layer (matrix ***H*** transform. + *tanh* activ.)

4th: *Optional* linear layer w. matrix ***W*** from input to softmax layer (dashed lines)

5th: Softmax layer (matrix ***U*** transform. + softmax)



Overall: input to softmax

- $y = \boldsymbol{b} + \boldsymbol{Wx} + \boldsymbol{U}tanh(\boldsymbol{d} + \boldsymbol{Hx})$ where ***b,d*** are biases
- Then $\boldsymbol{\Theta} = (\boldsymbol{b}, \boldsymbol{d}, \boldsymbol{W}, \boldsymbol{U}, \boldsymbol{H}, \boldsymbol{C})$

Language Modeling with FNNs (4)

- How did they train?
  - They estimated **Θ** that maximized likelihood of training data (i.e. **MLE**)
- Specifically:
  - $L = 1/T \sum_m \log f(w_1, w_2 \ldots, w_{m-1}; \boldsymbol{\Theta}) + R(\boldsymbol{\Theta})$
  - Here, T is number of words in training corpus
- Note a few things:
  - They average loss over the training set (empirical risk)
  - Penalizing term $R(\boldsymbol{\Theta})$ is regularization (weight decay on ***C, W, H, U***)
- They trained in a **self-supervised** manner
  - I.e. yes, there are labels, but *no manual labeling* required
  - Instead, they used sequences of length *m* in the training corpus
  - For each sequence, first *m-1* words are input, word *m* is target
- **Many of these components still in use today!**
  - Softmax layer, MLE, empirical risk, self-supervision

Summary: DL Basics

- **FNNs:** framework for constructing parameterized real-valued functions
  - We design and learn these functions (algebraic circuits)
- Information flows in both directions on these circuits
  - **Forward pass:** model expressivity
  - **Backward pass:** model "learnability"
- **Backpropagation:** training deep networks, chain rule
- **Important: we can learn useful features with deep models!**
  - Hidden layers interpreted as *learned representations*
  - Learned representations useful in other settings, i.e. transfer learning
  - E.g. style transfer, pre-training word representations
- **Language modeling with FNNs** by Bengio et al. (2003)
  - Established many modeling and training components still used today
- **Later in the course**, other types of FNNs relevant for NLP
  - **RNNs, transformers**

Word Embeddings

About this Section

- **Goal:** introduce concepts and methods about static word embeddings
- **Outline:**
    1. Sparse vs Dense Word Representations
    2. Word2Vec
    3. Properties of Word Vectors
    4. Evaluation of Word Vectors

What do we want to represent? (1)

- Words? I.e. strings of symbols such as *cat* or *dog*?
  - No, those are representations themselves!
  - What do they represent?
- **Meaning!**
  - We map strings such as *cat* or *dog* to concepts (or *senses*)
  - We refer to this mapping as *meaning*, what those words refer to
- **Is meaning a 1-to-1 mapping?**
  - **No**, words can have many meanings (remember ambiguity?)
- How do we, humans, handle word ambiguity? **Context!**
  - “I work at a bank.”
  - What’s more likely? A financial institution or by the river?
  - Again, context. Does the person work in Frankfurt? Are they fishermen?
- So, **we want to represent the relation between words and senses!**

What do we want to represent? (2)

- What do we want from such representations? Ideally usefulness, e.g.
  - Tell us if words have similar meanings, e.g. cat and dog
  - Tell us if words are antonyms, e.g. hot and cold
  - Tell us if they have positive or negative connotations, e.g. happy or sad
- **"Generally, a model of word meaning should allows us to draw inferences to address meaning-related tasks like question answering."** Jurafsky and Martin, 2023
- Perhaps we should represent senses, then, not words!
  - Use something like WordNet, expert-crafted database of senses
- Databases like WordNet have not worked well in practice
  - Missing nuance, "good" is synonym of "proficient" only in some cases
  - New meanings missing, e.g. *genius*, *wicked*, *wizard*
- In practice, **dealing with word representations works better**
  - We focus on their relation to meaning (based on context)
  - We don't commit to a given representation of sense

Sparse Word Representations

- Traditional approach: sparse word vectors
- Example: **one-hot vectors**
  - Given vocabulary *V*, represent each word with vector of size *|V|* using one-hot encoding
- For example, given $V = \{cat, dog, airplane\}$
  - $V_{cat} = [1, 0, 0]$
  - $V_{dog} = [0, 1, 0]$
  - $V_{airplane} = [0, 0, 1]$
- **Problem 1:** *|V|* can be very large
  - Recall Bengio's work from 2003: $|V| = 17K$
- **Problem 2:** No natural notion of similarity from such vectors
  - They are all equally different, e.g. with a dot product
  - But *cat* and *dog* are more similar to each other than to *airplane*
- Generally, **difficult to draw inferences from such representations**
  - Let's look at another approach that addresses these issues

Distributed Representations

- Idea from deep learning community (Geoffrey Hinton in 1986)
- Sparse representations:
  - Each representational component maps to single represented object
  - E.g. each feature in vector corresponds to one word
- **Distributed (or dense) representations:**
  - Many-to-many relation between representational components and represented objects
  - E.g. each word is represented with a set of (distributed) features
- **PROs:** model has freedom to use features “at will“ during learning
- **CONs:** Less interpretable than, e.g. one-hot encoding
  - Given V, we know what word is represented by [0, 1, 0]
  - But what word is this? [0.23, 0.447, 0.02]
- Using **vectors to represent words** also has a **history in linguistics**
  - Osgood et al. (1957) represented words with 3-dimensional vectors
  - Each dimension encoded known useful (non-learned) features

Distributional Semantics (1)

- Idea from linguistics (Joos, 1950; Harris, 1954; Firth, 1957)
- Not the same concept!
  - *Distributed* representations != *distributional* semantics
  - In NLP, the latter can be seen as a special case of the former
- Let's guess what the word *foobar* means
  1. *Foobar* is often played in teams of 7.
  2. A *foobar* match is divided into 3 thirds of 25 minutes each, called runs.
  3. A *foobar* match is won by scoring 10 points.
- **What would you guess the word *foobar* refers to? Why?**
  - A type of food?
  - A sport?
  - A country?
- **Distributional hypothesis:**
  - *"A word is characterized by the company that it keeps."* (Firth, 1957)

Distributional Semantics (2)

- Put differently:
  - We define the meaning of a word by its *distribution* in a language
  - I.e. by its neighboring words, grammatical environment
- **Words whose neighboring words are similar, have similar meanings.**
- We can use this idea to build useful **vector representations** of words!
- For example: **co-occurrence matrices**
  - Given defined *context*, e.g. a document, a sentence, a word window
  - Construct matrix C of size V x V where V is given vocabulary
  - Rows in matrix are words, columns are also words
  - m_{ij} denotes number of times word i appears in same *context* as word j (so, sparse)
- Co-occurrence matrices useful in many applications
  - E.g. latent semantic analysis (LSA) --> singular value decomposition of C
  - Extracts vector representations from (truncated) left–singular vectors
  - Similar count-based methods covered in basic Text Analytics course
- **Word embeddings: distributional hypothesis to learn dense vectors**

Word Embeddings

- **Idea:** to *embed* higher dimensional vectors into lower dimensional vector space such that (some of) the relative properties of the higher dimensional space are kept
  - Hence, **embedding** --> (**relatively) low-dimensional vector representation**
  - Low-dimensional compared to, e.g. $|V|$
  - Size typically between 50 and 1000 dimensions
  - Vectors are dense (distributed) representations
- Historically important approach: **word2vec** (Mikolov et al. 2013)
  - Training based on distributional semantics
  - Done in the context of deep learning (though this model is not deep)
- Intuition:
  - **Instead of counting words for co-occurrence, let's train a classifier to predict whether two words are likely to appear in the same context.**
- Two approaches proposed for learning such a classifier
  - Skip-gram and continuous bag of words (CBOW) (we focus on skip-gram)

The Skip-Gram Algorithm

- Algorithm for learning dense word vectors
- **Basic steps**
  1. Treat target words and neighboring context words as **positive examples**
  2. Randomly sample other words in vocabulary to get **negative examples**
  3. Train a classifier to predict whether two given words appear together
  4. Use learned weights as word embeddings
- **Why negative examples?**
  - In some tasks, negative examples often needed to avoid trivial solutions
  - E.g. push all vectors to same point, thus all vectors similar
  - If training objective is poorly designed, such a solution may "reach objective"
- Let's have a look at:
  1. **The task**
  2. **The model**
  3. **The training objective**

Skip-Gram: The Task

- Say we have the sentence:
  - *… known as [ramen, a Japanese dish that] has recently become…*

    c_1 c_2 w c_3 c_4
- Here, *w* denotes **center word** and c_i denotes **context words**
  - I.e. context window is plus/minus a few words in this case
  - More context --> usually more useful, not always, more later
  - E.g. plus/minus 1 gives us *[a; ???; dish]*
  - Plus/minus 2 gives us *[ramen; a; ???; dish; that]* (assumes no punctuation)
- **Task:** given tuple *(w,c)*, predict if *c* appears in context of *w* or not
  - So, binary classification
- Probabilistically, *P(True|w,c)*
  - Similarly, *P(False|w,c) = 1 - P(True|w,c)*
- How can we model these probabilities?

Skip-Gram: The Model (1)

- How to get probability *P(True|w,c)*?
  - **Key idea:** based on similarity of corresponding word embeddings
  - I.e. a word *c* is likely to occur near a target word *w* if their corresponding embeddings $\boldsymbol{e}_c$ and $\boldsymbol{e}_w$ are similar
  - Probabilistically, *w* and *c* are independent given their embeddings $\boldsymbol{e}_c$, $\boldsymbol{e}_w$
  - Let $\boldsymbol{\Theta} = [\boldsymbol{e}_{1, \dots,} \boldsymbol{e}_{|V|}]$. We have $P(True|w,c,\boldsymbol{\Theta})$ ($\boldsymbol{\Theta}$ often omitted for brevity)
- How can we represent that similarity?
  - Skip-gram went with **dot product**, i.e. $\boldsymbol{e}_c^T \boldsymbol{e}_w$
- But didn't we need a probability?
  - Ok, then $P(True|w,c) = \sigma(\boldsymbol{e}_c^T \boldsymbol{e}_w)$ where σ is the **sigmoid function**
  - Similarly, $P(False|w,c) = 1 - \sigma(\boldsymbol{e}_c^T \boldsymbol{e}_w) = \sigma(-\boldsymbol{e}_c^T \boldsymbol{e}_w)$
  - The last equality results from a property of the logistic function
- This is the probability of word *c* being in context of word *w*.
  - **What if I want to provide more context?** More context is better, right?

Skip-Gram: The Model (2)

- How do we compute $P(True|w,c_{1:L})$ where L is size of context window?
  - **Assumption: context words are independent of each other**
  - Computing joint distribution of words in context becomes much simpler
- Thus, we have

$$P(True|w,c_{1:L}) = \prod_{i\ in\ L} \sigma(\mathbf{e}_i^T \mathbf{e}_w) = \sum_{i\ in\ L} \log \sigma(\mathbf{e}_i^T \mathbf{e}_w).$$

- **How is this model parameterized?**
  - It actually learns *two* representations for each word
  - One for words as targets, one for words as context (just one vocabulary V)
  - Thus, parameterized by matrices $\boldsymbol{W}$ and $\boldsymbol{C}$, both of size $|V| \times d$ where d is a hyperparameter (embedding size)
  - *In other words,* $\boldsymbol{\Theta} = [\boldsymbol{W}, \boldsymbol{C}]$
- How was this model trained?

Skip-Gram: The Training Objective (1)

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- As with Bengio's LM from 2003: **self-supervision**
- Given text corpus (i.e. large sequence of text)
  - Iterate over sequences of size *n = L*2 + 1* (L is size of context window)
  - For each sequence, set center word as target, other words as context
- **For example:**

1. *… [known as ramen, a Japanese] dish that has recently become…*

   c_1 c_2 w c_3 c_4

2. *… known [as ramen, a Japanese dish] that has recently become…*

   c_1 c_2 w c_3 c_4

3. *… known as [ramen, a Japanese dish that] has recently become…*

   c_1 c_2 w c_3 c_4

- Thus, **for sequences 1, 2 and 3, you get positive examples *(w,c)***:

1. (ramen, known), (ramen, as), (ramen, a), (ramen, Japanese)
2. (a, as), (a, ramen), (a, Japanese), (a, dish)
3. (Japanese, ramen), (Japanese, a), (Japanese, dish), (Japanese, that)

Skip-Gram: The Training Objective (2)

- We also need **negative examples!**
- **Given *w*, we randomly sample words from *V* as candidates for *c***
  - *For example:* (ramen, ostrich), (ramen, the), (ramen, Alaska), …
  - They generated k (hyperparameter) negative examples per positive
- Then, given (w,c_{pos}), corresponding $(w,c_{neg\ i})$, and cross entropy (CE), we have:

$$
\begin{aligned}
L_{CE} &= -\log\left[P(+|w,c_{pos})\prod_{i=1}^{k}P(-|w,c_{neg_i})\right] \\
&= -\left[\log P(+|w,c_{pos}) + \sum_{i=1}^{k}\log P(-|w,c_{neg_i})\right] \\
&= -\left[\log P(+|w,c_{pos}) + \sum_{i=1}^{k}\log\left(1-P(+|w,c_{neg_i})\right)\right] \\
&= -\left[\log \sigma(c_{pos}\cdot w) + \sum_{i=1}^{k}\log \sigma(-c_{neg_i}\cdot w)\right]
\end{aligned}
$$

- Generally, they applied empirical risk over all training examples
  - Since this is negative log likelihood (NLL), they did MLE of the training set

Task vs Goal

- **Task:** *P(True|w,c)*
  - What do we use that task for?
  - *Real-world uses?*
- **Goal:** learn word embeddings!
  - Task designed to force model to capture properties (distributional semantics)
  - Original task was different, $P(w_{t+j}|w_t)$
- Other models have tasks and goals
  - E.g. GPT-style LLMs are autoregressive language models
  - BERT-style models are masked language models
- **Sometimes we want to use the model to make predictions**
  - **Other times, we just want their learned weights**

Other Types of Static Embeddings

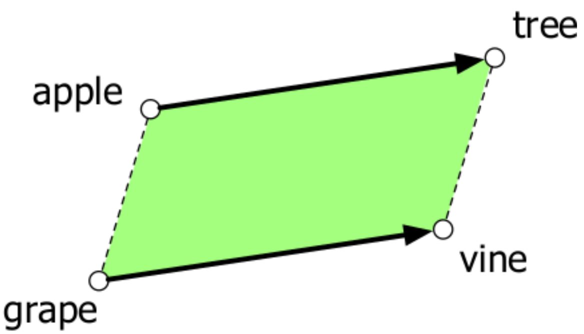
- **Word2vec is useful model for intuition/learning about embeddings**
  - It was also successful and is historically important
  - But there are other ways of learning word embeddings
  - These are often used in practice as well
- **GloVe:** stands for Global Vectors (Pennington et al., 2014)
  - Combines count-based models with methods like word2vec
  - Constructs co-occurrence matrix
  - Learns matrix factorization with least squares objective
  - Covered in basic Text Analytics course
- **fasttext:** addresses problem of unknown words in word2vec
  - That is, new words in test that were not present in training
  - For this, they used representations based on subwords
  - Also covered in basic Text Analytics course
- Static embeddings have some **nice and useful properties!**
  - At the time, they were quite impressive!

Properties of Word Embeddings (1)

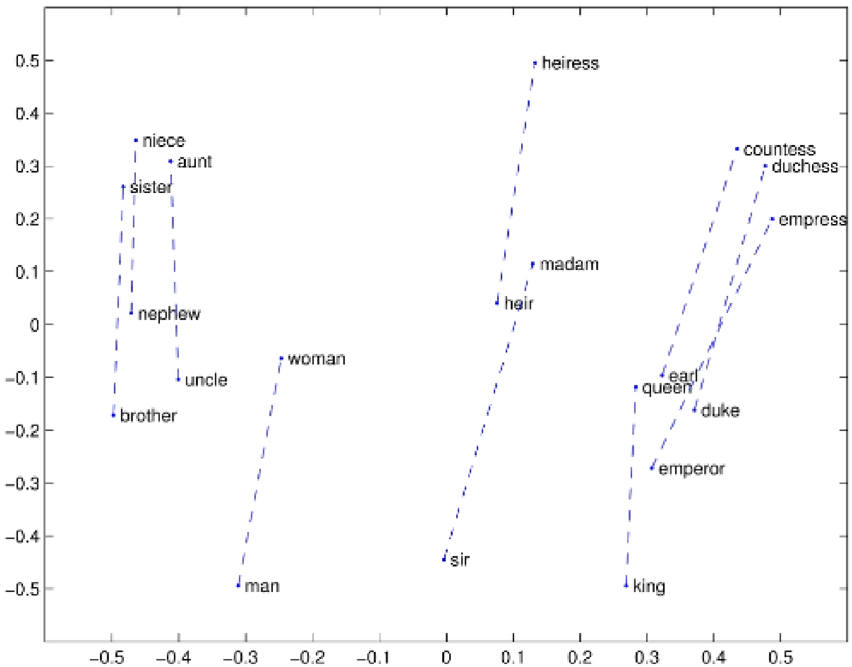
- **Size of context window is important!** (Levy and Goldberg, 2014)
  - It affects the representations we learn!
- **Smaller sizes tend to provide more static embeddings**
  - Depends on immediate neighborhood
  - Similarity tends to be based on same parts of speech
- E.g. what is similar to Hogwarts with small context window?
  - Other fictional schools, e.g. Sunnydale (from Buffy the Vampire Slayer)
- **Larger sizes tend to focus on similarity based on topic**
  - So words with same part of speech not necessarily related
- E.g. what is similar to Hogwarts with larger context window?
  - Dumbledore, Malfoy, half-blood
- **This was observed for skip-gram**
  - Different methods may differ in such properties
  - But it is safe to assume the context window has an impact

Properties of Word Embeddings (2)

- They capture relational similarity
  - Parallelogram model from cognitive science



- Example from GloVe
  - Skip-gram known to capture this
  - It’s not as smooth all the time



Evaluating Word Embeddings

- **Intrinsic evaluation:**
  - Correlation between similarities of representations with expert-given similarity scores
- Examples:
  - WordSim353 (2002): 353 noun pairs, e.g. *(plane, car)*
  - SimLex-999 (2015): more difficult, e.g. *(cup, mug)*
- Another approach: **analogy task** (e.g. *man + woman – king = queen*)
  - Word2vec used this in their original work
- **Most important evaluation: extrinsic!**
  - I.e. using the representations in NLP *downstream tasks* and see if performance improves

Summary: Word Embeddings

- **Vector representations** of words
- **Sparse vs dense** (distributed) representations
  - PROs and CONs to each
  - Different methods to learn them (each a different task)
- **Distributional semantics**
  - Important principle
  - Commonly used to learn word embeddings, language models
  - Spoiler: also used when training LLMs
- Classic way to learn dense word representations: **word2vec**
  - Specifically, the skip-gram task, model and training objective
  - Resulting learned representations have **nice properties**
- **In the future**, other tasks for other goals
  - E.g. masked and autoregressive language models

References

- Natural Language Processing: A Machine Learning Perspective, Zhang et al, 2018
  - Chapters 2, 3, 4 and 13
- Speech and Language Processing, Jurafsky et al., 2023
  - Chapter 3, 6
- Natural Language Processing, Eisenstein, 2018
  - Chapter 14