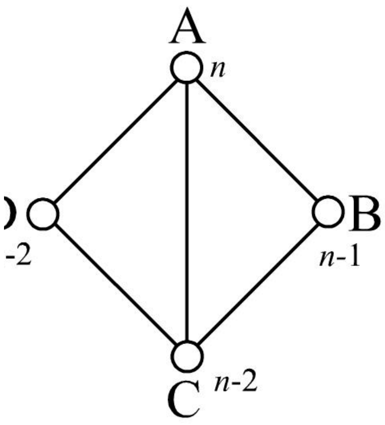
# Colouring a Network

We can solve the first two problems using simple multiplication.
We colour vertices from A to D.




Vertex A: no restrictions. $n$ ways to colour.

Vertex B: Cannot be same colour as A. $n – 1$ ways

Vertex C: Cannot be same colour as A or B. A and B are always coloured different because they are connected by a line. $n – 2$ ways

Vertex D: Cannot be same colour as A or C, which are always differently coloured because they are connected by a line. $n – 2$ ways

Total: $n(n – 1)(n – 2)(n – 2)$

# Colouring a Network

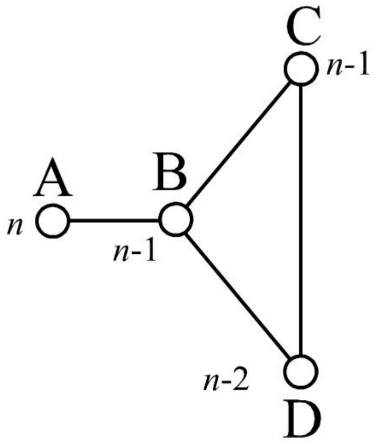
Network 2:

Vertex A: no restrictions. $n$ ways to colour.

Vertex B: Cannot be same colour as A. $n-1$ ways

Vertex C: Cannot be same colour as B. $n-1$ ways

Vertex D: Cannot be same colour as B or C, which are always differently coloured because they are connected by a line. $n-2$ ways




Total: $n(n-1)(n-1)(n-2)$

# Colouring a Network

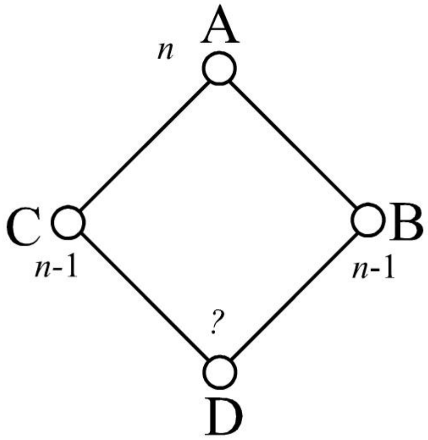
Network 3:

Vertex A: no restrictions. $n$ ways to colour.

Vertex B: Cannot be same colour as A. $n - 1$ ways

Vertex C: Cannot be same colour as A. $n - 1$ ways.

Vertex D: Cannot be same colour as B or C, but they could be the same colour or different.



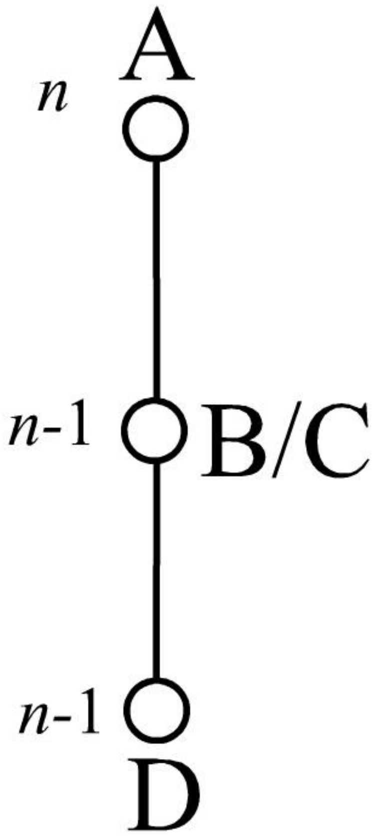
# Colouring a Network

When we set two nodes in a network to be the same colour, we are merging the two nodes.

When we set two nodes to be different colours, we are drawing a line between the nodes.

We split the problem into 2 cases.

If B and C are the same colour, then the # of ways to colour A-C is $n(n-1)$. D cannot be the same colour as B or C, which are the same colour. So the # of ways is $n(n-1)(n-1)$.



# Colouring a Network

- If B and C are different colours, then the # of ways to colour A-C is $n(n-1)(n-2)$. D cannot be the same colour as B or C, which are different. So the # of ways is $n(n-1)(n-2)(n-2)$.

Total: $n(n-1)(n-1) + n(n-1)(n-2)(n-2)$
$= n(n-1)(n-1+(n-2)(n-2))$
$= n(n-1)(n^2-3n+3)$
$= n^4-4n^3+6n^2-3n$
$= (n-1)^4+(n-1)$