AM $\geq$ GM

For a set of n numbers $x_1 \ldots x_n$:

Arithmetic Mean (AM) = $(x_1+x_2+x_3+ \ldots +x_n)/n$

Geometric Mean (GM) = $\sqrt[n]{(x_1 \times x_2 \times x_3 \times \ldots x_n)}$

One of the most well-known inequalities states that

AM is always $\geq$ GM

It is a bit dry and hard to imagine when stated this way. It is possible to visualize this inequality in a tangible and more intuitive way…

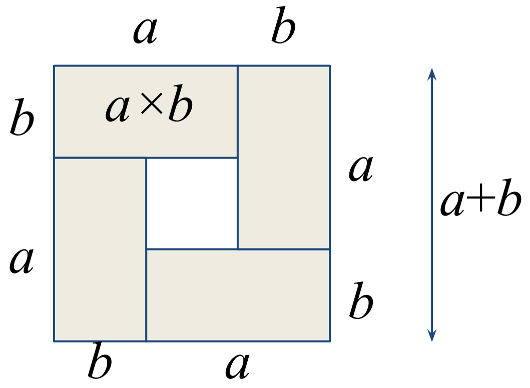
AM $\geq$ GM : 2D Case

Given 2 numbers a, b:

$$(a+b)/2 \geq \sqrt{(a \times b)}$$

$$\Leftrightarrow \quad (a+b)^2/4 \geq a \times b$$

$$\Leftrightarrow \quad (a+b)^2 \geq 4a \times b$$



This could be interpreted as that the area of a square with side-length $(a+b)$ is always $\geq$ the total area of 4 $a \times b$ rectangles. The above figure shows that this is true indeed.

AM ≥ GM : 3D Case

Given 3 numbers a, b, c:

$$(a+b+c)/3 \geq \sqrt[3]{(a\times b\times c)}$$

$$\Leftrightarrow \quad (a+b+c)^3/27 \geq a\times b\times c$$

$$\Leftrightarrow \quad (a+b+c)^3 \geq 27a\times b\times c$$

This can also be interpreted as saying that the volume of a cube with side-length $(a+b+c)$ is always ≥ the total volume of 27 $a\times b\times c$ rectangular blocks.

It turns out that we can indeed fit 27 blocks of arbitrary dimensions $a\times b\times c$ into a cube of side-length $(a+b+c)$ – Tricky!

😕 *Really?*

Packing Blocks

You could build your own set of bricks using cardboard, but it's a big undertaking to make 27 identical boxes with accurate dimensions – it took me many many hours…

If you don't have the patience to build paper blocks, you can make them out of lego!