Mining Massive Datasets

Lecture 10

Artur Andrzejak
http://pvs.ifi.uni-heidelberg.de

RUPRECHT-KARLS-
UNIVERSITÄT
HEIDELBERG

Note on Slides

A substantial part of these slides come (either verbatim or in a modified form) from the book
Mining of Massive Datasets
by Jure Leskovec, Anand Rajaraman, Jeff Ullman
(Stanford University).
For more information, see the website accompanying the book: http://www.mmds.org.

Association Rule Discovery

Supermarket shelf management: Market-basket model:

- **Goal:** Identify items that are bought together by sufficiently many customers
- **Approach:** Process the sales data collected with barcode scanners to find dependencies among items
- **A classic rule:**
  - If someone buys diaper and milk, then he/she is likely to buy beer

The Market-Basket Model

- A large set of **items**
  - e.g., things sold in a supermarket
- A large set of **baskets**
  - Each basket is a small subset of items
  - E.g., the things one customer buys on one day
- Goal: discover **association rules**
  - People who bought {x,y,z} tend to buy {v,w}
    - Amazon!

Input:

TID	*Items in a basket*
1	Bread, Coke, Milk
2	Beer, Bread
3	Beer, Coke, Diaper, Milk
4	Beer, Bread, Diaper, Milk
5	Coke, Diaper, Milk

Output:

Rules Discovered:

{Milk} --> {Coke}

{Diaper, Milk} --> {Beer}

Applications

- **Items** = products; **Baskets** = sets of products someone bought in one trip to the store
- **Real market baskets:** Chain stores keep TBs of data about what customers buy together
  - Tells how typical customers navigate stores, lets them position tempting items
  - Suggests tie-in “tricks”, e.g., run sale on diapers and raise the price of beer
- **Amazon’s people who bought *X* also bought *Y***

Outline

First: Define

- Frequent itemsets
- Association rules: Confidence, Support, Interestingness

Then: Algorithms for finding frequent itemsets

- Finding frequent pairs
- A-Priori algorithm
- PCY algorithm + refinements

Frequent Itemsets

- **Simplest question:** Find sets of items that appear together "frequently" in baskets
- **Support** for itemset I: Number of baskets containing all items in I
  - (Often expressed as a fraction of the total number of baskets)
- Given a ***support threshold s***, ***frequent itemsets*** are sets of items that appear in at least s baskets

TID	*Items*
1	**Bread, Coke, Milk**
2	**Beer, Bread**
3	**Beer, Coke, Diaper, Milk**
4	**Beer, Bread, Diaper, Milk**
5	**Coke, Diaper, Milk**

Support of {Beer, Bread} = 2

Example: Frequent Itemsets

- **Items** = {milk, coke, pepsi, beer, juice}
- **Support threshold = 3 baskets**

$\mathbf{B_1}$ = {m, c, b}	$\mathbf{B_2}$ = {m, p, j}
$\mathbf{B_3}$ = {m, b}	$\mathbf{B_4}$ = {c, j}
$\mathbf{B_5}$ = {m, p, b}	$\mathbf{B_6}$ = {m, c, b, j}
$\mathbf{B_7}$ = {c, b, j}	$\mathbf{B_8}$ = {b, c}

- **Frequent itemsets:** {m}, {c}, {b}, {j},
  {m,b} , {b,c} , {c,j}.

Association Rules

- **Association Rule:**
  <u>If-then rule</u> about the contents of baskets
- $\{i_1, i_2, \ldots, i_k\} \rightarrow J$ means: "if a basket contains all of $i_1, \ldots, i_k$ then it is *likely* to contain $J = \{j\}$"
- In practice there are many rules, want to find significant/interesting ones
- **Confidence** of this association rule is the <u>probability</u> of $J = \{j_1, .., j_p\}$ given $I = \{i_1, \ldots, i_k\}$

$$\mathrm{conf}(I \rightarrow j) = \frac{\mathrm{support}(I \cup J)}{\mathrm{support}(I)}$$

Interesting Association Rules

- Not all high-confidence rules are interesting
  - The rule $X \rightarrow milk$ may have high confidence for many itemsets X, because milk is just purchased very often (independent of X) and the confidence is high
- **Interest** of an association rule $I \rightarrow J$: difference between its confidence and the fraction of baskets that contain $J = \{j_1, .., j_p\}$

$$\text{Interest}(I \rightarrow J) = \text{conf}(I \rightarrow J) - \Pr[J]$$

  - Interesting rules are those with high positive or negative interest values (usually above 0.5)

Example: Confidence and Interest

$B_1 = \{m, c, b\}$ $B_2 = \{m, p, j\}$
$B_3 = \{m, b\}$ $B_4 = \{c, j\}$
$B_5 = \{m, p, b\}$ $B_6 = \{m, c, b, j\}$
$B_7 = \{c, b, j\}$ $B_8 = \{b, c\}$

- **Association rule: $\{m, b\} \rightarrow c$**
  - **Confidence** = 2/4 = 0.5
  - **Interest** = |0.5 – 5/8| = 1/8
    - Item *c* appears in 5/8 of the baskets
    - Rule is not very interesting!

Set	m,b	m,b,c	c
B1	1	1	1
B2			
B3	1		
B4			1
B5	1		
B6	1	1	1
B7			1
B8			1

Finding Association Rules

$$\text{conf}(I \rightarrow j) = \frac{\text{support}(I \cup j)}{\text{support}(I)}$$

- **Problem:** Find all association rules with support $\geq s$ and confidence $\geq c$
  - **Note: Support of an association rule** is the fraction of transactions containing both $\boldsymbol{I}$ and $\boldsymbol{J}$, i.e. **support(I ∪ J)** ([link](), Sec. 3.1)

- **Hard part:** Finding the frequent itemsets!
  - If $\{i_1, i_2,\ldots, i_k\} \rightarrow J$ has high support and confidence, then both $\{i_1, i_2,\ldots, i_k\}$ and $\{i_1, i_2,\ldots,i_k, j_1,\ldots,j_p\}$ will be "frequent"

Mining Association Rules

- **Step 1:** Find all frequent itemsets I
  - (we will explain this next)
- **Step 2: Rule generation**
  - For every subset A of I, generate a rule $A \to I \setminus A$
    - Since I is frequent, A is also frequent
    - **Variant 1:** Single pass to compute the rule confidence
      - confidence(***A,B***→***C,D***) = support(**A,B,C,D**) / support(**A,B**)
    - **Variant 2:**
      - **Observation:** If **A,B,C**→**D** is below confidence, so is **A,B**→**C,D**
      - Can generate "bigger" rules from smaller ones!
  - Output the rules above the confidence threshold

Example

B_1 = {m, c, b} B_2 = {m, p, j}

B_3 = {m, c, b, n} B_4= {c, j}

B_5 = {m, p, b} B_6 = {m, c, b, j}

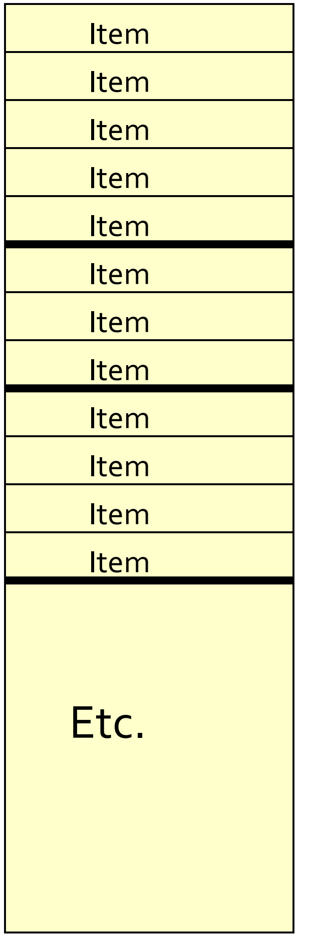
B_7 = {c, b, j} B_8 = {b, c}

- **Support threshold $s = 3$, confidence $c = 0.75$**
- **1) Frequent itemsets:**
  - **Singletons and {b,m} {b,c} {c,m} {c,j} {m,c,b}**
- **2) Generate rules:**
  - ~~**b**$\rightarrow$**m**: c=4/6~~ **b**$\rightarrow$**c**: c=5/6 ~~**b,c**$\rightarrow$**m**: c=3/5~~
  - **m**$\rightarrow$**b**: c=4/5 … **b,m**$\rightarrow$**c**: c=3/4
  - ~~**b**$\rightarrow$**c,m**: c=3/6~~

Finding Frequent Itemsets

Itemsets: Computation Model

- **Back to finding frequent itemsets**
- Typically, data is kept in flat files rather than in a database system:
  - Stored on disk
  - Stored basket-by-basket
  - Baskets are **small** but we have many baskets and many items
    - Expand baskets into pairs, triples, etc. as you read baskets
    - Use ***k*** nested loops to generate all sets of size ***k***



Items are positive integers, and boundaries between baskets are –1.

Note: We want to find frequent itemsets. To find them, we have to count them. To count them, we have to generate them.

Computation Model

- The true cost of mining disk-resident data is usually the **number of disk I/Os**

- In practice, association-rule algorithms read the data in ***passes*** – all baskets read in turn

- We measure the cost by the **number of passes** an algorithm makes over the data

Main-Memory Bottleneck

- For many frequent-itemset algorithms, **main-memory** is the critical resource
  - As we read baskets, we need to count something, e.g., occurrences of pairs of items
  - The number of different things we can count is limited by main memory
  - Swapping counts in/out is a disaster

Finding Frequent Pairs

- **The hardest problem often turns out to be finding the frequent pairs of items $\{i_1, i_2\}$**
  - **Why?** Freq. pairs are common, freq. triples are rare
    - **Why?** Probability of being frequent drops exponentially with size; number of sets grows more slowly with size
- **Let's first concentrate on pairs, then extend to larger sets**
- **The approach:**
  - We always need to generate all the itemsets
  - But we would only like to count (keep track) of those itemsets that in the end turn out to be frequent

Naïve Algorithm

- **Naïve approach to finding frequent pairs**
- Read file once, counting in main memory the occurrences of each pair:
  - From each basket of ***n*** items, generate its ***n(n-1)/2*** pairs by two nested loops
- **Fails if (#items)^2 exceeds main memory**
  - **Remember:** #items can be 100K (Wal-Mart) or 10B (Web pages)
    - Suppose 10^5 items, counts are 4-byte integers
    - Number of pairs of items: $10^5(10^5-1)/2 = 5*10^9$
    - Therefore, $2*10^{10}$ (20 gigabytes) of memory needed

Counting Pairs in Memory

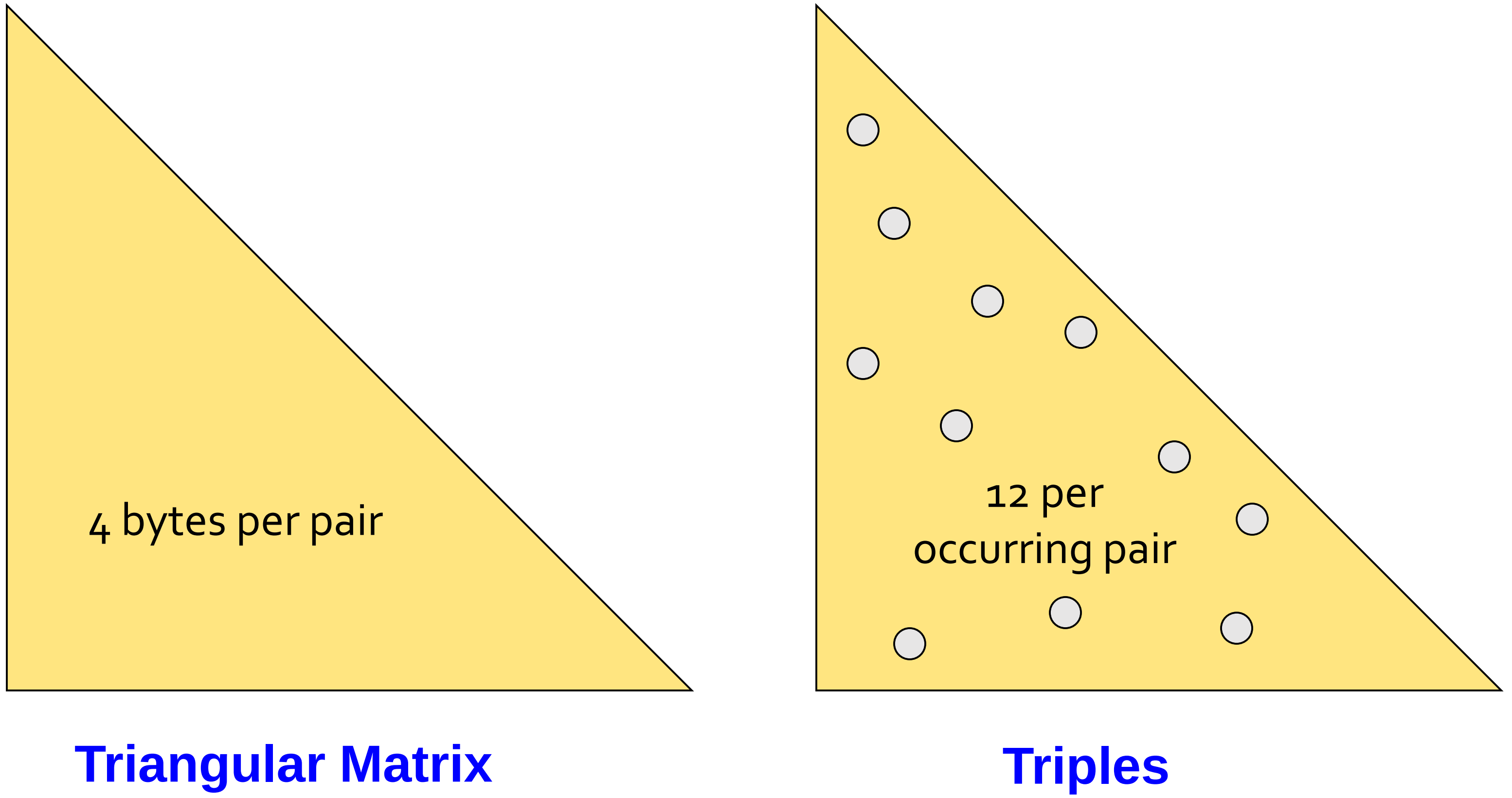
Two approaches:

- **Approach 1:** Count all pairs using a matrix
- **Approach 2:** Keep a table of triples [*i*, *j*, *c*] = "the count of the pair of items {*i*, *j*} is *c*."
  - If integers and item ids are 4 bytes, we need approximately 12 bytes for pairs with count > 0
  - Plus some additional overhead for the hashtable

Note:

- **Approach 1** only requires 4 bytes per pair
- **Approach 2** uses 12 bytes per pair (but only for pairs with count > 0)

Comparing the 2 Approaches



Comparing the two approaches

- **Approach 1: Triangular Matrix**
  - **n** = total number items
  - Count pair of items $\{i, j\}$ only if $i<j$
  - Keep pair counts in lexicographic order:
    - $\{1,2\}, \{1,3\},\ldots, \{1,n\}, \{2,3\}, \{2,4\},\ldots,\{2,n\}, \{3,4\},\ldots$
  - Pair $\{i, j\}$ is at position $(i-1)(n-i/2)+j-1$
  - Total number of pairs $\boldsymbol{n(n-1)/2}$; total bytes= $\boldsymbol{2n^2}$
  - **Triangular Matrix** requires 4 bytes per pair
- **Approach 2** uses **12 *bytes*** per occurring pair *(but only for pairs with count > 0)*
  - Beats Approach 1 if less than **1/3** of possible pairs actually occur

Comparing the two approaches

- **Approach 1: Triangular Matrix**
  - **n** = total number items
  - Co
  - K
  - P
  - T **$2n^2$**
  - **T**
- **Ap**

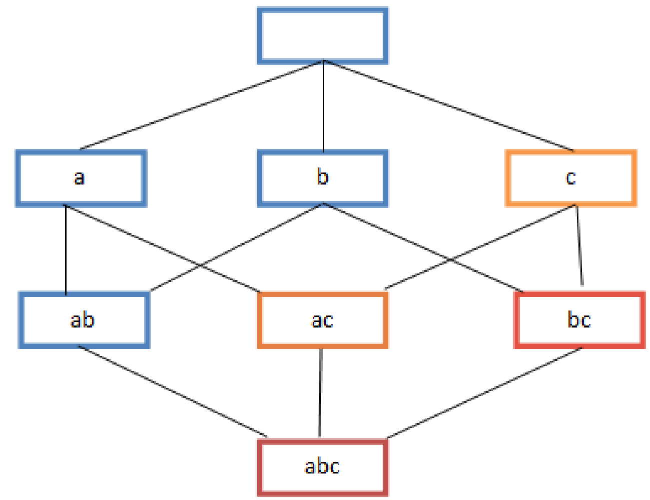
  (bu
  - Beats Approach 1 if less than **1/3** of possible pairs actually occur

Problem is if we have too many items so the pairs do not fit into memory.

Can we do better?

A-Priori Algorithm

A-Priori Algorithm – (1)



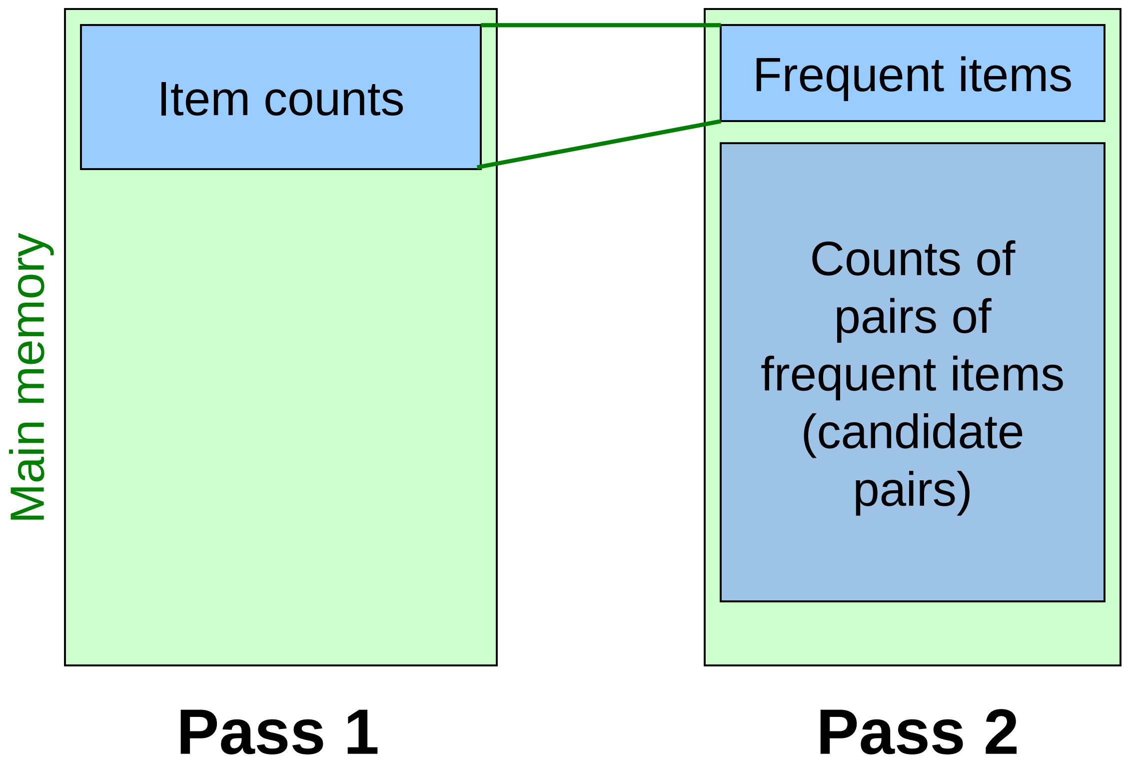
- A **two-pass** approach called ***A-Priori*** limits the need for main memory
- **Key idea: *monotonicity***
  - If a set of items I appears at least s times, so does every **subset** J of I
- **Contrapositive for pairs:**
  If item i does not appear in s baskets, then no pair including i can appear in s baskets

- **So, how does A-Priori find freq. pairs?**

A-Priori Algorithm – (2)

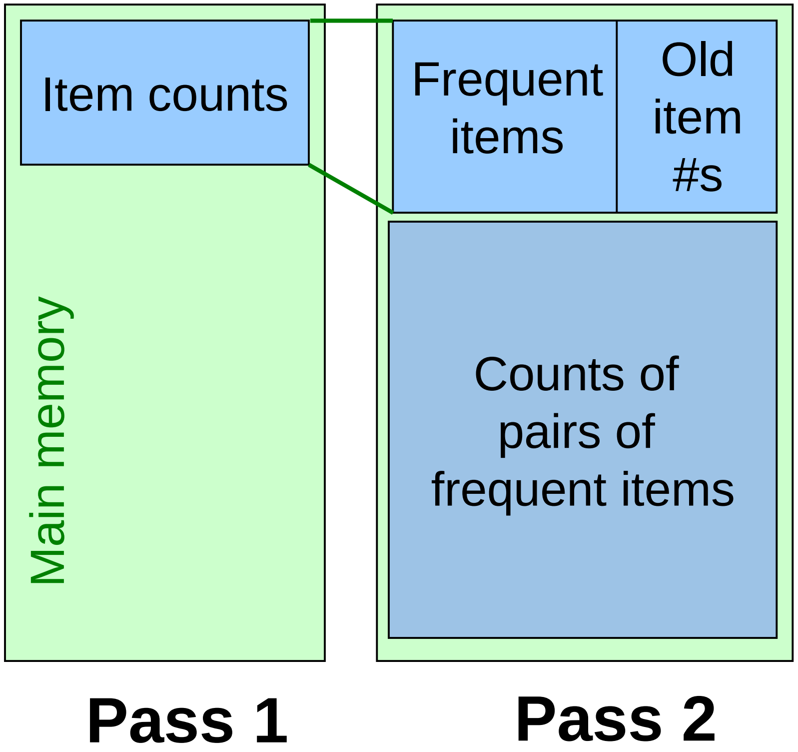
- **Pass 1:** Read baskets and count in main memory the occurrences of each **individual item**
  - Requires only memory proportional to #items
- **Items that appear $\geq s$ times are the frequent items**
- **Pass 2:** Read baskets again and count in main memory only those pairs where both elements are frequent (from Pass 1)
  - Requires memory proportional to square of **frequent** items only (for counts)
  - Plus a list of the frequent items (so you know what must be counted)

Main-Memory: Picture of A-Priori



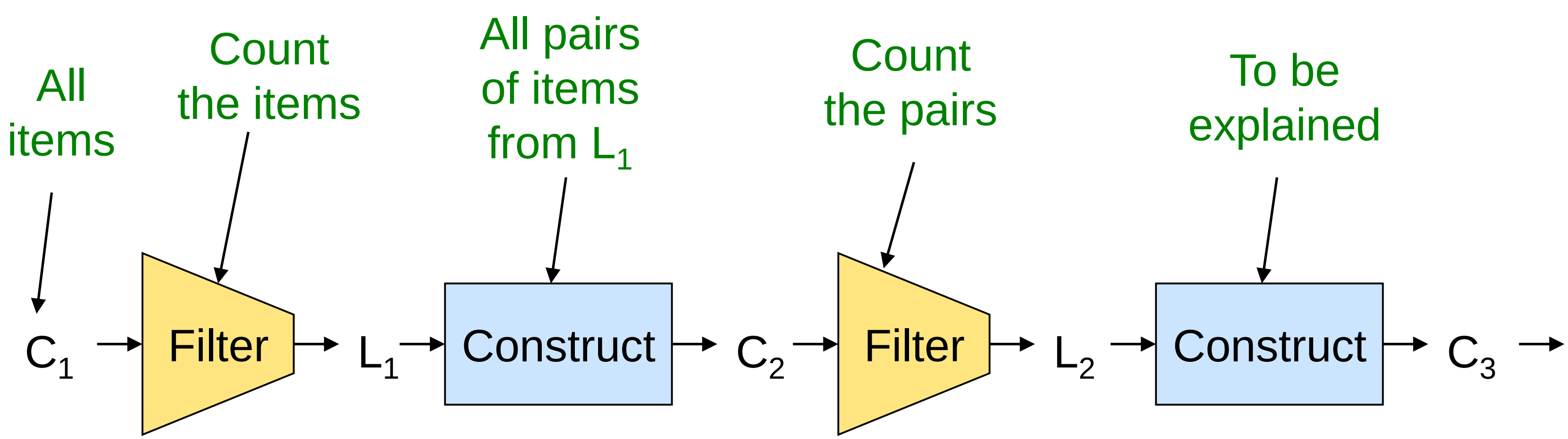
Detail for A-Priori

- You can use the triangular matrix method with ***n*** = number of frequent items
  - May save space compared with storing triples
- **Trick:** re-number frequent items 1,2,… and keep a table relating new numbers to original item numbers



Frequent Triples, Etc.

- **For each *k*, we construct two sets of *k-tuples*** (sets of size *k*):
  - C_k = ***candidate k-tuples*** = those that might be frequent sets (support $\geq$ **s**) based on information from the pass for ***k*–1**
  - L_k = the set of truly frequent ***k***-tuples



Example

** Note here we generate new candidates by generating C_k from L_{k-1} and L_1.
But that one can be more careful with candidate generation. For example, in C_3 we know {b,m,j} cannot be frequent since {m,j} is not frequent

- **Hypothetical steps of the A-Priori algorithm**
  - C_1 = { {b} {c} {j} {m} {n} {p} }
  - Count the support of itemsets in C_1
  - Prune non-frequent: L_1 = { b, c, j, m }
  - Generate C_2 = { {b,c} {b,j} {b,m} {c,j} {c,m} {j,m} }
  - Count the support of itemsets in C_2
  - Prune non-frequent: L_2 = { {b,m} {b,c} {c,m} {c,j} }
  - Generate C_3 = { {b,c,m} <u>{b,c,j} {b,m,j} {c,m,j}</u> } **
  - Count the support of itemsets in C_3
  - Prune non-frequent: L_3 = { {b,c,m} }

A-Priori for All Frequent Itemsets

- One pass for each ***k*** (itemset size)
- Needs room in main memory to count each candidate ***k***–tuple
- For typical market-basket data and reasonable support (e.g., 1%), ***k* = 2** requires the most memory

- **Many possible extensions:**
  - Association rules with intervals:
    - For example: Men over 65 have 2 cars
  - Association rules when items are in a taxonomy
    - Bread, Butter $\longrightarrow$ FruitJam
    - BakedGoods, MilkProduct $\longrightarrow$ PreservedGoods
  - Lower the support ***s*** as itemset gets bigger

PCY (Park-Chen-Yu) Algorithm

PCY (Park-Chen-Yu) Algorithm

- **Observation:**
  In pass 1 of A-Priori, most memory is idle
  - We store only individual item counts
  - **Can we use the idle memory to reduce memory required in pass 2?**
- **Pass 1 of PCY:** In addition to item counts, maintain a hash table with as many buckets as fit in memory
  - Keep a **count** for each bucket into which **pairs** of items are hashed
    - **For each bucket just keep the count, not the actual pairs that hash to the bucket!**

PCY Algorithm – First Pass

```
FOR (each basket) :
    FOR (each item in the basket) :
        add 1 to item's count;
    FOR (each pair of items) :
        hash the pair to a bucket;
        add 1 to the count for that bucket;
```

New in PCY

- **Few things to note:**
  - Pairs of items need to be generated from the input file; they are not present in the file
  - We are not just interested in the presence of a pair, but we need to see whether it is present at least ***s*** (support) times

Observations about Buckets

- **Observation:** If a bucket contains a frequent pair, then the bucket is surely frequent
- However, even without any frequent pair, a bucket can still be frequent ☹
  - So, we cannot use the hash to eliminate any member (pair) of a "frequent" bucket
- **But, for a bucket with total count less than s, none of its pairs can be frequent ☺**
  - Pairs that hash to this bucket can be eliminated as candidates (even if the pair consists of 2 frequent items)
- **Pass 2:**
  Only count pairs that hash to frequent buckets

PCY Algorithm – Between Passes

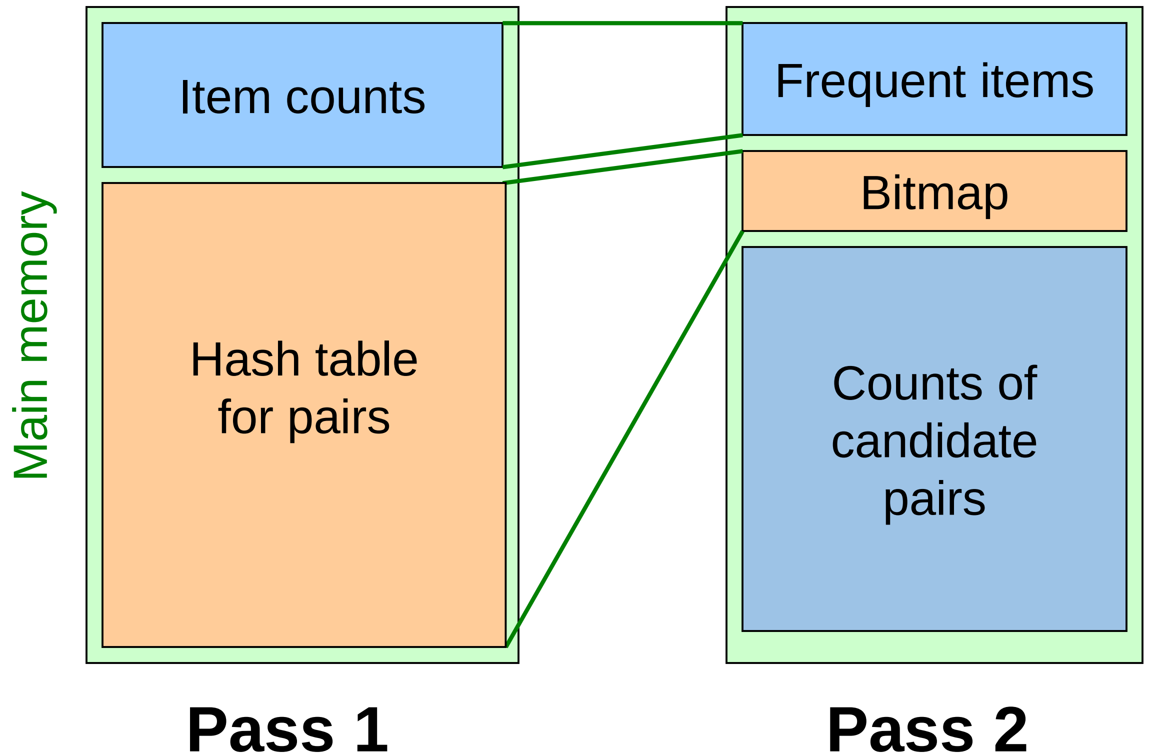
- Replace the buckets by a bit-vector:
  - **1** means the bucket count exceeded the support s (call it a frequent bucket); **0** means it did not
- 4-byte integer counts are replaced by bits, so the bit-vector requires 1/32 of memory
- Also, decide which items are frequent and list them for the second pass

PCY Algorithm – Pass 2

- Count all pairs ***{i, j}*** that meet the conditions for being a **candidate pair**:
  1. Both ***i*** and ***j*** are frequent items
  2. The pair ***{i, j}*** hashes to a bucket whose bit in the bit vector is **1** (i.e., a **frequent bucket**)

- Both conditions are necessary for the pair to have a chance of being frequent

Main-Memory: Picture of PCY



Main-Memory Details

- Buckets require a few bytes each:
  - **Note:** we do not have to count past s
  - #buckets is *O(main-memory size)*
- On second pass, a table of (item, item, count) triples is essential (we cannot use triangular matrix approach)
  - Thus, hash table must eliminate approx. 2/3 of the candidate pairs for PCY to beat A-Priori

Thank you.

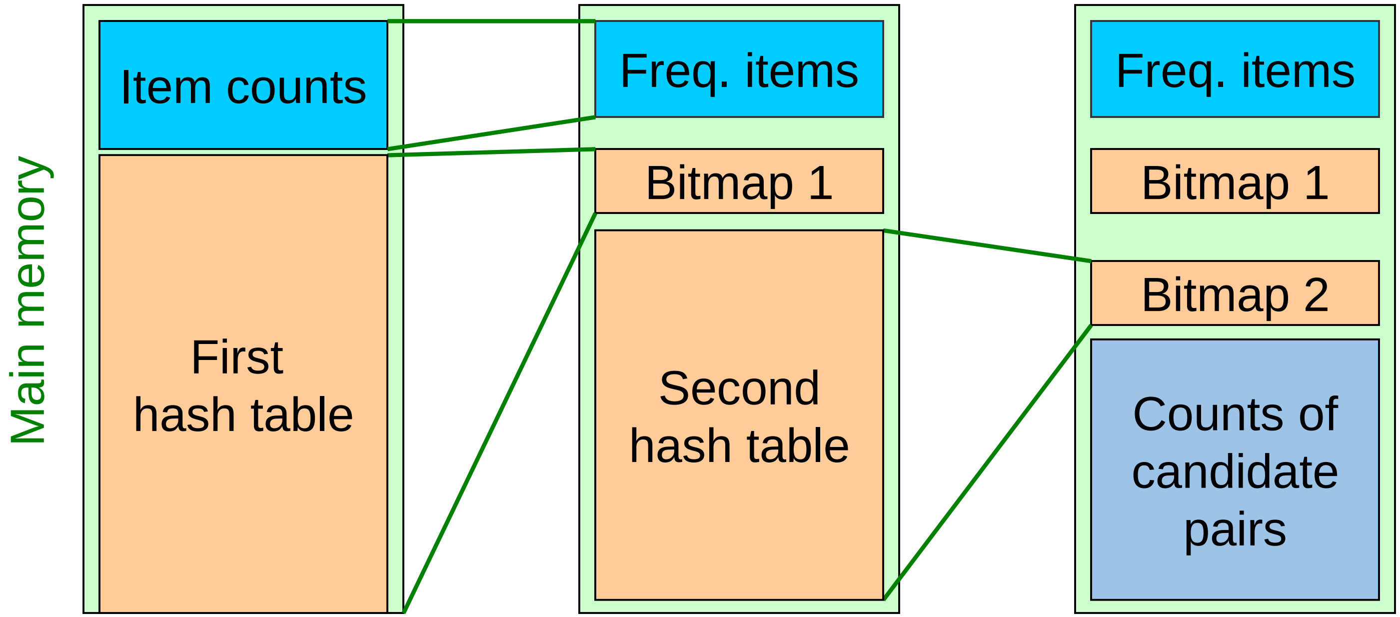
Questions?

Additional Slides

Refinement: Multistage Algorithm

- **Limit the number of candidates to be counted**
  - **Remember:** Memory is the bottleneck
  - Still need to generate all the itemsets but we only want to count/keep track of the ones that are frequent
- **Key idea:** After Pass 1 of PCY, rehash only those pairs that **qualify** for Pass 2 of PCY
  - ***i*** and ***j*** are frequent, and
  - ***{i, j}*** hashes to a frequent bucket from **Pass 1**
- On middle pass, fewer pairs contribute to buckets, so fewer ***false positives***
- **Requires 3 passes over the data**

Main-Memory: Multistage



Pass 1

Count items
Hash pairs {i,j}

Pass 2

Hash pairs {i,j}
into Hash2 iff:
i,j are frequent,
{i,j} hashes to
freq. bucket in B1

Pass 3

Count pairs {i,j} iff:
i,j are frequent,
{i,j} hashes to
freq. bucket in B1
{i,j} hashes to
freq. bucket in B2

Multistage – Pass 3

- **Count only those pairs $\{i, j\}$ that satisfy these candidate pair conditions:**
  1. Both i and j are frequent items
  2. Using the first hash function, the pair hashes to a bucket whose bit in the first bit-vector is **1**
  3. Using the second hash function, the pair hashes to a bucket whose bit in the second bit-vector is **1**

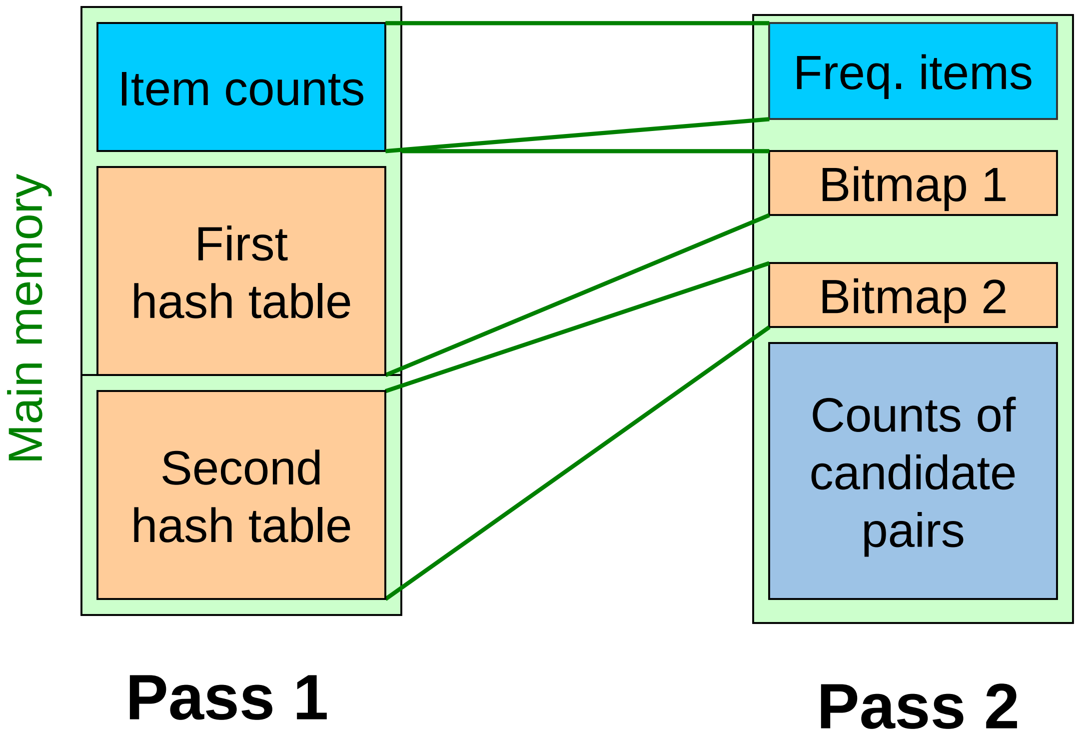
Important Points

1. **The two hash functions have to be independent**
2. **We need to check both hashes on the third pass**
   - If not, we would end up counting pairs of frequent items that hashed first to an infrequent bucket but happened to hash second to a frequent bucket

Refinement: Multihash

- **Key idea:** Use several independent hash tables on the first pass
- **Risk:** Halving the number of buckets doubles the average count
  - We have to be sure most buckets will still not reach count s
- If so, we can get a benefit like multistage, but in only 2 passes

Main-Memory: Multihash



PCY: Extensions

- Either **multistage** or **multihash** can use more than two hash functions
- In **multistage**, there is a point of diminishing returns, since the bit-vectors eventually consume all of main memory
- For **multihash**, the bit-vectors occupy exactly what one PCY bitmap does, but too many hash functions makes all counts $\geq s$