Control Theory Homework 3

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Given non-linear system:

$$\begin{cases} (M+m)\ddot{x} - mlcos(\theta)\ddot{\theta} + mlsin(\theta)\dot{\theta}^2 = F \\ -cos(\theta)\ddot{x} + l\ddot{\theta} - gsin(\theta) = 0 \end{cases}$$

I have chosen next coefficients:

$$M = 4, m = 1, l = 0.8$$

Solution:

- From previous assignment we have linearized nonlinear dynamics of the systems around equilibrium point $\bar{z} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$:

$$\delta\dot{z} = A\delta z + B\delta u$$

$$\delta\dot{z} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{mg}{M} & 0 & 0 \\ 0 & \frac{(M+m)g}{Ml} & 0 & 0 \end{bmatrix} \delta z + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \\ \frac{1}{Ml} \end{bmatrix} \delta u$$

- Let find matrix C

$$\delta y = C\delta z$$

$$C = \frac{\partial h(z)}{\partial z} = \left. \begin{bmatrix} \frac{\partial h_1}{\partial z_1} & \frac{\partial h_1}{\partial z_2} & \frac{\partial h_1}{\partial z_3} & \frac{\partial h_1}{\partial z_4} \\ \frac{\partial h_2}{\partial z_1} & \frac{\partial h_2}{\partial z_2} & \frac{\partial h_2}{\partial z_3} & \frac{\partial h_2}{\partial z_4} \end{bmatrix} \right|_{z=\bar{z}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

- To prove that it is possible to design state observer linearized system we need to check observability. For figuring out that we need to check the rank of matrix $O = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix}$

$$rank(O) = 4 \Rightarrow \text{Linearized system is } \textbf{observable}.$$

- For open loop state observer, the error dynamics is not stable:

$$\text{Real system: } \dot{z} = Az + Bu$$

$$\text{Imaginary system: } \dot{\xi} = A\xi + Bu$$

$$\text{Error dynamics: } e = \xi - x$$

$$\dot{e} = A(\xi - x) = Ae$$

  For figuring out the stability of error dynamics we need to check eigenvalues of A matrix:

$$\text{Eigenvalues: } \lambda_1 = 0, \lambda_2 = 0, \lambda_3 = \sqrt{\frac{(M+m)g}{Ml}} > 0, \lambda_4 = -\sqrt{\frac{(M+m)g}{Ml}} < 0$$

  As we can see the error dynamics is **unstable**, because there are non-negative eigenvalues are present.

- Design Luenberger observer for linearized system:

$$\text{Real system: } \dot{z} = Az + Bu,\ y = Cz$$

$$\text{Imaginary system: } \dot{\xi} = A\xi + Bu + L(y - \hat{y}),\ \hat{y} = C\xi$$

$$\text{Error dynamics: } e = \xi - x$$

$$\dot{e} = A\xi - Az + L(Cz - C\xi) = Ae - LCe = (A - LC)e$$

$$eig(A - LC) = eig(A^T - C^T L^T)$$

So, we can find matrix L the same way we found K matrix.

- Luenberg observer designed by pole placement method:

Poles are -1, -2, -3, -4.

$$L = \begin{bmatrix} 5 & 1 \\ 1 & 5 \\ 5.5 & 4.95 \\ 2.5 & 20.8125 \end{bmatrix}$$

$$\dot{\xi} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{mg}{M} & 0 & 0 \\ 0 & \frac{(M+m)g}{Ml} & 0 & 0 \end{bmatrix} \xi + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \\ \frac{1}{Ml} \end{bmatrix} u + \begin{bmatrix} 5 & 1 \\ 1 & 5 \\ 5.5 & 4.95 \\ 2.5 & 20.8125 \end{bmatrix} (y - C\xi)$$

- Luenberg observer designed by LQR method:

$$Q = 100 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$L = \begin{bmatrix} 10.9719193 & 0.21374809 \\ 0.21374809 & 12.92495096 \\ 10.21435066 & 3.74210706 \\ 1.36580322 & 33.5500228 \end{bmatrix}$$

$$\dot{\xi} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{mg}{M} & 0 & 0 \\ 0 & \frac{(M+m)g}{Ml} & 0 & 0 \end{bmatrix} \xi + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{M} \\ \frac{1}{Ml} \end{bmatrix} u + \begin{bmatrix} 10.9719193 & 0.21374809 \\ 0.21374809 & 12.92495096 \\ 10.21435066 & 3.74210706 \\ 1.36580322 & 33.5500228 \end{bmatrix} (y - C\xi)$$

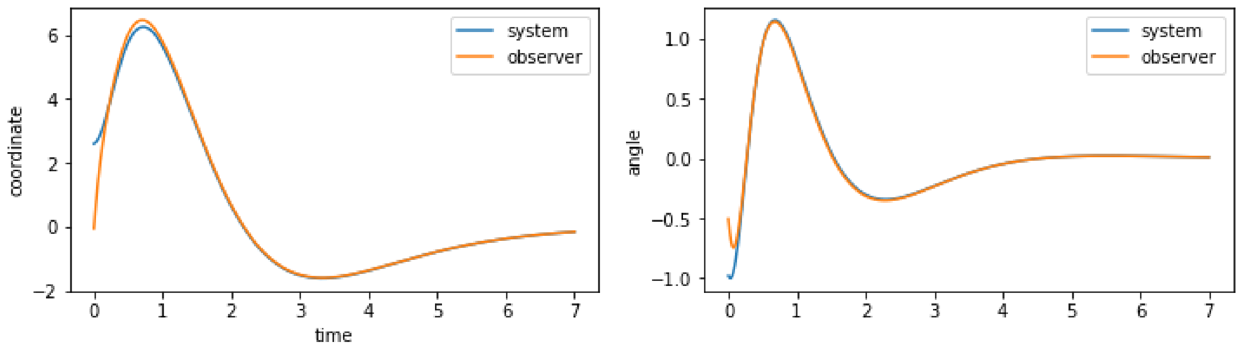
- State feedback controller for linearized system was used from previous homework(LQR one):

$$Q = 100 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},\ R = \begin{bmatrix} 1 \end{bmatrix}$$

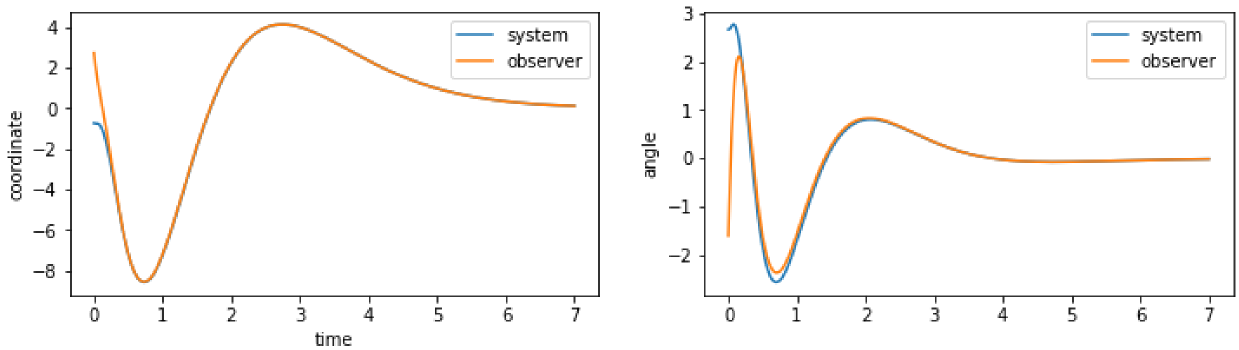
$$K = \begin{bmatrix} -10 & 189.41311337 & -19.66106239 & 52.13000272 \end{bmatrix}$$

- Simulate nonlinear system with Luenberger observer and state feedback controller that uses estimated states ($u = K\hat{z}$). Initial z and $\hat{z}$ were chosen randomly. Simulation was tested a lot-always converges. Here are some of examples:

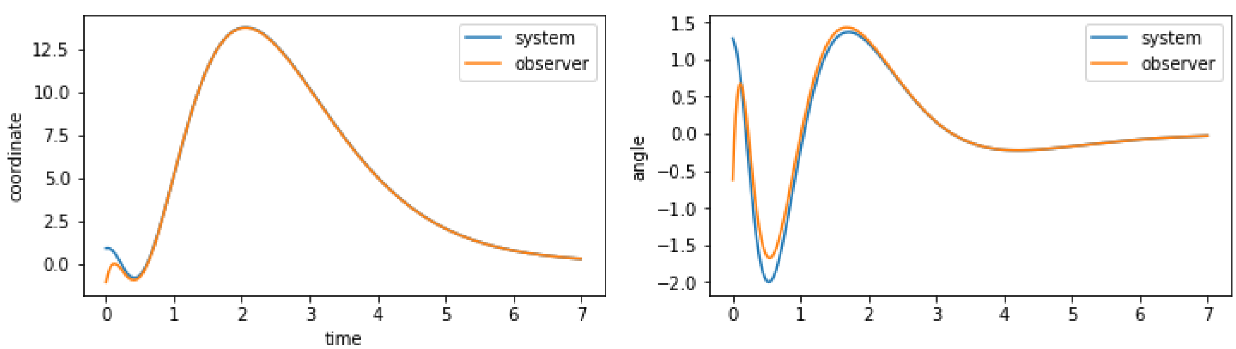
$$z = \begin{bmatrix} 2.59986281 & -0.98053523 & 0.9768121 & -1.3574642 \end{bmatrix}^T$$
$$\hat{z} = \begin{bmatrix} -0.0573398 & -0.505249 & 2.12582228 & -1.18692937 \end{bmatrix}^T$$



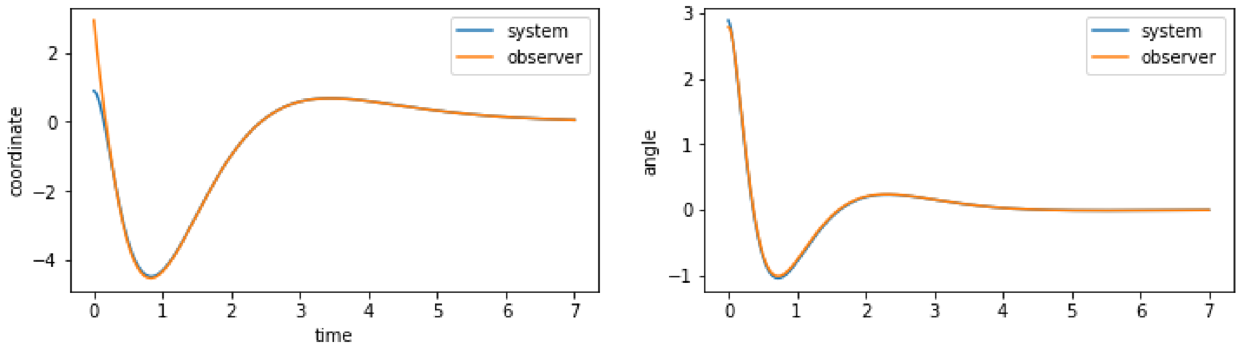
$$z = \begin{bmatrix} -0.7287425 & 2.66919078 & -1.59777559 & -0.61083029 \end{bmatrix}^T$$
$$\hat{z} = \begin{bmatrix} 2.73195345 & -1.60243462 & 2.58018783 & -0.24703558 \end{bmatrix}^T$$



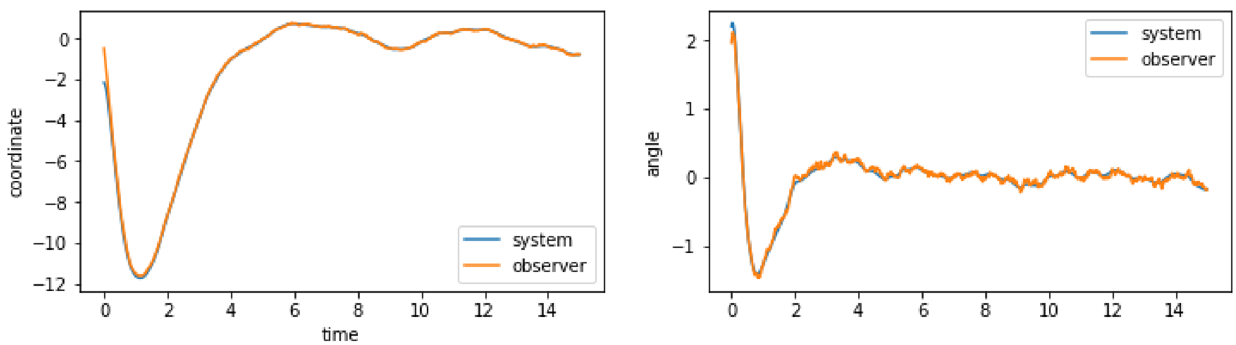
$$z = \begin{bmatrix} 0.89434729 & 1.28001835 & 0.48259297 & -2.58896538 \end{bmatrix}^T$$
$$\hat{z} = \begin{bmatrix} -1.08018145 & -0.62369521 & -2.74591982 & 2.07311268 \end{bmatrix}^T$$



$$z = \begin{bmatrix} 0.88791035 & 2.88543178 & -0.59597022 & -2.09810452 \end{bmatrix}^T$$
$$\hat{z} = \begin{bmatrix} 2.93473614 & 2.78270008 & 0.68536234 & -0.6962794 \end{bmatrix}^T$$



- Some white gaussian noise($\mu = 0, \sigma = 0.1$) was added to the output to simulate real situation. State estimation starts looking really ugly and useless. But finally it's values are 'jumping' around stable position(x=0,$\theta = 0$).



- Some white gaussian noise($\mu = 0, \sigma = 0.1$) to the dynamics was added. Control system and state estimation now have the same manner of jumping values. It's really bad situation. Finally neither observer nor system goes to stable position(x=0,$\theta = 0$).