Control Theory Homework 3

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- Coefficients are chosen to be:

$$l_1 = 2, l_2 = 2, l_{c1} = 1, l_{c2} = 1, m_1 = 1, m_2 = 1$$

- Simulate the system $M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$, where(matrices were taken from the book, p.206):

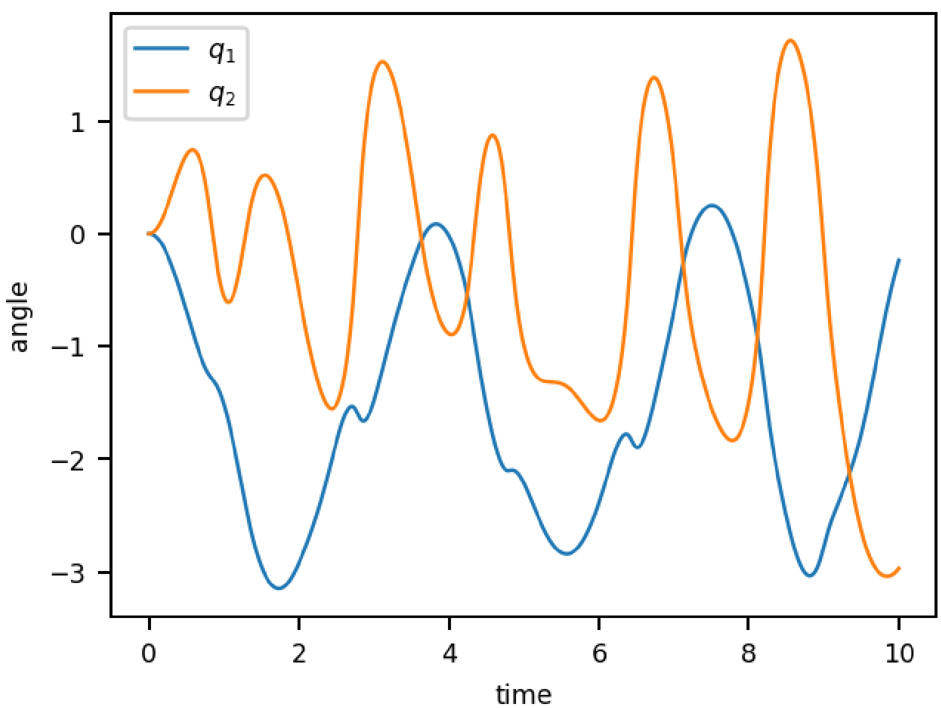
$$M(q) = \begin{bmatrix} m_1 l_{c1}^2 + m_2(l_1^2 + l_{c2}^2 + 2 l_1 l_{c2} cos q_2) + I_1 + I_2 & m_2(l_{c2}^2 + l_1 l_{c2} cos q_2) + I_2 \\ m_2(l_{c2}^2 + l_1 l_{c2} cos q_2) + I_2 & m_2 l_{c2}^2 + I_2 \end{bmatrix}$$

$$C(q,\dot{q}) = \begin{bmatrix} -m_2 l_1 l_{c2} sin q_2 \dot{q}_2 & -m_2 l_1 l_{c2} sin q_2 (\dot{q}_2 + \dot{q}_1) \\ m_2 l_1 l_{c2} sin q_2 \dot{q}_1 & 0 \end{bmatrix}$$

$$g(q) = \begin{bmatrix} (m_1 l_{c1} + m_2 l_1) g cos q_1 + m_2 l_{c2} g cos(q_1 + q_2) \\ m_2 l_{c2} g cos(q_1 + q_2) \end{bmatrix}$$

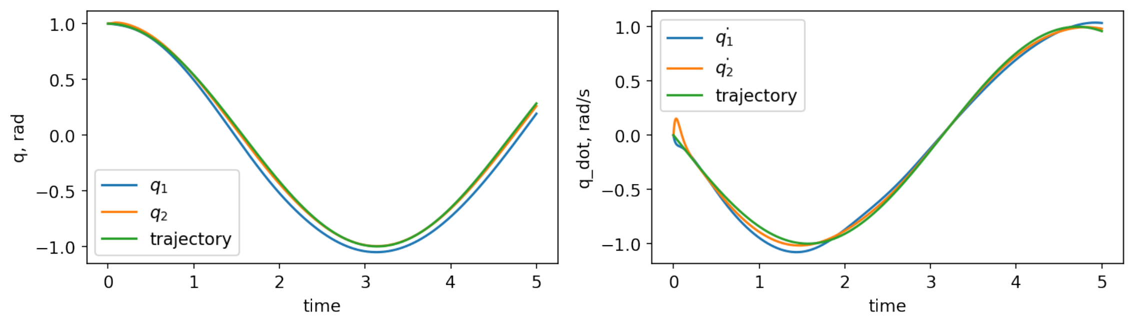
$$I_i = \frac{m_i(l_{ci}^3 + (l_i - l_{ci})^3)}{3 l_i}$$

System without any control, $u = 0$



- Make the robot to track $q^*(t) = [cos(t)\ cos(t)]^T$ for 5 seconds using following PD controller: $\tau = K_p(q^* - q) + K_d(\dot{q}^* - \dot{q})$, where $K_p = 400, K_d = 30$

System with PD-controller, $u = 0$



As we can see our system with controller follows the trajectory in a good manner.

- Our equation in regressor form $Y(q, \dot{q}, \ddot{q})\pi = \tau$, where(again matrices were taken from the book):

$$Y(q,\dot{q},\ddot{q}) = \begin{bmatrix} \ddot{q}_1 & \cos(q_2)(2\ddot{q}_1 + \ddot{q}_2) + \sin(q_2)(\dot{q}_1^2 - 2\dot{q}_1\dot{q}_2) & \ddot{q}_2 & g\cos(q_1) & g\cos(q_1+q_2) \\ 0 & \cos(q_2)\ddot{q}_1 + \sin(q_2)\dot{q}_1^2 & \ddot{q}_2 & 0 & g\cos(q_1+q_2) \end{bmatrix}$$

And systems parameters are:

$$\pi = \begin{bmatrix} \pi_1 \\ \pi_2 \\ \pi_3 \\ \pi_4 \\ \pi_5 \end{bmatrix} = \begin{bmatrix} m_1\ell_{c1}^2 + m_2(\ell_1^2 + \ell_{c2}^2) + I_1 + I_2 \\ m_2\ell_1\ell_{c2} \\ m_2\ell_1\ell_{c2} \\ m_1\ell_{c1} + m_2\ell_1 \\ m_2\ell_2 \end{bmatrix}$$

- On this step we estimate the system's parameters using least squares method:

$$\tau = Y\pi$$

$$\tilde{\pi} = (Y^T Y)^{-1} Y^T \tau$$

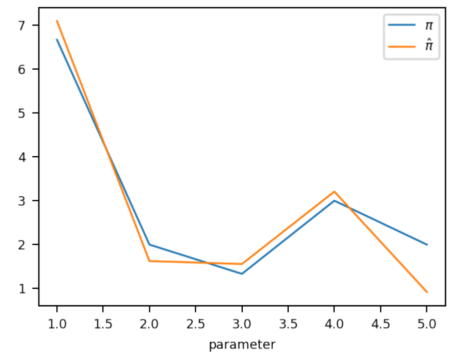
- Our "ideal" parameters:

$$\pi = \begin{bmatrix} 6.66666667 \\ 2. \\ 1.33333333 \\ 3. \\ 2. \end{bmatrix}$$

- Our estimated parameters:

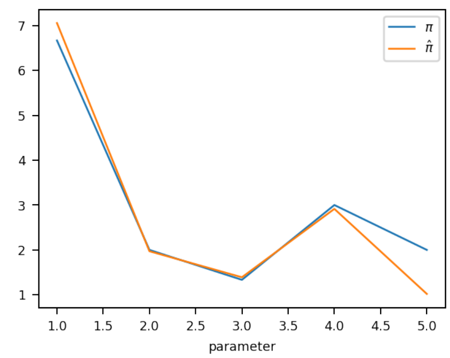
  For trajectory $q^*(t) = [cos(t)\ cos(t)]^T$:

$$\hat{\pi} = \begin{bmatrix} 7.05311994 \\ 1.65225701 \\ 1.53138347 \\ 3.10170406 \\ 0.96687331 \end{bmatrix}$$



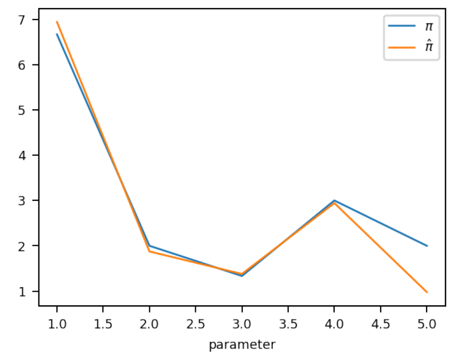
  For trajectory $q^*(t) = [sin(t)\ cos(t)]^T$:

$$\hat{\pi} = \begin{bmatrix} 7.05520537 \\ 1.97036377 \\ 1.39027737 \\ 2.9141718 \\ 1.01960572 \end{bmatrix}$$



  For trajectory $q^*(t) = [sin(t)\ sin(t)]^T$:

$$\hat{\pi} = \begin{bmatrix} 6.97775212 \\ 1.95317755 \\ 1.39307542 \\ 2.8832854 \\ 0.93205818 \end{bmatrix}$$



As we can see all parameters are estimated quite good except the last one, so it seems that matrix for ideal π from the book was not fully correct...

The source code can be found here.