Control Theory Homework 1

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1 Part

Consider the system:

$$3.2316y^{(6)} + 2.2279y^{(5)} + 2.4488y^{(4)} + 0.9344y^{(3)} + 3.9760y^{(2)} + 1.8276y = b_0$$

Let introduce new variables $x_1(t)$ .. $x_6(t)$:

$$x_1(t) = y(t)$$

$$x_2(t) = \dot{y}(t)$$

$$...$$

$$x_6(t) = y^{(5)}(t)$$

After introducing new variables we can rewrite our equation:

$$\dot{x}_6 = -\frac{1.8276}{3.2316}x_1 - \frac{0}{3.2316}x_2 - \frac{3.9760}{3.2316}x_3 - \frac{0.9344}{3.2316}x_4 - \frac{2.4488}{3.2316}x_5 - \frac{2.2279}{3.2316}x_6 + \frac{1}{3.2316}b_0$$

Our equation in a vector-matrix form:

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -\frac{1.8276}{3.2316} & 0 & -\frac{3.9760}{3.2316} & -\frac{0.9344}{3.2316} & -\frac{2.4488}{3.2316} & -\frac{2.2279}{3.2316} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{3.2316} \end{bmatrix} b_0$$

The output equation can be written as

$$\dot{\mathbf{y}} = C\mathbf{x}$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix}$$

Answers to the questions:

- ODE is unstable due to the fact that not all real parts of eigenvalues of the matrix A are negative. Works for both b_0, because the same matrix A:

Numerical values of eigenvalues.

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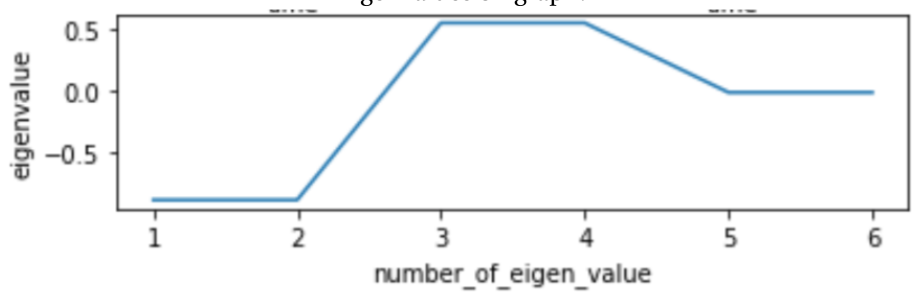
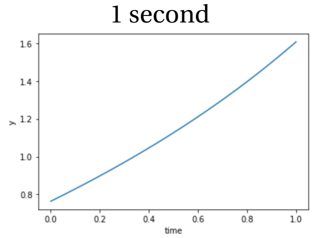
Eigenvalues of matrix A: [-0.88310149+0.76221959j -0.88310149-0.76221959j  0.55078504+0.71738984j
  0.55078504-0.71738984j -0.01238896+0.7126587j  -0.01238896-0.7126587j ]
```

Eigenvalues on graph.

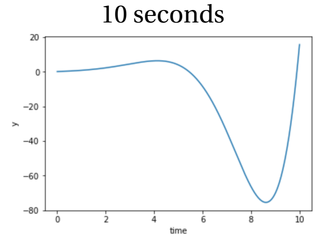


- Let consider the system $\dot{\mathbf{x}} = A\mathbf{x}$. solution of the system is $x = Ce^{At}$, where C - const, A - our matrix. We can decompose our A into WDW^{-1}, where D-diagonal matrix with our eigenvalues. Then our solution will look like $x_i = ce^{\lambda_i t}$, where λ_i our i-th eigenvalue of matrix A. Then let take a look on $\lim_{t\to\infty} ce^{\lambda_i t}$. The limit of our solution will exist(so solution will converge) only in case when all eigenvalues are less than zero. **So, the system diverges according to the fact that ODE is unstable**. Let plot y(t) for different amount of seconds and see how the system behaves(first 4 pictures for $b_0 = 0$, second 4 pictures for $b_0 = 0.7094$:

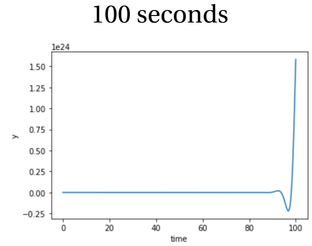
1 second



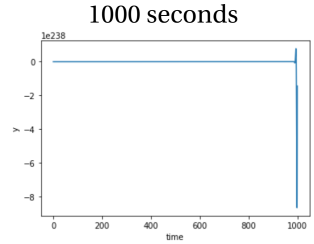
10 seconds

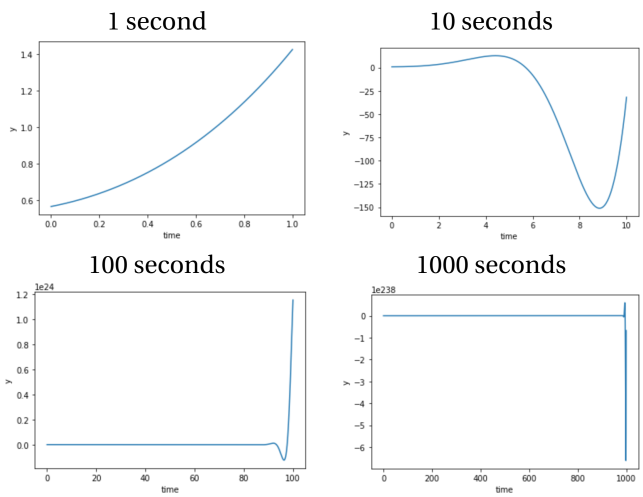


100 seconds



1000 seconds





- LTV system means that matrix A depends on time. That means if matrix is stable in each moment of time(we could check it, just LTI), then all ODE is stable.

2 Part

Our differential equation for the system is

$$m\ddot{x} + b\dot{x} + kx = F(t)$$

Let introduce new variables $y_1(t)$ and $y_2(t)$:

$$y_1(t) = x(t)$$

$$y_2(t) = \dot{x}(t)$$

After introducing new variables we can rewrite our equation:

$$\dot{y}_2 = -\frac{k}{m}y_1 - \frac{b}{m}y_2 + \frac{1}{m}F(t)$$

Our equation in a vector-matrix form:

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} F(t)$$

The output equation can be written as

$$x = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

Transfer function for this equation:

$$\text{Let } p = \frac{d}{dt}$$

$$mp^2x + bpx + kx = F(t)$$

$$(mp^2 + bp + k)x = F(t)$$

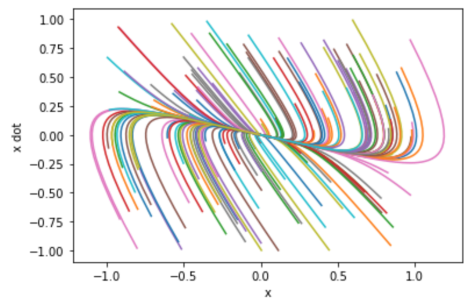
$$x(t) = \frac{1}{mp^2+bp+k}F(t) = G(p)F(t)$$

Transfer function:

$$G(p) = \frac{1}{mp^2+bp+k}$$

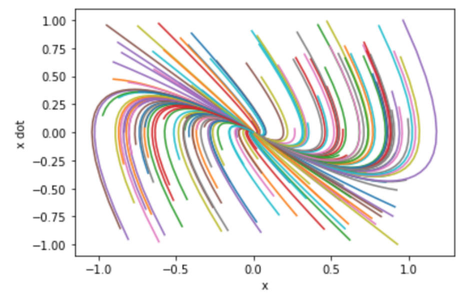
Phase Portraits:

Node



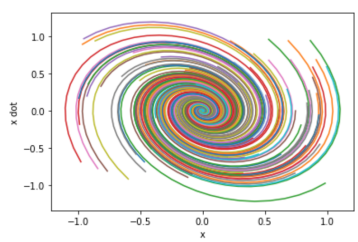

```
Coefficients: k=2, m=4, b=8
Eigenvalues for phase portrait: [-0.29289322 -1.70710678]
```

Node(Degenerate)



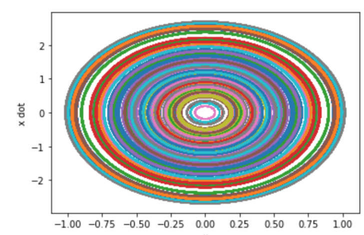

```
Coefficients: k=1, m=1, b=2
Eigenvalues for phase portrait: [-1. -1.]
```

Spiral



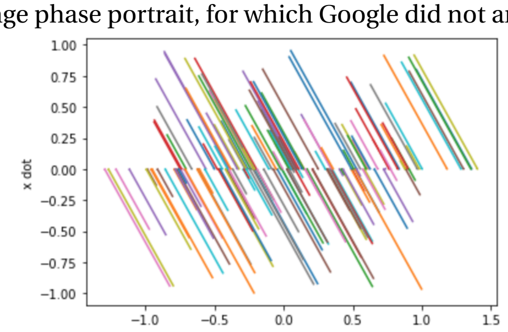

```
Coefficients: k=2, m=1.3, b=0.7
Eigenvalues for phase portrait: [-0.26923077+1.2107751j -0.26923077-1.2107751j]
```

Center




```
Coefficients: k=7, m=1, b=0
Eigenvalues for phase portrait: [0.+2.64575131j 0.-2.64575131j]
```

Strange phase portrait, for which Google did not answer




```
Coefficients: k=0, m=4, b=8
Eigenvalues for phase portrait: [ 0. -2.]
```

Conclusion: $D = b^2 - 4k$

Node: D > 0

Node(Degenerate): D = 0

Spiral: D < 0, b != 0

Center: D < 0, b = 0