Control Theory Homework 2

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Our equation in a vector-matrix form:

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \mathbf{u}(t)$$

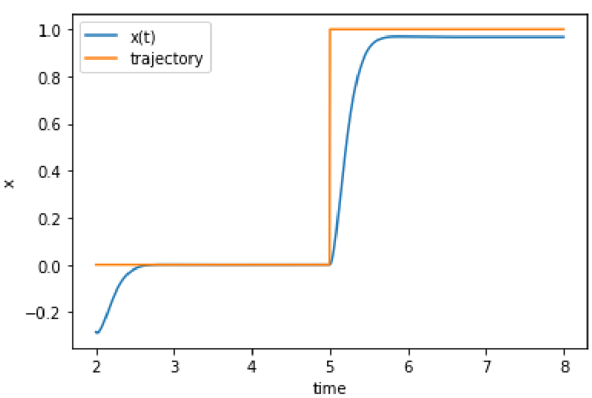
We design PD controller => $u(t) = K_p e(t) + K_d \dot{e}(t) = K_p(x_{desired}(t) - x(t)) + K_d(\dot{x}_{desired}(t) - \dot{x}(t))$

Coefficients k = 2, m = 1, b = 2 were chosen at random.

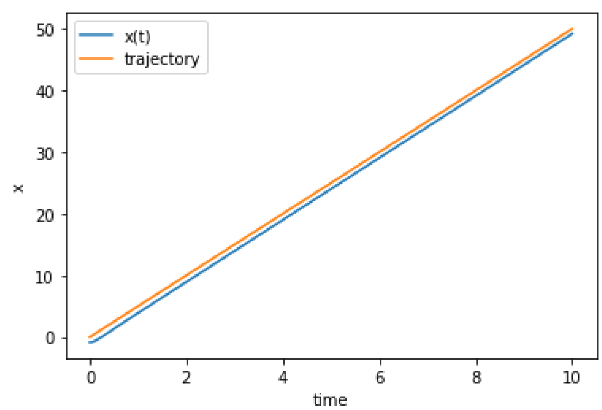
Coefficients $K_p = 60, K_d = 12$. As we can see on the step input signal, there is no any oscillations and overshoot.

Different Trajectories:

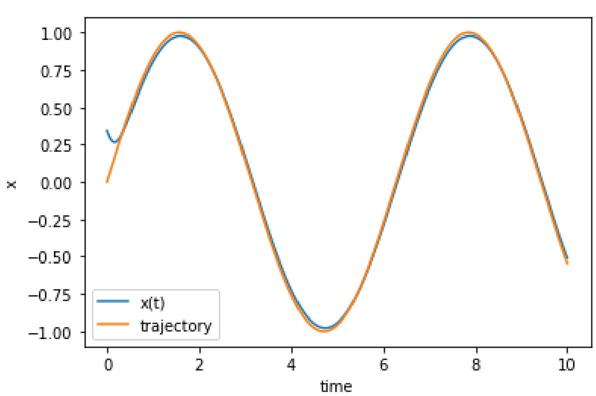
Step input signal



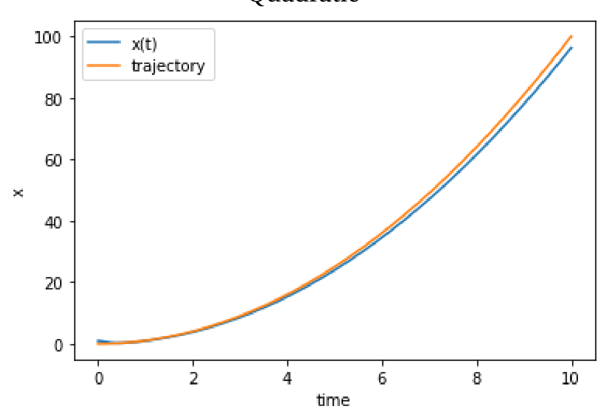
Linear



Sinus



Quadratic



Description of different trajectories:

Step input signal: $x_{desired} = \begin{cases} 0, if\ time < 5; \\ 1, otherwise;. \end{cases}$

Linear: $x_{desired} = 5t$

Sinus: $x_{desired} = sin(time)$

Quadratic: $x_{desired} = t^2$

Stability:

A: $\begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}$ B: $\begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$ K: $\begin{bmatrix} K_p \\ K_d \end{bmatrix}$ e: $\begin{bmatrix} e(t) & \dot{e}(t) \end{bmatrix}$

System $\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u}$ is stable <=> All real parts of eigenvalues of $A - BK$ less than 0.
Eigenvalues of $A - BK$ matrix are [-7.+3.60555128j -7.-3.60555128j], so system is **stable**.

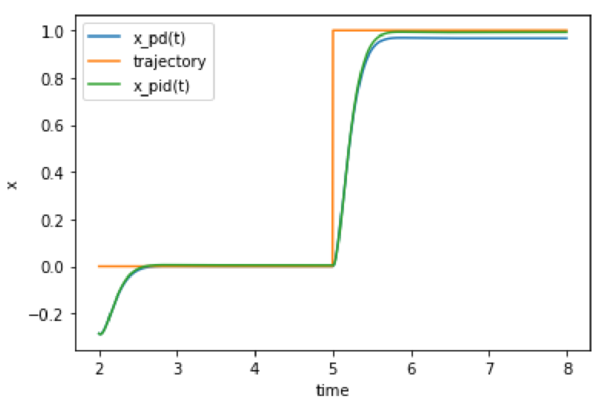
Comparison PD with PID controller :

PID controller => $u(t) = K_p e(t) + K_d \dot{e}(t) + K_i \int e(t) dt$

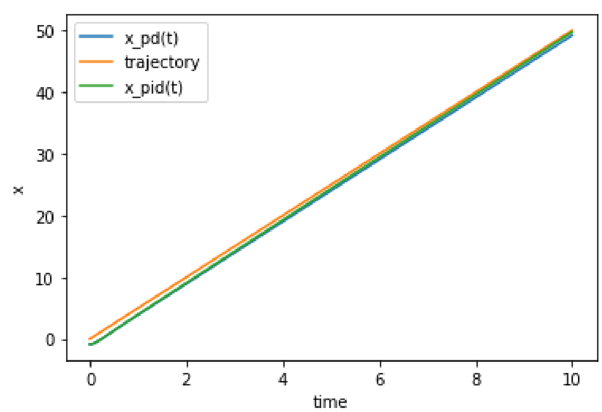
The coefficient K_i was chosen to be 5. K_p and K_d remain the same values.

PID controller makes the system a little bit more accurate than PD controller as we can see on plots.

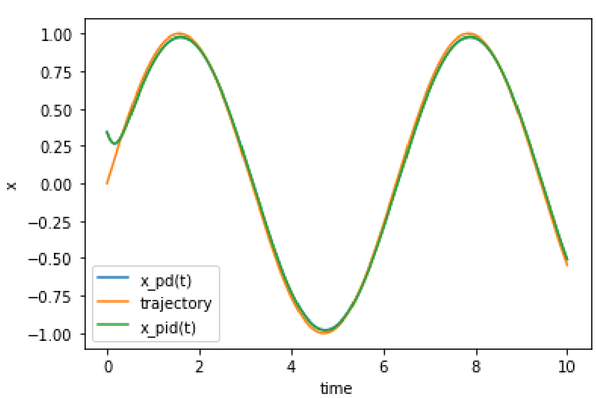
Step input signal



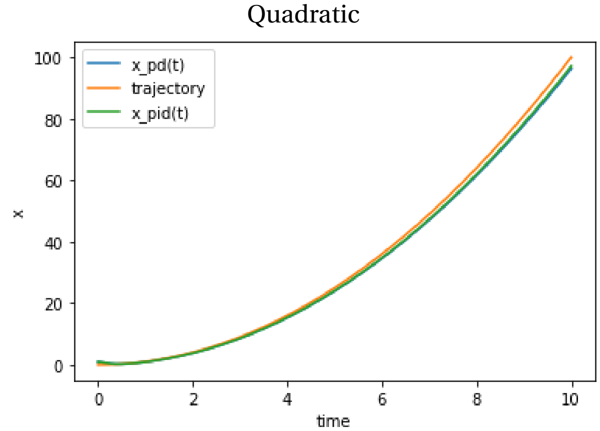
Linear



Sinus



Quadratic



The code of the assignment can be found here.