

Reg.No: 24BCE0554
Name : Partha Pratim Gogoi
Topic : Huffman Coding

Algorithm:

```
1 text = "message" [T.C: O(1)]
2 freq = [] [T.C: O(1)]
3 struct Node [T.C: O(1)]
  3.1 char data
  3.2 unsigned freq
  3.3 Node *left, *right
  3.4 Node(data, freq)
    3.4.1 this->data = data
    3.4.2 this->freq = freq
    3.4.3 left = right = NULL
4 calcFrequencies(string text) [T.C: O(n)]
  4.1 for char in text O(n)
  4.2 freq[char]++
5 buildHuffmanTree(unordered_map(char, int) freq) [T.C: O(n+mlogm)]
  5.1 priority_queue(Node) minHeap = [] O(1)
  5.2 for char,freqValue in freq O(m)
  5.3 minHeap.push(new Node(char, freqValue)) O(log(m))
  5.4 while minHeap.size > 1 O(m-1)
  5.5 Node left = minHeap.top()
  5.6 minHeap.pop() O(log(m))
  5.7 Node right = minHeap.top()
  5.8 minHeap.pop() O(log(m))
  5.9 Node internal = new Node('$',left.freq + right.freq)
  5.10 internal.left = left
  5.11 internal.right = right
  5.12 minHeap.push(internal) O(log(m))
  5.13 return minHeap.top()
6 storeCodes(
  Node rootNode,
  string code,
  unordered_map(char, int) codes
) [T.C: O(m*L)]
  6.1 if rootNode == NULL
  6.2 return
  6.3 if rootNode.left == NULL and rootNode.right == NULL
  6.4 codes[rootNode.data] = code
  6.5 storeCodes(rootNode.left, code+"0", codes) O(L)
  6.6 storeCodes(rootNode.right, code+"1", codes) O(L)
```

Time Complexity:

IMPORTANT CLARIFICATION

- A Balanced tree can have $m \neq n$ OR $m = n$
 - 'abcdefgh' \rightarrow balanced tree w/ $m = n$ (special case, all unique)
 - 'aaaabbbbcccc' \rightarrow balanced tree with $m \neq n$
- A Skewed tree can have $m \neq n$
 - 'bbbbbbcdca' \rightarrow generates skewed tree with $m \neq n$
- It is not possible to get a Skewed tree with $m = n$

1. buildHuffmanTree()

A temporary data structure 'minHeap' used to create the Huffman Tree.

Time Complexity:

$$O(m \log(m)) + O((m-1) * 3 \log(m)) = O(m \log(m))$$

where m = no. of unique characters

- Repeated chars ($m \neq n$) = $O(m \log(m))$
 - Same for skewed and balanced tree
- When all characters unique ($m = n$) = $O(n \log(n))$

2. storeCodes()

- Total nodes visited : $2m-1$ 'm' original + 'm-1' internal
- At each node : $O(L)$ String concatenation
- $m \neq n$ Case $\rightarrow O((2m-1) * L) : O(m * L)$ L = Huffman Tree Height
 - Balanced Tree $\rightarrow O(m \log(m))$
 - Skewed Tree $\rightarrow O(m^2)$
- $m = n$ Case $\rightarrow O((2n-1) * L) : O(n * L)$

Where,

Huffman Tree Height (L) is determined by
no. of unique characters (m)

NOTE: calcFrequency() is $O(n)$ and not dependent on 'm' or 'L'

So, including calcFrequency() the total time complexity will be:

$$\text{Total Time Complexity} = O(n) + O(m \log(m)) + O(m * L)$$

- Balanced Tree (Typical/Expected Structure):

$$O(n) + O(m \log(m)) + O(m \log(m))$$

Total Time Complexity

$$\text{Repeated chars } (m \neq n) = O(n + m \log m)$$

$$\text{All chars unique } (m = n) = O(n \log n) \quad (\text{special case})$$

- Skewed Tree (Pathological Structure):

$$O(n) + O(m \log(m)) + O(m^2)$$

Only one case

$$\text{Repeated chars} = O(m^2)$$

Source Code:

```
#include <iostream>
#include <queue>
#include <unordered_map>
#include <vector>
#include <string>
using namespace std;

struct Node {
    char data;
    unsigned freq;
    Node *left, *right;

    Node(char data, unsigned freq) {
        this->data = data;
        this->freq = freq;
        left = right = nullptr;
    }
};

struct Compare {
    bool operator()(Node* l, Node* r) {
        return l->freq > r->freq;
    }
};

// Non-Recursive Function
Node* buildHuffmanTree(unordered_map<char, int>& freq) {
    // Define minHeap
    priority_queue<Node*, vector<Node*>, Compare> minHeap;

    // Populate minHeap
    for (auto pair: freq) {
        minHeap.push(new Node(pair.first, pair.second));
    }
    // Build Huffman Tree
    while (minHeap.size() > 1){
        // Select the two elements from the minHeap
        // having least frequencies
        Node* left = minHeap.top();
        minHeap.pop();
        Node* right = minHeap.top();
        minHeap.pop();

        // Generate non-leaf node
        // Generated node chracter -> '$'
        Node* internal = new Node('$', left->freq + right->freq);
        internal->left = left;
        internal->right = right;

        // Push generated node to tree
        minHeap.push(internal);
    }
    return minHeap.top();
}

// Recursive Function
```

```

// Tree is traversed top-to-bottom
void storeCodes(Node* root, string code, unordered_map<char, string>& codes) {
    if (!root) return;
    if (!root->left && !root->right) {
        codes[root->data] = code;
        return;
    } // If at leaf node, character code is complete
    storeCodes(root->left, code+"0", codes); // If left edge, append 0
    storeCodes(root->right, code+"1", codes); // If right edge, append 1
}

int main() {
    // 1. Get text input
    string text;
    cout << "Enter text to encode: ";
    getline(cin, text);

    // 2. Calculate frequencies
    cout << endl << "Calculating frequencies..." << endl;
    unordered_map<char, int> freq;
    for (char c: text) {
        freq[c]++;
    }
    // 3. Build the tree
    cout << "Building Huffman tree..." << endl;
    Node* root = buildHuffmanTree(freq);

    // 4. Generate the huffman codes
    cout << "Generating Huffman encoding table..." << endl;
    unordered_map<char, string> codes;
    storeCodes(root, "", codes);
    cout << endl;

    // 4. Print out encoding table
    cout << "Huffman Encoding Table" << endl;
    for (auto pair: codes) {
        cout << pair.first << ": " << pair.second << endl;
    }
    cout << endl;

    // 5. Print out compression results
    int originalBits = 8*text.length();
    int compressedBits = 0;
    string compressedMessage = "";

    for (char c: text) {
        compressedBits += codes[c].length();
        compressedMessage += codes[c];
    } // Calculating new length of compressed message

    cout << "Compressed Message: " << compressedMessage << endl << endl;
    cout << "No. of bits required to store original message: " << originalBits << endl;
    cout << "No. of bits required to store compressed version: " << compressedBits <<
endl;
    cout << "Size reduction: " << (originalBits-compressedBits)*100/originalBits << "%"
<< endl;

    return 0;
}

```

Sample Output:

```
kug@Oxide [Algorithm-Code]>> ./a.out
Enter text to encode: Hello World

Calculating frequencies...
Building Huffman tree...
Generating Huffman encoding table...

Huffman Encoding Table
l: 10
.: 011
W: 1111
d: 010
o: 110
H: 001
e: 1110
r: 000

Compressed Message: 0011110101011001111111000010010

No. of bits required to store original message: 88
No. of bits required to store compressed version: 32
Size reduction: 63%
kug@Oxide [Algorithm-Code]>> ./a.out
Enter text to encode: She sells sea shells by the sea shore

Calculating frequencies...
Building Huffman tree...
Generating Huffman encoding table...

Huffman Encoding Table
t: 10110
.: 111
a: 1010
e: 110
r: 01111
l: 100
y: 01110
o: 10111
p: 01101
S: 01100
h: 010
s: 00

Compressed Message: 01100010110111001100100001110011010111000101101001000011110111011110110010110111001101011100010011010111110

No. of bits required to store original message: 296
No. of bits required to store compressed version: 118
Size reduction: 60%
```