$$x = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\ddot{x} = \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix}$$

Notation:

$C_n \equiv \cos(\theta_n)$

$S_n \equiv \sin(\theta_n)$

Position:

$\hat{\imath}: \quad r_1 C_1 + r_2 C_2 = -r_3 C_3 - r_4 C_4$

$\hat{\jmath}: \quad r_1 S_1 + r_2 S_2 = -r_3 S_3 - r_4 S_4$

Velocity:

$\hat{\imath}: \quad -r_1 S_1 \dot{\theta}_1 - r_2 S_2 \dot{\theta}_2 = r_3 S_3 \dot{\theta}_3$

$\hat{\jmath}: \quad r_1 C_1 \dot{\theta}_1 + r_2 C_2 \dot{\theta}_2 = -r_3 C_3 \dot{\theta}_3$

* θ_4 is constant

$\therefore \dot{\theta}_4 = 0$

$\therefore A\dot{x} = b:$

$$\begin{bmatrix} -r_1 S_1 & -r_2 S_2 \\ r_1 C_1 & r_2 C_2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} r_3 S_3 \dot{\theta}_3 \\ -r_3 C_3 \dot{\theta}_3 \end{bmatrix}$$

Acceleration:

*$\dot{e}_3$ constant
$\therefore \ddot{e}_3 = 0$

$\hat{i}: \quad -r_1(s_1\ddot{\theta}_1 + c_1\dot{\theta}_1^2) - r_2(s_2\ddot{\theta}_2 + c_2\dot{\theta}_2^2) = r_3(s_3\overset{0}{\ddot{\theta}_3} + c_3\dot{\theta}_3^2)$

$\rightarrow \quad -r_1 s_1 \ddot{\theta}_1 - r_2 s_2 \ddot{\theta}_2 - r_1 c_1 \dot{\theta}_1^2 - r_2 c_2 \dot{\theta}_2^2 = r_3 c_3 \dot{\theta}_3^2$

$\Rightarrow \quad -r_1 s_1 \ddot{\theta}_1 - r_2 s_2 \ddot{\theta}_2 = r_1 c_1 \dot{\theta}_1^2 + r_2 c_2 \dot{\theta}_2^2 + r_3 c_3 \dot{\theta}_3^2$

$\hat{j}: \quad r_1(c_1\ddot{\theta}_1 - s_1\dot{\theta}_1^2) + r_2(c_2\ddot{\theta}_2 - s_2\dot{\theta}_2^2) = -r_3(c_3\overset{0}{\ddot{\theta}_3} - s_3\dot{\theta}_3^2)$

$\rightarrow \quad r_1 c_1 \ddot{\theta}_1 + r_2 c_2 \ddot{\theta}_2 - r_1 s_1 \dot{\theta}_1^2 - r_2 s_2 \dot{\theta}_2^2 = r_3 s_3 \dot{\theta}_3^2$

$\Rightarrow \quad r_1 c_1 \ddot{\theta}_1 + r_2 c_2 \ddot{\theta}_2 = r_1 s_1 \dot{\theta}_1^2 + r_2 s_2 \dot{\theta}_2^2 + r_3 s_3 \dot{\theta}_3^2$

$\therefore \quad A\ddot{x} = b:$

$$\begin{bmatrix} -r_1 s_1 & -r_2 s_2 \\ r_1 c_1 & r_2 c_2 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} r_1 c_1 \dot{\theta}_1^2 + r_2 c_2 \dot{\theta}_2^2 + r_3 c_3 \dot{\theta}_3^2 \\ r_1 s_1 \dot{\theta}_1^2 + r_2 s_2 \dot{\theta}_2^2 + r_3 s_3 \dot{\theta}_3^2 \end{bmatrix}$$