ME 326 - Term Project: Bicycle Dynamics Analysis

Written By: Jason Davis

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Honor Statement:

As a student of Cal Poly University, I, Jason Davis , pledge that I have not knowingly given nor received any inappropriate assistance regarding the term project in accordance with the syllabus.

Background:

This live script represents the primary source code employed to analyze the kinematics and kinetics of a person pedaling a bicycle as described in the ME 326 lab manual.

Resources:

- MATLAB documentation
- ME 326 Lab Manual
- Google

Required Files:

- tp_final.mlx - Term Project live script where analysis, code, figures, and hand calculations are presented.
- vector_loop_alt.slx - Simulink model
- formatKineticsPlot.m - Custom function to format the plots related to kinetics analysis to preserve code readability in the live script.
- formatSubplot.m - Custom function to format the subplots related to kinematic analysis to preserve code readability in the live script.
- indexOf.m - Helper function to locate the index of a value in an array
- maxAbsValue.m - Helper function to locate the max value in an array, irrespective of sign
- numCols.m - Helper function to return the number of columns in an array so I don't make needless indexing errors
- numRows.m - Helper function to return the number of rows in an array so I don't make needless indexing errors
- partitionArray.m - Partitions an array into roughly equal halves and returns them
- relA.m - Performs relative acceleration analysis assuming non rotating axes

Table of Contents:

Problem Statement

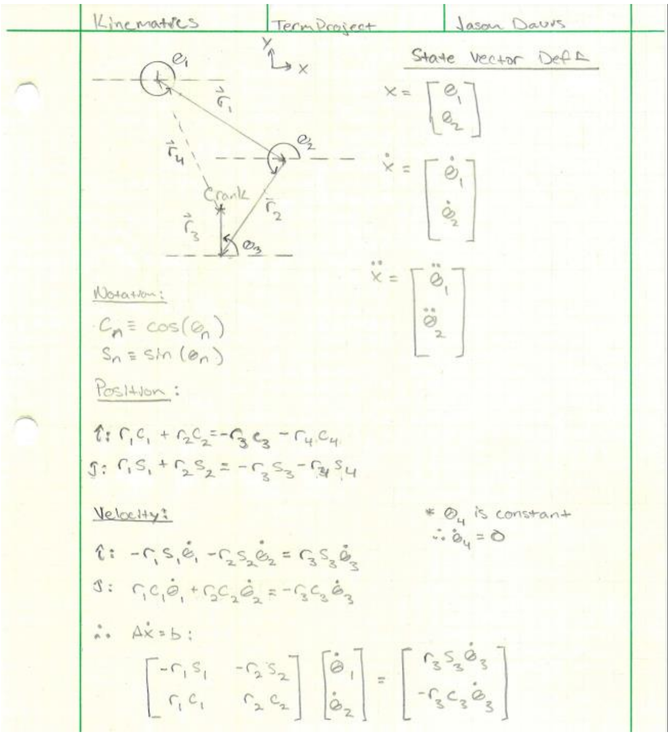
The objectives of this exercise are to:

1. Investigate the kinematics and kinetics of a planar linkage mechanism
2. Derive state-space expressions for angular velocity and angular acceleration for dependent linkage elements
3. Use Simulink to solve derived differential equations
4. Present data graphically according to lab manual
5. Analyze graphical results and present conclusion

Kinematics

Derivation

All equations were derived by hand. Note that all angles are defined as displacements above the positive x-axis.



Kinematics | Term Project | Jason Davis

State Vector Def^{n}

$$x = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \quad \dot{x} = \begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \end{bmatrix} \quad \ddot{x} = \begin{bmatrix} \ddot\theta_1 \\ \ddot\theta_2 \end{bmatrix}$$

Notation:

$C_n \equiv \cos(\theta_n)$

$S_n \equiv \sin(\theta_n)$

Position:

$\hat{i}: r_1c_1 + r_2c_2 = -r_3c_3 - r_4c_4$

$\hat{j}: r_1s_1 + r_2s_2 = -r_3s_3 - r_4s_4$

Velocity:

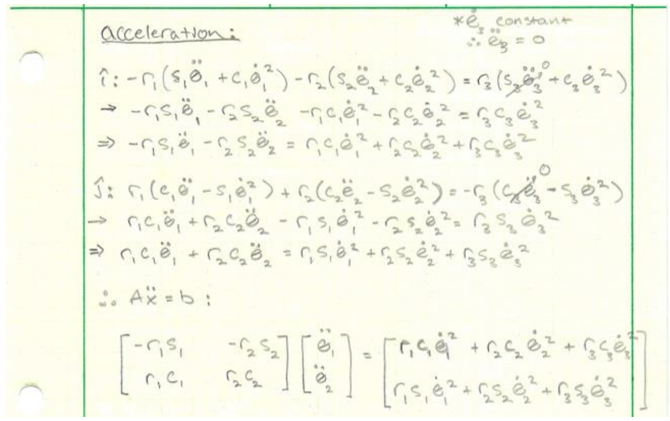
* θ_4 is constant $\therefore \dot\theta_4 = 0$

$\hat{i}: -r_1s_1\dot\theta_1 - r_2s_2\dot\theta_2 = r_3s_3\dot\theta_3$

$\hat{j}: r_1c_1\dot\theta_1 + r_2c_2\dot\theta_2 = -r_3c_3\dot\theta_3$

$\therefore A\dot{x} = b$:

$$\begin{bmatrix} -r_1s_1 & -r_2s_2 \\ r_1c_1 & r_2c_2 \end{bmatrix} \begin{bmatrix} \dot\theta_1 \\ \dot\theta_2 \end{bmatrix} = \begin{bmatrix} r_3s_3\dot\theta_3 \\ -r_3c_3\dot\theta_3 \end{bmatrix}$$



Acceleration:

* $\dot\theta_3$ constant $\therefore \ddot\theta_3 = 0$

$\hat{i}: -r_1(s_1\ddot\theta_1 + c_1\dot\theta_1^2) - r_2(s_2\ddot\theta_2 + c_2\dot\theta_2^2) = r_3(\cancel{s_3\ddot\theta_3}^{0} + c_3\dot\theta_3^2)$

$\rightarrow -r_1s_1\ddot\theta_1 - r_2s_2\ddot\theta_2 - r_1c_1\dot\theta_1^2 - r_2c_2\dot\theta_2^2 = r_3c_3\dot\theta_3^2$

$\Rightarrow -r_1s_1\ddot\theta_1 - r_2s_2\ddot\theta_2 = r_1c_1\dot\theta_1^2 + r_2c_2\dot\theta_2^2 + r_3c_3\dot\theta_3^2$

$\hat{j}: r_1(c_1\ddot\theta_1 - s_1\dot\theta_1^2) + r_2(c_2\ddot\theta_2 - s_2\dot\theta_2^2) = -r_3(\cancel{c_3\ddot\theta_3}^{0} - s_3\dot\theta_3^2)$

$\rightarrow r_1c_1\ddot\theta_1 + r_2c_2\ddot\theta_2 - r_1s_1\dot\theta_1^2 - r_2s_2\dot\theta_2^2 = r_3s_3\dot\theta_3^2$

$\Rightarrow r_1c_1\ddot\theta_1 + r_2c_2\ddot\theta_2 = r_1s_1\dot\theta_1^2 + r_2s_2\dot\theta_2^2 + r_3s_3\dot\theta_3^2$

$\therefore A\ddot{x} = b$:

$$\begin{bmatrix} -r_1s_1 & -r_2s_2 \\ r_1c_1 & r_2c_2 \end{bmatrix} \begin{bmatrix} \ddot\theta_1 \\ \ddot\theta_2 \end{bmatrix} = \begin{bmatrix} r_1c_1\dot\theta_1^2 + r_2c_2\dot\theta_2^2 + r_3c_3\dot\theta_3^2 \\ r_1s_1\dot\theta_1^2 + r_2s_2\dot\theta_2^2 + r_3s_3\dot\theta_3^2 \end{bmatrix}$$

Original Configuration

Measurements

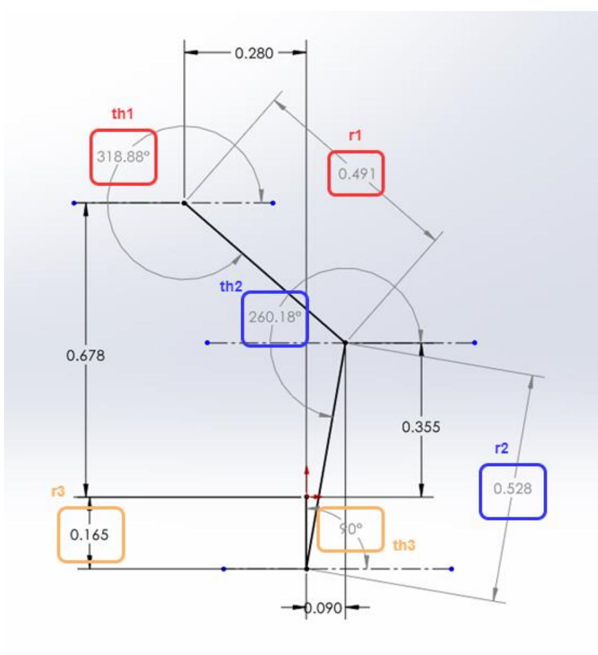


Figure 1: Scale diagram of each link with magnitudes and angles measured in Solidworks. All links and angles have units of meters and degrees, respectively.

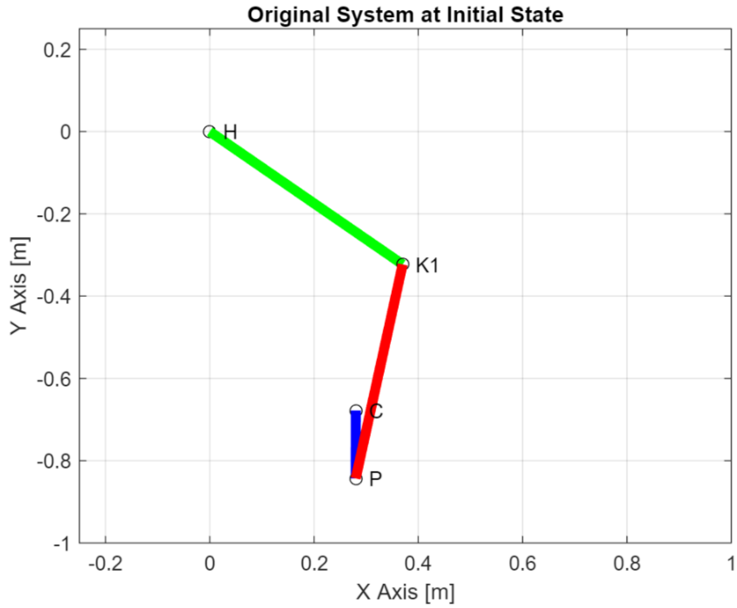


Figure 2: Plot of the system linkage for the original length crank at its initial state with the pedal at bottom dead center (BDC).

Shortened Crank Configuration

Measurements

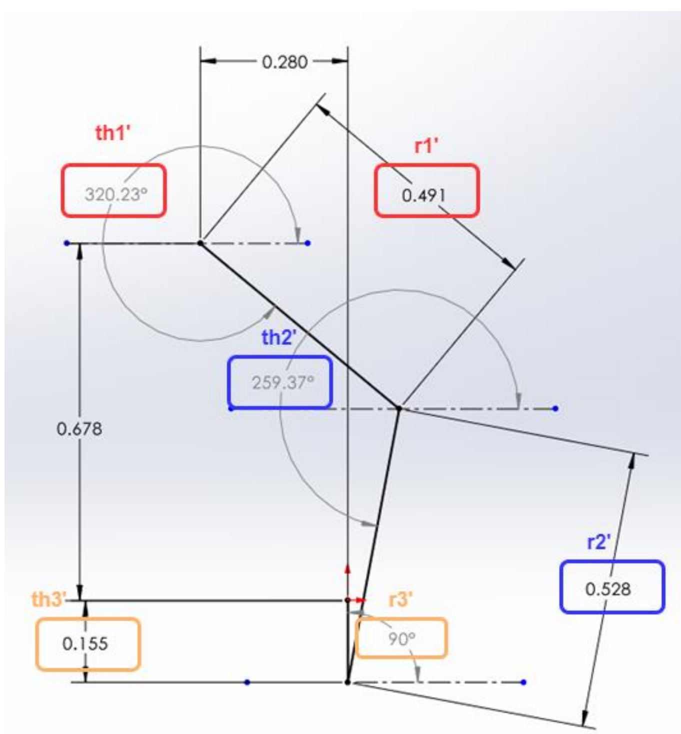


Figure 3: Diagram of the new linkage configuration with the crank shortened by 10mm. All links and angles have units of meters and degrees, respectively.

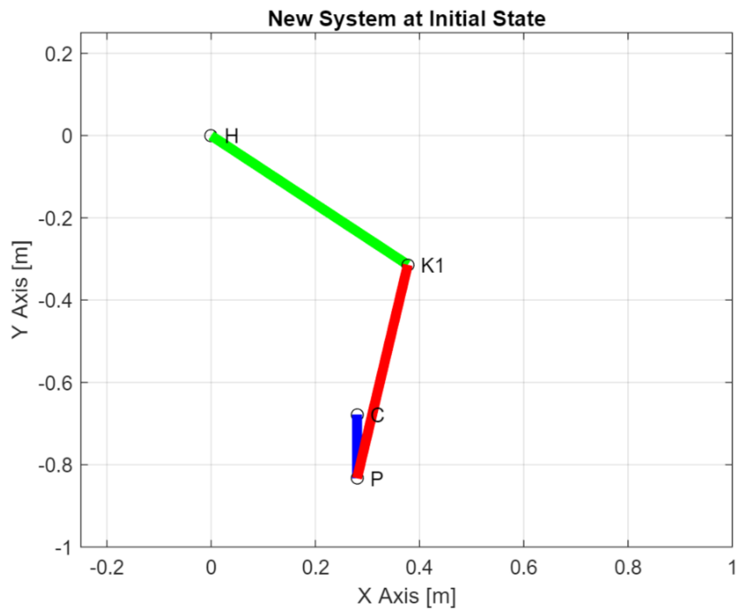


Figure 4: Plot of the system linkage for the shortened length crank at its initial state with the pedal at bottom dead center (BDC).

Plotting Results

Hip and knee angles were plotted against both time and crank angle over two revolution for each configuration in the figures below. Global extrema were annotated on each plot for ease of analysis.

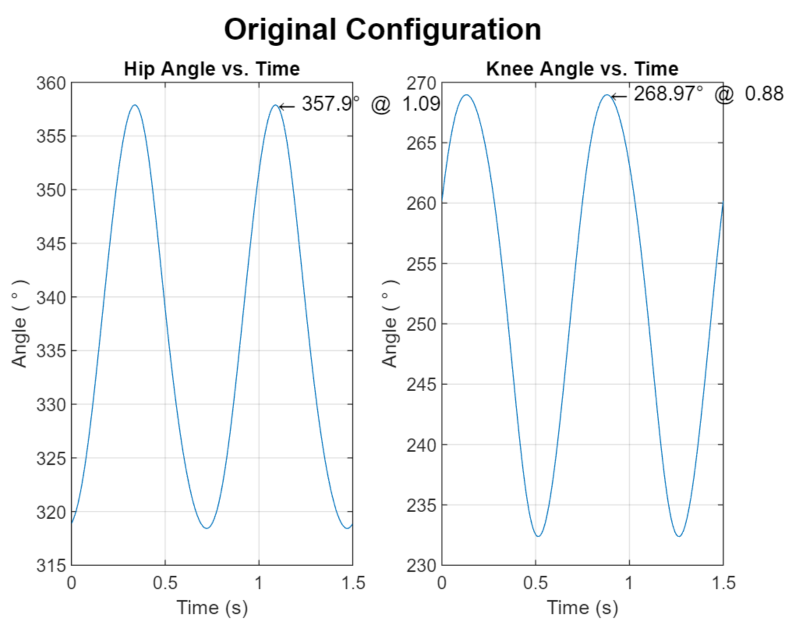


Figure 5: Subplots of the hip and knee angles against time in seconds in the system's original configuration. All angles are plotted in degrees relative to the positive x-axis.

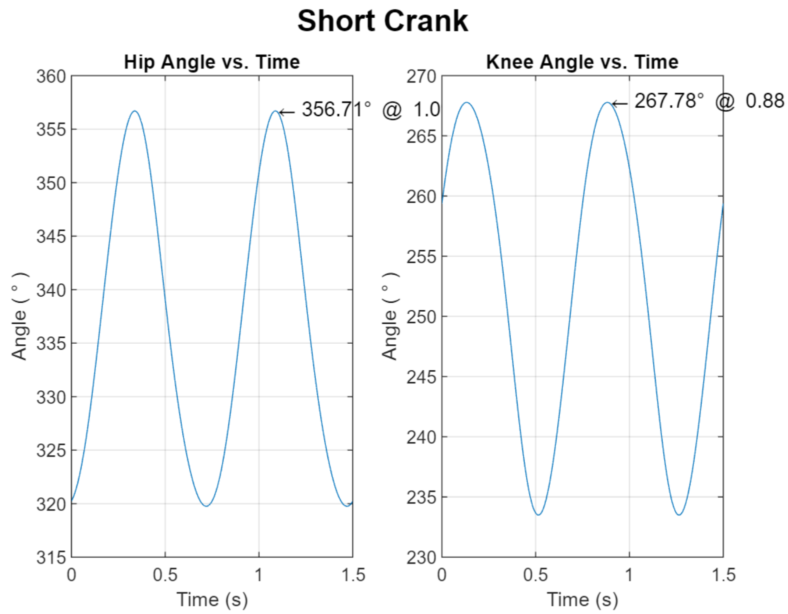


Figure 6: Subplots of the hip and knee angles against time in seconds in the system's modified configuration. All angles are plotted in degrees relative to the positive x-axis.

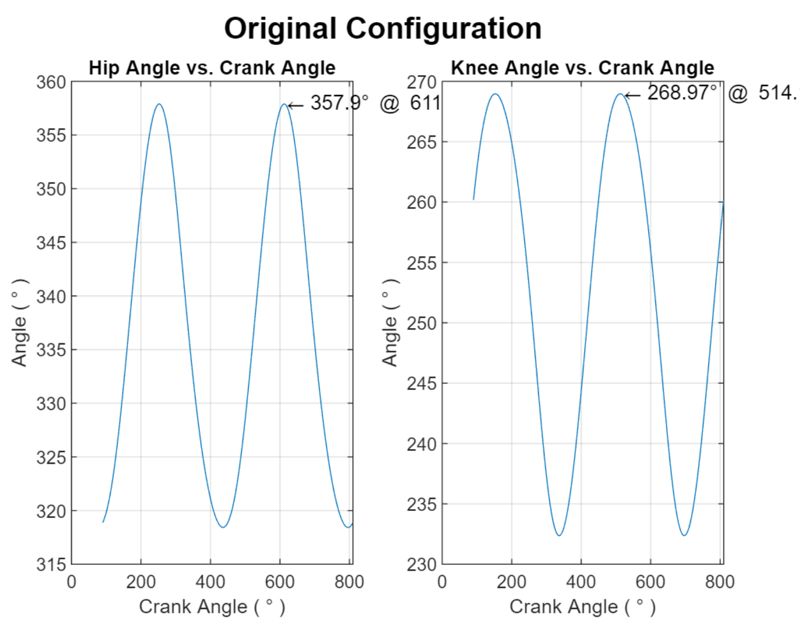


Figure 7: Subplots of the hip and knee angles against the crank angle in the system's original configuration. All angles are plotted in degrees relative to the positive x-axis.

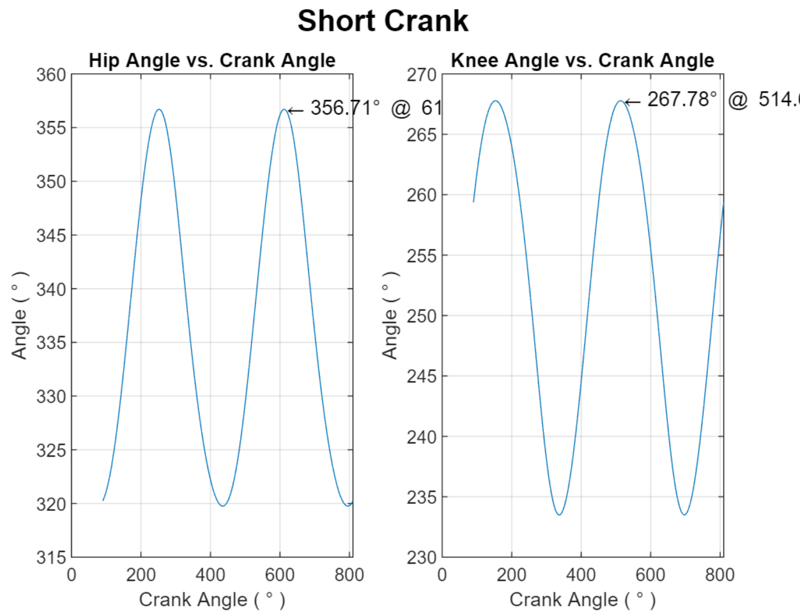


Figure 8: Subplots of the hip and knee angles against the crank angle in the system's modified configuration. All angles are plotted in degrees relative to the positive x-axis.

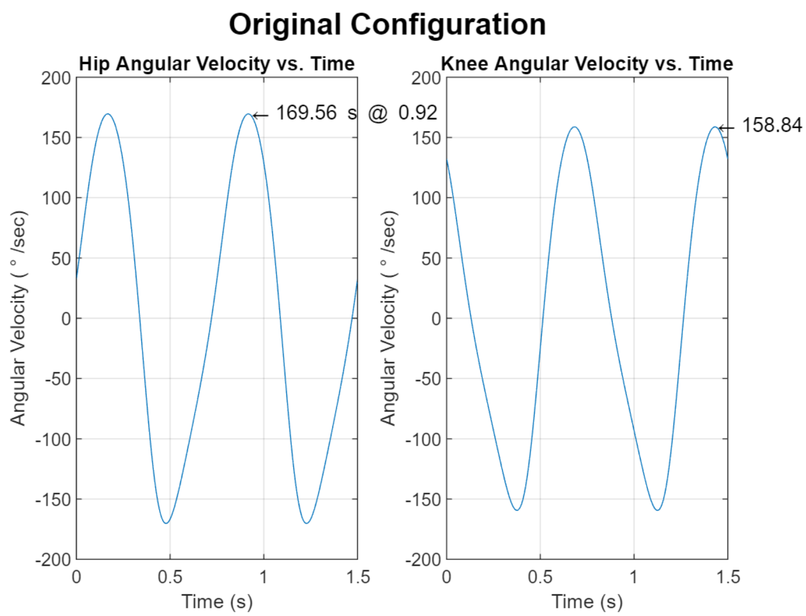


Figure 9: Subplots of the hip and knee angular velocities against time in the system's original configuration. All angular velocities and times are plotted in degrees per second relative to the positive x-axis and seconds, respectively.

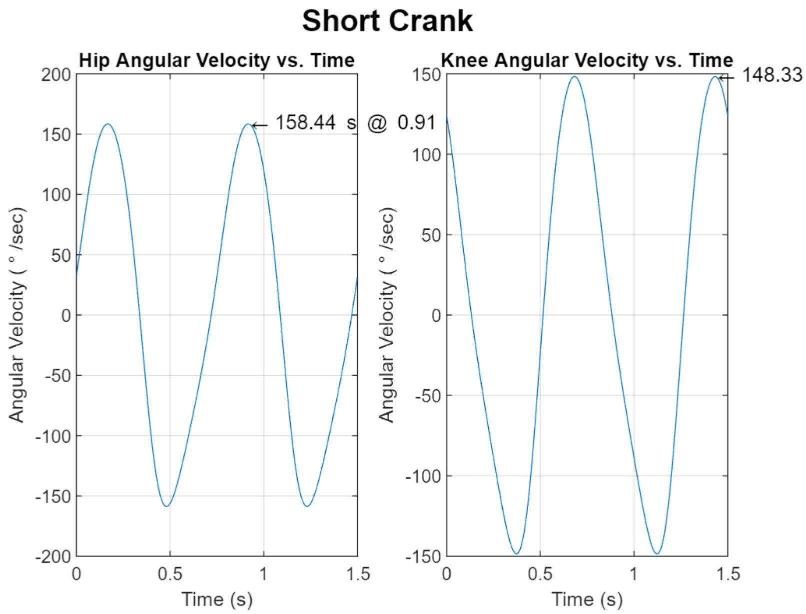


Figure 10: Subplots of the hip and knee angular velocities against time in the system's modified configuration. All angular velocities and times are plotted in degrees per second relative to the positive x-axis and seconds, respectively.

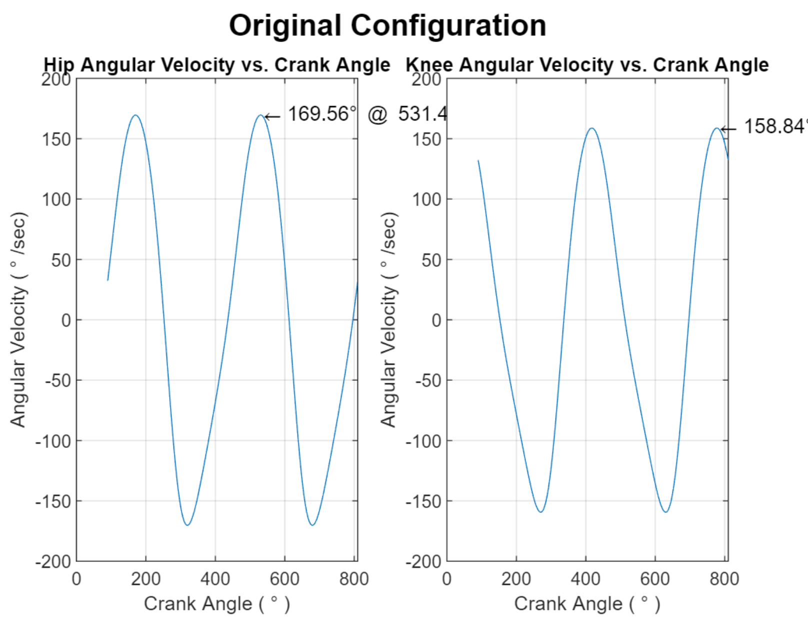


Figure 11: Subplots of the hip and knee angular velocities against crank angle in the system's original configuration. All angular velocities and angles are plotted in degrees per second and degrees, respectively, relative to the positive x-axis.

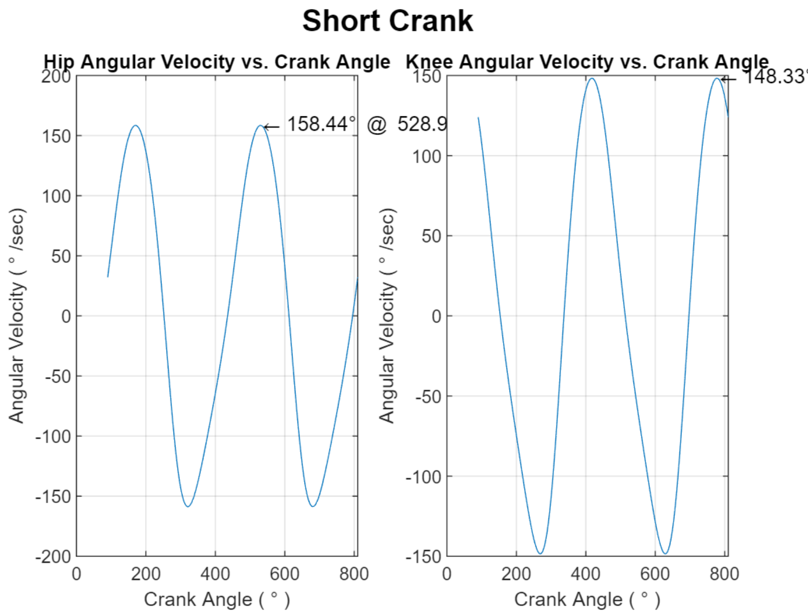


Figure 12: Subplots of the hip and knee angular velocities against crank angle in the system's modified configuration. All angular velocities and angles are plotted in degrees per second and degrees, respectively, relative to the positive x-axis.

Discussion Questions

Question 1:

For each configuration...

What crank angle is the maximum hip angle at (relative to the x-axis) after one revolution? What is the value of this hip angle?

```
hip1_max = 357.8968
crank1_max = 611.9079
hip2_max = 356.7064
crank2_max = 611.1850
```

For the original configuration after one crank revolution, the maximum hip angle of 357.90 degrees occurs at a crank angle of **611.91 degrees.**

For the modified configuration after one crank revolution, the maximum hip angle of 356.71 degrees occurs at a crank angle of **611.19 degrees.**

Question 2:

What crank angle is the maximum knee angle at (relative to the x-axis) after one revolution? What is the value of this knee angle?

```
knee1_max = 268.9666
crank1_max = 514.1296
knee2_max = 267.7775
crank2_max = 514.6165
```

For the original configuration after one crank revolution, the maximum knee angle of 268.97 degrees occurs at a crank angle of **514.13 degrees**.

For the modified configuration after one crank revolution, the maximum knee angle of 267.78 degrees occurs at a crank angle of **514.62 degrees**.

Question 3:

When comparing the hip and the knee vs. crank plots, describe the behavior. What do the peaks represent?

The peaks of figures 7-8 represent the crank positions at which the maximum and minimum hip and knee angles occur.

Referencing figure 7 as an example, we know that a crank angle at 270 degrees corresponds to the system at TDC. In this state, the rider's upper leg is at its highest elevation in the cycle. Consequently, the angle constructed by the rider's upper leg and the horizontal datum coincident with the rider's hip joint is at a global maximum which repeats every time the TDC condition occurs. It follows that, for the BDC condition, the rider's upper leg is at its lowest position in the cycle. Consequently, the angle constructed by the rider's upper leg and the horizontal datum coincident with the rider's hip joint is at a global minimum which repeats near every occurrence of the BDC condition.

Continuing to use figure 7 as an example, we observe that a maximum knee angle occurs when the system is near BDC and a minimum knee angle at TDC. This is functionally opposite what we see with the hip angle. It is worth noting that, for a well-adjusted bicycle, the maximum knee angle should occur *at* TDC, however this is not the case in our example. Consequently, the angle constructed by the rider's lower leg and the horizontal datum coincident with the rider's knee joint is at a global minimum which repeats near every occurrence of the BDC condition.

Question 4:

What crank angle is the maximum angular velocity for the hip at one revolution? What is the value for velocity?

```
hip1_max = 169.5575
crank1_max = 531.4181
hip2_max = 158.4408
crank2_max = 528.9800
```

For the original configuration after one crank revolution, the crank angle associated with maximum hip angular velocity of 169.56 degrees per second is **531.42 degrees**.

For the modified configuration after one crank revolution, the crank angle associated with maximum hip angular velocity of 158.44 degrees per second is **528.98 degrees**.

Question 5:

What crank angle is the maximum angular velocity for the knee at one revolution? What is the value for velocity?

```
knee1_max = 158.8404
crank1_max = 777.0514
knee2_max = 148.3279
crank2_max = 777.6484
```

For the original configuration after one crank revolution, the crank angle associated with maximum knee angular velocity of 158.84 degrees per second is **777.05 degrees**.

For the modified configuration after one crank revolution, the crank angle associated with maximum hip angular velocity of 148.33 degrees per second is **777.65 degrees**.

<u>Question 6:</u>

Compare the results of the original crank with the shorter crank. Is there a noticeable difference with the angle? Is there a noticeable difference with the angular velocity? Why do you think this is?

Regarding the maximum hip and knee angles, there is a small (likely unnoticeable) difference in the crank angles between each configuration. As one would expect, a longer crank length corresponds to a larger maximum hip / knee angle, however the results differ by several degrees at most and would likely go unnoticed by the rider.

Regarding the maximum hip and knee angular velocities, the difference between configurations is small, however it is possible that the rider would notice this difference.

These differences can be explained by the relationship between translational velocity, angular velocity, and the crank radius. Since the angular velocity of the crank is unchanged, by shortening the crank radius we will inevitably lower the translational velocity of the pedal. This, in turn, lowers the angular velocity of the shank since its length is unchanged.

<u>Question 7:</u>

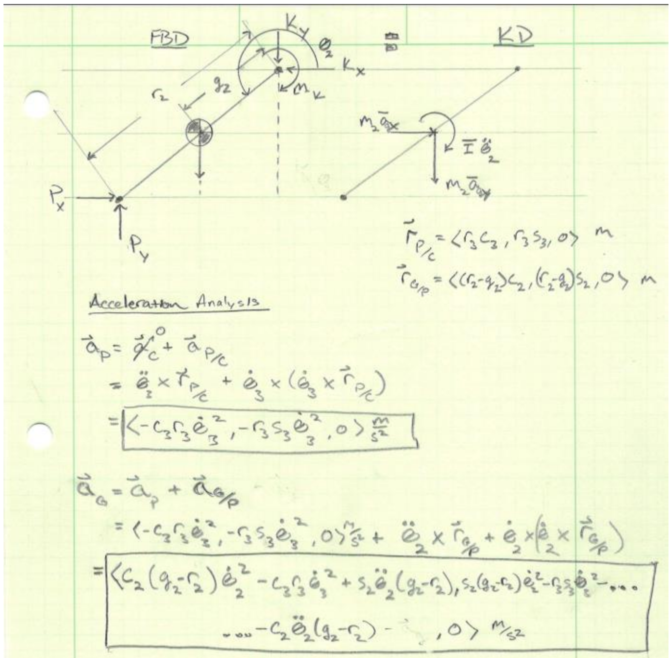
With this behavior in mind there is a trend, but why do the maximums slightly differ in value in the data sets? What can be done to align these closer together?

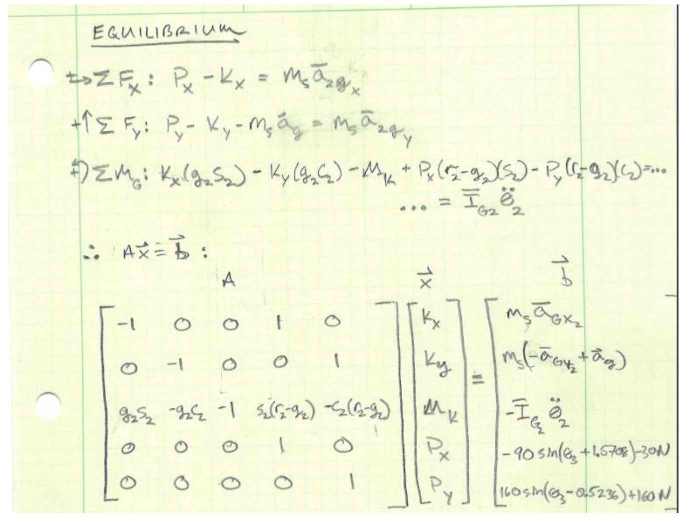
We can apply a phase offset in the definition of the sines and cosines of each angle to align the maximum values with the angles at which they should be occurring.

Kinetics

Derivation

All equations were derived by hand. Note that all angles are defined as displacements above the positive x-axis.





Plotting Results

Forces and moments at the knee were plotted over two revolution for each configuration and plotted in the figures below. Global extrema were annotated on each plot for ease of analysis.

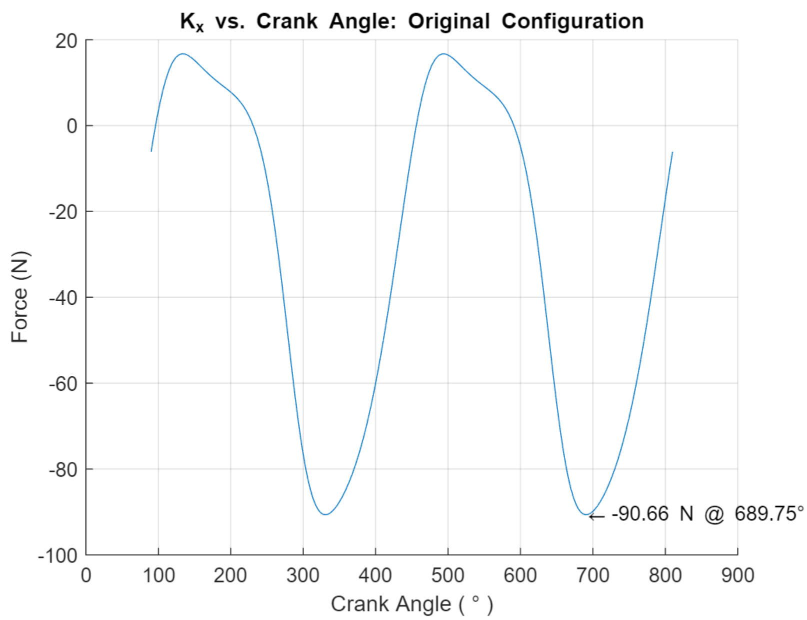


Figure 13: Plot of horizontal knee forces against the crank angle over two full revolutions of the original crank configuration.

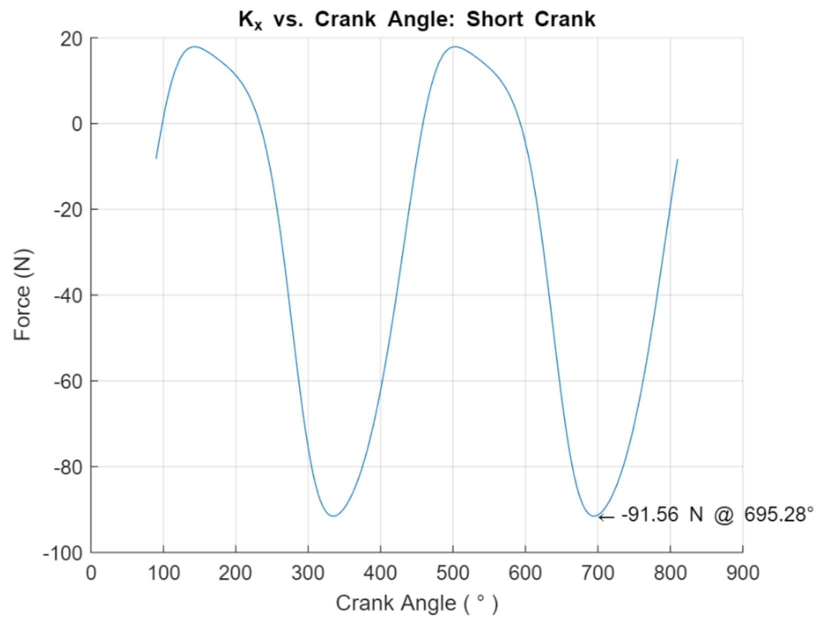


Figure 14: Plot of horizontal knee forces against the crank angle over two full revolutions of the shortened crank configuration.

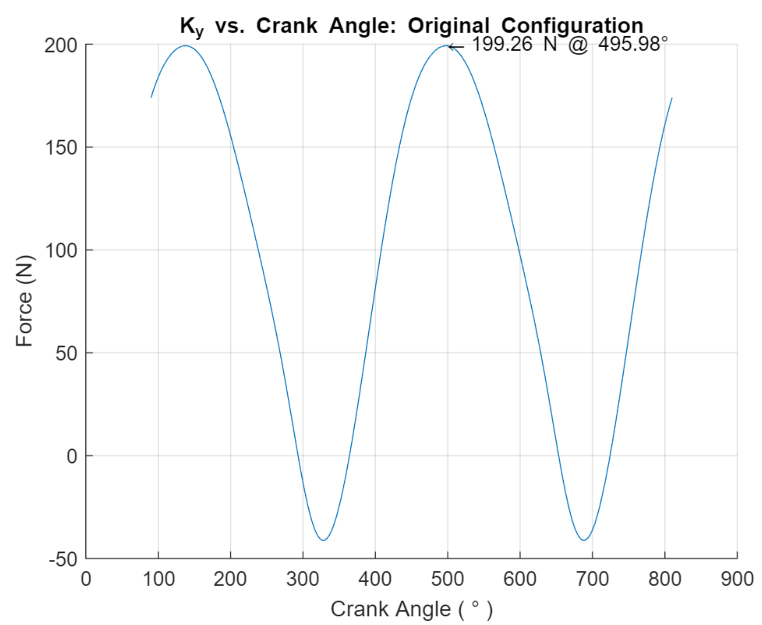


Figure 15: Plot of vertical knee forces against the crank angle over two full revolutions of the original crank configuration.

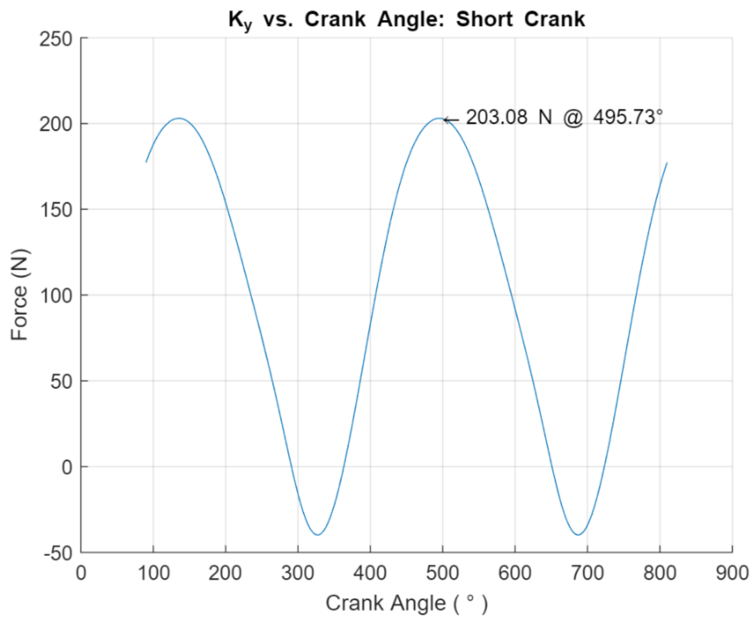


Figure 16: Plot of vertical knee forces against the crank angle over two full revolutions of the shortened crank configuration.

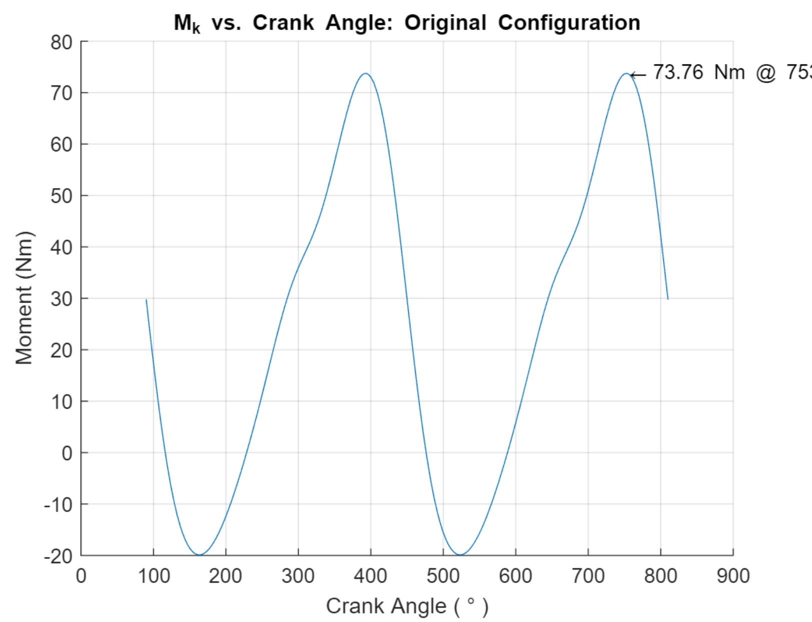


Figure 17: Plot of applied torque at the knee against the crank angle over two full revolutions of the original crank configuration.

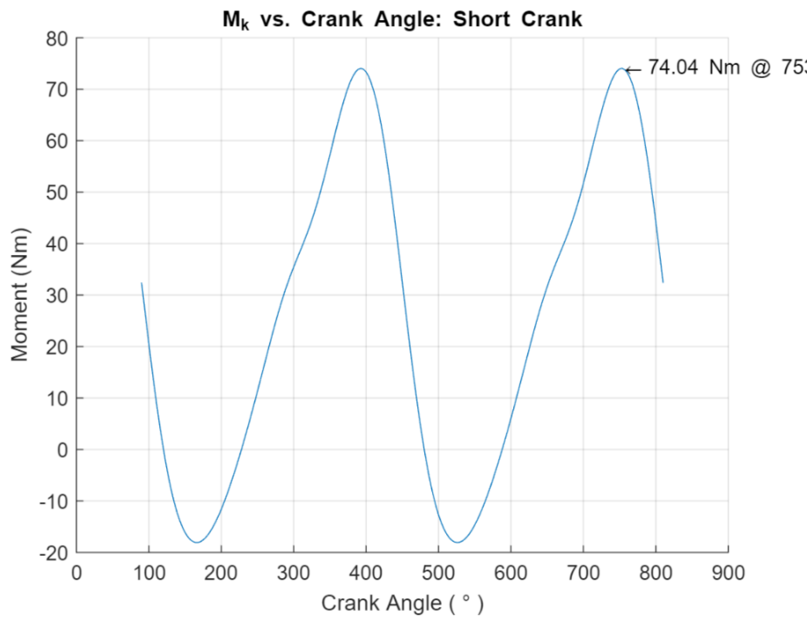


Figure 18: Plot of applied torque at the knee against the crank angle over two full revolutions of the shortened crank configuration.

Discussion Questions

Question 1:

For each configuration, what percentage of the body weight are the maximum Kx and Ky forces?

```
percentBW_kx_reg = 13.7700
percentBW_ky_reg = 30.2600
percentBW_kx_short = 13.9000
percentBW_ky_short = 30.8400
```

Regarding the original configuration, the maximum **horizontal force** at the knee accounts for **13.77%** of total body weight, while the maximum **vertical force** at the knee accounts for **30.26%** of total body weight.

Regarding the shortened configuration, the maximum **horizontal force** at the knee accounts for **13.90%** of total body weight, while the maximum **vertical force** at the knee accounts for **30.84%** of total body weight.

Question 2:

When are the quadriceps the dominant muscle groups (for which direction of the knee couple) and when are the hamstrings the dominant muscle group?

When the quadriceps muscles contract, they work to extend the knee, thereby inducing a positive moment (counter-clockwise direction as defined in the above derivation) about the knee. Extension at the knee elongates the leg, which starts when the crank is at TDC (top dead center) and concludes when the crank is at BDC (bottom dead center). Thus, the quadriceps muscles are dominant when the cyclist is pushing down on the pedal.

Since the hamstrings directly oppose the quadriceps in terms of force application at the knee, it follows that they are dominant beginning when the crank is at BDC and ending at TDC, thereby inducing a negative moment (clockwise direction) about the knee. Thus the hamstrings are dominant when the cyclist is pulling up on the pedal.

Question 3:

Discuss whether the knee couple is intuitive. Are the magnitude of the knee forces considerably large? Compare the maximum knee forces to the maximum pedal forces.

The knee couple is intuitive. The figure below shows the pedal forces plotted with the knee forces:

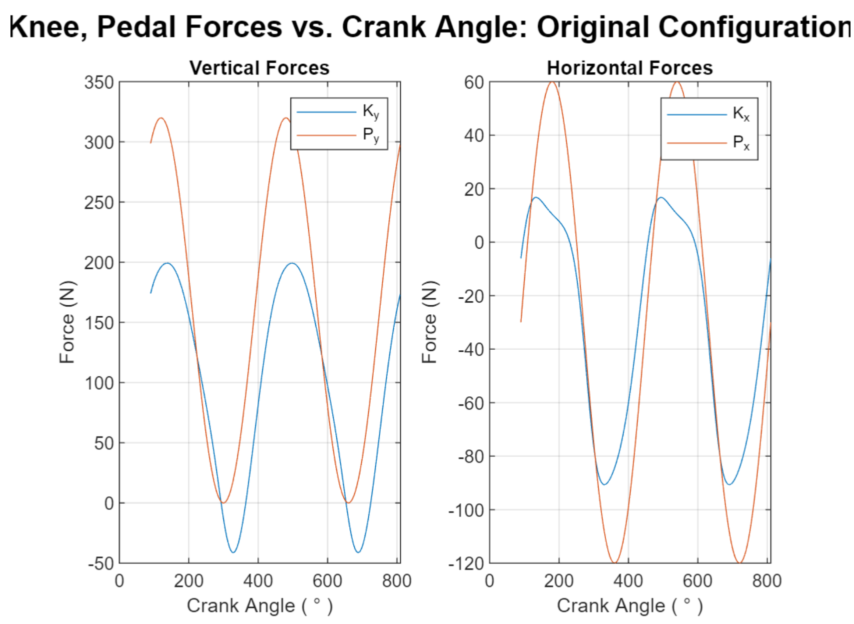


Figure 19: Subplots with pedal forces plotted against knee forces for the original configuration.

When examining the graph of the combined vertical forces, we can directly visualize Newton's Second Law with each progressive crank angle. For example, just past 270 degrees of the crank, the net force applied in the vertical direction is negative, which corresponds to the pedal being at TDC and travelling downward. Conversely, at BDC, the pedal moves upward, corresponding to a positive net force applied by the pedal.

Question 4:

How do the knee forces and couple change by changing the crank length? Is more or less torque required to pedal the bike?

```
kx_max_short = -91.5573
kx_max_reg = -90.6558
ky_max_short = 203.0801
ky_max_reg = 199.2565
mk_max_short = 74.0440
mk_max_reg = 73.7613
```

As expected, **the magnitude of the knee forces must increase as the crank length is reduced** to produce the same torque at the crank. By summing moments about the crank, we can observe that for a given reduction in the moment arm length (ie the crank radius), we must proportionately increase our applied force at the pedal. The required torque at the crank remains the same, however because of the change in system geometry, we must **increase the applied torque at the knee**.

<u>Question 5:</u>

How could you improve the accuracy of this analysis?

Bicycle fitting is a service that many bicycle shops provide at a cost to the rider for good reason: optimizing the bicycle's adjustments to the rider's body leads to better overall performance and efficiency. It follows that the pedal forces are likely influenced by many factors, such as the rider's position on the seat, seat post elevation, tire inflation pressures, and even bearing quality at the crank and pedals. Since many factors can influence the force that a rider could experience at the pedals, the following changes could be made to improve the accuracy of the analysis:

1. Use force transducers to experimentally collect pedal force data
2. Place IR trackers at the pedal, crank, knee, and hip and use IR cameras to acquire better measurements for the rider

While likely a more expensive solution, these changes would provide better quality data and lead to better insights with regard to analysis.

Supplemental: Simulink Model