Heuristic Optimization Project-3

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1 Introduction

In this assignment we will look at three important optimization algorithms mainly as follows:

- Particle Swarm Optimization
- Bayesian Optimization
- Ant Colony Optimization

2 Part A: Particle Swarm Optimization

2.1 A. Coding

The PSO Algorith as discussed above was implemented in Python using python-class. The PSO Implementation had following user defined parameters.

- Number of particles or population size = 100
- No of Decision Variables = 3
- The inertial coefficient(α) = 0.5
- The individual local search coefficient (β_1) = 1.49
- The social or global search coefficient (β_1) 1.49
- Maximum allowed number of Iterations = 25

2.2 B. Solving Benchmark Minimization Problems

In this section lets look at the various minimization benchmark problems. This problems essentially be solved using above design PSO. All of them have 3 decision variables and total number of particles be 100.

2.2.1 1. De Jong's Function

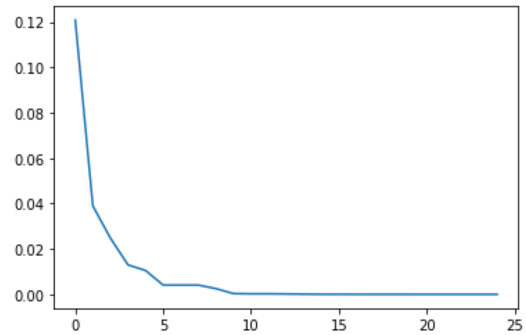
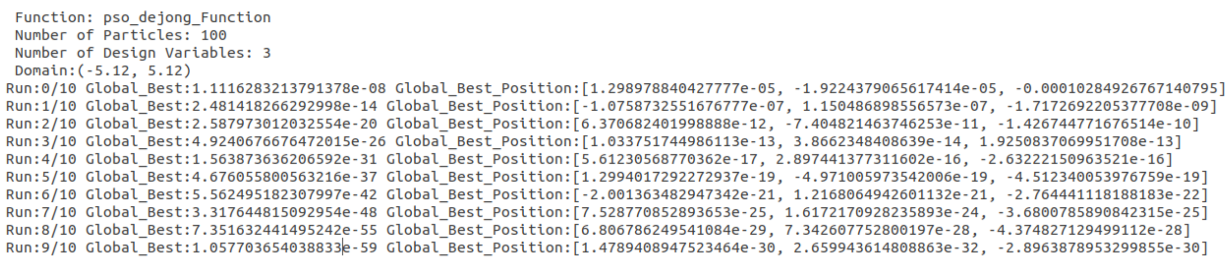
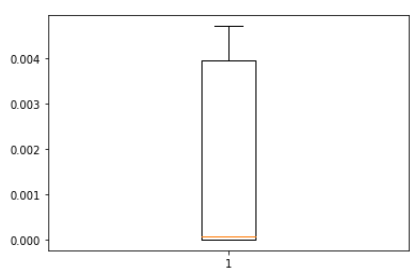


Figure 1: De Jong's Convergence

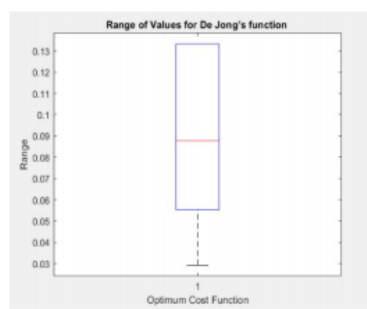


```
 Function: pso_dejong_Function
 Number of Particles: 100
 Number of Design Variables: 3
 Domain:(-5.12, 5.12)
Run:0/10 Global_Best:1.1116283213791378e-08 Global_Best_Position:[1.298978840427777e-05, -1.9224379065617414e-05, -0.00010284926767140795]
Run:1/10 Global_Best:2.481418266292998e-14 Global_Best_Position:[-1.0758732551676777e-07, 1.150486898556573e-07, -1.7172692205377708e-09]
Run:2/10 Global_Best:2.587973012032554e-20 Global_Best_Position:[6.370682401998888e-12, -7.404821463746253e-11, -1.426744771676514e-10]
Run:3/10 Global_Best:4.9240676676472015e-26 Global_Best_Position:[1.033751744986113e-13, 3.8662348408639e-14, 1.9250837069951708e-13]
Run:4/10 Global_Best:1.563873636206592e-31 Global_Best_Position:[5.61230568770362e-17, 2.897441377311602e-16, -2.63222150963521e-16]
Run:5/10 Global_Best:4.676055800563216e-37 Global_Best_Position:[1.2994017292272937e-19, -4.971005973542006e-19, -4.512340053976759e-19]
Run:6/10 Global_Best:5.562495182307997e-42 Global_Best_Position:[-2.001363482947342e-21, 1.2168064942601132e-21, -2.764441118188183e-22]
Run:7/10 Global_Best:3.317644815092954e-48 Global_Best_Position:[7.528770852893653e-25, 1.6172170928235893e-24, -3.6800785890842315e-25]
Run:8/10 Global_Best:7.351632441495242e-55 Global_Best_Position:[6.806786249541084e-29, 7.342607752800197e-28, -4.374827129499112e-28]
Run:9/10 Global_Best:1.057703654038833e-59 Global_Best_Position:[1.4789408947523464e-30, 2.659943614808863e-32, -2.8963878953299855e-30]
```

Figure 2: Best Values in 10 Runs



(a) PSO Box Plot



(b) GA Box Plot

Figure 3: De Jong's Function

2.2.2 2. Rosenbrock Function

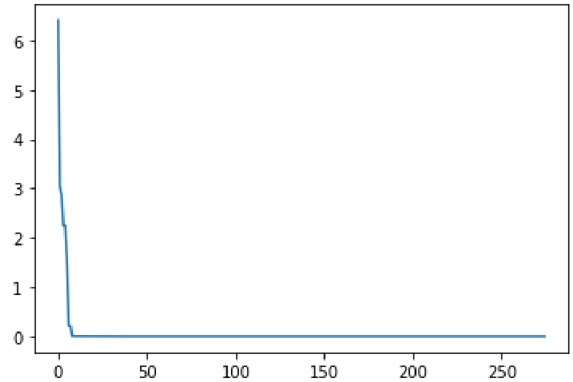
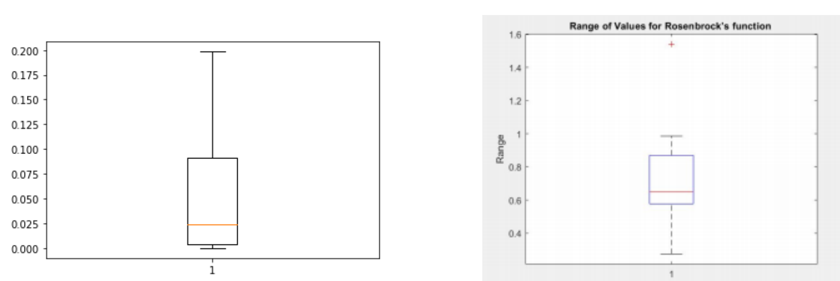


Figure 4: Rosenbrock's Convergence

```
 Function: Rosebrock_Function
 Number of Particles: 100
 Number of Design Variables: 3
 Domain:(-2.048, 2.048)
Run:0/10 Global_Best:1.1661121371489689e-05 Global_Best_Position:[0.9991016477315589, 0.9980097233355596, 0.9961998889596547]
Run:1/10 Global_Best:3.805396066017321e-11 Global_Best_Position:[1.0000003149728356, 1.0000000295700942, 0.999999920981472]
Run:2/10 Global_Best:5.979086746612417e-17 Global_Best_Position:[1.0000000019945456, 1.0000000038900958, 1.0000000084102694]
Run:3/10 Global_Best:9.480831384569767e-23 Global_Best_Position:[0.9999999999958045, 0.9999999999914778, 0.9999999999831247]
Run:4/10 Global_Best:5.6107731883844465e-28 Global_Best_Position:[1.0000000000000062, 1.0000000000000142, 1.0000000000000282]
Run:5/10 Global_Best:0.0 Global_Best_Position:[1.0, 1.0, 1.0]
Run:6/10 Global_Best:0.0 Global_Best_Position:[1.0, 1.0, 1.0]
Run:7/10 Global_Best:0.0 Global_Best_Position:[1.0, 1.0, 1.0]
Run:8/10 Global_Best:0.0 Global_Best_Position:[1.0, 1.0, 1.0]
Run:9/10 Global_Best:0.0 Global_Best_Position:[1.0, 1.0, 1.0]
Run:10/10 Global_Best:0.0 Global_Best_Position:[1.0, 1.0, 1.0]
```

Figure 5: Rosenbrock's Best Values in 10 Runs



(a) PSO Box Plot

(b) GA Box Plot

Figure 6: Rosenbrock's Function

2.2.3 3. Rastrigin Function

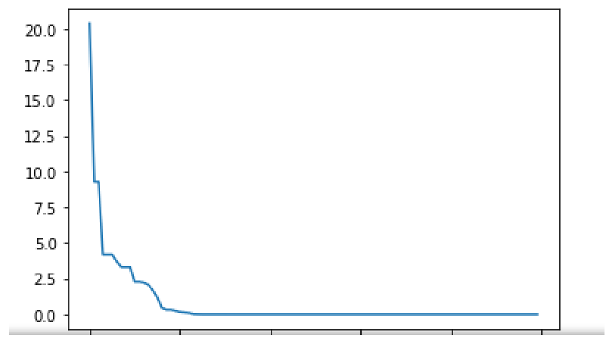
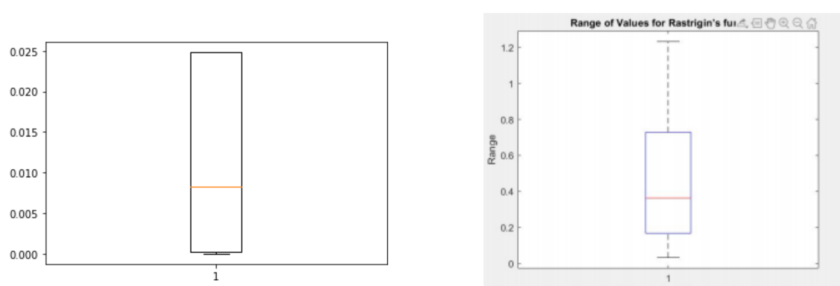


Figure 7: Rastrigin's Convergence

```
 Function: Rastrigin_function_Function
 Number of Particles: 100
 Number of Design Variables: 3
 Domain:(-5.12, 5.12)
Run:0/10 Global_Best:0.007474015727346739 Global_Best_Position:[0.003483618363703367, 0.0038629624599273438, 0.003258290439602844]
Run:1/10 Global_Best:2.0535061651116848e-08 Global_Best_Position:[-7.797354616597907e-06, 3.3543149405345707e-06, 5.608680651285159e-06]
Run:2/10 Global_Best:4.973799150320701e-14 Global_Best_Position:[8.21352938063838e-09, 6.163474056767671e-10, -1.3755952553121801e-08]
Run:3/10 Global_Best:0.0 Global_Best_Position:[5.178658817211388e-10, -4.942621891813629e-10, 8.098968461339825e-10]
Run:4/10 Global_Best:0.0 Global_Best_Position:[5.178658817211388e-10, -4.942621891813629e-10, 8.098968461339825e-10]
Run:5/10 Global_Best:0.0 Global_Best_Position:[5.178658817211388e-10, -4.942621891813629e-10, 8.098968461339825e-10]
Run:6/10 Global_Best:0.0 Global_Best_Position:[5.178658817211388e-10, -4.942621891813629e-10, 8.098968461339825e-10]
Run:7/10 Global_Best:0.0 Global_Best_Position:[5.178658817211388e-10, -4.942621891813629e-10, 8.098968461339825e-10]
Run:8/10 Global_Best:0.0 Global_Best_Position:[5.178658817211388e-10, -4.942621891813629e-10, 8.098968461339825e-10]
Run:9/10 Global_Best:0.0 Global_Best_Position:[5.178658817211388e-10, -4.942621891813629e-10, 8.098968461339825e-10]
```

Figure 8: Rastrigin's Best Values in 10 Runs



(a) PSO Box Plot

(b) GA Box Plot

Figure 9: Rastrigin's Function

2.2.4 4. Griewangk's function

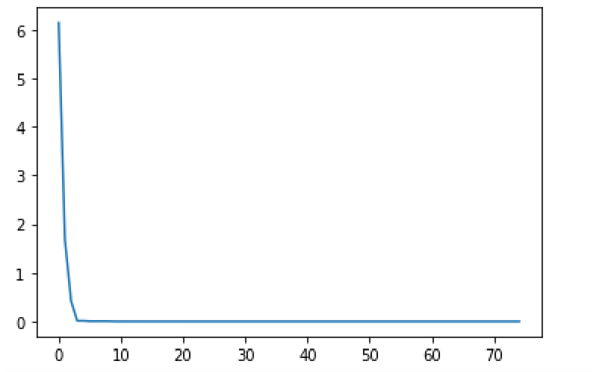
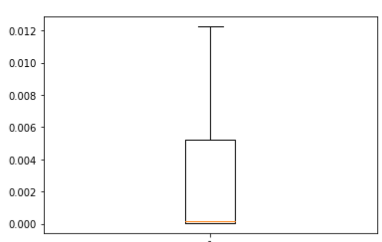


Figure 10: Griewangk's function Convergence

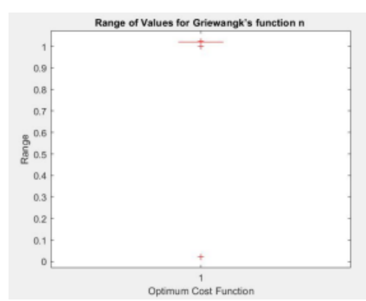
```
 Function: griewangk_function_Function
 Number of Particles: 100
 Number of Design Variables: 3
 Domain:(-600, 600)

Run:0/10 Global_Best:3.228722434502818e-07 Global_Best_Position:[-0.0028374549021350226, -0.01004290817268963, 0.03438862920958614]
Run:1/10 Global_Best:6.686517704538928e-14 Global_Best_Position:[1.3884094931671171e-05, 8.271744669248888e-06, -2.5041677732637216e-06]
Run:2/10 Global_Best:3.967596248911736e-19 Global_Best_Position:[-9.041564983905837e-09, -1.1463054417237245e-08, -3.70659815145095e-08]
Run:3/10 Global_Best:4.611793253397123e-25 Global_Best_Position:[1.579835726449996e-11, 3.89297662828367e-11, -8.922023664269321e-12]
Run:4/10 Global_Best:3.484992694219291e-30 Global_Best_Position:[8.556196532249892e-14, 8.109875357988488e-14, 6.489455664211526e-15]
Run:5/10 Global_Best:1.7064022770095737e-37 Global_Best_Position:[1.3832462310053142e-17, 4.746315761288591e-18, -2.1649396849310426e-17]
Run:6/10 Global_Best:7.150498078590415e-43 Global_Best_Position:[-2.5756442816344986e-20, -3.4070926073169433e-20, 3.218659474689953e-20]
Run:7/10 Global_Best:8.536995162353257e-48 Global_Best_Position:[-1.6351013719370942e-22, 1.2178493456825597e-23, 8.522968955375474e-23]
Run:8/10 Global_Best:5.878970101285497e-53 Global_Best_Position:[-2.1658859050195827e-26, -4.242501900077675e-25, 2.33883462763909e-25]
Run:9/10 Global_Best:1.328698997020648e-58 Global_Best_Position:[-3.76710429979056e-29, -5.126666278201482e-28, 5.169462448332774e-28]
```

Figure 11: Griewangk's function Best Values in 10 Runs



(a) PSO Box Plot



(b) GA Box Plot

Figure 12: Griewangk's Function

2.2.5 5. Schwefel's function

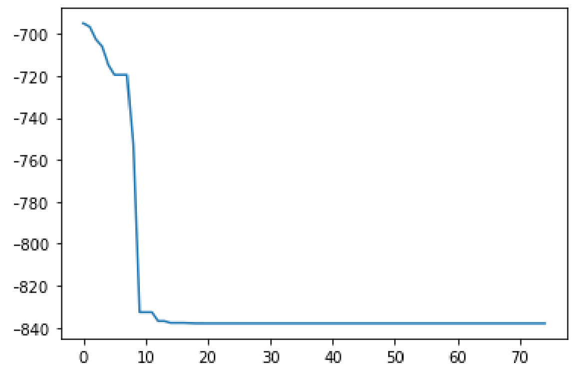


Figure 13: Schwefel's function Convergence

```
 Function: Schwefel_Function
 Number of Particles: 100
 Number of Design Variables: 2
 Domain:(-500, 500)
Run:0/10 Global_Best:-837.9648060082079 Global_Best_Position:[421.05628319565574, 420.9651874445524]
Run:1/10 Global_Best:-837.9657745447378 Global_Best_Position:[420.96873988794823, 420.9687777350301]
Run:2/10 Global_Best:-837.9657745448675 Global_Best_Position:[420.96874599796126, 420.96874663378384]
Run:3/10 Global_Best:-837.9657745448676 Global_Best_Position:[420.9687460501275, 420.9687464690474]
Run:4/10 Global_Best:-837.9657745448676 Global_Best_Position:[420.9687460501275, 420.9687464690474]
Run:5/10 Global_Best:-837.9657745448676 Global_Best_Position:[420.9687460501275, 420.9687464690474]
Run:6/10 Global_Best:-837.9657745448676 Global_Best_Position:[420.9687460501275, 420.9687464690474]
Run:7/10 Global_Best:-837.9657745448676 Global_Best_Position:[420.9687460501275, 420.9687464690474]
Run:8/10 Global_Best:-837.9657745448676 Global_Best_Position:[420.9687460501275, 420.9687464690474]
Run:9/10 Global_Best:-837.9657745448676 Global_Best_Position:[420.9687460501275, 420.9687464690474]
Run:10/10 Global_Best:-837.9657745448676 Global_Best_Position:[420.9687460501275, 420.9687464690474]
```

Figure 14: Schwefel's function Best Values in 10 Runs

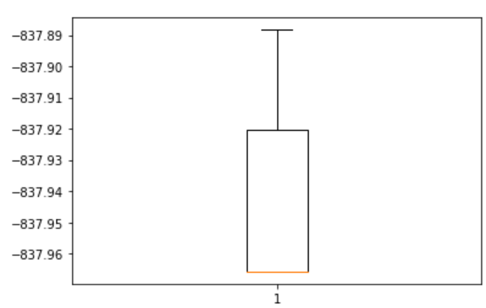


Figure 15: Schwefel's function Box Plot

2.2.6 6. Ackley function

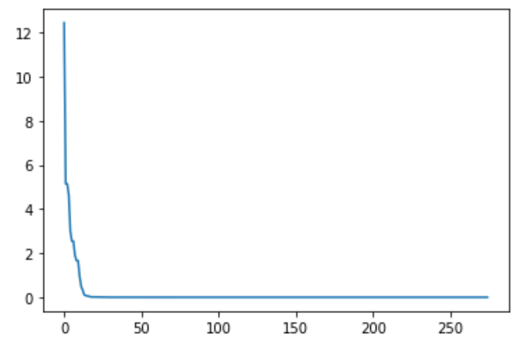
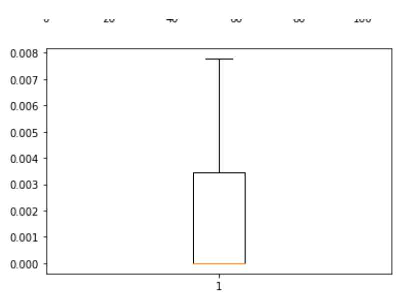


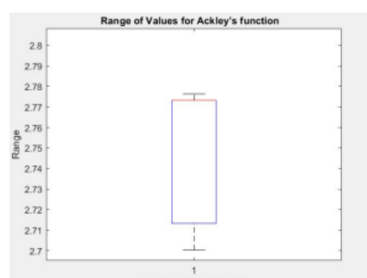
Figure 16: Ackley function Convergence

```
 Function: Ackley_Function
 Number of Particles: 100
 Number of Design Variables: 3
 Domain:(-32.768, 32.768)
Run:0/10 Global_Best:0.005028850842201837 Global_Best_Position:[-0.0006358373449282511, 0.0008265199155299326, 0.0018713457440547752]
Run:1/10 Global_Best:1.2771464565908275e-06 Global_Best_Position:[-2.8157192550075285e-07, 3.792619046206657e-07, 2.8758804341660674e-07]
Run:2/10 Global_Best:1.264091498143216e-09 Global_Best_Position:[-3.4007358795028254e-10, 1.7469089462714043e-10, -3.917201377895759e-10]
Run:3/10 Global_Best:6.366907001620348e-12 Global_Best_Position:[-2.3460505614271488e-12, 1.4360796198105595e-12, -1.7426786376633092e-13]
Run:4/10 Global_Best:3.9968028886505635e-15 Global_Best_Position:[-2.0024182417609437e-15, 1.1254300515815232e-15, 5.250052411014805e-16]
Run:5/10 Global_Best:4.440892098500626e-16 Global_Best_Position:[1.5152242958615736e-16, 3.502703982886276e-16, 1.3109897050833673e-16]
Run:6/10 Global_Best:4.440892098500626e-16 Global_Best_Position:[6.8851602240075e-17, -9.324363190235212e-17, 1.1041785300455516e-16]
Run:7/10 Global_Best:4.440892098500626e-16 Global_Best_Position:[6.8851602240075e-17, -9.324363190235212e-17, 1.1041785300455516e-16]
Run:8/10 Global_Best:4.440892098500626e-16 Global_Best_Position:[6.8851602240075e-17, -9.324363190235212e-17, 1.1041785300455516e-16]
Run:9/10 Global_Best:4.440892098500626e-16 Global_Best_Position:[6.8851602240075e-17, -9.324363190235212e-17, 1.1041785300455516e-16]
```

Figure 17: Ackley function Best Values in 10 Runs



(a) PSO Box Plot



(b) GA Box Plot

Figure 18: Ackley's Function

2.2.7 7. Michalewicz's function

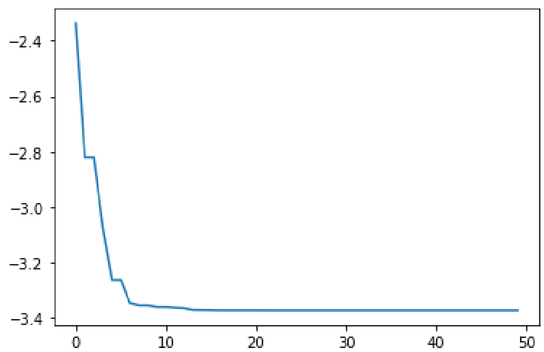


Figure 19: Michalewicz's function Convergence

```
 Function: Michalewicz's_Function
 Number of Particles: 100
 Number of Design Variables: 5
 Domain:(0, 3.141592653589793)
Run:0/10 Global_Best:-3.3725212348739646 Global_Best_Position:[2.286859719660791, 2.20293893083416, 1.5707375390849345, 1.284741144932034,
2.48197902609157]
Run:1/10 Global_Best:-3.3725257815399052 Global_Best_Position:[2.2868091693007075, 2.202905202774212, 1.5707964625174184, 1.2849913766873808,
2.4820332721634486]
Run:2/10 Global_Best:-3.372525781554729 Global_Best_Position:[2.2868080776884177, 2.202905515523092, 1.570796327826958, 1.2849915722699123,
2.482033475886799]
Run:3/10 Global_Best:-3.372525781554729 Global_Best_Position:[2.2868080776884177, 2.202905515523092, 1.570796327826958, 1.2849915722699123,
2.482033475886799]
Run:4/10 Global_Best:-3.372525781554729 Global_Best_Position:[2.2868080780807003, 2.2029055188961375, 1.5707963289020412, 1.2849915710286073,
2.4820334763478074]
Run:5/10 Global_Best:-3.372525781554729 Global_Best_Position:[2.2868080828342157, 2.202905515812781, 1.5707963266699643, 1.2849915698072152,
2.4820334769248102]
Run:6/10 Global_Best:-3.372525781554729 Global_Best_Position:[2.2868080828342157, 2.202905515812781, 1.5707963266699643, 1.2849915698072152,
2.4820334769248102]
Run:7/10 Global_Best:-3.372525781554729 Global_Best_Position:[2.2868080828342157, 2.202905515812781, 1.5707963266699643, 1.2849915698072152,
2.4820334769248102]
Run:8/10 Global_Best:-3.372525781554729 Global_Best_Position:[2.2868080828342157, 2.202905515812781, 1.5707963266699643, 1.2849915698072152,
2.4820334769248102]
Run:9/10 Global_Best:-3.372525781554729 Global_Best_Position:[2.2868080828342157, 2.202905515812781, 1.5707963266699643, 1.2849915698072152,
2.4820334769248102]
Run:10/10 Global_Best:-3.372525781554729 Global_Best_Position:[2.2868080828342157, 2.202905515812781, 1.5707963266699643, 1.2849915698072152,
2.4820334769248102]
```

Figure 20: Michalewicz's function Best Values in 10 Runs

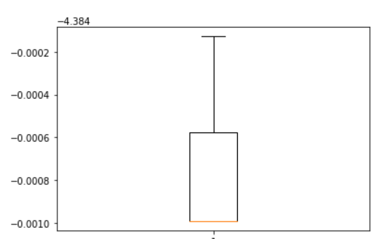


Figure 21: Michalewicz's function Box Plot

3 Bayesian Optimization

The convergence history of the Baysian optimization is as shown below. It is minimum objective function value vs Number of function evaluations.

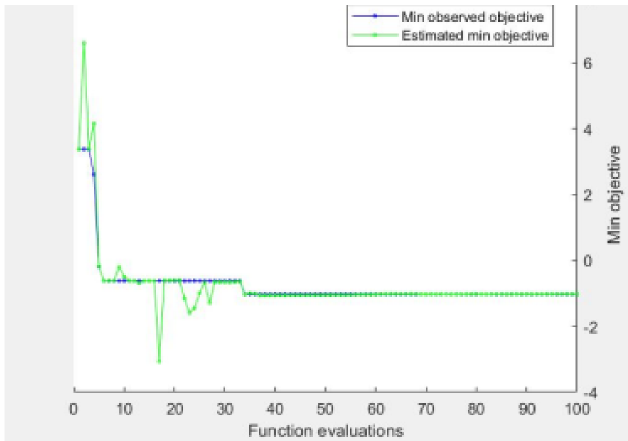


Figure 22: BO Convergence History

The optimum design variables and the optimum objective function values are as follows.

```
                 ObjectiveFcn: @(x)objective_function(x)
         VariableDescriptions: [1×2 optimizableVariable]
                      Options: [1×1 struct]
                 MinObjective: -1.0302
              XAtMinObjective: [1×2 table]
        MinEstimatedObjective: -1.0305
     XAtMinEstimatedObjective: [1×2 table]
      NumObjectiveEvaluations: 100
             TotalElapsedTime: 154.8645
                    NextPoint: [1×2 table]
                       XTrace: [100×2 table]
               ObjectiveTrace: [100×1 double]
             ConstraintsTrace: []
                UserDataTrace: {100×1 cell}
 ObjectiveEvaluationTimeTrace: [100×1 double]
           IterationTimeTrace: [100×1 double]
                   ErrorTrace: [100×1 double]
             FeasibilityTrace: [100×1 logical]
  FeasibilityProbabilityTrace: [100×1 double]
          IndexOfMinimumTrace: [100×1 double]
        ObjectiveMinimumTrace: [100×1 double]
EstimatedObjectiveMinimumTrace: [100×1 double]
```

Figure 23: BO Optimum Design Variables and Optimum Function Values

4 Ant Colony Optimization

Following are the user defined parameters for the ACO Algorithm.

- Number of ants or population size = 100
- No of Decision Variables = 3
- Pheromone exponent parameter(α) = 0.9
- Heuristic exponent parameter (β_1) = 1.9
- Pheromone reward factor/constant (Q) = 90
- Pheromone evaporation rate (ρ) = 0.5
- Initial pheromone deposit between all cities = 1
- Maximum allowed number of iterations = 100
- Runs = 10

The Initial Scatter Plot of the cities is mapped here. There are total 52 Cities and they are numbered according to the file given in the homework. It had (x,y) Co-ordinates according to which it is plotted below.

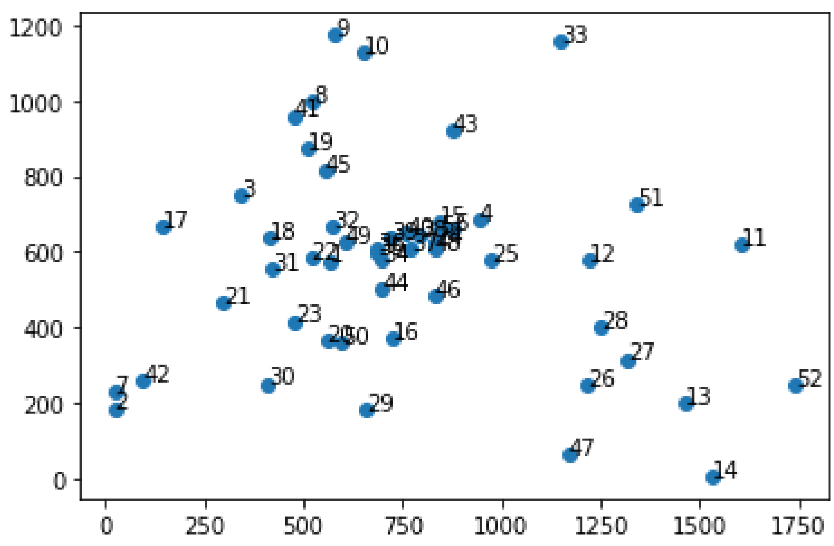
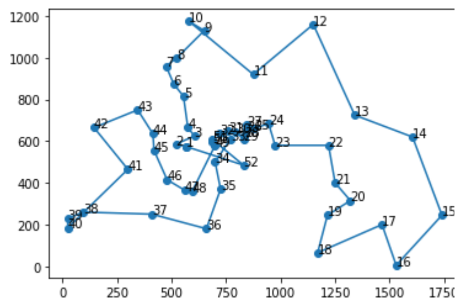
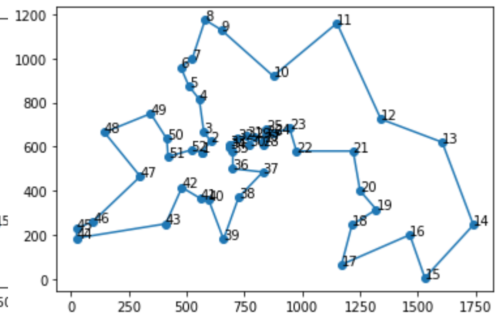


Figure 24: ACO Scattered Cities Plot

Lets look at the comparison between best path and the optimum path of Berlin-52 problem. The best path is achieved after coding the algorithm and running it several time.



(a) Best Path After Coding



(b) Optimum Path After Coding

Figure 25: Comparison Between Best Path and Optimum Path

After running the algorithm for 10 runs, following values were achieved. Where the best trajectory length of 8211 was obtained.

```
Ant Colony Optimization Berlin-52 Problem

Run 0/10 Length of best path: 8360.86603938313
Run 0/10 Length of best path: 8794.434452114805
Run 1/10 Length of best path: 9154.795899596313
Run 2/10 Length of best path: 8211.86603938313
Run 3/10 Length of best path: 8494.108099335235
Run 4/10 Length of best path: 8516.800280898815
Run 5/10 Length of best path: 8325.416305603427
Run 6/10 Length of best path: 8909.800280898815
Run 7/10 Length of best path: 9097.092560861063
Run 8/10 Length of best path: 8827.815696155272
```

Figure 26: ACO 10 Runs Result

The trajectory(Path) of least score 8211 is as follows.

```
best path : [ 1. 22. 49. 32. 45. 19. 41.  8. 10.  9. 43. 33. 51. 11. 52. 14. 13. 47.
 26. 27. 28. 12. 25.  4.  6.  5. 15. 24. 48. 38. 40. 39. 37. 44. 16. 29.
 30. 42.  7.  2. 21. 17.  3. 18. 31. 23. 20. 50. 34. 35. 36. 46.  1.]
cost of the best path 8211.86603938313
```

Figure 27: Best Trajectory

The Convergence plot of the ACO best score over the 100 iterations is shown below, the ants keep on exploring better path over the period of 60 iterations and after that the path is converged and pretty much achieves same results.

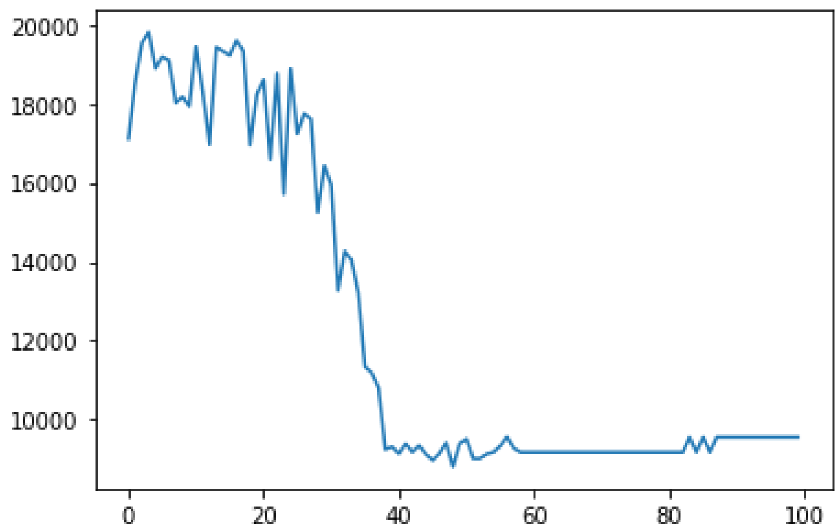


Figure 28: Trajectory Cost Convergence of ACO

5 Conclusion

In this assignment three important optimization algorithms are implemented. The PSO Implementations is done using a class, which I further used in the other projects also. The programming of these assignments was helpful as my conceptual doubts were cleared during coding these problems from scratch.
Observations on PSO:
PSO is a very strong optimization algorithm. The computation time for the pso is lot more less compared to the genetic algorithm. The results achieved using PSO were also much better than the binary genetic algorithm and more importantly better results were achieved in less computation power. The reults of PSO are more accurate than the binary genetic algorithms.
Observations on ACO:
In this assignment we solved Berlin-52 problem, the aco algorithm can be used to solve TSP. The ACO is also a very robust and strong algorithm and determined to give best results no matter how complicated the problem is.