21cm and Detailed Balance

Abstract

Notes on the 21cm line and the principle of detailed balance. In particular, we are interested to know if we can write down the usual 21cm spin temperature equation for neutrino interactions, and the conditions that are required to do so.

21cm Physics

The 21cm line is a hyperfine transition between the two states of the $1s$ of neutral hydrogen. The energy difference between these states is $\Delta E_{21} = 5.9 \times 10^{-6}$ eV. The lifetime of the excited state is long, $\tau \sim 10^7$ years.

Three processes determine the spin temperature of the 21cm line:

1. Absorption/emission of CMB photons
2. Collisions with H atoms, free electrons, and protons
3. ~~Absorption/emission of Lyman-α photons~~ (not relevant for redshifts relevant for neutrinos)

The rate of these processes is fast compared to the lifetime of the 21cm line, and so the populations of the states are in equilibrium. In this limit, the spin temperature is given by the expression (which we will derive):

$$T_S^{-1} = \frac{T_\gamma^{-1} + x_c T_K^{-1} + x_\alpha T_K^{-1}}{1 + x_c + x_\alpha}$$

where we will drop the α term from now on. T_γ is the CMB temperature, T_K is the kinetic temperature of the gas, and x_c is the collisional coupling coefficient. The spin temperature becomes strongly coupled to the kinetic temperature when $x_c + x_\alpha \gg 1$ and relaxes to T_γ when $x_c + x_\alpha \ll 1$. The collisional coupling coefficient is given in the Pritchard and Loeb review arxiv:1109.6012 eqn. 10 and references therein. The rest of the review talks in detail about Ly-α coupling, which is not relevant for neutrinos.

From the P&L review § 3.1: An important feature of T_S is that the dependence saturates at some point. For example, when the Ly-α flux is strong enough, any further increase in the Ly-α flux does not increase the spin temperature. This leads to conceptually distinct regimes for the spin temperature: the Ly-α coupling dominated regime, the collisional coupling dominated regime, and the CMB coupling dominated regime.

I assume this also holds for the gas collisional couplings from the following statement, but I have not explicitly checked this

From the Furlanetto review arxiv:1909.13740 In the presence of CMB alone, the spin states reach equilibrium on a time-scale of $\sim 10^5$ years. In the presence of collisions, the spin states reach equilibrium on a time-scale of $\sim 10^3$ years. In the presence of Ly-α photons, the spin states reach equilibrium on a time-scale of $\sim 10^2$ years. [^life] [^life]: Verify this!

Thus, we conclude that all relevant processes adjust on time-scales much shorter than the lifetime of the 21cm line, and so the populations of the states are in equilibrium.

Detailed Balance

Since the populations of each state, n_1 and n_0, are in equilibrium, we can write down the rate equations for the transitions between the states. The rate equation for n_1 (similarly for n_1)is:

$$\frac{dn_0}{dt} = (A_{10} + B_{10}I_{CMB} + C_{10})n_1 - (B_{01}I_{CMB} + C_{01})n_0$$

where A_{10} is the spontaneous emission rate, B_{10} is the stimulated emission rate, B_{01} is the stimulated absorption rate, C_{10} is the collisional excitation rate, and C_{01} is the collisional de-excitation rate. I_{CMB} is the intensity of the CMB at the frequency of the 21cm line.

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