Technical Report: Merton Structural Credit Model
with Exponential Smoothing Improvement

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December 25, 2025

1 Introduction

This report presents an implementation and improvement of the Merton (1974) structural credit model. The baseline model treats equity as a call option on firm assets and calibrates unobservable asset values and volatilities from market-observable equity prices (stock prices) and volatilities. After implementing and diagnosing the baseline model on API-fetched 2020 data for five U.S. firms (Apple, JPMorgan Chase, Tesla, ExxonMobil, and Ford), we identified default probability (PD) instability as a primary weakness: all firms exhibit coefficient of variation (CV) > 1.0, making the model unsuitable for daily credit risk monitoring.

To approach this issue we implement exponential smoothing as a minimal improvement that reduces PD volatility by 15–20% and daily changes by 25–50% while preserving the model's ability to properly capture credit deterioration during crisis periods.

2 Model Formulation

The Merton model views a firm's equity as a European call option on its assets, with the strike price being the face value of its debt. The key equations and assumptions of the baseline Merton model are outlined below.

2.1 Baseline Merton Model

2.1.1 Key Assumptions

1. Firm asset value V_t follows geometric Brownian motion:
$$dV_t = \mu V_t \, dt + \sigma_V V_t \, dW_t \tag{1}$$
2. Firm has zero-coupon debt with face value D maturing at time T
3. Default occurs only at maturity T if $V_T < D$
4. Equity is a European call option on firm assets: $E = \max(V_T - D, 0)$
5. Markets are frictionless (no transaction costs), trading is continuous, no arbitrage(no riskless profit opportunities)

2.1.2 Key Equations/ Mathematical Formulation

Equity Value Under the Merton model, equity value is given by the Black-Scholes call option formula:

$$E = V \cdot \Phi(d_1) - D \cdot e^{-r(T-t)} \cdot \Phi(d_2) \tag{2}$$

where

$$d_1 = \frac{\ln(V/D) + (r + \sigma_V^2/2)(T-t)}{\sigma_V \sqrt{T-t}} \tag{3}$$

$$d_2 = d_1 - \sigma_V \sqrt{T-t} \tag{4}$$

and $\Phi(\cdot)$ denotes the standard normal cumulative distribution function.

Equity Volatility The relationship between equity volatility σ_E and asset volatility σ_V is:

$$\sigma_E \cdot E = \Phi(d_1) \cdot \sigma_V \cdot V \tag{5}$$

where $\Phi(d_1)$ is the option delta, representing the sensitivity of equity value to changes in asset value.

Default Probability The risk-neutral probability of default at maturity is:

$$\mathrm{PD} = \Phi(-d_2) = \Phi\left(-\frac{\ln(V/D) + (r - \sigma_V^2/2)T}{\sigma_V \sqrt{T}}\right) \tag{6}$$

Distance-to-Default The distance-to-default metric is:

$$\mathrm{DD} = \frac{\ln(V/D) + (r - \sigma_V^2/2)T}{\sigma_V \sqrt{T}} \tag{7}$$

2.1.3 Calibration Approach

The Merton model calibration involves solving the following system of equations for the unobservable asset value V and asset volatility σ_V, given the observable equity value E, equity volatility σ_E, debt face value D, time to maturity $T - t$, and risk-free rate r:

$$E = \mathrm{BlackScholes}(V, D, T-t, r, \sigma_V) \tag{8}$$

$$\sigma_E \cdot E = \Phi(d_1) \cdot \sigma_V \cdot V \tag{9}$$

2.2 Improved Model

2.2.1 Modification

We apply post-processing exponential smoothing to default probabilities. For each firm's time series of raw PDs $\{\mathrm{PD}_t^{\mathrm{raw}}\}$ we applied the following smoothing formula:

$$\mathrm{PD}_t^{\mathrm{smooth}} = \alpha \cdot \mathrm{PD}_t^{\mathrm{raw}} + (1-\alpha) \cdot \mathrm{PD}_{t-1}^{\mathrm{smooth}} \tag{10}$$

where $\alpha = 0.1$ is the smoothing parameter.