# Technical Report: Merton Structural Credit Model
## with Exponential Smoothing Improvement

Ruhani Rekhi

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# 1 Introduction

This report presents an implementation and improvement of the Merton (1974) structural credit model for estimating default probabilities. The baseline model treats equity as a call option on firm assets and calibrates unobservable asset values and volatilities from market-observable equity prices (stock prices) and volatilities. After implementing and diagnosing the baseline model on 2020 data for five U.S. firms (Apple, JPMorgan Chase, Tesla, Exxon-Mobil, and Ford), we identified **default probability (PD) instability** as the primary weakness: all firms exhibit coefficient of variation (CV) > 1.0, making the model unsuitable for daily credit risk monitoring.

To approach this issue we implement **exponential smoothing** ($\alpha = 0.1$) as a minimal improvement that reduces PD volatility by 15–20% and daily changes by 25–50% while preserving the model's ability to properly capture credit deterioration during crisis periods.

# 2 Model Formulation

The Merton model views a firm's equity as a European call option on its assets, with the strike price being the face value of its debt. The key equations and assumptions of the baseline Merton model are outlined below.

## 2.1 Baseline Merton Model

### 2.1.1 Key Assumptions

1. Firm asset value $V_t$ follows geometric Brownian motion:

$$dV_t = \mu V_t \, dt + \sigma_V V_t \, dW_t \tag{1}$$

2. Firm has zero-coupon debt with face value $D$ maturing at time $T$

3. Default occurs only at maturity $T$ if $V_T < D$

4. Equity is a European call option on firm assets: $E = \max(V_T - D, 0)$

5. Markets are frictionless (no transaction costs), trading is continuous, no arbitrage(no riskless profit opportunities)

### 2.1.2 Key Equations/ Mathematical Formulation

**Equity Value**  Under the Merton model, equity value is given by the Black-Scholes call option formula:

$$E = V \cdot \Phi(d_1) - D \cdot e^{-r(T-t)} \cdot \Phi(d_2) \tag{2}$$

where

$$d_1 = \frac{\ln(V/D) + (r + \sigma_V^2/2)(T-t)}{\sigma_V \sqrt{T-t}} \tag{3}$$

$$d_2 = d_1 - \sigma_V \sqrt{T-t} \tag{4}$$

and $\Phi(\cdot)$ denotes the standard normal cumulative distribution function.

**Equity Volatility**  The relationship between equity volatility $\sigma_E$ and asset volatility $\sigma_V$ is:

$$\sigma_E \cdot E = \Phi(d_1) \cdot \sigma_V \cdot V \tag{5}$$

where $\Phi(d_1)$ is the option delta, representing the sensitivity of equity value to changes in asset value.

**Default Probability**  The risk-neutral probability of default at maturity is:

$$PD = \Phi(-d_2) = \Phi\left(-\frac{\ln(V/D) + (r - \sigma_V^2/2)T}{\sigma_V \sqrt{T}}\right) \tag{6}$$

**Distance-to-Default**  The distance-to-default metric is:

$$DD = \frac{\ln(V/D) + (r - \sigma_V^2/2)T}{\sigma_V \sqrt{T}} \tag{7}$$

### 2.1.3 Calibration Approach

The Merton model calibration involves solving the following system of equations for the unobservable asset value $V$ and asset volatility $\sigma_V$, given the observable equity value $E$, equity volatility $\sigma_E$, debt face value $D$, time to maturity $T-t$, and risk-free rate $r$:

$$E = \text{BlackScholes}(V, D, T-t, r, \sigma_V) \tag{8}$$

$$\sigma_E \cdot E = \Phi(d_1) \cdot \sigma_V \cdot V \tag{9}$$

## 2.2 Improved Model

### 2.2.1 Modification

We apply post-processing exponential smoothing to default probabilities. For each firm's time series of raw PDs $\{\text{PD}_t^{\text{raw}}\}$ we applied the following smoothing formula:

$$\text{PD}_t^{\text{smooth}} = \alpha \cdot \text{PD}_t^{\text{raw}} + (1 - \alpha) \cdot \text{PD}_{t-1}^{\text{smooth}} \tag{10}$$

where $\alpha = 0.1$ is the smoothing parameter.