Bulanık Mantık ve Bulanık Sistemler

Hazırlayan ve Sunan:
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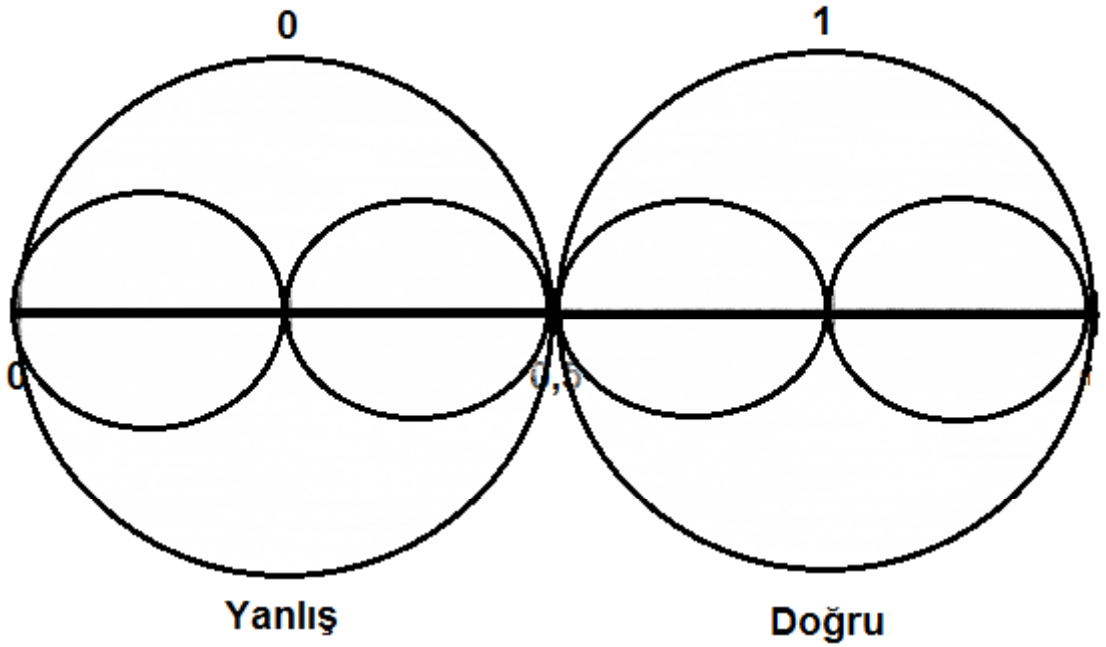
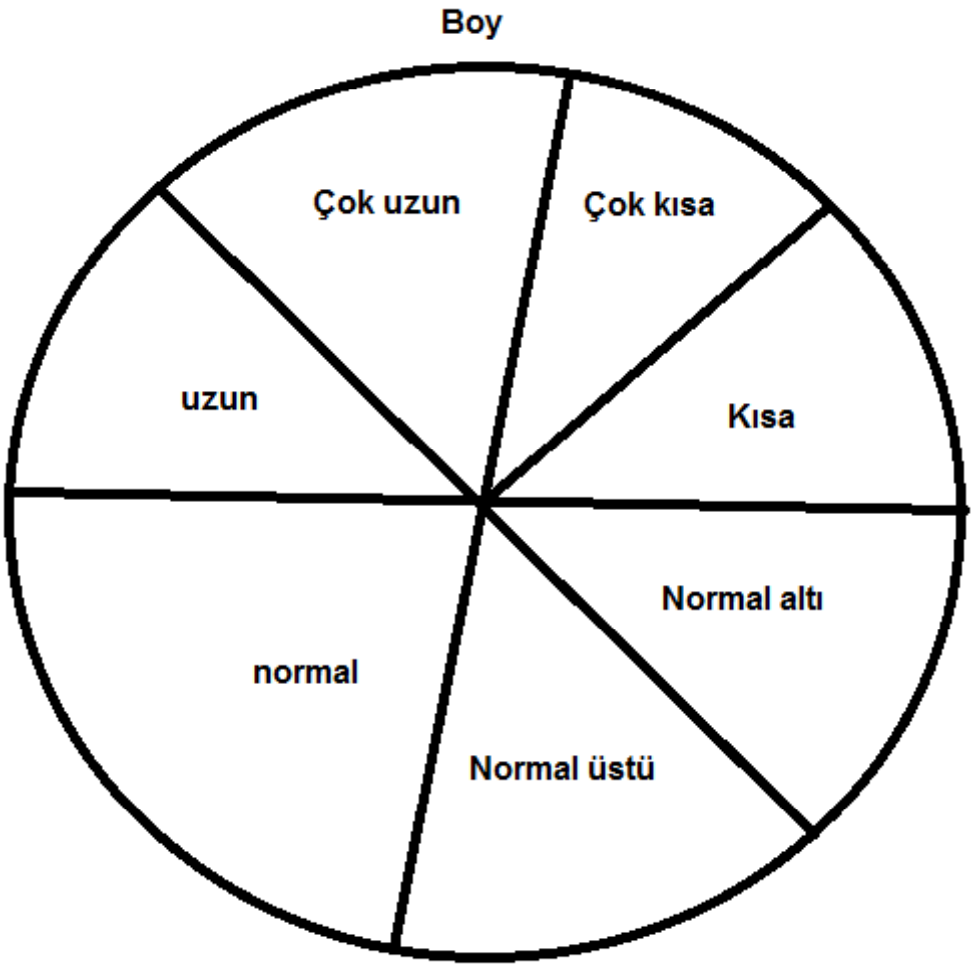
Bulanık Küme



Bulanık Küme



Bulanık Küme ve Üyelik Değeri

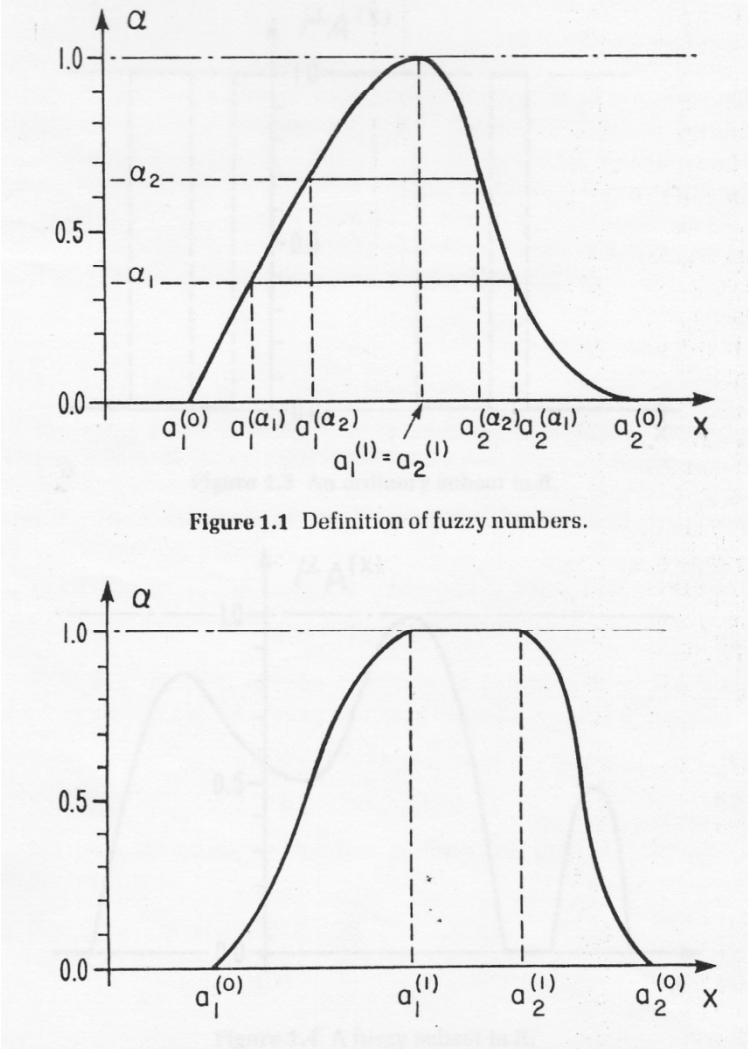
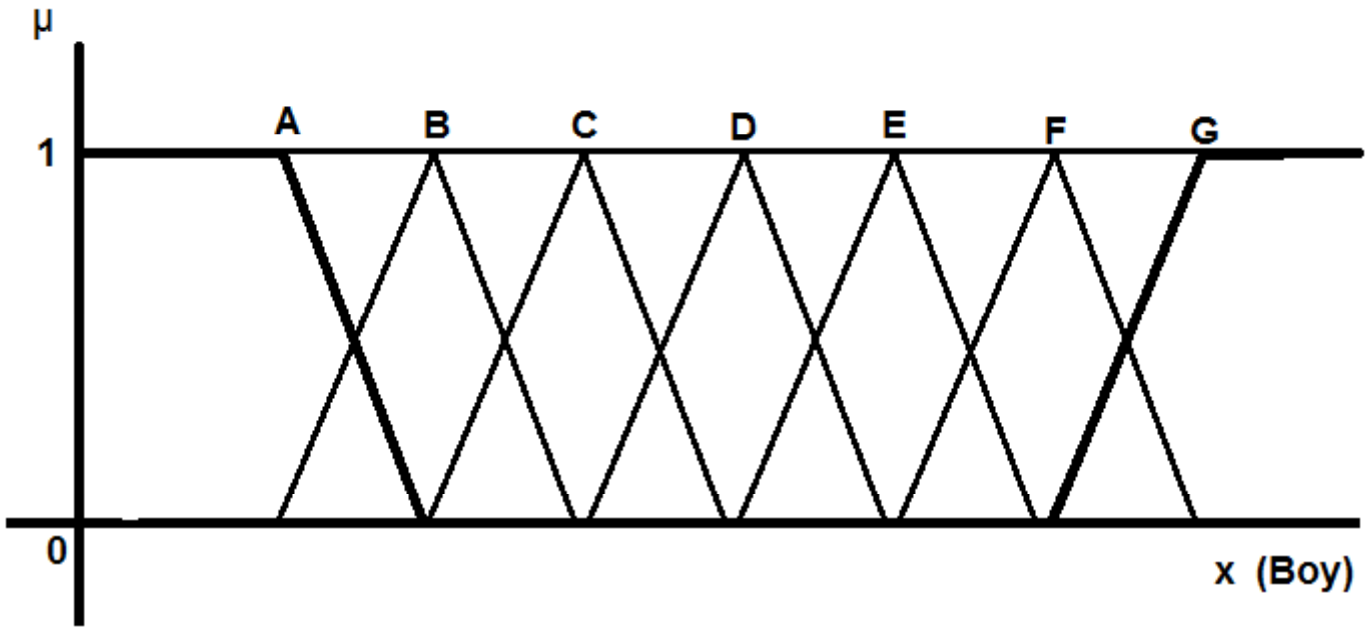
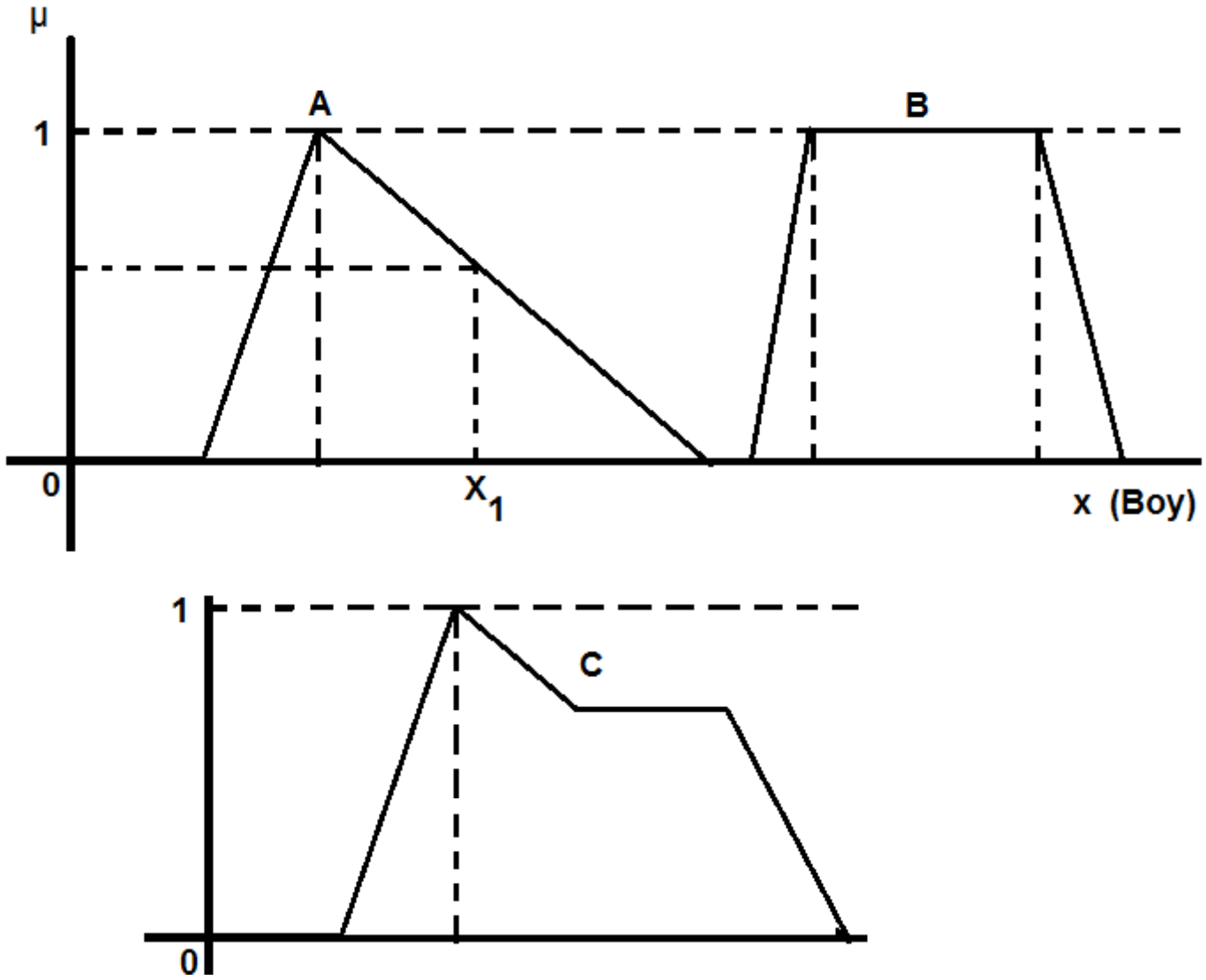


Figure 1.1 Definition of fuzzy numbers.

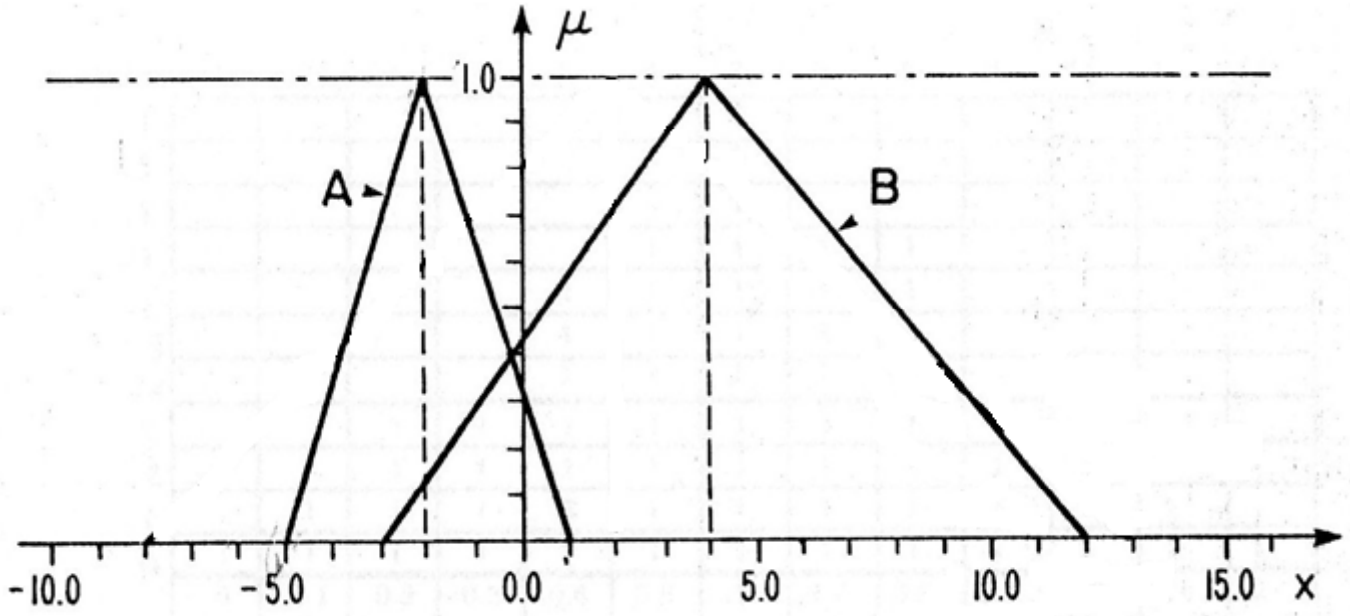

μ
1
0
A
B
C
D
E
F
G
x (Boy)


μ
1
A
B
0
X1
x (Boy)
1
C
0

Bulanık Kümelerde Toplama İşlemi

$$A_\alpha = \{x \mid \mu_{\mathbf{A}}(x) \geq \alpha\},$$
$$B_\alpha = \{x \mid \mu_{\mathbf{B}}(x) \geq \alpha\}.$$

$$\begin{aligned} A_\alpha (+) B_\alpha &= [a_1(\alpha), a_2(\alpha)] (+) [b_1(\alpha), b_2(\alpha)] \\ &= [a_1(\alpha) + b_1(\alpha), a_2(\alpha) + b_2(\alpha)]. \end{aligned}$$


μ
1.0
A
B
-10.0
-5.0
0.0
5.0
10.0
15.0
X

Bulanık Kümelerde Toplama İşlemi

$\forall\ x \in R$:

$$\mu_A(x) = \begin{cases} 0, & x \le -5, \\ x/3 + 5/3, & -5 \le x \le -2, \\ -x/3 + 1/3, & -2 \le x \le 1, \\ 0, & x \ge 1, \end{cases}$$

$$\mu_B(x) = \begin{cases} 0, & x \le -3, \\ x/7 + 3/7; & -3 \le x \le 4, \\ -x/8 + 12/8, & 4 \le x \le 12, \\ 0, & x \ge 12. \end{cases}$$

$$\alpha = a_1(\alpha)/3 + 5/3$$

$$\alpha = b_1(\alpha)/7 + 3/7$$

$$\alpha = -a_2(\alpha)/3 + 1/3.$$

$$\alpha = -b_2(\alpha)/8 + 12/8.$$

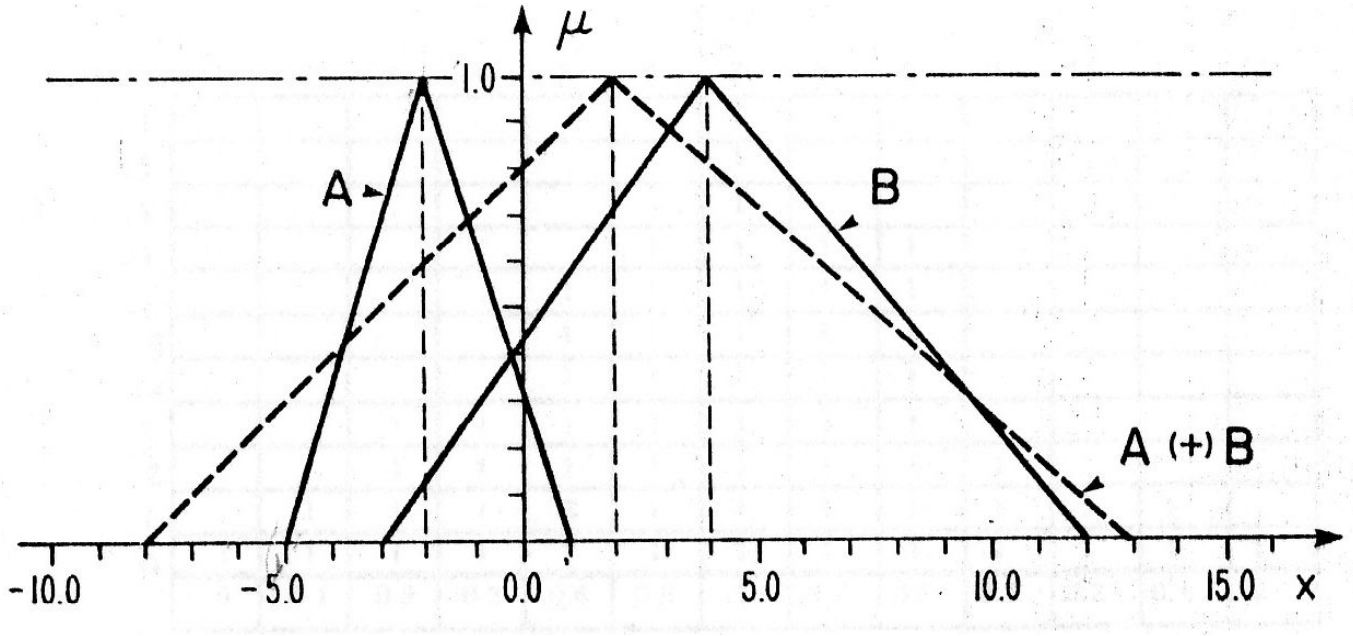
$$A_\alpha = [a_1(\alpha), a_2(\alpha)] = [3\alpha - 5, -3\alpha + 1].$$

$$B_\alpha = [b_1(\alpha), b_2(\alpha)] = [7\alpha - 3, -8\alpha + 12],$$

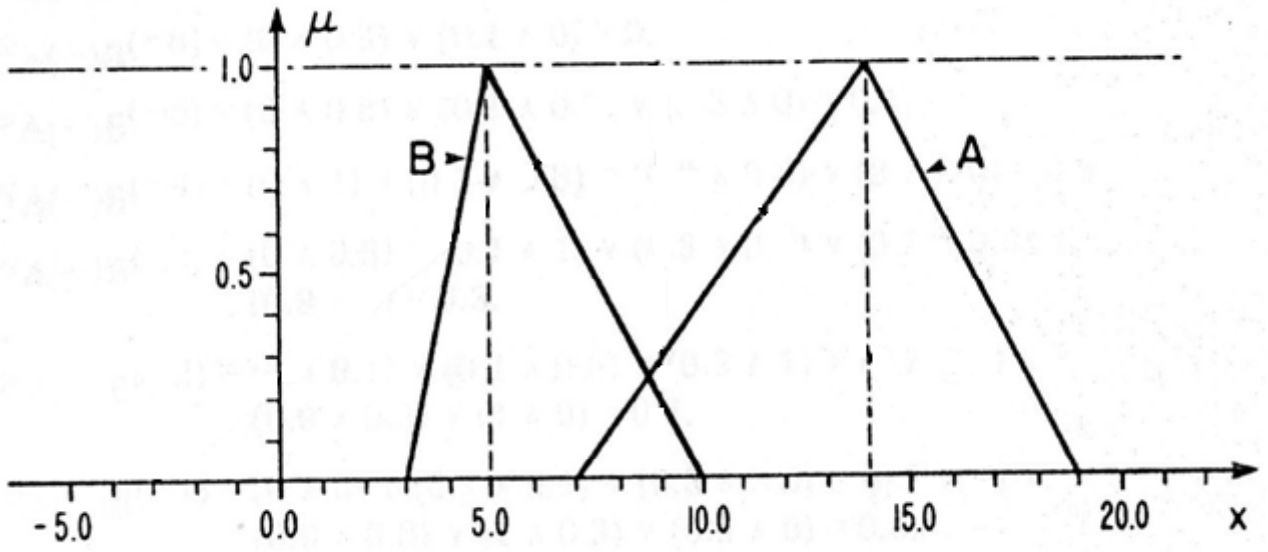
$$A_\alpha\ (+)\ B_\alpha = [a_1(\alpha) + b_1(\alpha), a_2(\alpha) + b_2(\alpha)] = [3\alpha - 5, -3\alpha + 1]\ (+)\ [7\alpha - 3, -8\alpha + 12] = [10\alpha - 8, -11\alpha + 13].$$

Bulanık Kümelerde Toplama İşlemi

$$\mu_{\mathbf{A}\,(+)\,\mathbf{B}}(x) = 0, \qquad x \le -8,$$
$$= x/10 + 8/10, \qquad -8 \le x \le 2,$$
$$= -x/11 + 13/11, \qquad 2 \le x \le 13,$$
$$= 0, \qquad x \ge 13.$$



Bulanık Kümelerde Çıkarma İşlemi



Bulanık Kümelerde Çıkarma İşlemi

$$\mathbf{A}\,(-)\,\mathbf{B} = [a_1(\alpha), a_2(\alpha)]\,(-)\,[b_1(\alpha), b_2(\alpha)]$$
$$= [a_1(\alpha) - b_2(\alpha), a_2(\alpha) - b_1(\alpha)],$$

$$\mu_{\mathbf{A}}(x) = 0, \qquad x \le 7,$$
$$= x/7 - 1, \qquad 7 \le x \le 14,$$
$$= -x/5 + 19/5, \qquad 14 \le x \le 19,$$
$$= 0, \qquad x \ge 19.$$

$$\mu_{\mathbf{B}}(x) = 0, \qquad x \le 3,$$
$$= x/2 - 3/2, \qquad 3 \le x \le 5,$$
$$= -x/5 + 10/5, \qquad 5 \le x \le 10,$$
$$= 0, \qquad x \ge 10.$$

$$\alpha = a_1(\alpha)/7 - 1,$$
$$\alpha = -a_2(\alpha)/5 + 19/5.$$

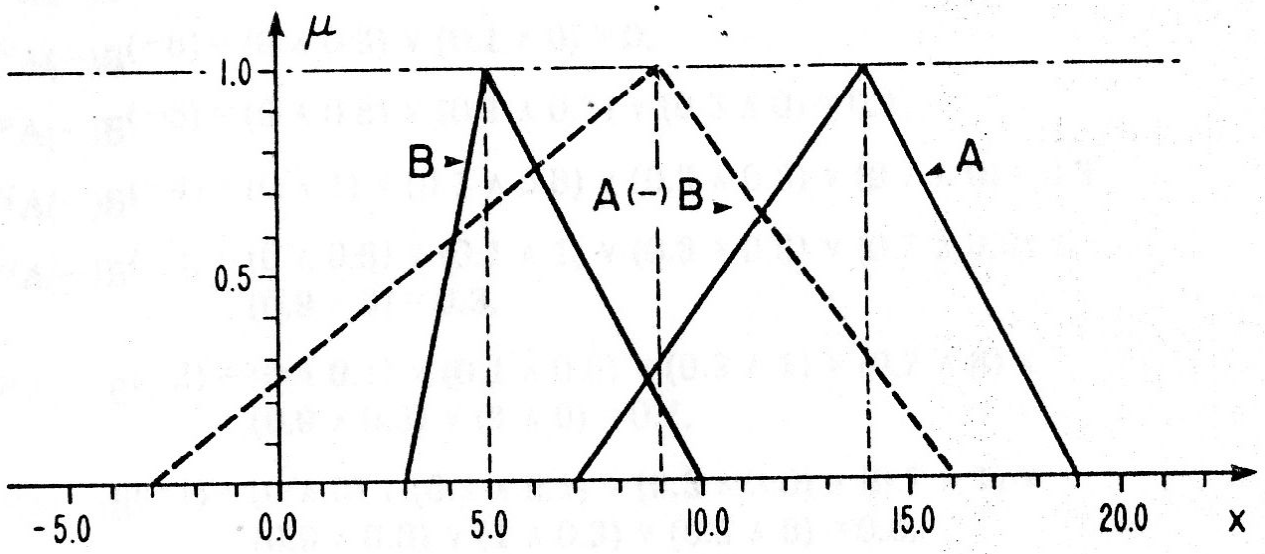
$$\alpha = b_1(\alpha)/2 - 3/2,$$
$$\alpha = -b_2(\alpha)/5 + 10/5,$$

$$A_\alpha = [a_1(\alpha), a_2(\alpha)] = [7\alpha + 7, -5\alpha + 19] \qquad B_\alpha = [b_1(\alpha), b_2(\alpha)] = [2\alpha + 3, -5\alpha + 10].$$

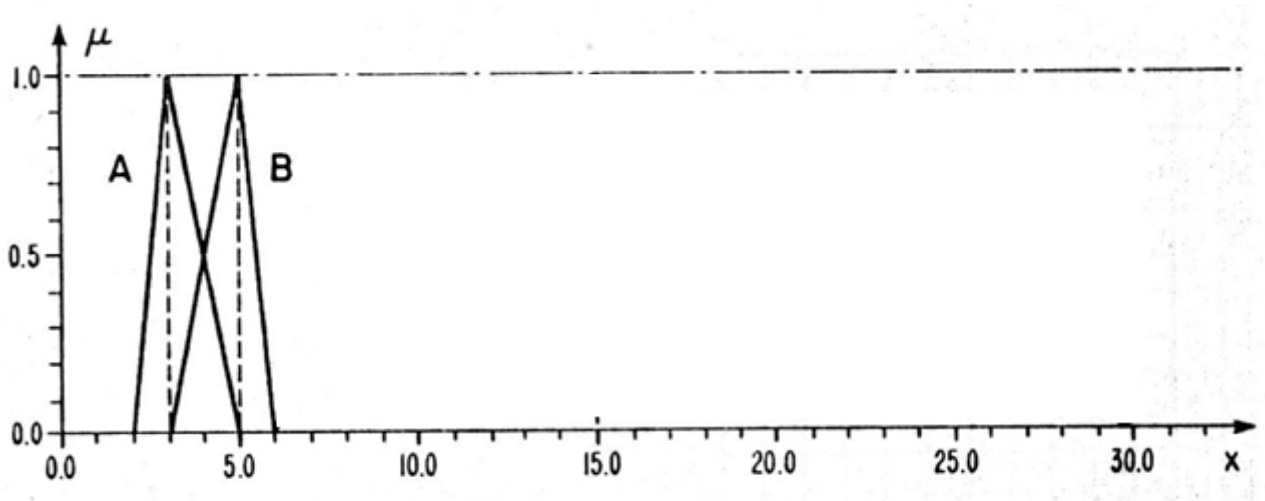
$$A_\alpha\,(-)\,B_\alpha = [7\alpha + 7, -5\alpha + 19]\,(-)\,[2\alpha + 3, -5\alpha + 10],$$
$$= [7\alpha + 7 - (-5\alpha + 10), -5\alpha + 19 - (2\alpha + 3)],$$
$$= [12\alpha - 3, -7\alpha + 16].$$

Bulanık Kümelerde Çıkarma İşlemi

$$
\begin{aligned}
\mu_{\mathbf{A}(-)\mathbf{B}}(x) &= 0, && x \le -3,\\
&= x/12 + 3/12, && -3 \le x \le 9,\\
&= -x/7 + 16/7, && 9 \le x \le 16,\\
&= 0, && x \ge 16.
\end{aligned}
$$



Bulanık Kümelerde Çarpma İşlemi



Bulanık Kümelerde Çarpma İşlemi

$$A_\alpha (\cdot) B_\alpha = [a_1(\alpha), a_2(\alpha)] (\cdot) [b_1(\alpha), b_2(\alpha)]$$
$$= [a_1(\alpha) \cdot b_1(\alpha), a_2(\alpha) \cdot b_2(\alpha)].$$

$$\mu_A(x) = 0, \quad x \le 2,$$
$$= x - 2, \quad 2 \le x \le 3,$$
$$= -x/2 + 5/2, \quad 3 \le x \le 5,$$
$$= 0, \quad x \ge 5.$$

$$\alpha = a_1(\alpha) - 2,$$

$$\alpha = -a_2(\alpha)/2 + 5/2.$$

$$A_\alpha = [\alpha + 2, -2\alpha + 5].$$

$$\mu_B(x) = 0, \quad x \le 3,$$
$$= x/2 - 3/2, \quad 3 \le x \le 5,$$
$$= -x + 6, \quad 5 \le x \le 6,$$
$$= 0, \quad x \ge 6.$$

$$\alpha = b_1(\alpha)/2 - 3/2$$

$$\alpha = -b_2(\alpha) + 6.$$

$$B_\alpha = [2\alpha + 3, -\alpha + 6].$$

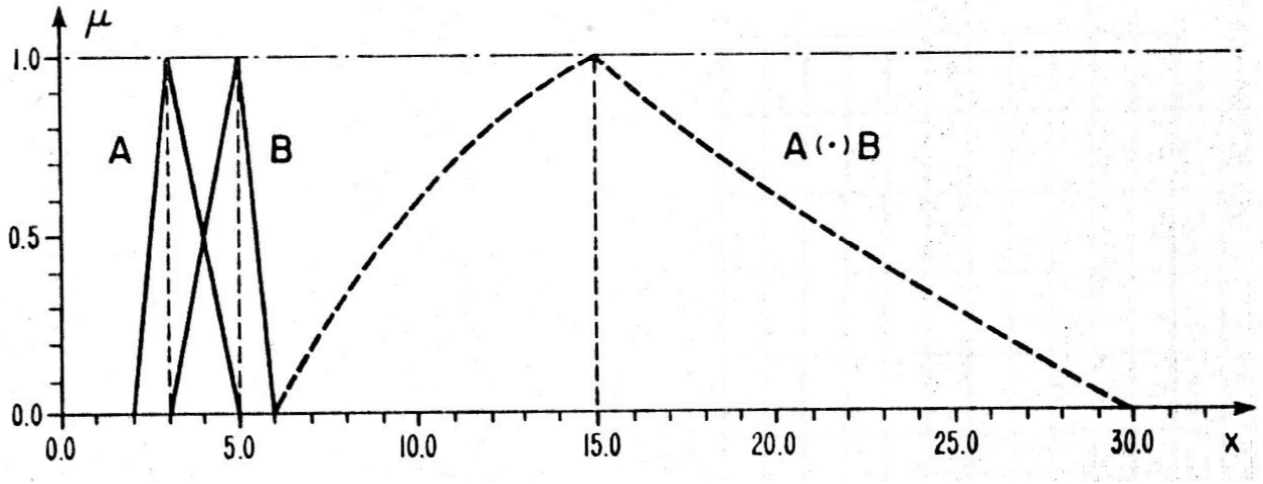
$$A_\alpha (\cdot) B_\alpha = [\alpha + 2)(2\alpha + 3), (-2\alpha + 5)(-\alpha + 6)]$$
$$= [2\alpha^2 + 7\alpha + 6, 2\alpha^2 - 17\alpha + 30].$$

$$2\alpha^2 + 7\alpha + 6 - x = 0 \qquad \alpha = (-7 + \sqrt{1 + 8x})/4,$$

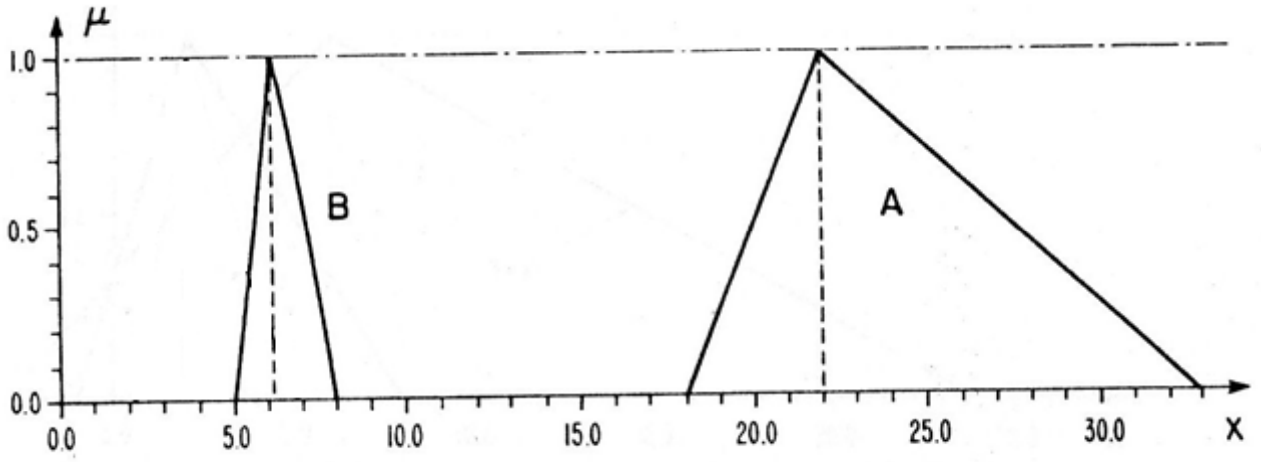
$$2\alpha^2 - 17\alpha + 30 - x = 0. \qquad \alpha = (17 - \sqrt{49 + 8x})/4.$$

Bulanık Kümelerde Çarpma İşlemi

$$\mu_{A(\cdot)B}(x) = 0 \qquad x \le 6,$$
$$= (-7 + \sqrt{1 + 8x})/4, \qquad 6 \le x \le 15,$$
$$= (17 - \sqrt{49 + 8x})/4, \qquad 15 \le x \le 30,$$
$$= 0, \qquad x \ge 30.$$



Bulanık Kümelerde Bölme İşlemi



Bulanık Kümelerde Bölme İşlemi

$$\mathbf{A}\,(:)\,\mathbf{B} = [a_1(\alpha), a_2(\alpha)]\,(:)\,[b_1(\alpha), b_2(\alpha)]$$
$$= [a_1(\alpha)/b_2(\alpha),\ a_2(\alpha)/b_1(\alpha)], \qquad b_2(\alpha) > 0,$$

$$\mu_{\mathbf{A}}(x) = \begin{cases} 0, & x \le 18, \\ x/4 - 18/4, & 18 \le x \le 22, \\ -x/11 + 3, & 22 \le x \le 33, \\ 0, & x \ge 33. \end{cases}$$

$$\mu_{\mathbf{B}}(x) = \begin{cases} 0, & x \le 5, \\ x - 5, & 5 \le x \le 6, \\ -x/2 + 4, & 6 \le x \le 8, \\ 0, & x \ge 8. \end{cases}$$

$$\alpha = a_1(\alpha)/4 - 18/4$$

$$\alpha = -a_2(\alpha)/11 + 3,$$

$$A_\alpha = [4\alpha + 18,\ -11\alpha + 33].$$

$$\alpha = b_1(\alpha) - 5$$

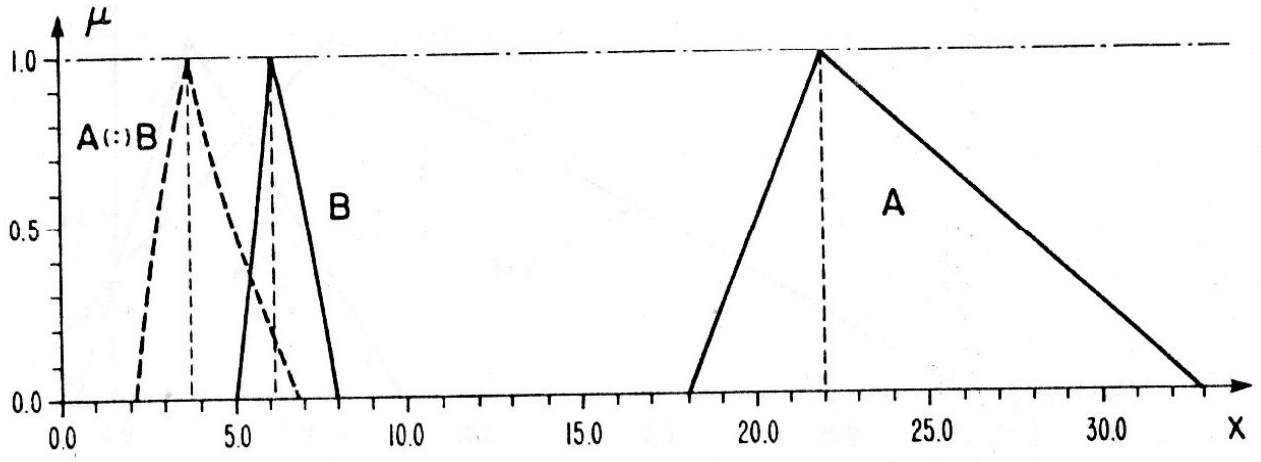
$$\alpha = -b_2(\alpha)/2 + 4,$$

$$B_\alpha = [\alpha + 5,\ -2\alpha + 8].$$

$$A_\alpha\,(:)\,B_\alpha = [4\alpha + 18,\ -11\alpha + 33]\,(:)\,[\alpha + 5,\ -2\alpha + 8]$$
$$= \left(\frac{4\alpha + 18}{-2\alpha + 8},\ \frac{-11\alpha + 33}{\alpha + 5}\right)$$

Bulanık Kümelerde Bölme İşlemi

$$\mu_{\mathbf{A}(:)\mathbf{B}}(x) = 0, \qquad x \le 9/4,$$
$$= \frac{8x - 18}{2x + 4} \qquad 9/4 \le x \le 11/3,$$
$$= \frac{-5x + 33}{x + 11}, \qquad 11/3 \le x \le 33/5,$$
$$= 0, \qquad x \ge 33/5.$$



Ardaşık İşlemler

Remark

Note that (A (:) B) (·) B ≠ A. Indeed,

$$A_\alpha\,(:)\,B_\alpha = \left[\frac{a_1(\alpha)}{b_2(\alpha)}, \frac{a_2(\alpha)}{b_1(\alpha)},\right]$$

$$(A_\alpha\,(:)\,B_\alpha)\,(\cdot)\,B_\alpha = \left[\frac{a_1(\alpha)}{b_2(\alpha)}, \frac{a_2(\alpha)}{b_1(\alpha)}\right](\cdot)\,[b_1(\alpha), b_2(\alpha)]$$

$$= \left[\frac{a_1(\alpha)b_1(\alpha)}{b_2(\alpha)}, \frac{a_2(\alpha)b_2(\alpha)}{b_1(\alpha)}\right]$$

$$\neq [a_1(\alpha), a_2(\alpha)]; \quad \text{i.e., } \neq A_\alpha.$$

Ardaşık İşlemler

This is also true for addition and subtraction.

$$(\mathbf{A}\ (-)\ \mathbf{B})\ (+)\ \mathbf{B} \neq \mathbf{A}.$$

That is,

$$A_\alpha\ (-)\ B_\alpha = [a_1(\alpha) - b_2(\alpha),\ a_2(\alpha) - b_1(\alpha)],$$

$$\begin{aligned}(A_\alpha\ (-)\ B_\alpha)\ (+)\ B_\alpha &= [a_1(\alpha) - b_2(\alpha),\ a_2(\alpha) - b_1(\alpha)]\ (+)\ [b_1(\alpha),\ b_2(\alpha)] \\ &= [a_1(\alpha) - b_2(\alpha) + b_1(\alpha),\ a_2(\alpha) - b_1(\alpha) + b_2(\alpha)] \\ &\neq [a_1(\alpha),\ a_2(\alpha)].\end{aligned}$$

Minimum ve Maksimum Kümeler

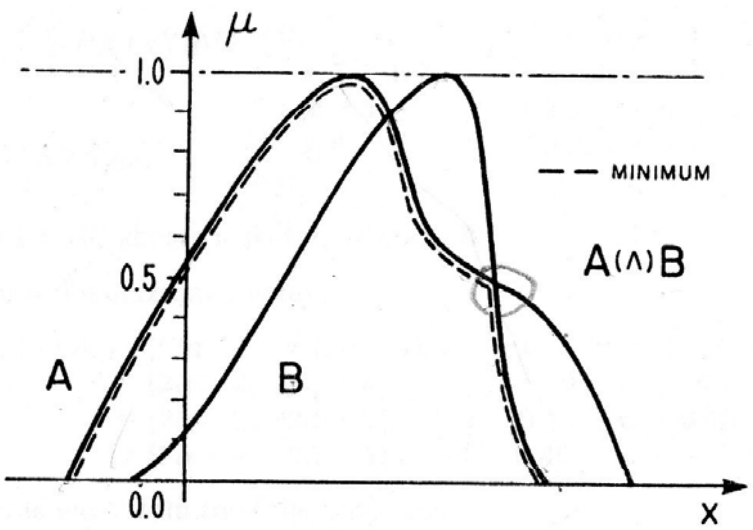


Figure 1.15 Minimum of fuzzy numbers **A** and **B**.

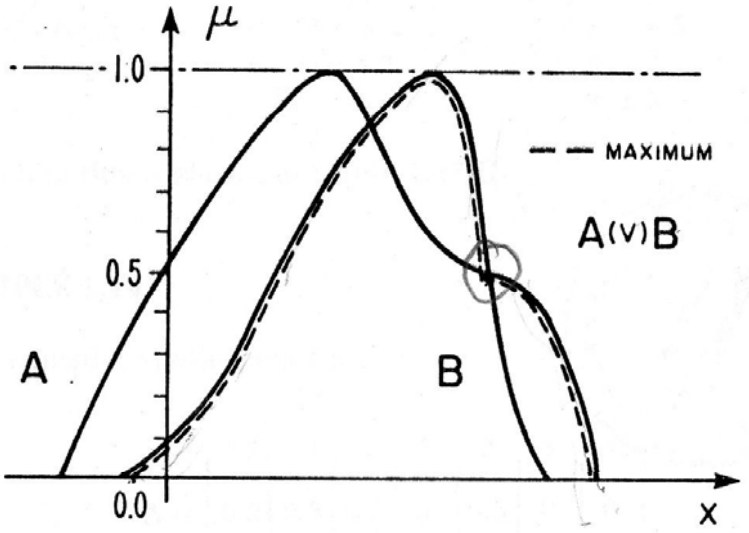


Figure 1.16 Maximum of fuzzy numbers **A** and **B**.

Minimum ve Maksimum Kümeler

$$\mu_{\mathbf{A}}(x) = 0, \qquad x \le -2,$$
$$= x/2 + 1, \qquad -2 \le x \le 0,$$
$$= -x/6 + 1, \qquad 0 \le x \le 6,$$
$$= 0, \qquad x \ge 6.$$

$$\mu_{\mathbf{B}}(x) = 0, \qquad x \le -4,$$
$$= x/7 + 4/7, \qquad -4 \le x \le 3,$$
$$= -x/2 + 5/2, \qquad 3 \le x \le 5,$$
$$= 0, \qquad x \ge 5.$$

$$A_\alpha = [2\alpha - 2, -6\alpha + 6]$$

$$B_\alpha = [7\alpha - 4, -2\alpha + 5].$$

$$A_\alpha (\wedge) B_\alpha = [(2\alpha - 2) \wedge (7\alpha - 4), (-6\alpha + 6) \wedge (-2\alpha + 5)],$$

$$A_\alpha (\wedge) B_\alpha = [7\alpha - 4, -2\alpha + 5], \qquad 0 \le \alpha \le 0.25,$$
$$= [7\alpha - 4, -6\alpha + 6], \qquad 0.25 \le \alpha \le 0.40,$$
$$= [2\alpha - 2, -6\alpha + 6], \qquad 0.40 \le \alpha \le 1.$$

$$A_\alpha (\vee) B_\alpha = [(2\alpha - 2) \vee (7\alpha - 4), \qquad (-6\alpha + 6) \vee (-2\alpha + 5)]$$

$$A_\alpha (\vee) B_\alpha = [2\alpha - 2, -6\alpha + 6], \qquad 0 \le \alpha \le 0.25,$$
$$= [2\alpha - 2, -2\alpha + 5], \qquad 0.25 \le \alpha \le 0.40,$$
$$= [7\alpha - 4, -2\alpha + 5], \qquad 0.40 \le \alpha \le 1.$$

Minimum ve Maksimum Kümeler

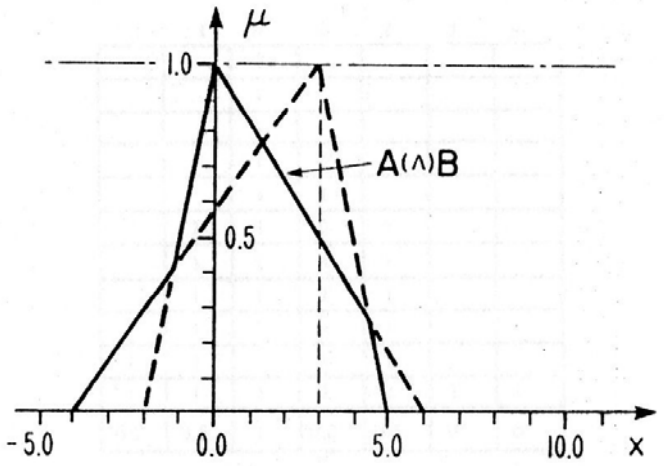


Figure 1.17(B) Minimum of fuzzy numbers **A** and **B** (Example 1.13).

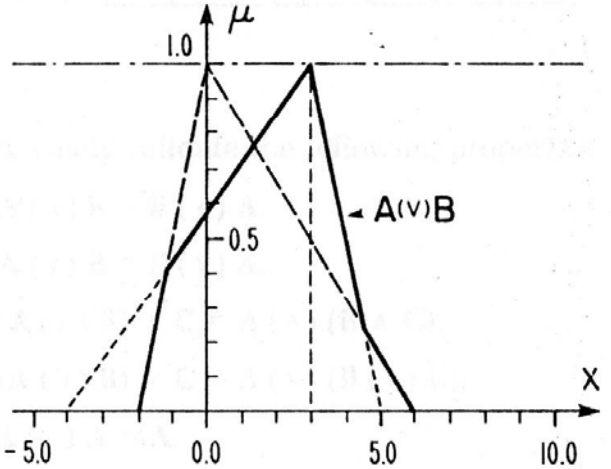


Figure 1.17(C) Maximum of fuzzy numbers **A** and **B** (Example 1.13).

Minimum ve Maksimum Kümeler

$$
\begin{aligned}
\mu_{\mathbf{A}\,(\wedge)\,\mathbf{B}}(x) &= 0, && x \le -4,\\
&= -x/7 + 4/7, && -4 \le x \le 1.2,\\
&= x/2 + 1, && -1.2 \le x \le 0,\\
&= -x/6 + 1, && 0 \le x \le 4.5,\\
&= x/2 + 5/2, && 4.5 \le x \le 5.\\
&= 0, && x \ge 5.
\end{aligned}
$$

$$
\begin{aligned}
\mu_{\mathbf{A}\,(\vee)\,\mathbf{B}}(x) &= 0, && x \le -2,\\
&= x/2 + 1, && -2 \le x \le -1.2,\\
&= x/7 + 4/7, && -1.2 \le x \le 3,\\
&= x/2 + 5/2, && 3 \le x \le 4.5,\\
&= -x/6 + 1, && 4.5 \le x \le 6,\\
&= 0, && x \ge 6.
\end{aligned}
$$