Modeling and Forecasting Daily Exchange Rates Volatility of the Canadian Dollar.

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1. Abstract

In this paper, different types of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models were used to model volatility and to predict one step ahead of the mean exchange rate returns and conditional volatility. The Canadian Dollar has been chosen as a primary currency and daily logarithmic exchange rate returns are constructed relative to the United State Dollar and Euro.

In one step ahead forecasting of the mean - logarithmic returns, GARCH (1,1) model via Gaussian normal distribution has outperformed the rest of the models in in-sample and out-of-sample forecasting by considering both currencies. The outperformance of GARCH (1,1) via Gaussian is proved to be statistically significant for USD returns, but not for Euro ones. When the volatility is forecasted in out-of-sample, EGARCH (1,1) via Gaussian normal distribution outperformed the rest of the models in both currencies. In in-sample forecasting, APGARCH (1,1) via Generalized Error Distribution (GED) was superior in Euro returns, while EGARCH (1,1) is the best for United State Dollar (USD) returns. The forecasting accuracy-test statistics do not enable us to say that these models are superior. However, APGARCH (1,1) by Student's t distribution is the worst forecasting model for USD returns.

Further, the leverage effect parameter has been positive and statistically significant in most estimations. Thus, past values of Canadian Dollar exchange rate returns allow to state that depreciation of the Canadian Dollar triggers more volatility than appreciation.

2. Introduction

In the globalized world, exchange rate return markets have been more volatile. So as to model volatility, many models have been developed and confirmed in empirical cases (Lunde 2005). I have used different types of Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models which is Generalized form of Autoregressive Conditional Heteroskedasticity (ARCH) to model logarithmic exchange rate returns (*for simplicity we will use USD or Euro returns instead of Logarithmic exchange rate returns of USD or Euro*) volatility from the Canadian dollar against USD and Euro.The ARCH and GARCH models were introduced by Robert Engle in 1982 and Tim Bollerslev in 1986 respectively. We need to mention that Taylor (1986) also employed GARCH model independently.

Until Engle (1982), traditional econometric models were estimated by assuming variance as a constant over time. The ARCH model was a new class of stochastic processes in time series. In this model, innovations or shocks are mean zero, serially uncorrelated processes with nonconstant variances. These variances are conditional on the past. But unconditional variance needs to be constant over time. One-period forecast variance is defined by recent past variances. Engle (1982) demonstrated the conditional variance as a simple quadratic function of its lagged values.

The ARCH model was proven useful in many economic cases, especially uncertainty of inflation rate in Engle (1983), Kraft (1983). According to Bollerslev (1986), the disadvantage of this model can be considered lag length. Thus, to predict volatility, it is needed to estimate a large number of parameters in the ARCH model, which is one of the major problems because of lag length. The lag length is still large. In contrast, GARCH models allow conditional variance to depend upon its lag and square of lagged shocks. This extension gives the latter model a huge advantage where GARCH model decreases the number of required ARCH lags while volatility is being measured.

Cyprian et al (2017) mentioned that the GARCH models cannot deal with skewed time series since this model responds equally to asymmetric shocks. The results are biased. When applying the GARCH models, one of the drawbacks can be considered how the GARCH models do not always fully embrace the heavy tails of high frequency financial time series. So as to succeed in dealing with the latter problem, Bollerslev (1986) used the Student's t distributions for error residuals.

I have chosen to employ the Canadian Dollar (CAD) daily exchange rates from United State Dollar, Great Britain Pound and Euro. First, they are independent currencies in the floating system as CAD. In these kinds of systems, exchange rates are usually determined by the market and monetary policy which work without consideration of exchange rates. The literature review shows that the APGARCH and GJR-GARCH models have not been employed to CAD in the last decade. Therefore, I got inspired to apply these models to CAD and compare them with mostly used GARCH and EGARCH models.

The rest of the paper is organised as following: In section 3, the general and specific literature review about ARCH and different GARCH family models are reviewed. The following section gives information about methodology, while section 5 consists of daily data and the source of the data and variables. Empirical results are discussed in section 6.1 and 6.2. Finally, section 7 is the conclusion part. Additional graphs and tables are included to section 9, and section 8 consists of references.

3. Literature review

After Engle (1982) and Bollerslev (1986), many models of the ARCH and GARCH have been initiated. One of the first examples of ARCH model is considered ARCH (q) Engle (1982) model where h_t variance is a function of q past squared returns. GARCH (q, p) is a more generalized form of ARCH and this model is created by adding p lags of past variance – σ_t^2. P = 1, q = 1 is famous GARCH (1, 1) model and it is proven to be successful in empirical results (Lunde 2005).

Another model from the GARCH universe is Exponential GARCH or simply EGARCH, which was introduced by Nelson (1991). It allows conditional variance to be in logarithmic form. The logarithmic form makes the equation easier because, in this case, there is no need to deal with negative variance by adding a constraint to the equation. With convenient conditioning of parameters, this specification captures the stylized fact that a negative shock makes a higher conditional variance than a positive shock does.

In addition to above-mentioned models, other GARCH family models are GJR-GARCH by Lawrence Glosten, Ravi Jagannathan, and David Runkle (1993) which mainly focus on nonsymmetrical dependencies and they are considered as nonlinear GARCH models. It is a simple extension of GARCH (1,1) model by adding indicator parameter I and γ - leverage effect parameter. In the methodology section we will discuss these parameters in detail. Even though EGARCH (1,1) and GJR-GARCH (1,1) are aimed to capture leverage effect or to deal with asymmetry, the structure of the variance equation is different. The leverage coefficient of the EGARCH is directly connected to the actual innovations. However, for the GJR-GARCH as given by equation (6) we see that the leverage coefficients are connected through an indicator variable I.

Another nonlinear GARCH model, which is prominent, is - the asymmetric power GARCH (APGARCH) model in the ARCH/GARCH family. Introduced by Ding et al (1993), the APGARCH model is then tested by Robert de Brooks et al (2000) that the APARCH and APGARCH models are applicable when we are talking about leverage effect. The idea behind the introduction of the power term is based on the fact that in modeling financial data, the assumption of normality restricts the power term to either unity or two. This is often unrealistic because of significant skewness and kurtosis. In APGARCH models, it allows a new power parameter to be estimated and not to be 1 or 2. Here 1 and 2 stands for power term of the standard deviation and the variance respectively which the variance of other models is restricted to power term 2.

The ARCH and GARCH families were well-known in terms of forecasting of volatility in inflation rates in the early years. Econometricians use broadly the GARCH models in order to forecast the time-varying volatility observed in exchange rate returns because they can be fitted to the financial data which have heavy-tailed distribution and volatility clustering (Bollerslev 1986). Economists began to use those models in order to measure volatility in many economic phenomena. The significance of volatility is undeniable as a determinant of inflation - Coulson and Robbins (1985), the term structure of interest rates – Engle, Hendry, and Trumble (1985), the volatility of stock markets – Engle, Lilien¸and Robins (1987), the behavior of foreign exchange markets – Domowitz and Hakkio (1985) and Bollerslev and Ghysels (1996). However, Keith Pilbeam et al (2014) criticized linear and nonlinear GARCH models not to forecast volatility precisely. He compared GARCH (1,1), EGARCH (1,1) and GJR-GARCH (1,1) with implied volatility series by Bloomberg. He concluded that these three GARCH models have poor exchange rates volatility forecast with comparison to the implied volatility. He also mentioned that these

GARCH models forecasted volatility better when volatility is low. He used Gaussian normal distribution for error residuals in United States Dollar/Euro, United States Dollar/Great Britain Pound.

The GARCH family was used for the first time by Hsieh (1988) in order to forecast exchange rate returns volatility. He employed three currencies – CAD, GBP and Deutsche mark against USD to capture volatility. He came to the realization that when we are talking about standardized residuals from GARCH models, exponential GARCH fits CAD forecast extremely well. This model fits Swiss franc and Deutsche mark quite well by considering non-normal distribution, however, there is no fit for the British pound.

Bollerslev (1990) proposes a multivariate time series model with time-varying conditional variances and covariances and in this model, heteroscedasticity was allowed. Conditional variances were taken as the GARCH process. He used five European currencies against the US dollar. According to Bollerslev, co-movements among the currencies are significantly higher after the inception of the European Monetary System. Tse and Tsui (1997) also used two currencies Singapore dollar and Malaysian ringgit against USD where both currencies are the managed-float systems. They worked on the new class of APARCH model to forecast future volatility based on present shocks. According to Tse and Tsui, depreciation shock in Malaysian ringgit against USD has more effect in future volatility rather than appreciation shock. The latter effect has not been observed for the Singapore dollar.

McKenzie et al (1997) used seventeen bilateral exchange rates including CAD, USD, Yen, GBP and so on. He concluded that the GARCH (1,1) model is preferred when the responses are symmetric. As McKenzie stated if asymmetry exists it is significant to include power term for asymmetric power ARCH model to capture leverage effect. To continue, Brook and Burke (1998) forecasted exchange rate volatility using conditional variance models - mostly from the GARCH family. The data stem from daily exchange rates of CAD, German mark and Yen against to USD exchange rate returns. All models were computed by quasi-maximum likelihood estimation. 1, 12, 24 steps ahead forecasts were compared. Brook and Burke also found that the GARCH (p, q) model outperforms others which simply uses historical means of squared returns assuming homoscedasticity. In contrast, Linear GARCH model and asymmetric ones were compared by Longmore and Robinson (2004) by using daily exchange rate returns from the Jamaican dollar to USD. In accordance with the authors, a long memory process was found for the exchange rate returns. They also illustrated that non-linear models were better than linear ones in terms of explanatory power. Non-linear models showed better in out-of-sample forecasts as well. Furthermore, Wayne Robinson et al (2004) worked on exchange rate returns from the Jamaican dollar against USD by using linear and non-linear GARCH models. According to Wayne, non-linear GARCH models did better than linear once considering the explanatory power. He showed that non-linear models provided a good forecast in terms of out-of-sample form. In addition, Engle, Hong, and Kane (1990), in the stock and foreign exchange market, compared out-of-sample performance. They discovered that the GARCH models are the best. As the GARCH models seem to dominate the other models, it is meaningful to develop any better model than the GARCH model and its extensions. The similar results were found by Kenneth et al (1994). Kenneth D. West and Dongchul Cho employed univariate homoscedastic, GARCH (EGARCH and IGARCH), autoregressive and nonparametric models for conditional variances. They chose five bilateral exchange rates for dollar. They concluded that for one-week horizon (On that time, exchange rates were announced weekly) GARCH models tend to show slightly better results than other models.

Cheong Vee et al (2011) assessed exchange rate volatility of Mauritanian rupee against USD by using the GARCH (1,1) model under Generalized Error Distribution (GED) and the Student's-t distribution. GARCH (1, 1) – GED was found slightly more applicable compare to latter one in terms of out-of-sample estimation. Additionally, to different distribution assumptions, Abdullah et al (2017) studied exchange rate volatility between Bangladeshi taka and USD under both normal and Student's-t distribution assumptions by using GARCH models such as APARCH, EGARCH, TGARCH and IGARCH. They found that applying Student's t-distribution improved prediction of volatility accuracy. For out-of-sample forecasts, AR (2) - GARCH (1, 1) outperformed the rest models. Student's t distribution has been employed by Omari et al (2017) who applied GARCH models in order to forecast volatility of exchange rate returns of USD/KES – Kenyan Shilling. They applied both symmetric and asymmetric models to capture volatility clustering and leverage effect. The performance of the symmetric GARCH (1, 1) and GARCH-M models as well as the asymmetric EGARCH (1, 1), GJR-GARCH (1, 1) and APARCH (1, 1) models with different residual distributions are employed to data. The asymmetric APARCH model, GJR-GARCH model and EGARCH model with Student's t-distribution were considered as the most acceptable models for assessing volatility of the exchange rates.

Zakaria (2012) used nineteen Arabic countries' currency against USD by estimating asymmetric and symmetric GARCH family models to capture volatility clustering and leverage effect. The empirical results demonstrated that the sum of relevant coefficients exceeds one for ten countries which implies volatility is an explosive process. Besides, EGARCH (1, 1) model was considered effective by defining that negative shocks bring more volatility than positive ones. Author concluded that GARCH family models can be considered for modelling exchange rate returns. On the contrary, Balaban (2004) found that the standard GARCH model was overall the most accurate forecast for monthly U.S. dollar-Deutschemark exchange rate volatility by using ARCH, GARCH, GJR-GARCH and EGARCH.

Despite of broad literature review, it is complicated to say which model is better for forecasting volatility.

4. Data

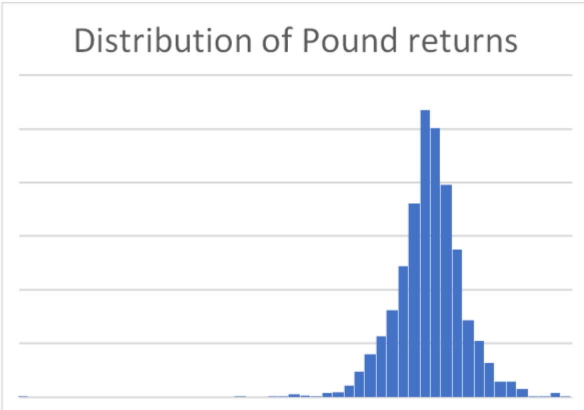
The daily exchange rates data was taken from OFX which is an Australian online foreign exchange and payments company and it covers the time frame from 05 January 2015 to 02 January 2020 with a daily basis as a nominal exchange rate, the sample size is 1593. In order to capture volatility and to get better results, Sundays and holidays are excluded from data.

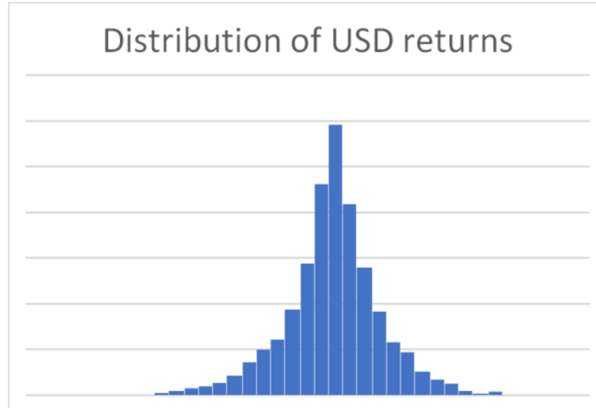
Exchange rate returns are calculated as the following formula:

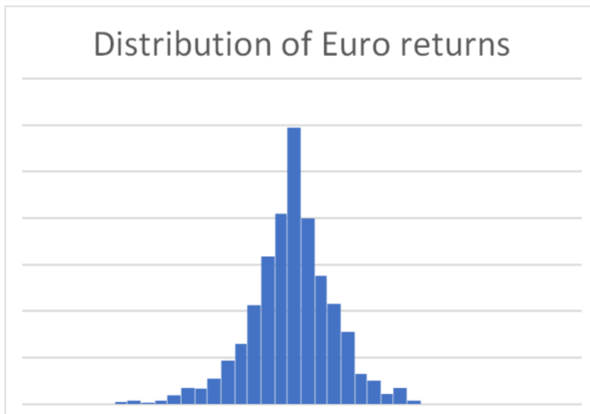
$$r_t = 100\% \times ln \left(\frac{x_{it}}{x_{it-1}}\right) \tag{1}$$

Where x_{it} is USD/CAD, Pound/CAD and Euro/CAD ("/" stands for division).

Where r_t is exchange rate returns and x_t and x_{t-1} are exchange rates from United States Dollar, Great Britain Pound, and Euro to Canadian Dollar in time t and t-1 respectively.







As we see from the charts, logarithmic returns of USD, Pound, and Euro are leptokurtic which implies that they have fatty tails and more concentrated around the mean.

All the estimations and the tests were implemented by using Stata 15.1 except for Diebold - Mariano test. The latter one was implemented by Microsoft Excel 2016.

5. Methods

The unconditional variance or standard deviation assessing methods of measuring volatility was not able to catch the features of financial time series. These features are leverage effect which means that - volatility is higher after deprecation with comparison to appreciation. This feature was first suggested by Black (1976) for stock returns. Memory means the volatility is highly persistent, excess kurtosis - fat tails, and volatility clustering - large changes tend to be followed by large, small changes by small volatility. Unlikely, the models, which are assumed to have a conditional variance, can capture these features like the ARCH and its generalized form GARCH models. Different GARCH models – GARCH (1, 1), GJR-GARCH (1, 1), APGARCH, and EGARCH are used in this paper in order to measure volatility and one-step ahead forecasting.

Volatility capturing models may be divided into two main fields, symmetric and asymmetric models. In the symmetric models, the conditional variance only depends on the magnitude, and not the sign, of the underlying asset, while in the asymmetric models the positive or negative shocks of the same magnitude have a different effect on the future volatility. Symmetric GARCH models are ARCH and GARCH, asymmetric ones are GJR-GARCH, APGARCH, and EGARCH models.

GARCH (p, q) model is defined by as follows:

The conditional variance depends on its lags. First, let's look at the general form of GARCH (p, q) model where p is ARCH, and q is the GARCH term.

$$\sigma_t^2 = \alpha_0 + \sum_i^q \alpha_i\, y_{t-i}^2 + \sum_j^p \beta_i\, \sigma_{t-j}^2 \tag{2}$$

is called the variance equation.

$r_t = \mu + y_t$ - mean equation.

$y_t = \sigma_t z_t$ where y_t is innovation or residual returns, μ – mean of exchange rate returns, σ_t^2 variance, z_t is standardized residual return with variance one and zero expected value.

When it comes to the GARCH (1, 1) model, the conditional variance is going to be

$$\sigma_t^2 = \alpha_0 + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{3}$$

and it is the function of the following parameters: α_0 – constant term, y_t – news about from previous volatility, σ_{t-1}^2 as a last period forecast variance. The unconditional variance or actual variance for the GARCH (1,1) model:

$$var(y_t) = \frac{\alpha_0}{1 - (\alpha + \beta)}$$

EGARCH (p, q) model:

As it is mentioned in the introduction part, Nelson's EGARCH (p, q) model introduced by Nelson (1993), the general conditional variance is written as follow:

$$Ln(\sigma_t^2) = \alpha_0 + \sum_i^q \alpha_i \left(\frac{y_{t-i}}{\sigma_{t-i}} - \sqrt{\frac{2}{\pi}}\right) + \sum_i^q \gamma_i \left|\frac{y_{t-i}}{\sigma_{t-i}}\right| + \sum_j^p \beta_j \, Ln(\sigma_{t-j}^2) \quad (4)$$

EGARCH is an asymmetric model, as it can capture leverage effect. It means that volatility treats different ways depending on the sign of past values. If negative shocks or bad news happen, in this circumstance, the magnitude of volatility raises more than the magnitude of positive shocks. For these features, the EGARCH model is considered one of the best volatilities capturing model in the GARCH family. The coefficient γ_i can be positive or negative depending on the past values. EGARCH (1, 1) model is offered as following:

$$Ln(\sigma_t^2) = \alpha_0 + \alpha \left(\frac{y_{t-1}}{\sigma_{t-1}} - \sqrt{\frac{2}{\pi}}\right) + \gamma \left|\frac{y_{t-1}}{\sigma_{t-1}}\right| + \beta \, Ln(\sigma_{t-1}^2). \quad (5)$$

Unconditional variance:

$$Ln(\sigma_t^2) = \frac{\alpha_0}{1 - \beta}$$

Another famous asymmetric GARCH model is GJR-GARCH which is introduced by Glosten, Jagannathan and Runkle in 1993. The conditional variance of GJR-GARCH (p, q) model was presented as below:

$$\sigma_t^2 = \alpha_0 + \sum_i^q \alpha_i \, y_{t-i}^2 + \sum_j^p \beta_j \, \sigma_{t-j}^2 + \sum_i^q \gamma_i I_{t-i} \, y_{t-i}^2 \quad (6)$$

where α_0, α, and β are constant parameters. However, I_t is a dummy variable or indicator function which has two types of value - zero and one depending on the sign of error term (y_i) positive or negative respectively. In this model, all three constant parameters are supposed to be positive and if all leverage coefficients are equal to zero, then GJR-GARCH (p, q) model changes to GARCH (p, q) model. The simple GJR - GARCH (1, 1) model is as follow:

$$\sigma_t^2 = \alpha_0 + \alpha\, y_{t-1}^2 + \beta\, \sigma_{t-1}^2 + \gamma\, I_t y_{t-1}^2 \tag{7}$$

Unconditional variance:

$$\mathrm{var}(y_t) = \frac{\alpha_0}{1-(\alpha+\beta+\frac{\gamma}{2})}$$

The asymmetric power ARCH or APARCH (p, q) is also one of the practical models in capturing volatility leverage in asymmetric GARCH family which introduced by Ding, Engle, and Granger (1993). The conditional variance equation can be demonstrated as below:

$$\sigma_t^{\delta} = \alpha_0 + \sum_{i=1}^{q} (\alpha_i |y_{t-i}| - \gamma_i y_{t-i})^{\delta} + \sum_{j}^{p} \beta_j\, \sigma_{t-j}^{\delta} \tag{8}$$

Where α_0, α, and β are constants and the two latter ones are ARCH and GARCH terms respectively. For the conditional variance, the following constraint needs to be included as well.

$\alpha_0 > 0$,

$\delta > 0$,

$\alpha, \beta \geq 0$

$-1 < \gamma < 1$.

As we see from the APARCH conditional variance equation, when $\delta = 1$, it is just standard deviation estimation, if $\delta = 2$ then it turns to classic GARCH (p, q) model.

There is a similarity between ARCH and GARCH model estimations, so the GARCH model also is estimated by maximum likelihood function. As Hsieh (1989) mentioned, there are several advantages of using maximum likelihood estimation. First of all, it allows joint estimation of parameters in the mean and variance equation. Secondly, it imposes the restriction that the GARCH term and variance greater than zero which is very tough to do in the least square estimation. Thirdly, it permits the likelihood ratio tests of restrictions of the model.

The standardized residuals of returns innovations are assumed to be three kinds of distributions: Gaussian - normal, Student's T and Generalized Error Distribution or simply GED. Thus, the correct form of the loglikelihood objective function can be defined depending on the parametric form of the innovation distribution. By maximizing the log-likelihood function parameter vectors can be attained:

$$\theta = [\alpha_0, \alpha, \beta, \gamma, \delta] \tag{9}$$

If innovation y_t^2 has distributed normally, then log-likelihood function is going to be as below:

$$Log\ L = \sum_{1}^{T} l_t = -\frac{T}{2} log\ [2\pi] - \frac{1}{2}\sum_{1}^{T} log\ \sigma_t^2 - \frac{1}{2}\sum_{1}^{T}\frac{y_t^2}{\sigma_t^2} \tag{10}$$

T is sample size and l_t is likelihood function is giving as following:

$$l_t = -\frac{1}{2} log\ [2\pi] - \frac{1}{2} log \sigma_t^2 - \frac{1}{2}\frac{y_t^2}{\sigma_t^2} \tag{11}$$

If we assume that the innovation residuals have Student's t distribution with degree of freedom $v > 2$, when v goes to infinity Student's t distribution is going to be closer the normal one. Then log-likelihood function may be offered as:

$$l_t = -\frac{1}{2} log \frac{\pi\ (v-2)\ \Gamma(\frac{v}{2})^2}{\Gamma(\frac{v+1}{2})^2} - \frac{1}{2} log\ \sigma_t^2 - \frac{v+1}{2}\ log\ [\ 1 + \frac{y_t^2}{\sigma_t^2\ (v-2)}\ ] \tag{12}$$

Where Γ is gamma function defined as $\Gamma\ (v) = (v - 1)!$, σ_t^2 is variance and y_t^2 innovation.

To continue with GED distribution, log - likelihood function becomes:

$$l_t = -\frac{1}{2}\ log \frac{\Gamma(\frac{\rho}{2})^3}{\Gamma(\frac{3}{\rho})\Gamma(\frac{\rho}{2})^2} - \frac{1}{2}\ log\, \sigma_t^2 - [\ \frac{\Gamma(\frac{3}{\rho})\ y_t^2}{\sigma_t^2\, \Gamma(\frac{1}{\rho})}]^{\frac{\rho}{2}} \qquad (13)$$

Where ρ is a tail parameter and is subjected to $\rho > 0$ constraint. If $\rho = 2$, then GED distribution simply transforms to Gaussian normal distribution. For fat - tailed distribution ρ is assumed lesser than two.

6. Empirical results

As we see from descriptive statistics of the returns from *Table a,* the mean value of all returns is approximately equal to zero. The number of observations differs in USD and Euro returns, even though they cover the period time of 05 January 2015 to 02 January 2020, since Sundays and holidays are excluded from the data sample. Although the returns from Pound range the most, the ARCH LM test results display that there is no ARCH effect in Pound returns.

Table a

Currencies returns	Minimum value	Maximum Value	Standard deviation	Mean value	Number of observations
USD return	-2.7621424	2.2174919	0.443151	0.006177	1592
Pound return	-6.4478679	2.1815202	0.555626	-0.003574	1590
Euro return	-3.0490911	3.197207	0.509766	0.001802	1584

Test statistics:

Lagrange multiplier test for ARCH effect:

H_0: no ARCH effect

H_1: ARCH (1) disturbance.

ARCH test *Table b* shows that we can reject the null hypothesis for both USD and Euro with 5% and 1% significant level respectively, however, we cannot reject the null for Pound returns. “Df” is a degree of freedom in this table and it implies the number of ARCH lags in this test. So, there is an ARCH effect for USD and Euro returns, on the contrary, it does not show the same patterns for Pound returns. Therefore, Pound returns are not employed in this paper.

Table b

ARCH test	**chi^2**	**df**	**prob>ch^2**
USD	5.443	1	0.0196
Pound	0.082	1	0.7751
Euro	15.310	1	0.0001

Jarque - Bera test for normality:

H_0: Normal

H_1: Not Normal

According to Jarque - Bera test statistics table *(Table c),* all exchange rate returns illustrate the same pattern and the null hypothesis is rejected at 1% significant level. All returns are not distributed normally. Table statistics also demonstrate that there are significant kurtosis and skewness at the level of 1% significant level for USD and Pound, 5% for Euro returns. Thus, the exchange rate returns are highly skewed. The distribution is also leptokurtic, signifying that its central peak is higher and sharper with longer and fatter tails. In addition to this, empirical results from GED estimation (all models) show that log shape from log-likelihood estimation is lesser than 2 which also implies fat-tailed distribution.

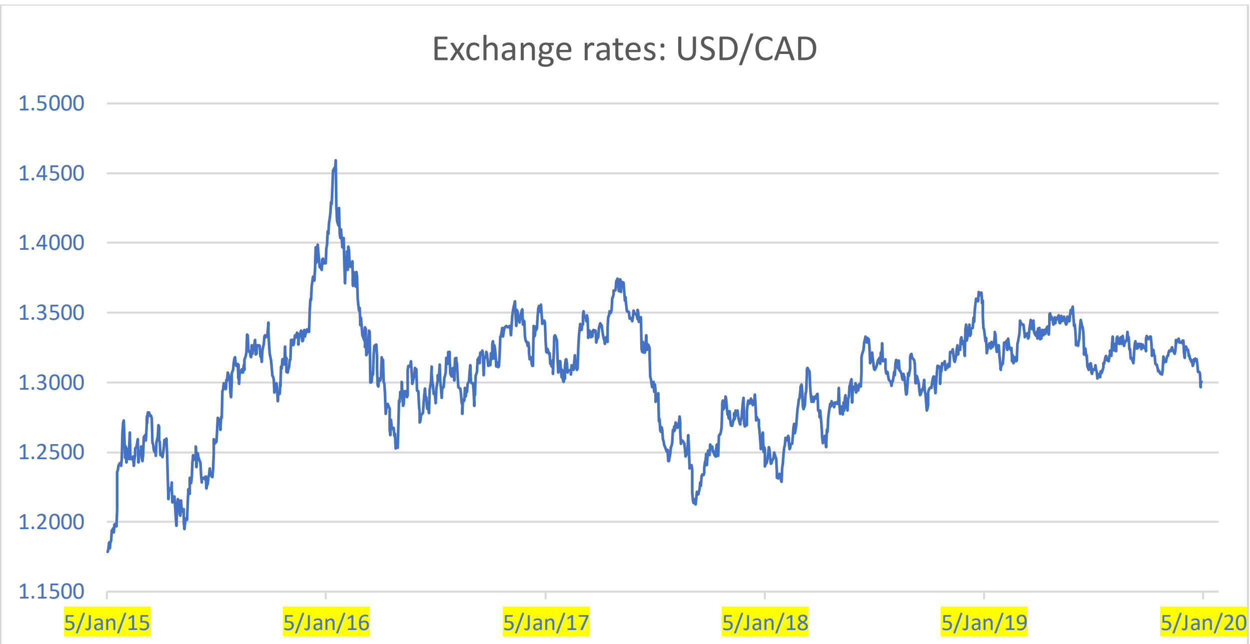
Table c

Jarque -Bera test	**Pr (skewness)**	**Pr (kurtosis)**	**Pr CHI^2**
USD returns	0.0000	0.0000	0.0000

Pound returns	0.0000	0.0000	0.0000
Euro returns	0.0483	0.0000	0.0000

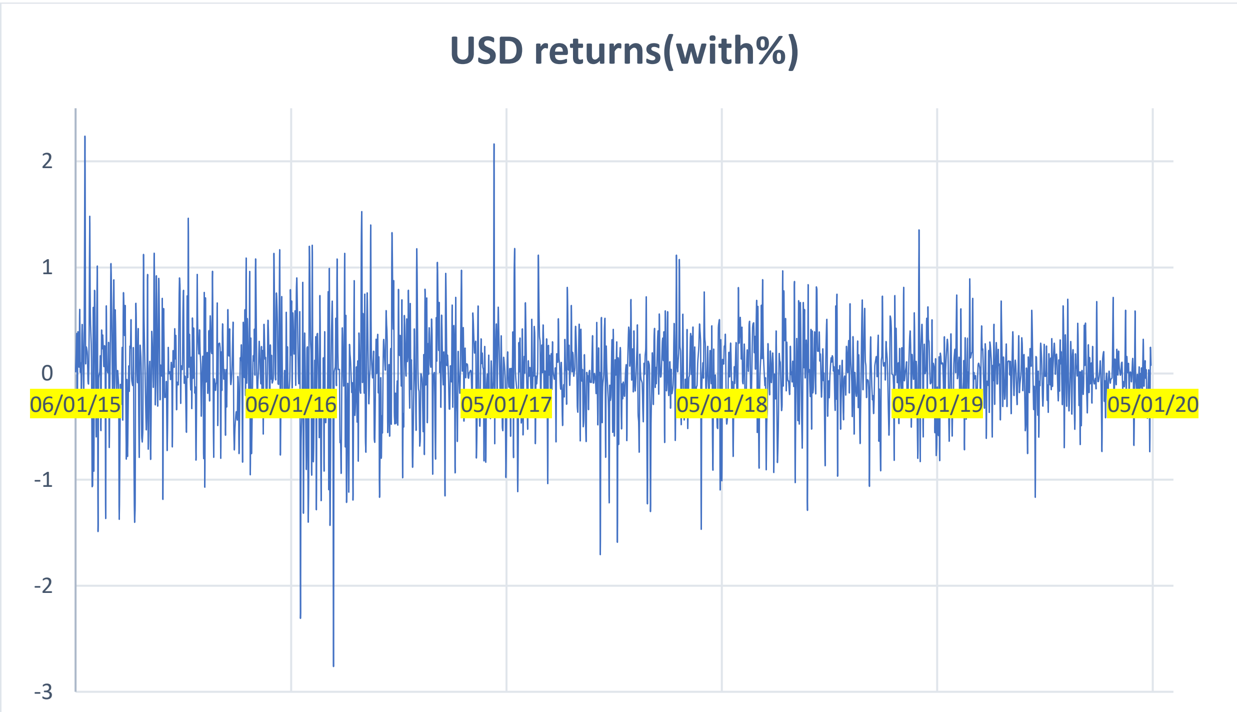
Graph 1 illustrates that the USD exchange rate to CAD increased to 1.4593 on 26 January 2016 or observation number 327 which is the highest point in the given time frame. After this hike, we can see a substantial decline to 1.2527 until 29 April 2016 or observation number 412. After April, we can see almost average 1.30 exchange rates which resemble steady despite instabilities except for 14 September 2017 or observation number 865 where CAD strengthened considerably compare to USD by 1.2127.

Graph 1 *(05 January 2015 - 02 January 2020)*



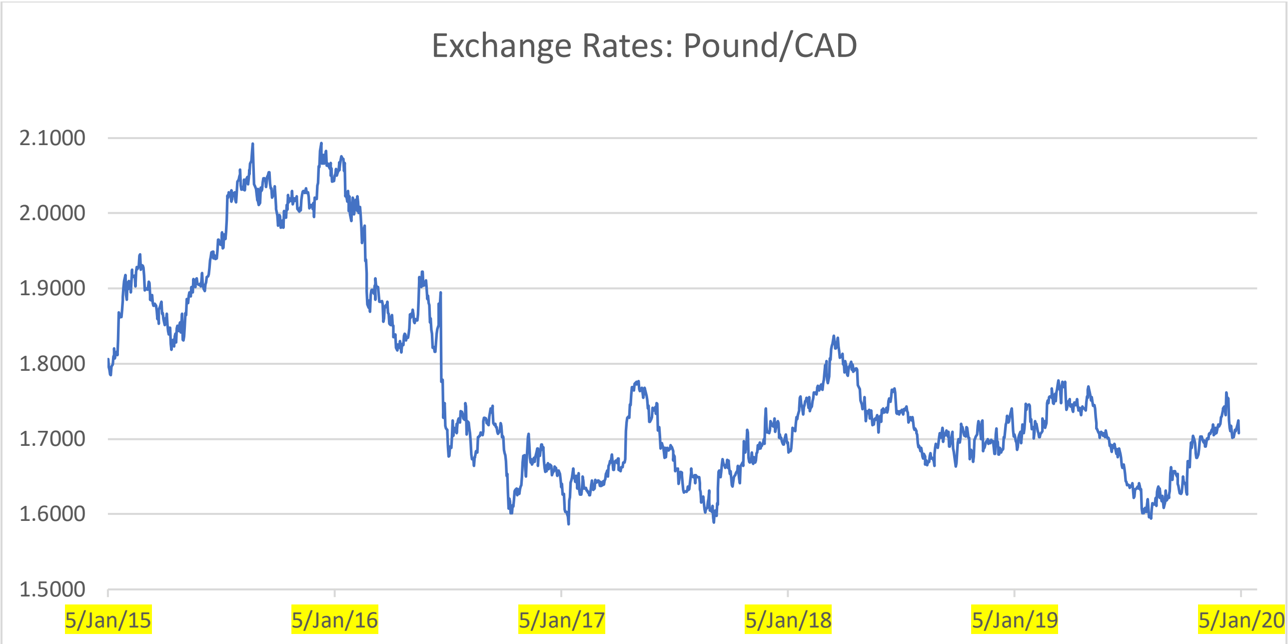
Time series line 1 clearly shows that in the first quarter of the years 2015 and 2016 USD exchange rate returns were more volatile in comparison to the rest of the observations. This happened because at the mentioned times USD exchange rate to CAD has reached its lowest and highest values. Without doing any estimation, it is clearly shown from *Time series line 1* and *Graph 1* that the substantial deviation from mean causes volatility and volatility is persistent. We can easily see volatility clustering here which means that big changes are followed by big, and small changes by small. A leverage effect also is observed in this line, implying negative values are followed by more volatile returns.

Time series line 1 *(05 January 2015 - 02 January 2020)*



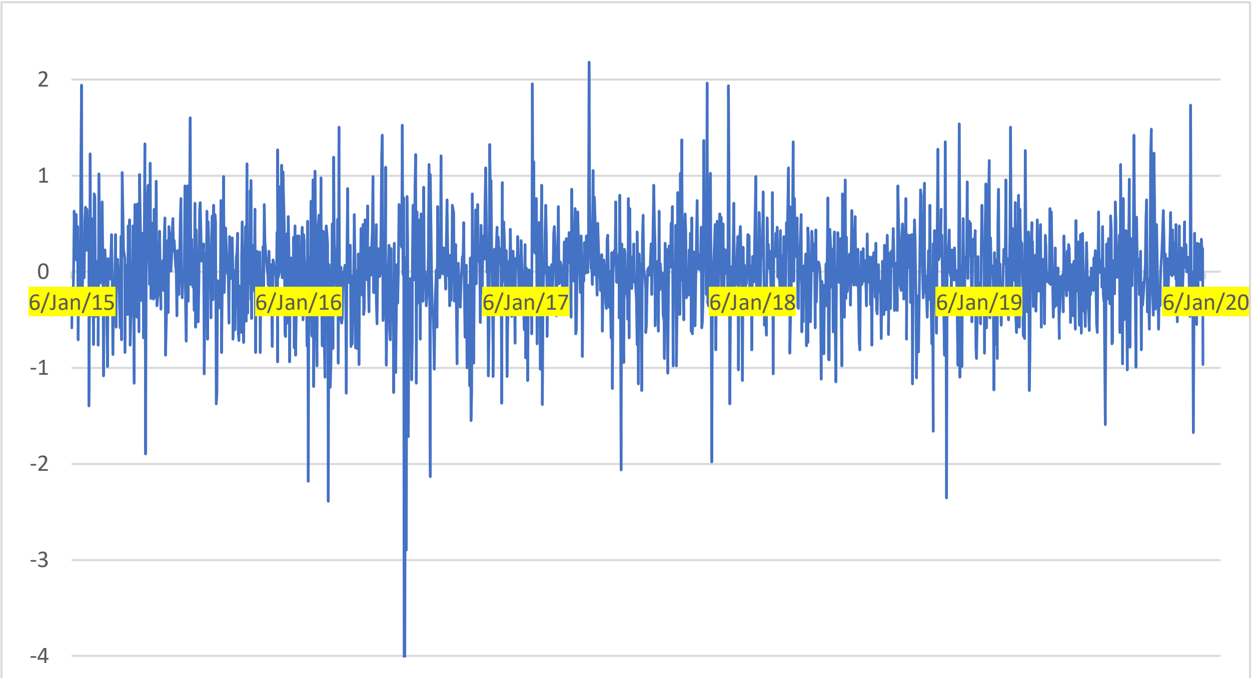
Great Britain Pound has been elevated its peak level in the second half of 2015 when 2.0935 exchange rate has been noted in the observation number 295 or December 14, 2015. After this apex, we can see a dramatic drop in Pound value until later 2016. The same patterns - apex, sharp decline then stability took place in USD exchange returns as well. Despite of trivial fluctuations, there have not been noteworthy changes from the beginning of 2017 to the beginning of 2020.

Graph 2 *(05 January 2015 - 02 January 2020)*



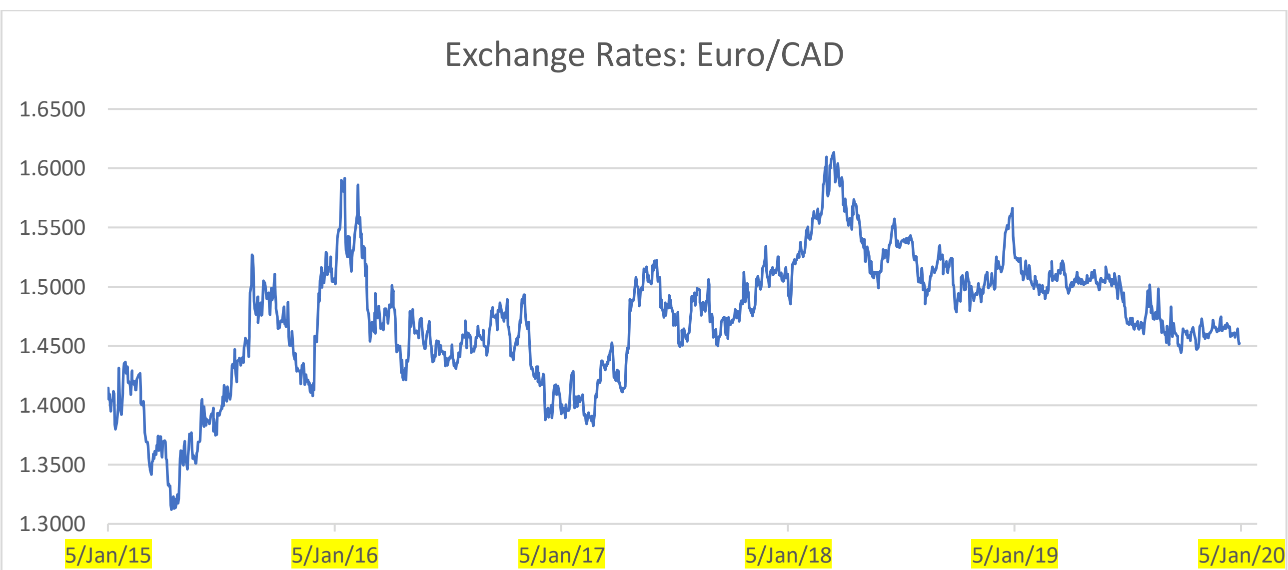
However, if we compare exchange rate returns between USD and Pound, we can easily observe from the *Time series line 1 and 2, 3* that the returns do not perform the same features. Thus, USD and Euro returns seem more volatile compared to Pound ones. The test statistics from the ARCH LM test, also exhibit that there is no ARCH effect in the Pound returns. This result was not expected.

Time series line 2 *(05 January 2015 - 02 January 2020)*



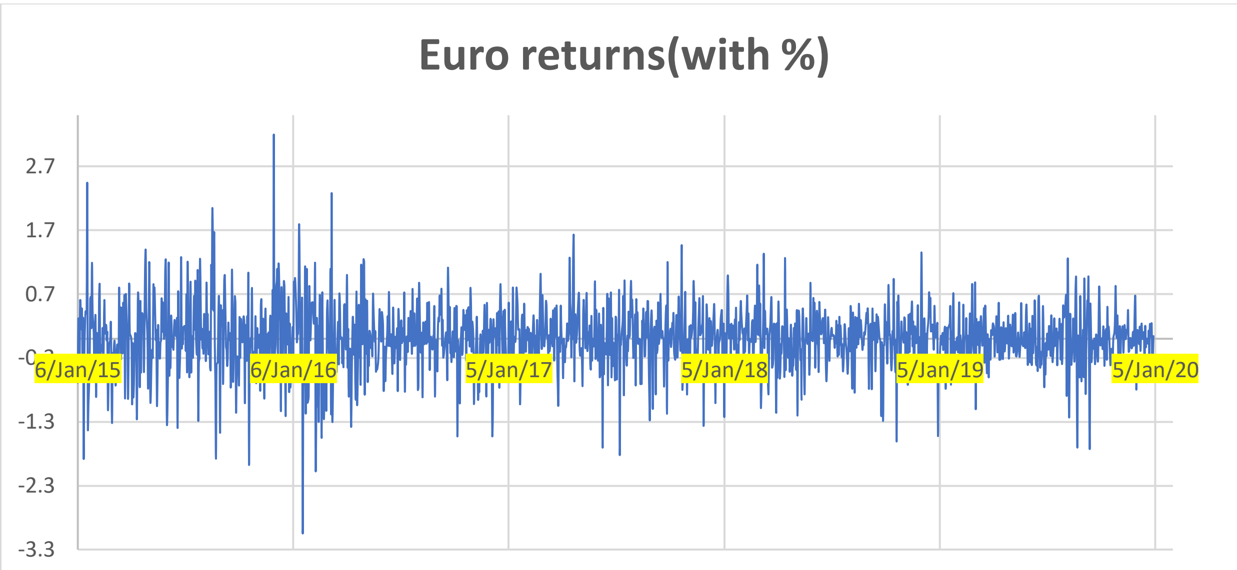
Speaking of exchange rates from Euro to CAD, although it shows the same features with USD and Pound exchange rates till the end of the first quarter of 2016, as we can see from *Graph 3*, Euro has gained value from the beginning of 2017 and it reached its summit on March 19, 2018, or observation number 1030. After the latter time, Euro began to devalue against CAD within a couple of months sharply and following by a slow decline till 2020.

Graph 3 *(05 January 2015 - 02 January 2020)*



Logarithmic exchange rate returns of Euro to CAD were volatile in the first quarter of the years 2015 and 2016 as same as USD returns following by less volatile trends. When we look through *Graph 3,* we would expect high volatility at the beginning of 2017 when CAD possessed its least value against Euro, nevertheless, it has not occurred. By looking at the time series line, we can observe volatility clustering.

Time series line 3 *(05 January 2015 - 02 January 2020)*



6.1 Modeling volatility:

In order to model volatility, symmetric and asymmetric GARCH family models have been used in this paper. GARCH (1,1) as a symmetric and EGARCH (1,1), GJR - GARCH (1,1) and APGARCH (1,1) as asymmetric models. Besides, the maximum likelihood estimation was applied to estimate the models. Considering the distribution of data, three distributions were chosen - Gaussian Normal, Student's t and GED.

Modeling of exchange rate returns, and volatility has been divided in two sections. The first one is full-sample estimation. In this occasion, all of the observations were included in GARCH estimations and last thirty observations were forecasted. On the other hand, sub-sample estimation is committed. In this case, the last thirty observations were excluded from GARCH estimations while they were kept in forecasting.

Table 1 and *Table 2* show GARCH (1,1) sub-sample estimations for USD and Euro returns respectively. The ARCH α is statistically significant at 1% in all distributions for both USD and Euro returns. The summation of α and β is less than one for all distributions in Euro returns which signifies that our variance is stationary. GARCH term - β is statistically significant at 10% for GED distribution in USD returns, but it is significant at 1% level in Euro returns. However, the mean μ is significant at 10% for GED and Student's t distribution in USD differing from Gaussian distribution. The mean is not statistically significant in Euro returns.

Table 1 *(05 January 2015 - 02 December 2019)*

GARCH (1,1)-USD (Sub-sample)	**Normal**	**Student's T**	**GED**
μ	0.0044113 *(0.0101517)*	0.016357* *(0.0091376)*	0.0152434* *(0.0081613)*
α	0.011348*** *(0.0015062)*	0.0190471*** *(0.0054928)*	0.1152371*** *(0.0448856)*
β	0.9887664*** *(0.0015062)*	0.9826068*** *(0.0049769)*	0.0148359* *(0.2317793)*
α_0	-0.00001358 *(0.000139)*	-0.0001861 *(0.0003316)*	0.1755404*** *(0.0473462)*
ML	-889.8008 *(11)*[1]	-827.246 *(12)*	-847.4636 *(10)*

Note:

**** - significant at 1%,*

*********- significant at 5%***

********- significant at 10%***

1- 11 is number of iterations in ML estimation

Table 13 (in Appendix) illustrating full-sample estimation can be compared to sub-sample estimation in *Table 1*. As we see from the tables, all parameters have the same significant level accordingly except for GARCH term in sub-ample estimation which is significant at 10% level, but full-sample estimation it is not statistically significant via GED distribution. Full-sample Euro returns estimation inferences for ARCH and GARCH term are statistically significant at 1% level

(See Table 14), in sub-sample estimation same patterns are observed and one case violates only in GARCH term by estimating Normal distribution. In sub-sample it is significant at 10% level, unlikely from full-sample which is 1%.

Table 2 *(05 January 2015 - 27 November 2019)*

GARCH (1,1) - Euro (Sub-sample)	**Normal**	**Student's T**	**GED**
μ	-0.0018399 *(0.0115409)*	0.0092706 *(0.0108356)*	0.0074591 *(0.0100441)*
α	0.0290886*** *(0.0049774)*	0.0306293*** *(0.0090776)*	0.029902*** *(0.0092203)*
β	0.9634834* *(0.0055388)*	0.9605539*** *(0.0115643)*	0.9609985*** *(0.0116693)*
α_0	0.0018449 *(0.0006282)*	0.0023626* *(0.0013505)*	0.0022125* *(0.0013285)*
ML	-1104.332 *(10)*	-1048.402 *(10)*	-1044.962 *(10)*

Note: *in sub-sample forecasting for Euro returns are until 27 November 2019 not 02 December 2019, this is because in Euro returns, there are more holidays which I have excluded from data to capture precise volatility.*

The empirical results of one of the famous and practical asymmetric GARCH family model- EGARCH (1,1) has been shown in *Table 3* and *Table 4* for USD and Euro returns sub-sample forecasting. The major difference between GARCH (1,1) and EGARCH (1,1) is that the latter one allows us to capture the leverage effect. If the sign of our parameter γ is positive, it means that during depreciation of currency, the magnitude of volatility changes more with comparison to a positive one. In our estimation, the asymmetric parameter is statistically significant at 1% level both USD and Euro returns. Thus, there is a leverage effect in Euro and USD returns and that volatility is persistent as Nelson (1991). Moreover, the ARCH and GARCH terms are also statistically significant for both Euro and USD returns at 1% significant level by considering all for Euro and Gaussian distribution for USD returns. On the contrary, Asemota (2013) examined exchange rate returns' volatility with GARCH models by using Nigeran Naira against USD, British pound and Euro. He mentioned that asymmetric models reject the existence of leverage effect except for the model with volatility breaks and the USD was least volatile.

Table 3 *(05 January 2015 - 02 December 2019)*

EGARCH (1,1)- USD (Sub-sample)	**Normal**	**Student's T**	**GED**
μ	0.0082031 *(0.0106861)*	0.0172286* *(0.0101472)*	.
α	0.0118214*** *(0.0036992)*	0.016205 *(0.0091806)*	.
γ	0.0293351*** *(0.006427)*	0.0505985*** *(0.0140692)*	.
β	0.9997297*** *(0.0009953)*	0.9998689*** *(0.0022647)*	.
α_0	0.0007631 *(0.001613)*	0.0036691 *(0.0041603)*	.
ML	-890.158 *(100)*	-827.3831 *(21)*	No uphill direction after 199 iterations.

EGARCH (1,1) sub-sample USD returns estimation could not have been calculated with the maximum likelihood estimation after 199 iterations via GED distribution. Student's T and Gaussian normal distribution in USD returns show the same significant level of leverage, ARHC and GARCH terms with full-sample estimation as well in Euro returns via all distributions. Thus, full-sample estimation also enables us to say that there is strong evidence for the leverage effect where past values play a crucial role by defining the future volatility *(Table 19 and 20)*.

Table 4 *(05 January 2015 - 27 November 2019)*

EGARCH (1,1) - EURO (Sub-sample)	**Normal**	**Student's T**	**GED**
μ	0.0083532 *(0.0118258)*	0.0139035 *(0.0108206)*	0.126058 *(0.0126058)*

α	0.0468999*** *(0.0.007486)*	0.0469021*** *(0.0141073)*	0.0464543*** *(0.0141037)*
γ	0.073667*** *(0.0122662)*	0.0780706*** *(0.0206415)*	0.0757585*** *(0.0213683)*
β	0.9920482*** *(0.0027073)*	0.9905995*** *(0.0049868)*	0.9910181*** *(0.0050974)*
α_0	-0.0075921** *(0.0036628)*	-0.008359 *(0.0072557)*	-0.0086145 *(0.0072003)*
ML	-1093.227 *(23)*	-1044.514 *(21)*	-1040.24 *(30)*

GJR - GARCH (1,1) is also an asymmetric model helping us to capture the leverage effect. This model is considered as an adjusted form TGARCH (1,1) model. The main difference between EGARCH (1,1) and GJR - GARCH (1,1) model is their variances. In the former one, the variance is in logarithmic form and there is no way to transform to GARCH (1,1) model. On the contrary, if the leverage parameters are equal to zero, GJR - GARCH (1,1) model transforms simply GARCH (1,1) model. According to *Table 5*, the leverage parameter is significant at 5% level in USD returns by considering Normal distribution, nonetheless, in Euro returns, the leverage parameter is statistically significant for all distributions at least 5% significant level. It implies that there is strong evidence that there is a leverage effect in USD and Euro returns.

Table 5 *(05 January 2015 - 02 December 2019)*

GJR-GARCH (1,1)-USD (Sub-sample)	**Normal**	**Student's T**	**GED**
μ	0.0057419 *(0.0104006)*	0.016692* *(0.0091918)*	0.0150941* *(0.0081941)*
α	0.0087262*** *(0.0018791)*	0.0161022*** *(0.0058718)*	0.0141849*** *(0.0054871)*
γ	0.0069179** *(0.0034846)*	0.0082747 *(0.0098905)*	0.007159 *(0.0091493)*
β	0.9877963*** *(0.0017286)*	0.9814748*** *(0.0056327)*	0.981959*** *(0.0057914)*

α_0	-0.0000647 *(0.000157)*	-0.0001056 *(0.0003653)*	0.0000571 *(0.0004091)*
ML	-888.8613 *(12)*	-826.8624 *(10)*	-814.2542 *(86)*

Table 5, 6 and *Table 17, 18* give information about full-sample and sub-sample estimation in USD and Euro returns. As we see from tables, the ARCH, GARCH, and the leverage parameters are at the same significant level except for ARCH term in full-sample estimation which is significant at 10% level, however, this term is not significant in sub-sample estimation.

Table 6 *(05 January 2015 - 27 November 2019)*

GJR GARCH (1,1) - Euro (Sub-sample)	**Normal**	**Student's T**	**GED**
μ	0.0061218 *(0.0118495)*	0.011855 *(0.10835)*	0.0104293 *(0.0100945)*
α	0.039877** *(0.0156583)*	0.012403 *(0.009096)*	0.0111589 *(0.0090099)*
γ	0.2095179*** *(0.0336721)*	0.0344199** *(0.014745)*	0.0338981** *(0.0139728)*
β	0.7324938*** *(0.0339764)*	0.9631929*** *(0.0104952)*	0.9645587*** *(0.0102923)*
α_0	0.0356611*** *(0.0061397)*	0.0022037* *(0.0011693)*	0.0020304* *(0.0011242)*
ML	-1106.145 *(9)*	-1045.2012 *(16)*	-1041.577 *(17)*

APGARCH (1,1) model also one of the applicable models to capture asymmetry as well. This model is considered a close model to TGARCH or GJR - GARCH model, the slight difference is in the variance equation is that the power parameter is added to variance and it is defined by the estimation. Power parameter δ can be any number, however, if it equals to one and two, our model becomes simply standard deviation and variance estimation respectively. To continue, from *Table 7 and 8* displaying USD and Euro returns respectively, it can be interpreted that power term is close to square in USD returns, nevertheless, it is less than one for Euro returns. APARCH (1,1) in-sample and out-of-sample estimations for USD and Euro returns have the same patterns in terms of a significant level of ARCH, GARCH and leverage effect terms except for USD returns in out-of-sample estimation in ARCH term via Student's T distribution which is statistically significant at 1%, nonetheless it is not significant for in-sample one. To continue with power term - δ is almost the same in Euro returns in-sample and out-of-sample estimation via Student's t distribution, in addition to this we could not compare Normal and GED distribution because in out-of-sample estimation they could have not been estimated via maximum likelihood estimation after 193 and 191 iterations respectively (*Tables 19*, *20*). Although power term is significant at 1% level for all distributions and greater than 2 or variance in USD returns full-sample mode, considering Student's t distribution estimation for sub-sample estimation in USD returns, it is not statistically significant. Besides that, it is lesser than standard deviation estimation around 0.98.

Table 7 *(05 January 2015 - 02 December 2019)*

APARCH (1,1) - USD (Sub-sample)	**Normal**	**Student's T**	**GED**
μ	0.0056455 *(0.0104108)*	0.0191642* *(0.0095412)*	0.015102* *(0.0081842)*
α	0.0102738*** *(0.0022965)*	0.1318586*** *(0.0450578)*	0.012934* *(0.007297)*
γ	0.1104538* *(0.647269)*	-0.1156944 *(0.22708)*	0.0515358 *(0.1040058)*
β	0.98818*** *(0.001247)*	0.1215749 *(0.2598157)*	0.9830064*** *(0.0052433)*
α_0	-0.0000754 *(0.0001247)*	0.3789624* *(0.0.2213663)*	-6.99e-07 *(0.0002359)*
δ	2.238915*** *(0.4037644)*	0.9810751 *(0.6572424)*	2.45621** *(0.7335198)*
LM	-888.9034	-865.1198	-814.22

	(40)	*(14)*	*(69)*

Not only the power term, but also ARCH, GARCH and the leverage parameters are statistically significant at 1% level for full-sample estimations of Euro returns by using all kinds of distribution. The power and GARCH term of USD full-sample estimation are statistically significant at 1% level, the leverage parameter is only significant at 10% by accounting Gaussian distribution. ARCH term is statistically significant at 1% and 10% level for Normal and GED estimation in USD returns. *(Tables - 7,8,19, and 20)*

Table 8 *(05 January 2015 - 27 November 2019))*

APARCH (1,1) - Euro (Sub-sample)	Normal	Student's T	GED
μ	.	0.016888 *(0.0108859)*	.
α	.	0.041949*** *(0.0098667)*	.
γ	.	0.748605*** *(0.191209)*	.
β	.	0.9608083*** *(0.0103761)*	.
α_0	.	0.0065175* *(0.00371310*	.
δ	.	0.6914194*** *(0.2836924)*	.
LM	No uphill direction after 193 iterations.	-1043.685 *(22)*	No uphill direction after 191 iterations.

6.2 Forecasting

The following Mean Square Error (MSE) formula is used to compare the final results of different GARCH model forecasting in terms of one-step forward predicted mean.

$$MSE = \frac{1}{T - M} \sum_{t=M}^{T} \left(\mu_j - r_t\right)^2 \qquad (14)$$

For in-sample forecasting, μ is predicted mean, r_i is actual returns, j is the number of models j = 1, ..., 4, M is the endpoint of the first sample used in forecasting, T is the endpoint of the last sample used in estimation and forecasting. In out-of-sample forecasting, M is the endpoint of the last sample used in the estimation and the first sample of forecasting and T is the endpoint of the last forecasting sample. Thus, in-sample forecasting, we include all observations to estimation, and we forecast the last 30 observations, on the other hand, out-of-sample forecasting, we exclude the last 30 observations from estimation, but we include them to forecasting.

MSE, out-of-sample forecasting for USD returns *(Table 9)*, shows that GARCH (1,1) via Gaussian - normal distribution model outperformed all other models, though the Jarque - Bera test statistics denies the existence of normality. As we see from the below table, the GARCH family Gaussian - normal distribution is better than both - Student's t distribution and Generalized Error Distribution (GED). Simultaneously, forecasting the mean by using GED distribution makes a little bit precise prediction compare to Student's t one. When talking about in-sample forecasting, the only difference between in-sample and out-of-sample forecasting is the number of the last 30 observations which excluded from GARCH estimation in the latter one. We would expect the same results for out-of-sample mean forecasting by considering the last 30 observations as less volatile. According to *Table 21 (see Appendix),* GARCH (1,1) via normal distribution carried out better forecasting than the rest models.

Table 9 *(USD MSE - out-of-sample: 05 January 2015 - 02 December 2019)*

Model/ Distribution	Normal	Student's T	GED
GARCH (1,1)	**0.0723503**	0.0742710	0.0740798
EGARCH (1,1)	0.0729290	0.0744223	-

GJR-GARCH (1,1)	0.0725501	0.0743289	0.0740544
APARCH (1,1)	0.0725355	0.0747637	0.0740558

Table 10, being out-of-sample forecasting for Euro returns, shows that GARCH (1,1) via normal distribution performs better than the rest models by all distributions similar to USD in out-of-sample forecasting. However, in this case, we cannot compare models with distributions, for example, GARCH (1,1) via GED gives more precise prediction than EGARCH (1,1) via normal or GARCH (1,1) via Student's t. Unlikely, APARCH (1,1) model cannot be estimated by using Gaussian - normal and GED distribution via maxim likelihood estimation. Here uphill direction has not been found. MSE, in in-sample forecasting for Euro returns (*Table 22)* also illustrates the same patterns where GARCH (1,1) model by Gaussian is the best forecasting tool.

Table 10 *(Euro MSE - out-of- sample: 05 January 2015 - 27 November 2020)*

Model/ Distribution	Normal	Student's T	GED
GARCH (1,1)	**0.0795628**	0.0802352	0.0801087
EGARCH (1,1)	0.0801703	0.0805885	0.0804852
GJR-GARCH (1,1)	0.0800195	0.0804270	0.0803195
APARCH (1,1)	-	0.0808388	-

Above, we have discussed that some models are slightly better in terms of MSE of the predicted means from different models. It is time to test whether there is a significant difference between forecasting of the different models. One of the ways to check it is Diebold - Mariano (1995) test which is given by as follows:

$$DM = \frac{\bar{d}}{\sqrt{\frac{\left(\varphi_0 + 2\sum_{k=1}^{h-1} \varphi_k\right)}{t}}} \qquad (15)$$

Where h equals to $h = t^{\frac{1}{3}} + 1$ (in our sample, it is 4 because t = 30), φ_k is an autocorrelation function at lag k, is given by:

$$\varphi_k = \frac{1}{t} \sum_{t=k+1}^{T} (d_t - \bar{d})(d_{t-k} - d_t) \qquad (16)$$

with t > k ≥ 1 constraint.

When talking about, $\overline{d}$ is the mean, d_t is a loss differential function which we assume as stationary:

$$d_t = \kappa_t^2 - \lambda_t^2 \quad (17)$$

where $\kappa_t = r_t - \mu_i$ (18) and $\lambda_t = r_t - \mu_j$ (19). Here r_t is returns and μ_j and μ_i are forecasted constant means. The number of models is given by $i \neq j = 1,2,3,4$.

Diebold - Mariano test:

H_0: E $[d_t] = 0$,

H_1: E $[d_t] \neq 0$.

DM ~ N (0, 1) is normally distributed and z_{crit} = NORM.S. DIST ($\frac{1-\alpha}{2}$, True) where z_{crit} is the two-tailed critical value for the standard normal distribution. Thus, there is a significant difference between the forecasts if $|DM| > z_{crit}$. Here α = 0.05 or 5%.

The right side of the diagonal is for out-of-sample forecasting, the left side is for in-sample forecasting in both tables. For instance, the intersection of column 3 (GARCH Student's t) and row 2 (GARCH normal) shows the critical value (DM) of the comparison of these two models. Since the critical value of DM equals 1.994 we can reject the null (the critical value of z equals 1.96, in case of α = 0.05, < 1.994) and it means that there is a significant difference between forecasting of these models. As we mentioned, GARCH (1,1) is the best in terms of MSE. The final results of the Diebold-Mariano test display that the superiority of GARCH (1,1) with normal distribution is also statistically significant where all critical values are greater than z_{crit} (1.96)

Table 9.1

Critical Value of DM: USD Out-of-sample Forecasting Comparisons of the Means

	GARCH Normal	GARCH Student's t	GARCH GED	EGARCH Normal	EGARCH Student's t	EGARCH GED	GJRGARCH Normal	GJRGARCH Student's t	GJRGARCH GED	APGARCH Normal	APGARCH Student's t	APGARCH GED
GARCH Normal		1.994	1.994	1.991	1.994	-	1.990	1.994	1.994	1.990	1.995	1.994
GARCH Student's t	1.992		0.003	0.004	1.997	-	0.005	1.997	0.003	0.005	1.997	0.003
GARCH GED	1.993	1.996		0.005	1.997	-	0.006	1.997	0.003	0.006	1.997	0.003
EGARCH Normal	1.987	0.007	0.006		1.995	-	0.008	1.995	1.995	0.008	1.996	1.995
EGARCH Student's t	1.992	1.996	0.003	1.993		-	0.005	0.002	0.003	0.005	1.997	0.003
EGARCH GED	1.992	1.996	0.003	1.993	1.996		-	-	-	-	-	-
GJRGARCH Normal	1.986	0.007	0.007	0.12	0.007	0.007		1.994	1.994	0.009	1.997	1.994
GJRGARCH Student's t	1.992	1.996	0.004	1.993	0.004	0.004	1.992		0.003	0.005	1.997	0.003
GJRGARCH GED	1.992	1.996	0.004	1.993	1.996	0.004	1.993	1.996		0.006	1.997	1.996

APGARCH Normal	1.986	0.007	0.007	0.012	0.007	0.007	0.013	0.008	0.007		1.995	1.994
APGARCH Student's t	1.992	1.996	0.004	1.993	0.004	0.004	1.993	1.996	0.004	1.993		0.003
APGARCH GED	1.992	1.996	0.004	1.993	1.996	0.004	1.993	1.996	1.996	1.993	1.996	
Critical Values of DM: USD In-sample Forecasting comparisons of the Means												

As in USD returns, GARCH (1,1) by Gaussian distribution is the greatest with regards to MSE in Euro returns (*Table 10.1*). Unlikely, here the final outcomes of the Diebold-Mariano test fail to reject the null hypothesise. Therefore, there is no statistical difference forecasting accuracy between GARCH (1,1) by Gaussian and the rest models.

Table 10.1

Critical Values of DM: Euro Out-of-sample Forecasting comparisons the Conditional Variances

	GARCH Normal	GARCH Student's t	GARCH GED	EGARCH Normal	EGARCH Student's t	EGARCH GED	GJRGARCH Normal	GJRGARCH Student's t	GJRGARCH GED	APGARCH Normal	APGARCH Student's t	APGARCH GED
GARCH Normal		1.931	1.9222	1.926	1.950	1.945	1.915	1.942	1.936	-	1.959	-
GARCH Student's t	1.927		0.036	0.034	1.978	1.976	0.040	1.974	1.971	-	1.982	-
GARCH GED	1.921	0.036		1.961	1.975	1.971	0.045	1.970	1.967	-	1.980	-
EGARCH Normal	1.925	0.0343	0.034		1.976	1.974	0.043	1.972	1.969	-	1.981	
EGARCH Student's t	1.947	1.977	1.977	1.976		0.017	0.028	0.018	0.020	-	1.988	-
EGARCH GED	1.945	1.976	1.976	1.975	0.016		0.031	0.020	0.022	-	1.987	-
GJRGARCH Normal	1.908	0.043	0.043	0.045	0.031	0.032		1.967	1.963	-	1.978	-
GJRGARCH Student's t	1.934	1.973	1.972	1.971	0.019	0.020	1.963		0.024	-	1.986	-
GJRGARCH GED	1.935	1.971	1.971	1.969	0.020	0.021	1.961	0.024		-	0.024	-
APGARCH Normal	1.955	1.981	1.981	1.980	1.987	1.986	1.974	1.984	1.983		-	-
APGARCH Student's t	1.953	1.980	0.036	1.979	1.986	1.985	1.973	1.983	1.982	0.011		-
APGARCH GED	1.947	1.977	1.977	1.976	0.016	1.983	1.969	1.981	1.979	0.013	1.986	

Critical Values of DM: Euro In-ample Forecasting comparisons of the Conditional Variances

One-step ahead forecasted conditional variance differs from the forecasted mean, so the former one changes over time while the latter one is constant. In order to compare which model captures the best volatility, we need to use MSE like the forecasted mean. MSE for modeling volatility is going to be similar to the mean MSE as follow:

$$MSE = \frac{1}{T - M} \sum_{t=M}^{T} (r_t^2 - \sigma_{tj}^2)^2 \tag{20}$$

where r_t^2 is actual variance and σ_{tj}^2 is one-step forward forecasted variance which is gotten from estimation. The rest T, M and J are the same in equation (14).

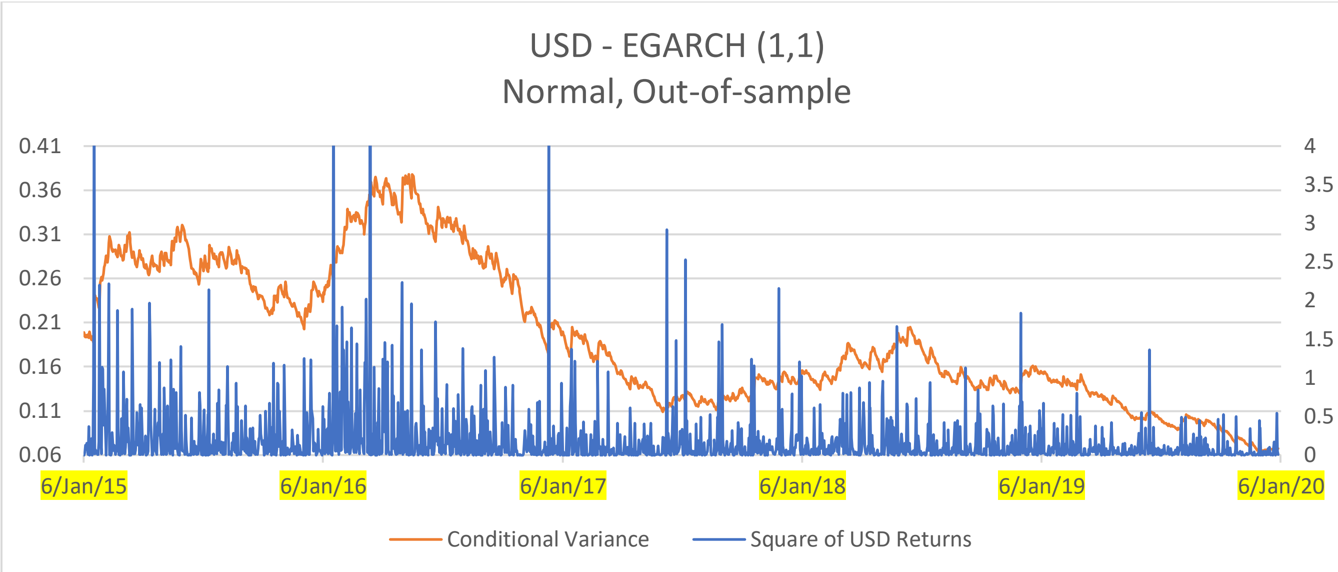
Table 11 is the final results of equation (20), as the table illustrates EGARCH (1,1) by normal distribution has the smallest value which makes it the best tool to predict the volatility in USD logarithmic returns out-of-sample forecast. When we compare the distributions, the models via normal distribution unexpectedly shows better prediction than Student's t and GED. It is a surprise since our logarithmic returns are leptokurtic. The same results were shown by in-sample forecasting of USD returns where EGARCH (1,1) via Gaussian outperformed the rest models via all distributions (*Table 23, see Appendix)*. It is the same results with Lee's (1991) conclusion that the linear GARCH models cannot generally outperform the nonlinear models in the RMSE or MSE criterion where one of the currencies he used was Canadian Dollar. On the other hand, David McMillan et al (2004) forecasted conditional variance by using different GARCH models where they found that CAD/USD logarithmic returns volatility had been forecasted better by GJR-GARCH model. Nonetheless, in my paper GJR-GARCH (1,1) model cannot outperform the rest models.

Table 11 *(USD MSE - out-of-sample: 05 January 2015 - 02 January 2020- The conditional Variance)*

Model/ Distribution	Normal	Student's T	GED
GARCH (1,1)	0.0181003	0.0186315	0.0344241
EGARCH (1,1)	**0.0178921**	0.0180131	-
GJR-GARCH (1,1)	0.018699	0.0197242	0.0193992
APGARCH (1,1)	0.0181977	0.0421658	0.0186415

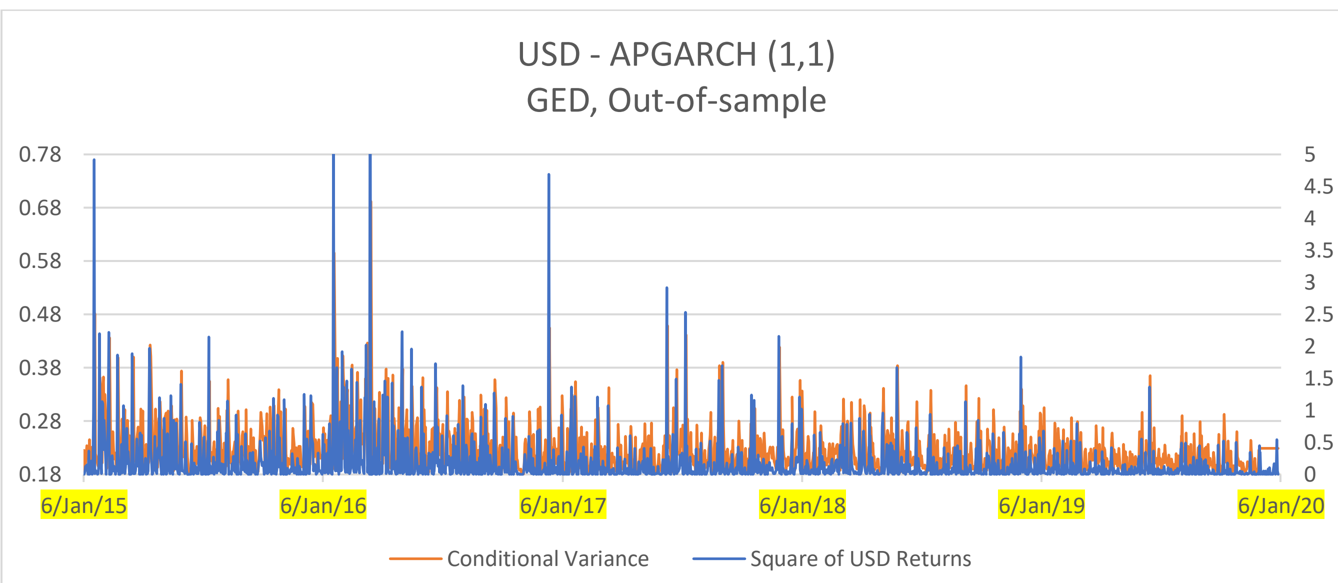
Time series line 4 shows USD squared returns, and one-step forward predicted conditional variance. More volatile periods are subjected to higher variance when logarithmic returns demonstrate less volatility the conditional variance is going to be closer to zero.

Time series line 4 *(05 January 2015 - 02 January 2020 right vertical axis for returns square, left one for conditional variance)*



Time series line 4.1 graph consists of the conditional variance of APGARCH (1,1) with GED distribution and USD returns square. Although here MSE is lesser than EGARCH (1,1) with Gaussian normal distribution, visually the latter one seems more precise when we compare graphs.

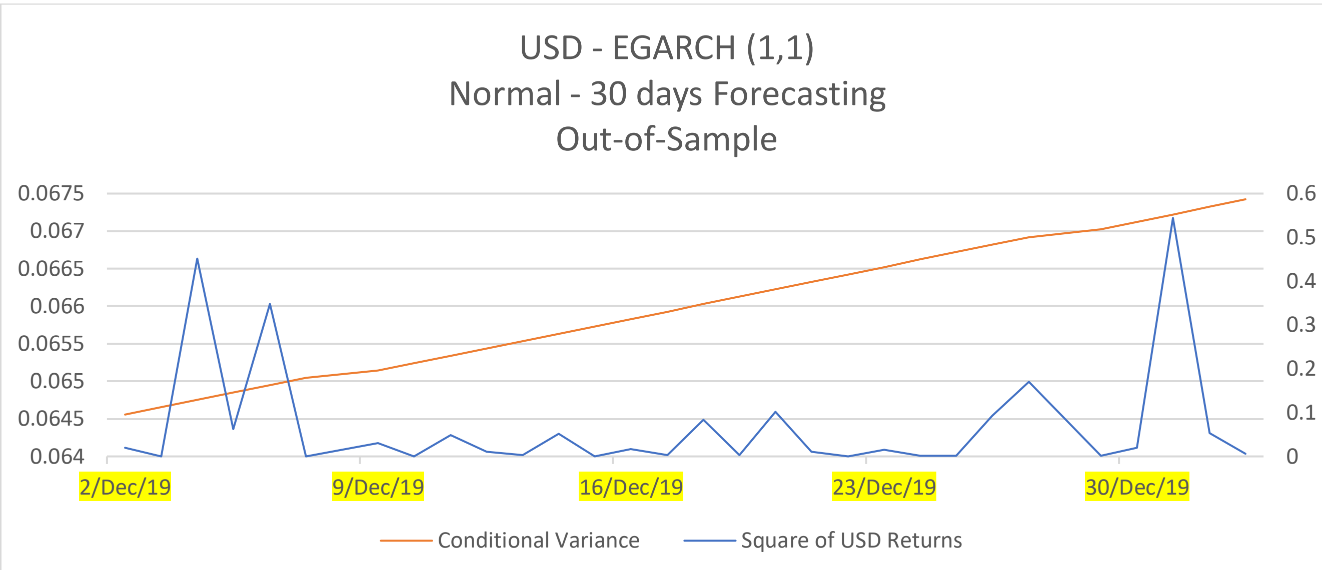
Time series line 4.1 *(05 January 2015 - 02 January 2020 right vertical axis for returns square, left one for conditional variance)*



The last 30 days forecasted conditional variance against actual returns squares are compared in *Time series line 4.2*. From the graph, it is difficult to say whether the model

(EGARCH (1,1) by Gaussian) with the most accurate MSE can capture volatility precisely or not. The conditional variance increases slowly by the time; however, the squares of the returns perform volatile patterns in the beginning and the end of time.

Time series line 4.2 *(05 January 2015 - 02 January 2020 right vertical axis for returns square, left one for conditional variance)*



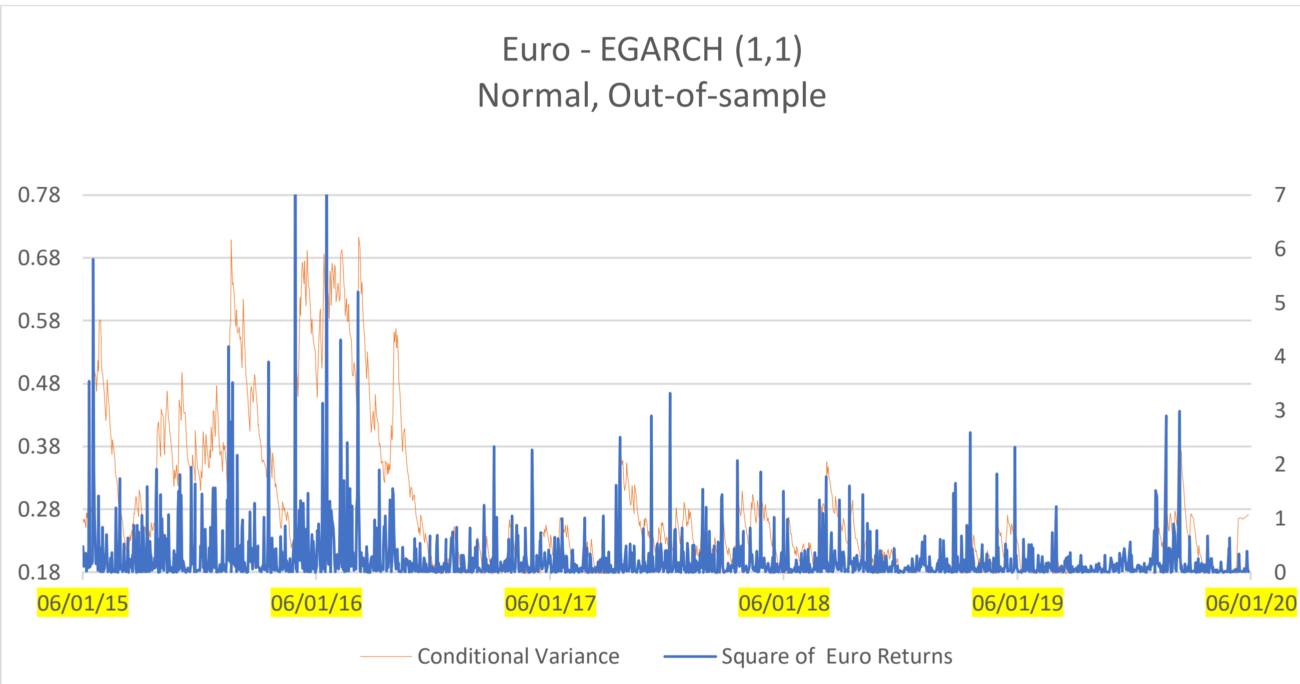
When we are talking about forecasting logarithmic Euro returns in out-of-sample prediction, EGARCH (1,1) via Gaussian distribution has the smallest MSE value which makes it superior to the rest models *(Table 12)*. However, in the in-sample forecasting of Euro returns (*Table 24, see Appendix*) APGARCH (1,1) by GED distribution is the best tool for prediction volatility.

Table 12 *(Euro MSE - out-of- sample: 05 January 2015 - 27 November 2020 The Conditional Variance)*

Model/ Distribution	Normal	Student's T	GED
GARCH (1,1)	0.0428161	0.0436527	0.0429289
EGARCH (1,1)	**0.0407045**	0.0412281	0.0408243
GJR-GARCH (1,1)	0.08908	0.0516731	0.0507574
APGARCH (1,1)	-	0.0439749	-

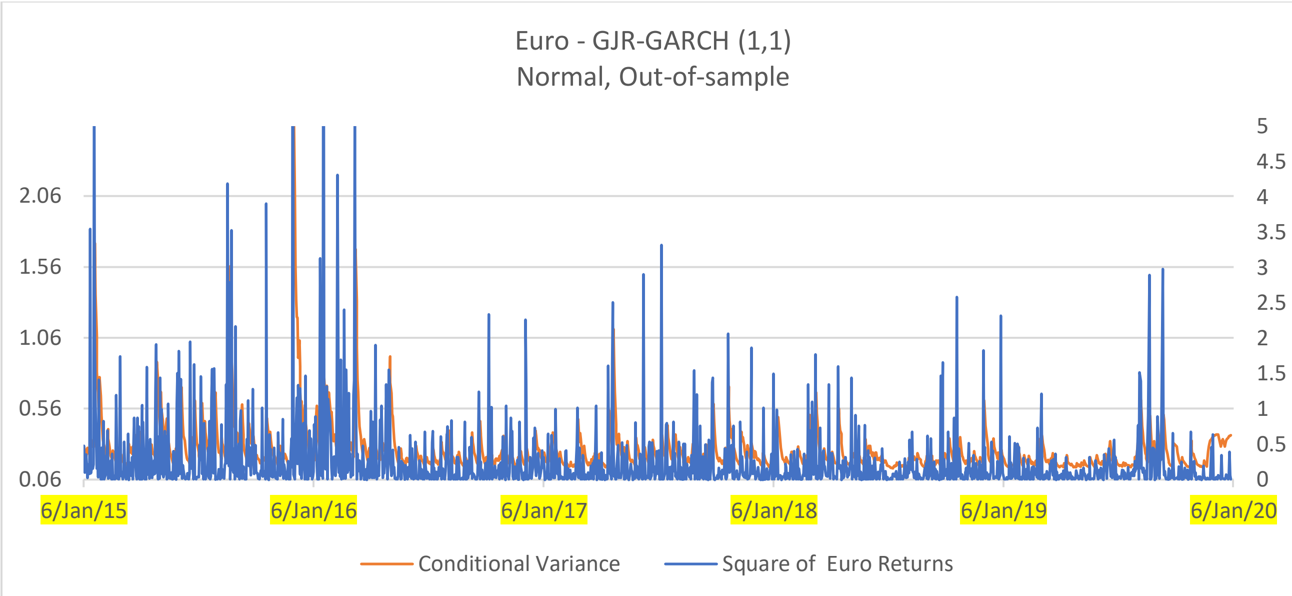
Time series line 5 clearly shows squares of Euro returns and its conditional variance which was gotten from EGARCH (1,1) via Gaussian Normal distribution estimation. From *Table 12*, this model has the smallest MSE. According to the graph, in the beginning, high squares of the returns is predicted by high conditional variance. When squares of Euro returns are less volatile, the conditional variance is close to zero and smaller fluctuations.

Time series line 5 *(05 January 2015 - 02 January 2020, right vertical axis for square of the returns, left one for conditional variance)*



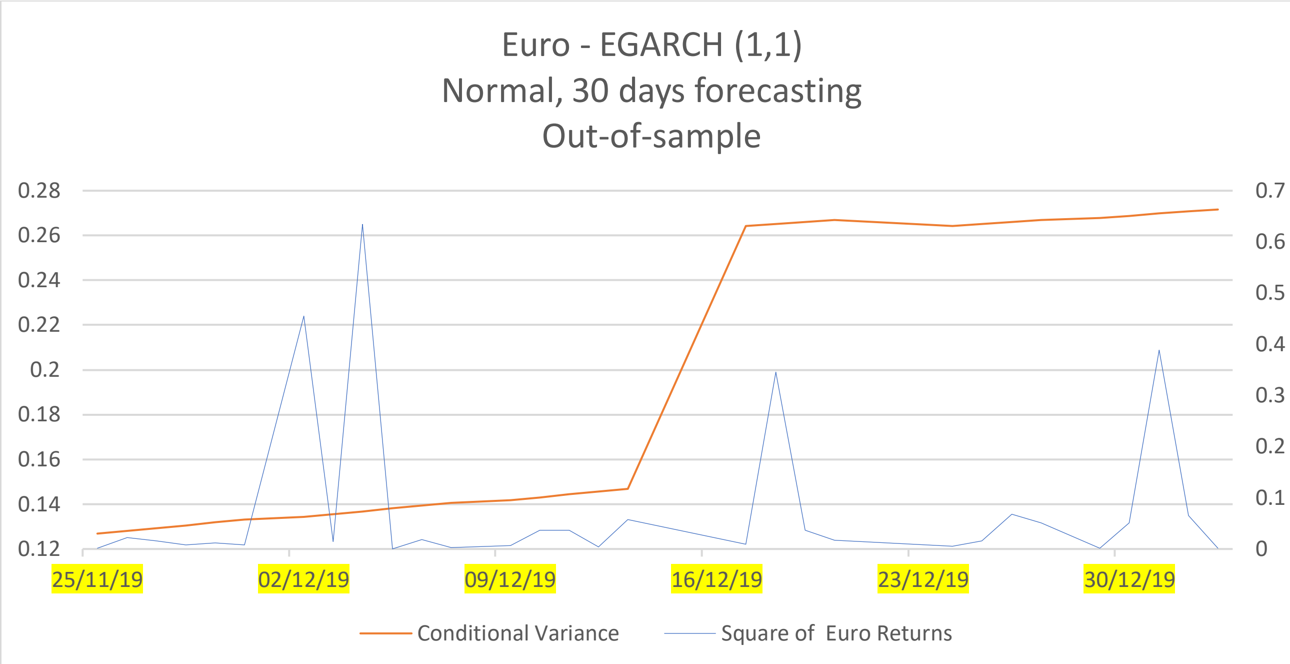
Time series line 5.1 gives information about out-of-sample forecasting of Euro returns by GJR-GARCH (1,1) model where Gaussian normal distribution was considered for error residuals. From first glance, the conditional variance coheres with the square of returns. Thus, similar to other models instabilities cause the conditional variance to be more explosive.

Time series line 5.1 *(05 January 2015 - 02 January 2020, right vertical axis for square of the returns, left one for conditional variance*



EGARCH (1,1) by normal distribution possesses the smallest MSE for the last 30 days in terms of the conditional variance. *Time series line 5.2* illustrates 30 days one-step forecasted conditional variance. In this graph, it is compared with the squares of the last 30 days of Euro returns. In this situation, it is hard to say that the conditional variance can predict the volatility future returns. The conditional variance has a gentle increasing tendency except for 13 December 2019 when the sharp rising is observed. On the contrary, the square of the Euro returns shows sharp hikes.

Time series line 5.2 *(05 January 2015 - 02 January 2020, right vertical axis for square of the returns, left one for conditional variance*



Diebold-Mariano test is also implemented for comparing of forecasting accuracy of the conditional variances as well. The equations (15), (16) and (17) are still the same, nevertheless (18) and (19) have adjusted to the conditional variances. Thus, latter ones will alter as follows:

$$\kappa_t = r_t - \sigma_{ti}$$

$$\lambda_t = r_t - \sigma_{tj}$$

where σ_{ti} and σ_{tj} are the conditional variances with $i \neq j = 1,2,3,4$.

The final results of Diebold - Mariano test is demonstrated in *Table 11.1 and 12.1* for USD and Euro returns respectively. In the previous paragraphs, we discussed that EGARCH (1,1) by Gaussian is superior to other models in terms of MSE. Let's give a close look whether the forecasting of this model differs from others with regards to statistical significance. *Table 11.1* illuminates that it is not statistically significant only against EGARCH (1,1) with Student's t and GED. In the rest circumstances, the forecasting accuracy of this model is significant. Moreover, if we look through *Table 11* and *Table 11.1*, without any forecasting significance test, MSE of APGARCH (1,1) via Student's t distribution differs from others considerably by having the biggest MSE which also shows itself in Diebold-Mariano test statistics. All critical values equal to approximately 2 which enables us to reject the null hypothesis. Therefore, APGARCH (1,1) by Student's t distribution is the worst forecasting model for USD returns.

Table 11.1

Critical Value of DM: USD Out-of-sample Forecasting Comparisons of the Conditional Variances

	GARCH Normal	GARCH Student's t	GARCH GED	EGARCH Normal	EGARCH Student's t	EGARCH GED	GJRGARCH Normal	GJRGARCH Student's t	GJRGARCH GED	APGARCH Normal	APGARCH Student's t	APGARCH GED
GARCH Normal		1.944	2.000	0.076	0.021	-	1.986	1.976	1.984	2.000	2.000	1.992
GARCH Student's t	0.007		2.000	0.066	0.032	-	2.000	1.987	1.997	0.130	2.000	1.990
GARCH GED	2.000	2.000		0.000	0.000	-	0.000	0.000	0.000	0.000	2.000	0.000
EGARCH Normal	1.152	1.978	0.000		0.000	-	1.964	1.962	1.968	1.956	2.000	1.967
EGARCH Student's t	0.003	0.018	0.000	0.000		-	1.983	1.977	1.982	1.987	2.000	1.986
EGARCH GED	0.005	0.106	0.000	0.000	2.000		-	-	-	-	-	-
GJRGARCH Normal	2.000	2.000	0.000	1.962	2.000	2.00		1.957	1.977	0.025	2.000	0.093
GJRGARCH Student's t	0.581	2.000	0.000	0.556	2.000	1.998	0.005		0.125	0.034	2.000	0.047
GJRGARCH GED	1.988	2.000	0.000	1.710	2.000	2.000	0.136	1.998		0.024	2.000	0.030
APGARCH Normal	1.999	2.000	0.000	1.943	2.000	2.000	0.214	1.989	1.780		2.000	1.983
APGARCH Student's t	1.704	2.000	0.000	1.379	2.000	2.000	0.049	1.999	0.027	0.082		0.000
APGARCH GED	2.000	2.000	0.000	1.852	2.000	2.000	0.770	1.997	1.987	0.937	1.988	

Critical Values of DM: USD In-sample Forecasting comparisons of the Conditional Variances

EGARCH (1,1) with Normal distribution has the least forecasting MSE in Euro returns. Nonetheless, the forecasting accuracy test shows that this is not statistically significant with compare to GACRH (1,1) models via all distributions. Hence, we cannot say that EGACH (1,1) by Student's t is the best tool to forecast Euro returns.

Table 12.1

Critical Values of DM: Euro Out-of-sample Forecasting comparisons the Conditional Variances

	GARCH Normal	GARCH Student's t	GARCH GED	EGARCH Normal	EGARCH Student's t	EGARCH GED	GJRGARCH Normal	GJRGARCH Student's t	GJRGARCH GED	APGARCH Normal	APGARCH Student's t	APGARCH GED
GARCH Normal		2.000	1.811	0.218	0.385	0.253	2.000	2.000	2.000	-	1.778	-
GARCH Student's t	1.999		0.000	0.118	0.217	0.139	2.000	2.000	2.000	-	1.216	-
GARCH GED	0.538	0.0000		0.375	0.250	2.000	2.000	2.000	2.000	-	1.706	
EGARCH Normal	0.011	0.000	0.010		2.000	1.972	2.000	2.000	2.000	-	1.988	
EGARCH Student's t	0.022	0.002	0.020	1.837		0.000	2.000	2.000	2.000	-	1.966	-
EGARCH GED	0.008	0.000	0.007	0.003	0.000		2.000	2.000	2.000	-	1.966	-
GJRGARCH Normal	1.341	1.271	1.350	1.546	1.541	1.569		0.000	0.000	-	0.000	-
GJRGARCH Student's t	2.000	2.000	2.000	2.000	2.000	2.000	0.861		0.000	-	0.000	-
GJRGARCH GED	2.000	1.938	2.000	2.000	2.000	2.000	0.793	0.000		-	0.000	-
APGARCH Normal	0.038	0.011	0.046	1.220	1.019	1.557	0.483	0.000	0.000		-	-
APGARCH Student's t	0.021	0.003	0.020	0.846	0.392	1.848	0.451	0.000	0.000	0.044		-
APGARCH GED	0.010	0.002	0.010	0.231	0.152	0.770	0.434	0.000	0.000	0.060	1.956	

Critical Values of DM: Euro In-ample Forecasting comparisons of the Conditional Variances

7. Conclusion

This paper has examined the daily exchange rates of the Canadian Dollar against the United State Dollar and Euro. The data is from 05 January 2015 to 02 January 2020. In sub-sample estimation, 30 days are excluded from estimation. The Canadian Dollar exchange rates are forecasted and modeled in terms of the mean equation and the conditional variance. First, the forecasted mean is predicted better by GARCH (1,1) model via Gaussian distribution in USD out-of-sample forecasting. GARCH (1,1) via Gaussian outperformed the rest models and it was proven by the Diebold-Marian test as well. Second, the conditional variance analysis results from the Diebold-Mariano test shows that the models with the least MSE are not statistically significant with compare to all models. Thus, none model outperformed all models. Nonetheless, APGARCH (1,1) via Student's t distribution in USD returns having the biggest MSE, was confirmed as the worst model in forecasting of out-of-sample conditional variance in USD returns.

The leverage parameters from asymmetric and nonlinear GARCH models have demonstrated that there is a leverage effect. Hence, negative shocks cause a higher next period conditional variance than positive shocks of the same magnitude in both in-sample and out-of-sample forecasting.

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10. Appendix

Additional graphs and tables:

Modeling volatility.

Full-sample results:

Table 13 *(05 January 2015 - 02 January 2020)*

GARCH (1,1)-USD (Full-Sample)	**Normal**	**Student's t**	**GED**
$\boldsymbol{\mu}$	0.0009415 *(0.0100308)*	0.0130743 *(0.0089715)*	0.0151181* *(0.0079443)*
$\boldsymbol{\alpha}$	0.0114151*** *(0.0015489)*	0.0187206*** *(0.0055156)*	0.1174958*** *(0.0079443)*
$\boldsymbol{\beta}$	0.9882049*** *(0.0016568)*	0.982486*** *(0.0051187)*	0.0869251 *(0.0478474)*
$\boldsymbol{\alpha_0}$	-0.0000421 *(0.0001314)*	-0.00003015 *(0.0003015)*	0.1589876*** *(0.0478474)*
ML	-893.6666 *(11[1])*	-829.6459 *(13)*	-851.3236 *(8)*

Table 14 *(05 January 2015 - 02 January 2020)*

GARCH (1,1) - Euro (Full-sample)	**Normal**	**Student's t**	**GED**
μ	-0.0022477 *(0.0114255)*	0.0090093 *(0.0106477)*	0.0077093 *(0.0098306)*
α	0.0315263*** *(0.0051383)*	0.0334757*** *(0.0095045)*	0.0324306*** *(0.0095344)*
β	0.9621037*** *(0.0055336)*	0.9600939*** *(0.0113992)*	0.9598902*** *(0.01162590*
α_0	0.001709*** *(0.0006296)*	0.0021534 *(0.001341)*	0.0020321 *(0.0013185)*
ML	-1112.497 *(10)*	-1053.867 *(11)*	-1050.13 *(11)*

Table 15 *(05 January 2015 - 02 January 2020)*

EGARCH (1,1)- USD (Full-sample)	**Normal**	**Student's t**	**GED**
μ	0.0037952 *(0.0103233)*	0.0139421 *(0.0090188)*	0.0148222* *(0.0079963)*
α	0.0113466*** *(0.0034817)*	0.0153091 *(0.0099896)*	0.0133995 *(0.0096522)*
γ	0.0252236*** *(0.0032308)*	0.479682*** *(0.0134511)*	0.0435496*** *(0.0126244)*
β	0.0252236*** *(0.0032308)*	0.9999473*** *(0.0021747)*	0.9985477*** *(0.0025402)*
α_0	0.000778 *(0.0013353)*	0.0028567 *(0.0038907)*	0.0001516 *(0.0041579)*
ML	-894.0778 *(31)*	-830.0044 *(31)*	-816.1488 *(23)*

Table 16 *(05 January 2015 - 02 January 2020)*

EGARCH (1,1)-Euro (Full-sample)	Normal	Student's t	GED
μ	0.0083781 *(0.0117201)*	0.0136594 *(0.0106444)*	0.0131231 *(0.0099121)*
α	0.0489731*** *(0.0079731)*	0.0490593*** *(0.0150155)*	0.04878*** *(0.0150395)*
γ	0.0824438*** *(0.0129333)*	0.0876261*** *(0.0216397)*	0.0853357*** *(0.0225088)*
β	0.9915903*** *(0.0029978)*	0.9906501*** *(0.005265)*	0.9907136*** *(0.005539)*
α_0	-0.0075269* *(0.0040527)*	-0.0067185 *(0.005265)*	-0.0080656 *(0.007803)*
ML	-1101.329 *(33)*	-1049.961 *(23)*	-1045.295 *(54)*

Table 17 *(05 January 2015 - 02 January 2020)*

GJR-GARCH (1,1)-USD (Full-sample)	Normal	Student's t	GED
μ	0.0023906 *(0.0102657)*	0.0134651 *(0.0090223)*	0.0145818* *(0.0079874)*
α	0.008592*** *(0.0018958)*	0.0157065*** *(0.005871)*	0.0138548** *(0.0071534)*
γ	0.0070825** *(0.0035068)*	0.0082441 *(0.0098661)*	0.0071534 *(0.0091461)*
β	0.9874084*** *(0.0017971)*	0.9814679*** *(0.0057227)*	0.9820024*** *(0.0058373)*
α_0	0.0000119 *(0.0001463)*	0.0000176 *(0.0003318)*	0.0001167 *(0.0003701)*
ML	-892.6842 *(12)*	-829.2639 *(9)*	-815.5628 *(15)*

Table 18 *(05 January 2015 - 02 January 2020)*

GJR-GARCH (1,1)-Euro (Full-sample)	Normal	Student's T	GED
μ	0.005238 *(0.0116431)*	0.0112767 *(0.0106592)*	0.0105524 *(0.0098831)*
α	0.0413724*** *(0.0155112)*	0.0170111* *(0.0101574)*	0.01578 *(0.0100104)*
γ	0.2110578*** *(0.0332763)*	0.0352149** *(0.0160472)*	0.0349037** *(0.0154728)*
β	0.7344472*** *(0.0330472)*	0.9597594*** *(0.0110804)*	0.9600364*** *(0.0112185)*
α_0	0.0342762*** *(0.0058647)*	0.0022053* *(0.0012656)*	0.0020754* *(0.001244)*
ML	-1115.107 *(9)*	-1051.121 *(17)*	-1047.146 *(17)*

Table 19 *(05 January 2015 - 02 January 2020)*

APARCH (1,1) - USD (Full-sample)	Normal	Student's T	GED
μ	0.0022669 *(0.0102692)*	0.0137684 *(0.0090131)*	0.0146329* *(0.0079775)*
α	0.0101268*** *(0.0023503)*	0.0124323 *(0.007577)*	0.0118604* *(0.0070995)*
γ	0.1112849* *(0.0641896)*	0.0371276 *(0.0878691)*	0.0449161 *(0.0051858)*
β	0.9877418*** *(0.0018812)*	0.9824538*** *(0.0051067)*	0.9831321*** *(0.0051858)*
α_0	-0.0000124 *(0.0001061)*	-0.0000343 *(0.0001454)*	0.0000284 *(0.0001876)*
δ	2.2623*** *(0.4093312)*	2.714718*** *(0.695353)*	2.555504*** *(0.7524095)*
ML	-892.7185 *(36)*	-829.0345 *(37)*	-815.4831 *(53)*

Table 20 *(05 January 2015 - 02 January 2020)*

APGARCH (1,1) - Euro (Full-sample)	**Normal**	**Student's T**	**GED**
μ	0.015904 *(0.0114107)*	0.0154316 *(0.0107449)*	0.0136424 *(0.0099789)*
α	0.0439776*** *(0.006184)*	0.0465074*** *(0.0108962)*	0.0453379*** *(0.0110951)*
γ	0.7705372*** *(0.1108408)*	0.6594358*** *(0.2054435)*	0.6942246*** *(0.2061667)*
β	0.9592976*** *(0.0065557)*	0.9575593*** *(0.011096)*	0.9580338*** *(0.0116934)*
α_0	0.0070104** *(0.0027873)*	0.0059479 *(0.0036516)*	0.006284 *(0.0041083)*
δ	0.5992976*** *(0.140715)*	0.7891935*** *(0.3214723)*	0.7194167*** *(0.2932009)*
LM	-1099.561 *(13)*	-1049.735 *(22)*	-1044.855 *(22)*

MSE for the mean: in-sample forecasting.

Table 21 *(USD MSE: in-sample, 05 January 2015 - 02 January 2020 - The mean)*

Model/Distribution	Normal	Student's t	GED
GARCH (1,1)	0.0718459	0.0737147	0.0740585
EGARCH (1,1)	0.072259	0.0738597	0.0740082
GJR-GARCH (1,1)	0.0720536	0.0737798	0.0739675
APARCH (1,1)	0.0720357	0.0738305	0.0739761

Table 22 *(Euro MSE: in-sample, 05 January 2015 - 02 January 2020- The mean)*

Model/Distribution	Normal	Student's t	GED
GARCH (1,1)	0.0795428	0.0802165	0.0801258
EGARCH (1,1)	0.0801721	0.0805688	0.0805260
GJR-GARCH (1,1)	0.0799626	0.0803829	0.0803286
APARCH (1,1)	0.0807543	0.0807145	0.0805675

MSE for the conditional variance analysis: in-sample forecasting

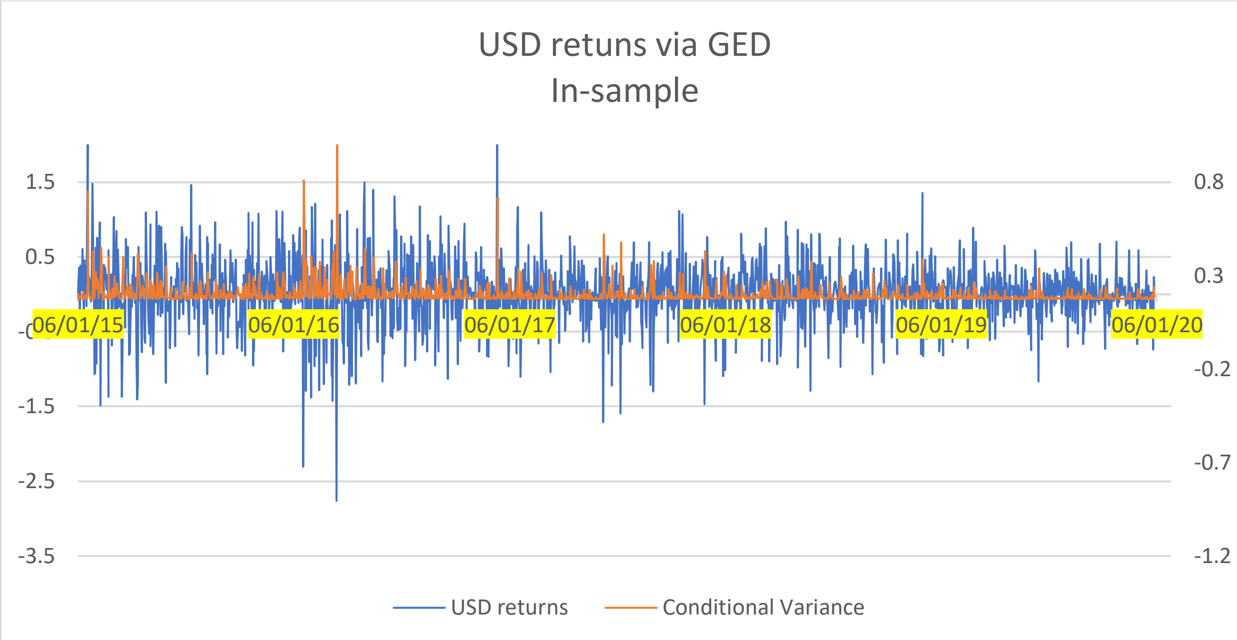
Table 23 *(USD MSE: in-sample, 05 January 2015 - 02 January 2020*
The conditional Variance)

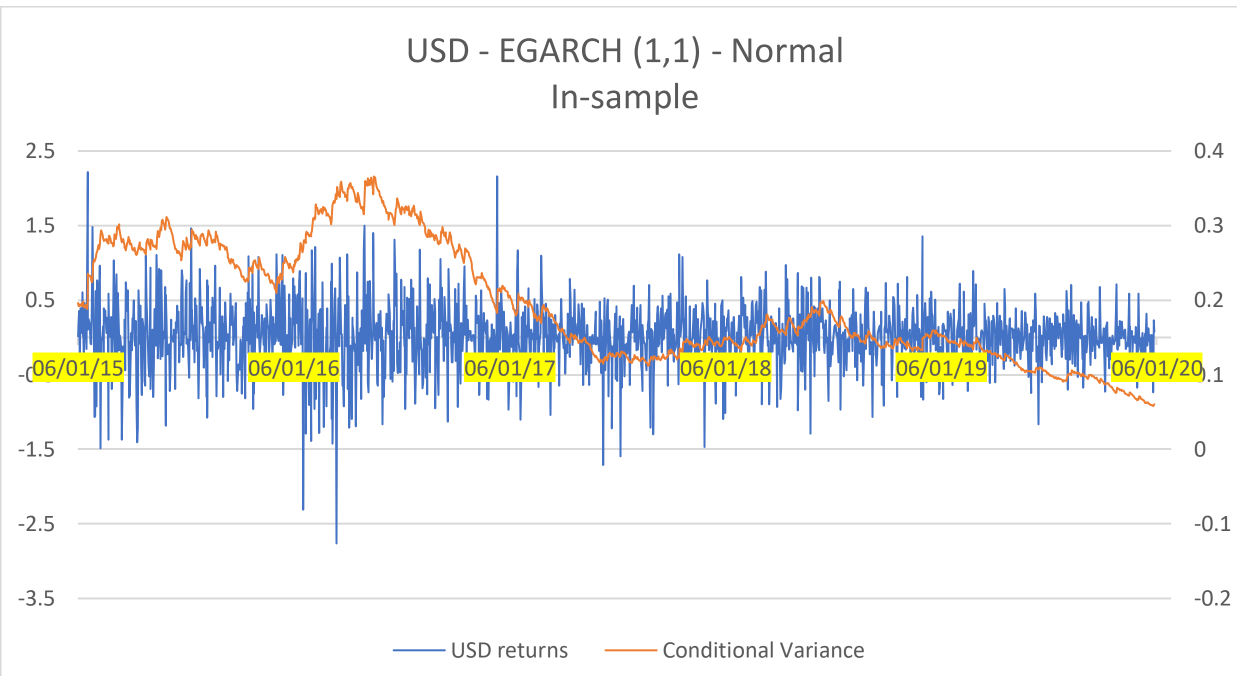
Model/Distribution	Normal	Student's t	GED
GARCH (1,1)	0.0181076	0.0182587	0.0309373
EGARCH (1,1)	**0.0180844**	0.0183426	0.0182715
GJR-GARCH (1,1)	0.0181085	0.0182636	0.0181977
APGARCH (1,1)	0.0180992	0.0182102	0.0181564

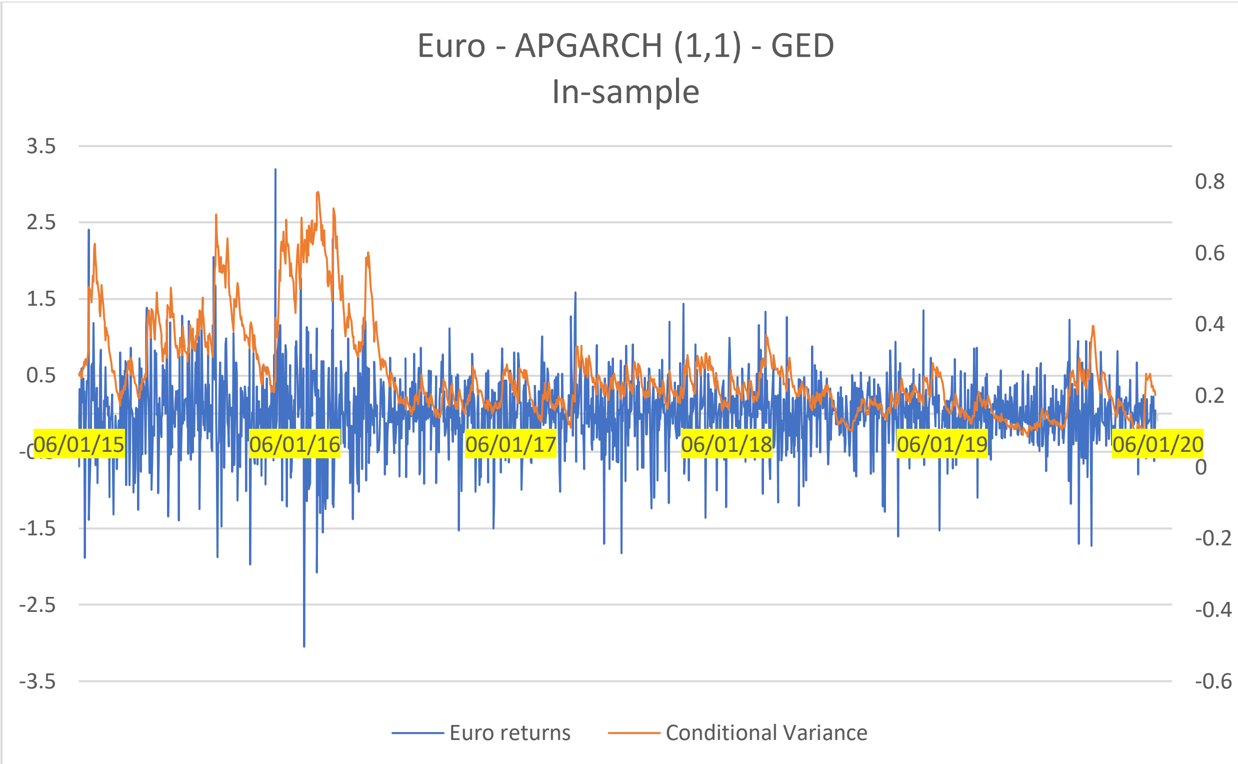
Table 24 *(Euro MSE: in-sample, 05 January 2015 - 02 January 2020*
The conditional Variance)

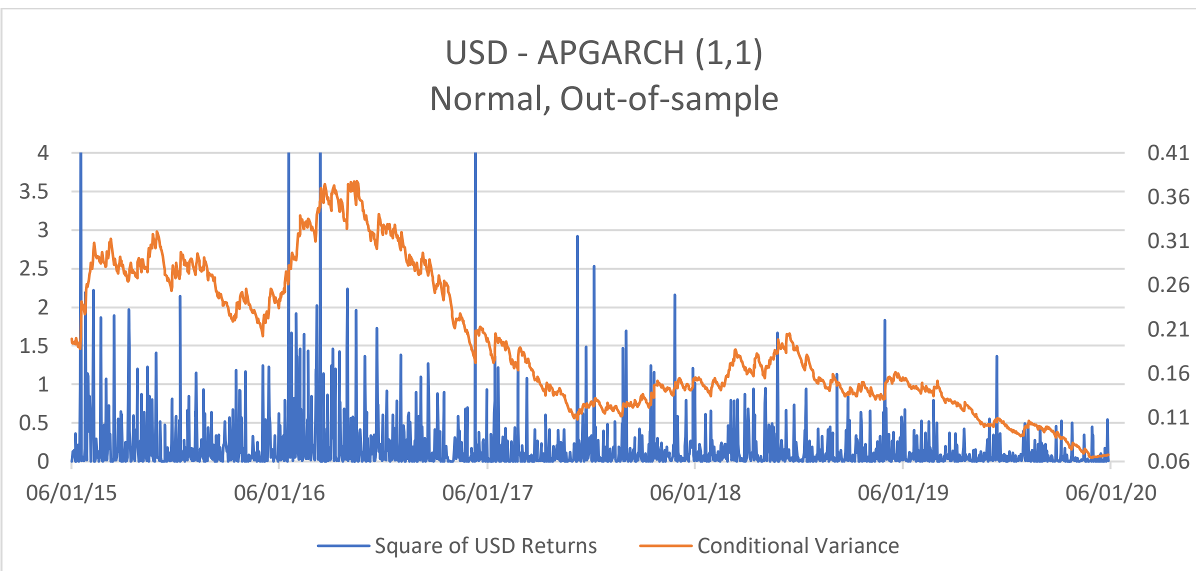
Model/Distribution	Normal	Student's t	GED
GARCH (1,1)	0.0371734	0.0376777	0.0372054
EGARCH (1,1)	0.0350696	0.0352086	0.0350284
GJR-GARCH (1,1)	0.0371484	0.0377392	0.0373534
APGARCH (1,1)	0.0347516	0.0349437	**0.0347349**

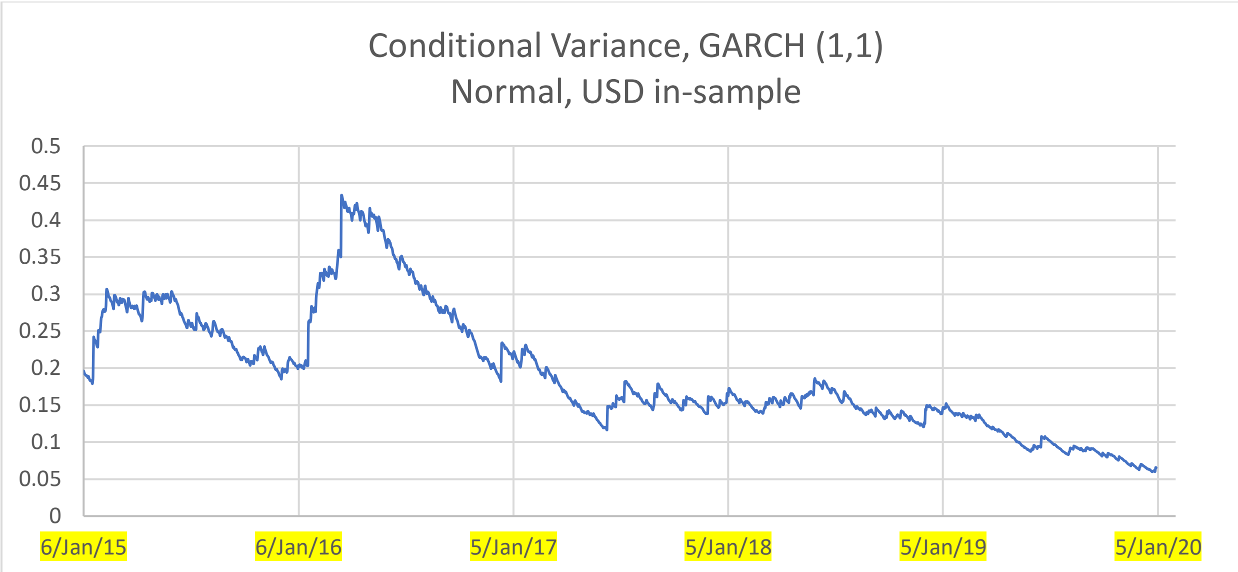
Time series line 6 *(05 January 2015 - 02 January 2020)*

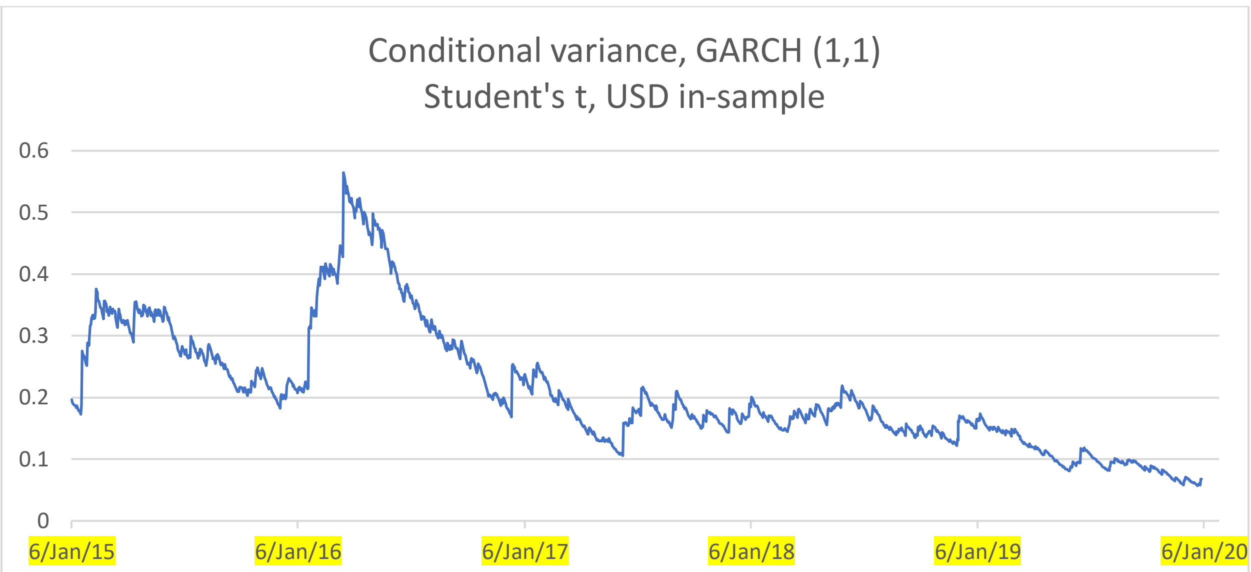

USD retuns via GED
In-sample
1.5
0.5
-1.5
-2.5
-3.5
0.8
0.3
-0.2
-0.7
-1.2
06/01/15
06/01/16
06/01/17
06/01/18
06/01/19
06/01/20
USD returns
Conditional Variance

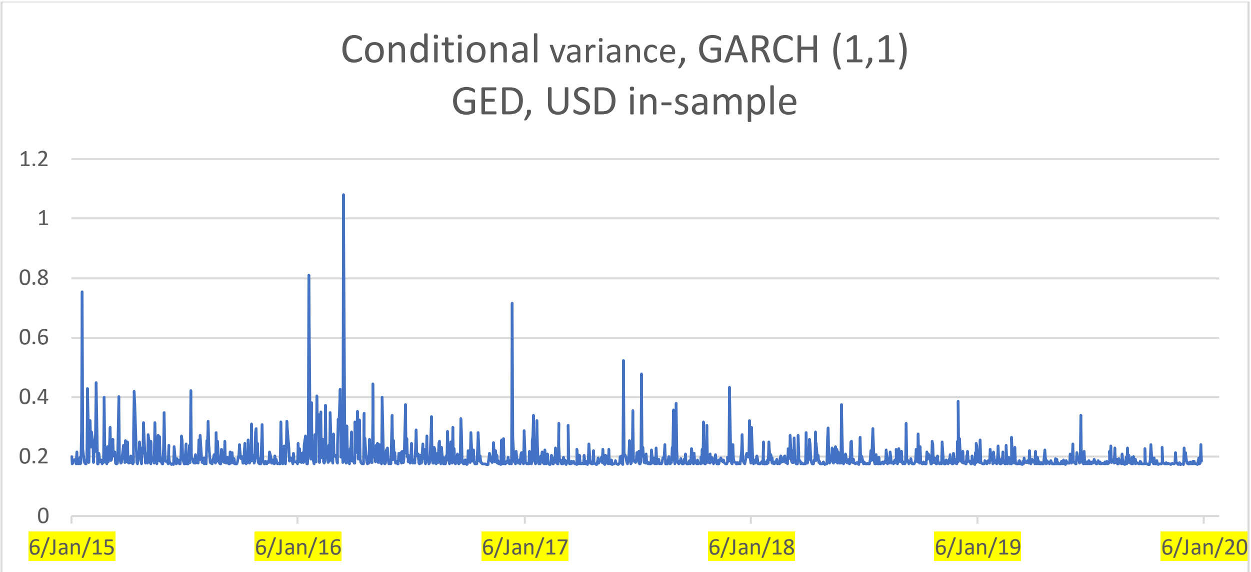

USD - EGARCH (1,1) - Normal
In-sample
2.5
1.5
0.5
-1.5
-2.5
-3.5
0.4
0.3
0.2
0
-0.1
-0.2
06/01/15
06/01/16
06/01/17
06/01/18
06/01/19
06/01/20
USD returns
Conditional Variance

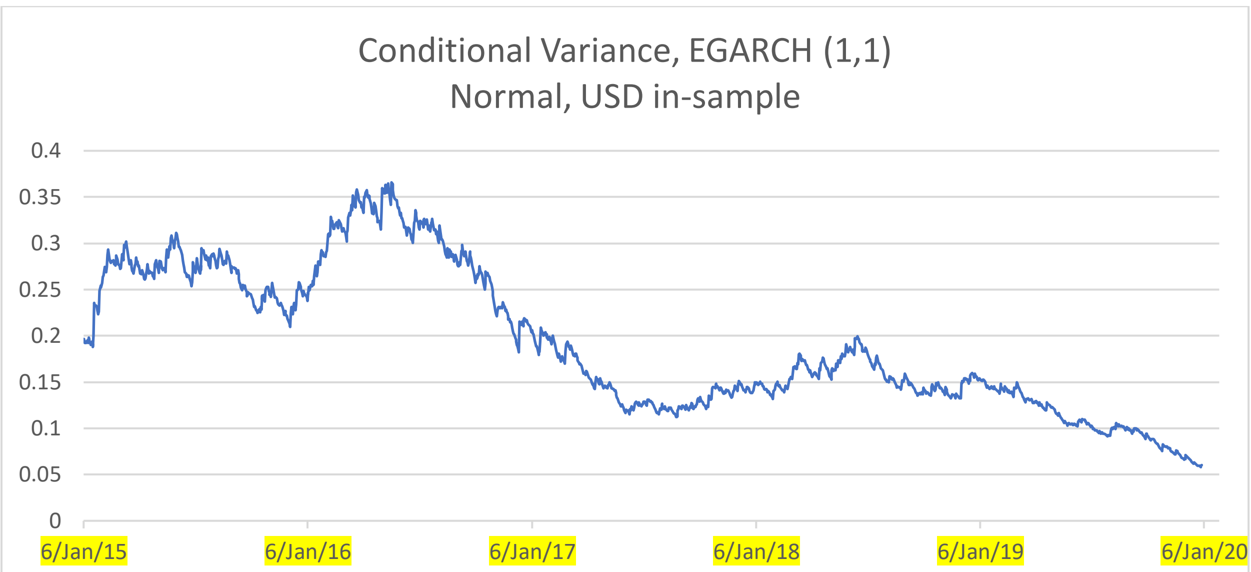

Euro - APGARCH (1,1) - GED
In-sample
3.5
2.5
1.5
0.5
-1.5
-2.5
-3.5
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
06/01/15
06/01/16
06/01/17
06/01/18
06/01/19
06/01/20
Euro returns
Conditional Variance

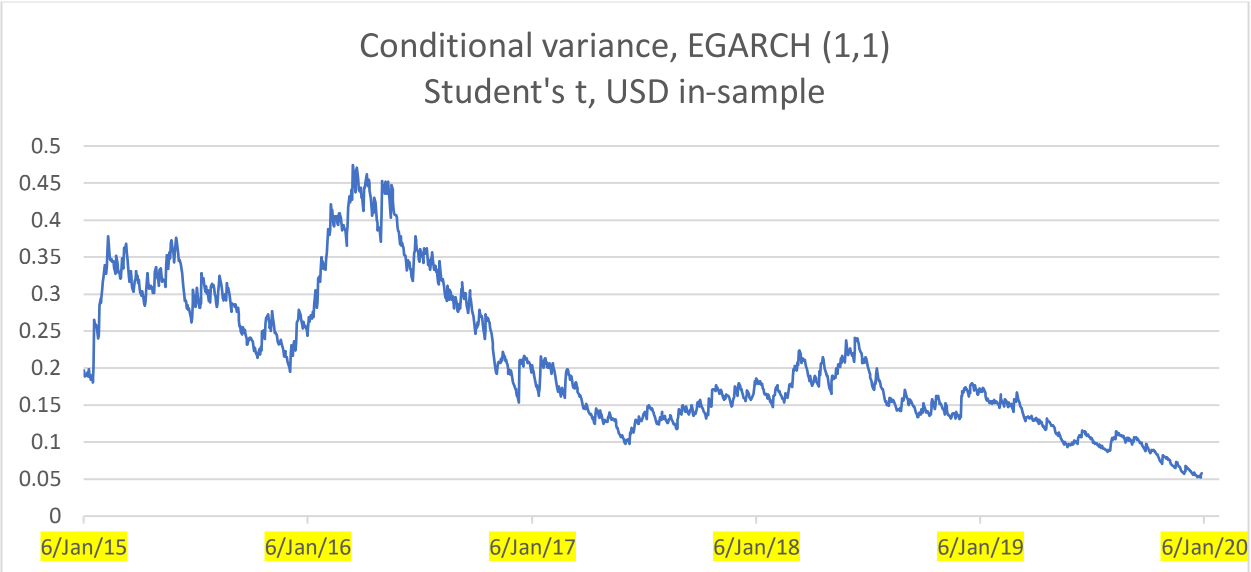

USD - APGARCH (1,1)
Normal, Out-of-sample
4
3.5
3
2.5
2
1.5
1
0.5
0
0.41
0.36
0.31
0.26
0.21
0.16
0.11
0.06
06/01/15
06/01/16
06/01/17
06/01/18
06/01/19
06/01/20
Square of USD Returns
Conditional Variance

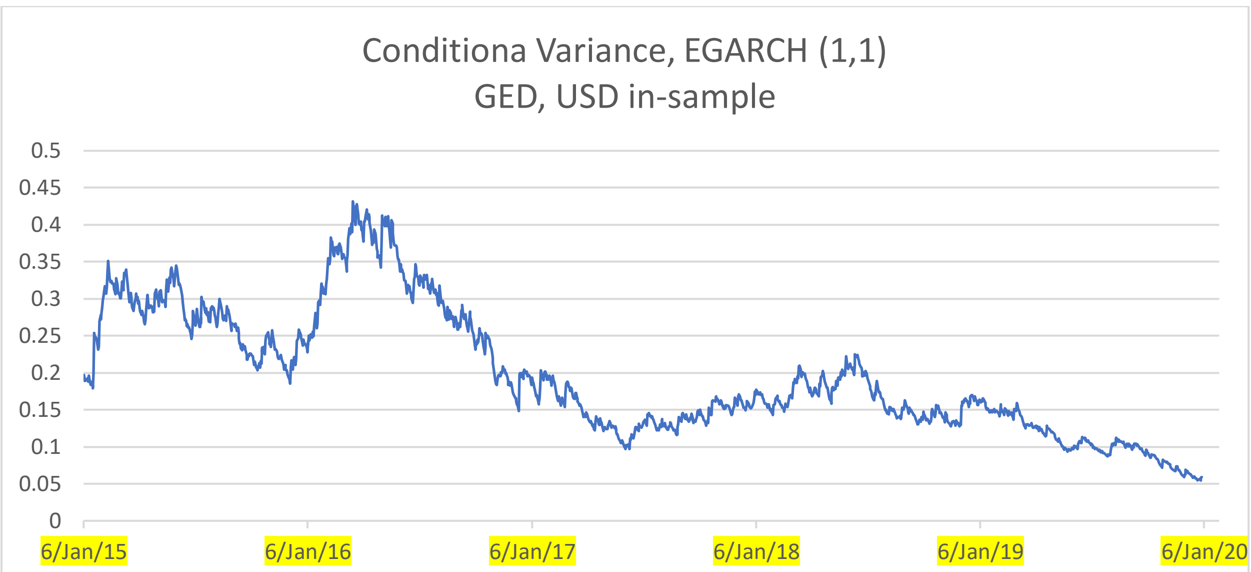

Conditional Variance, GARCH (1,1)
Normal, USD in-sample
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
6/Jan/15
6/Jan/16
5/Jan/17
5/Jan/18
5/Jan/19
5/Jan/20

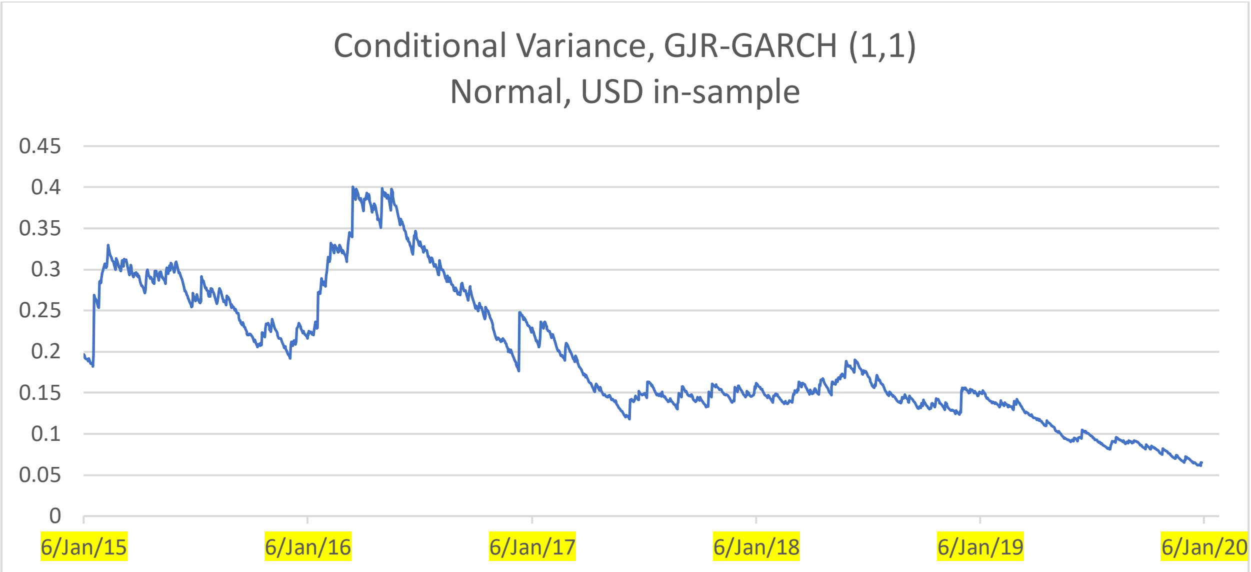

Conditional variance, GARCH (1,1)
Student's t, USD in-sample
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

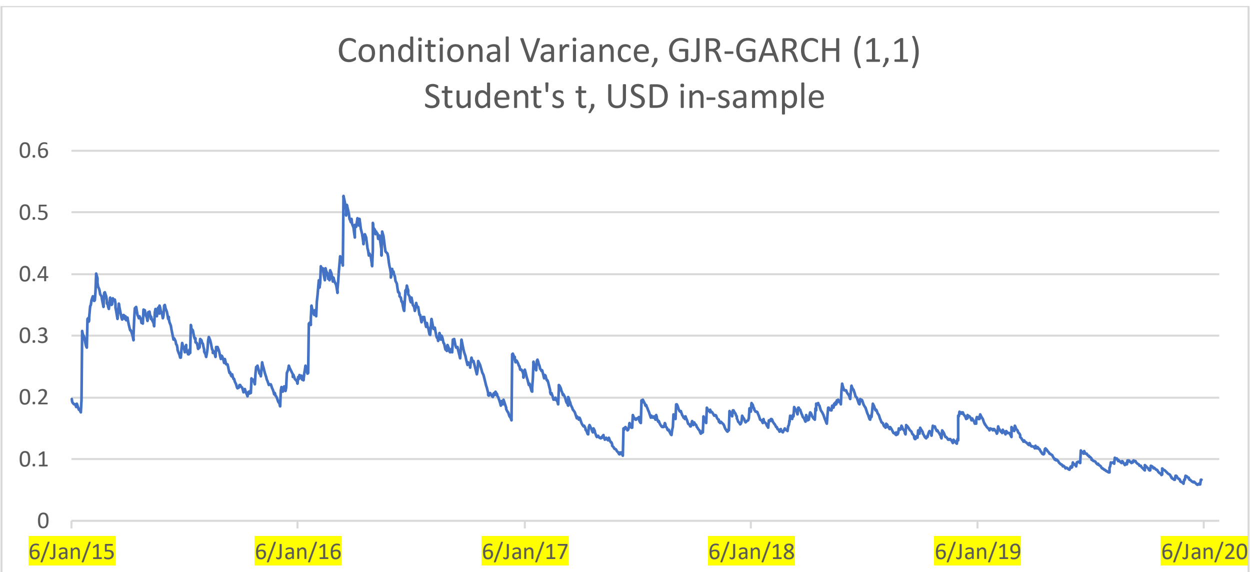

Conditional variance, GARCH (1,1)
GED, USD in-sample
1.2
1
0.8
0.6
0.4
0.2
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

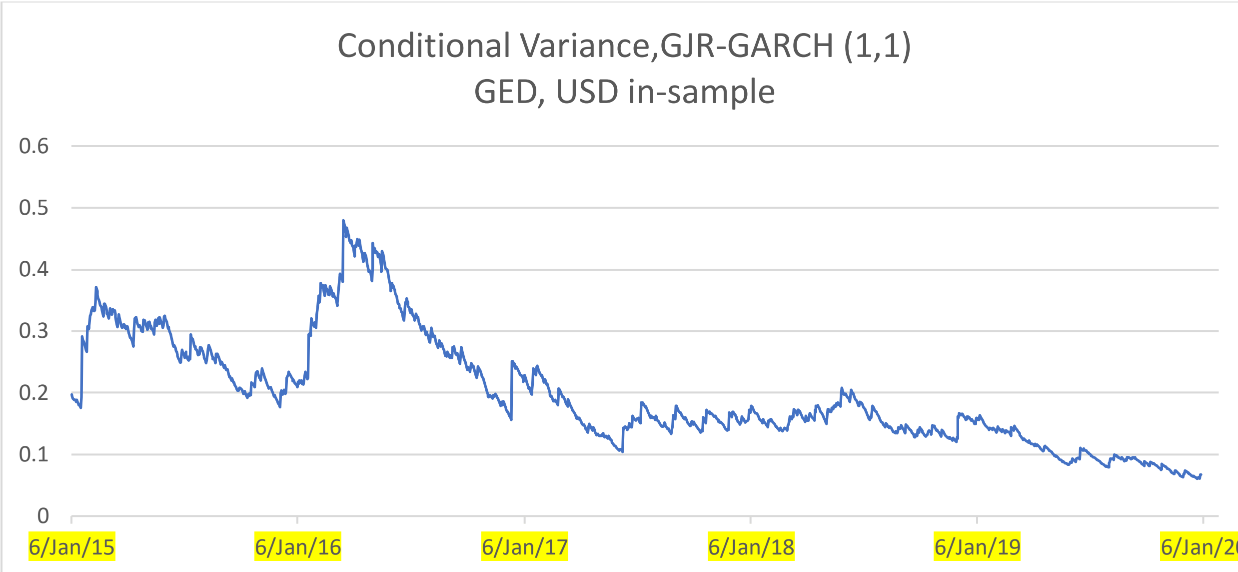

Conditional Variance, EGARCH (1,1)
Normal, USD in-sample
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20


Conditional variance, EGARCH (1,1)
Student's t, USD in-sample
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

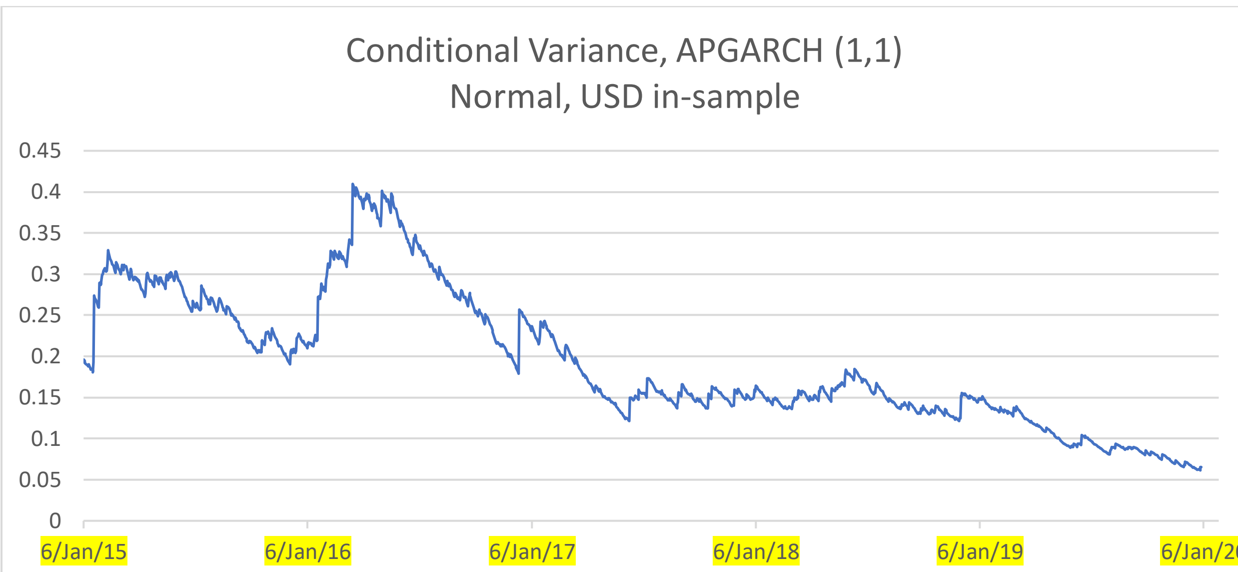

Conditiona Variance, EGARCH (1,1)
GED, USD in-sample
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20


Conditional Variance, GJR-GARCH (1,1)
Normal, USD in-sample
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

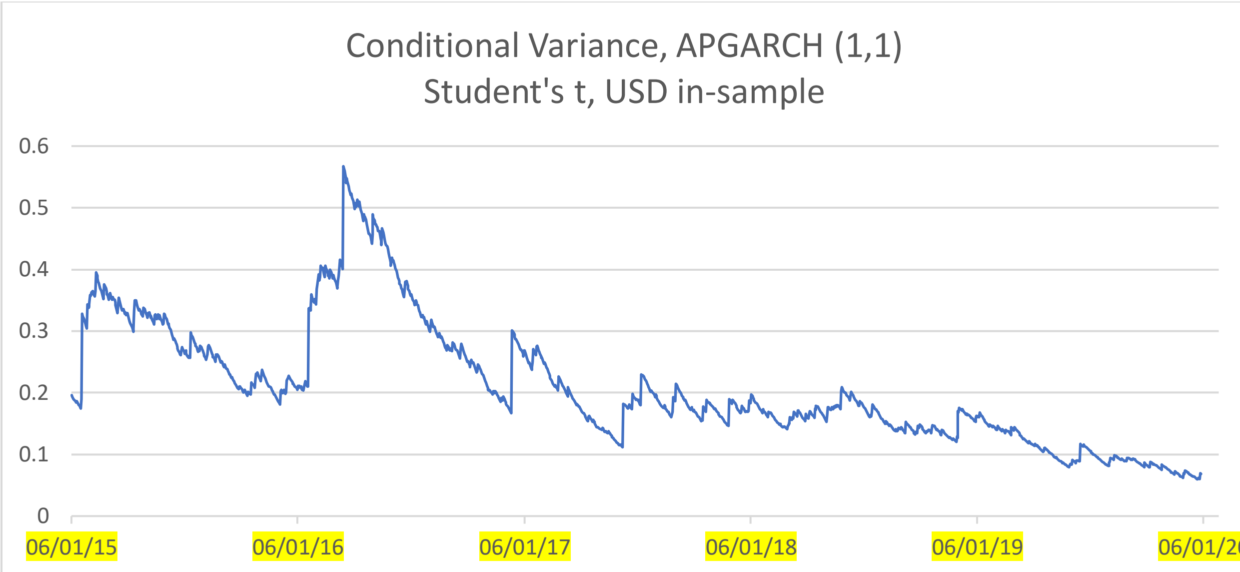

Conditional Variance, GJR-GARCH (1,1)
Student's t, USD in-sample
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

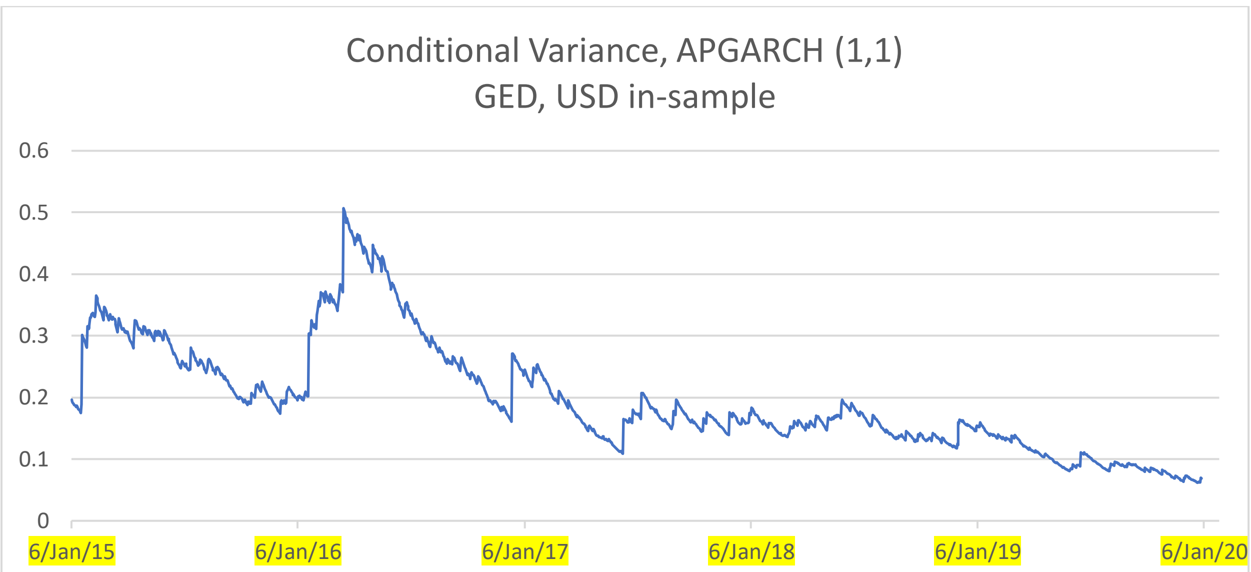


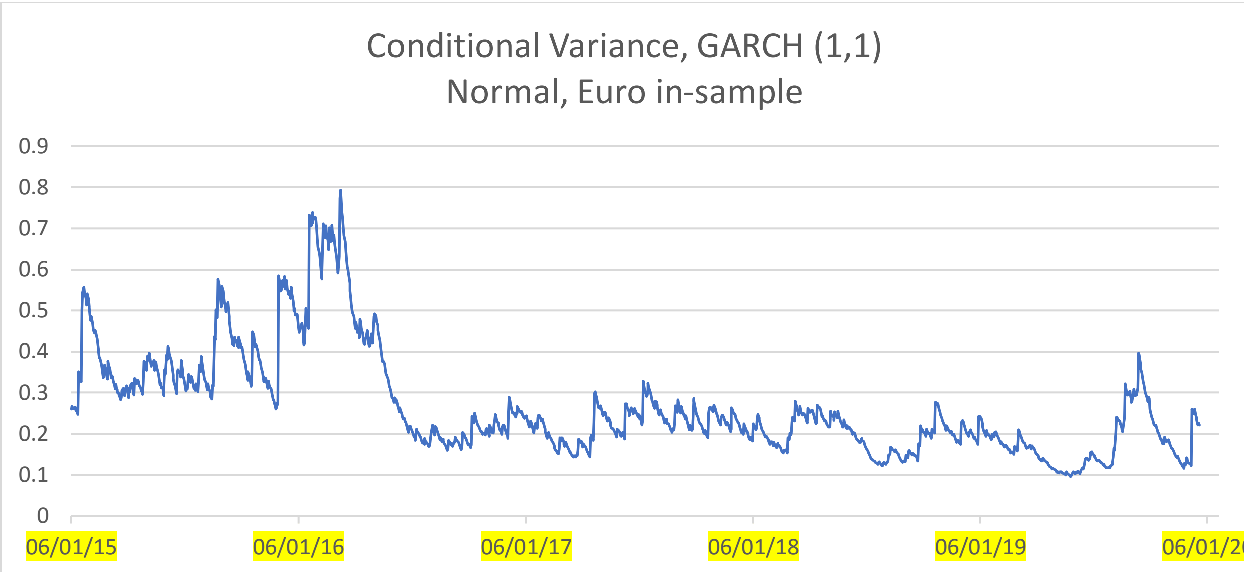
Conditional Variance,GJR-GARCH (1,1)
GED, USD in-sample
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

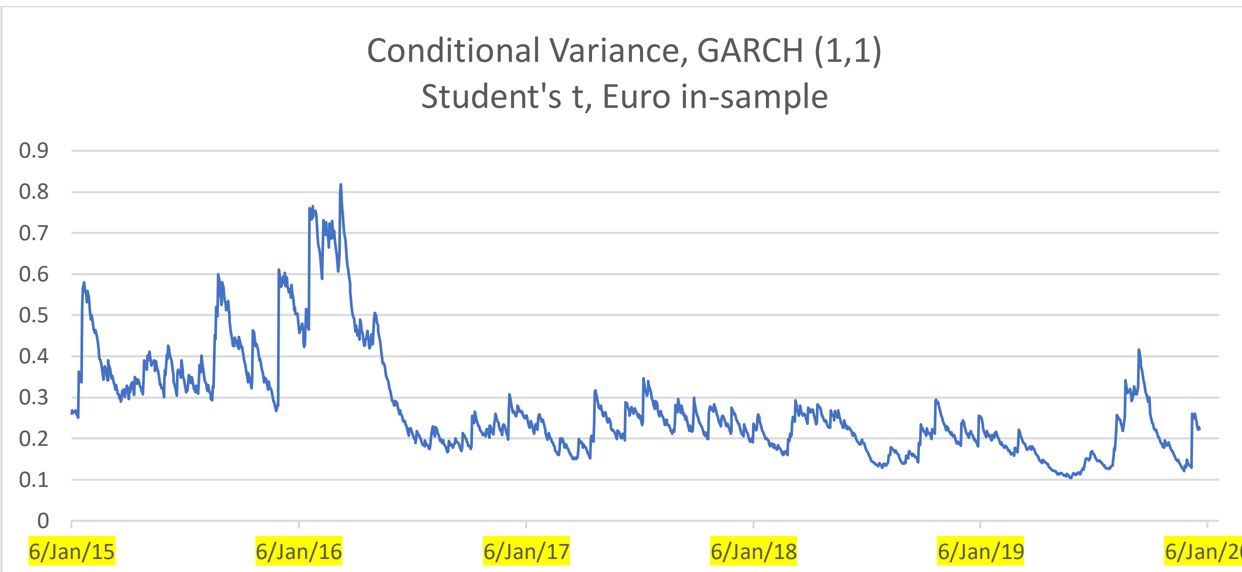


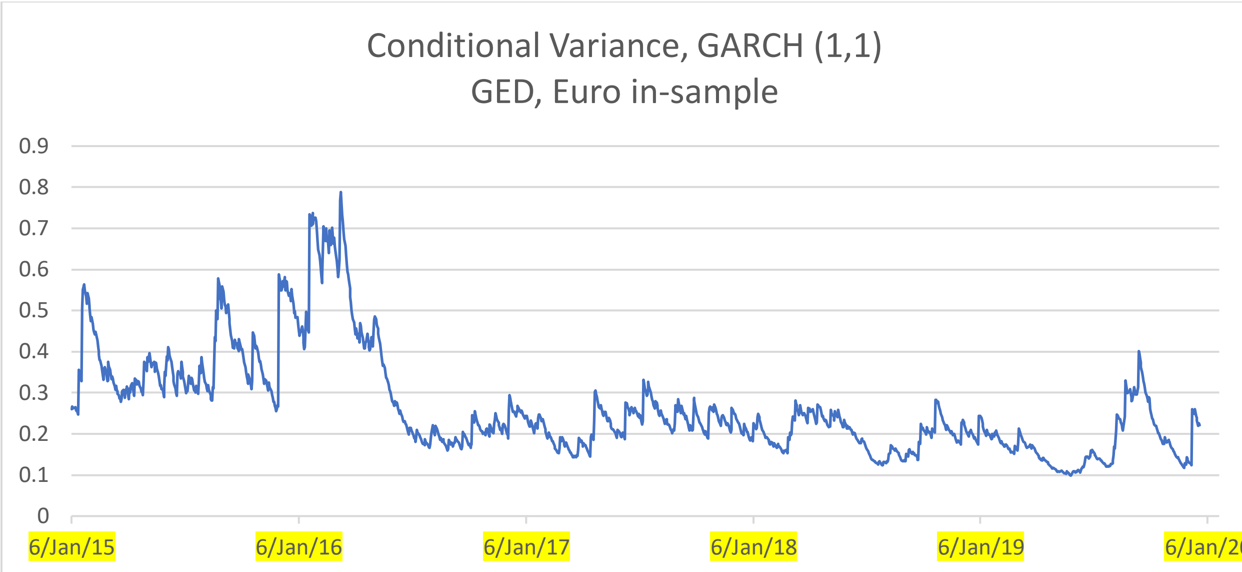
Conditional Variance, APGARCH (1,1)
Normal, USD in-sample
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

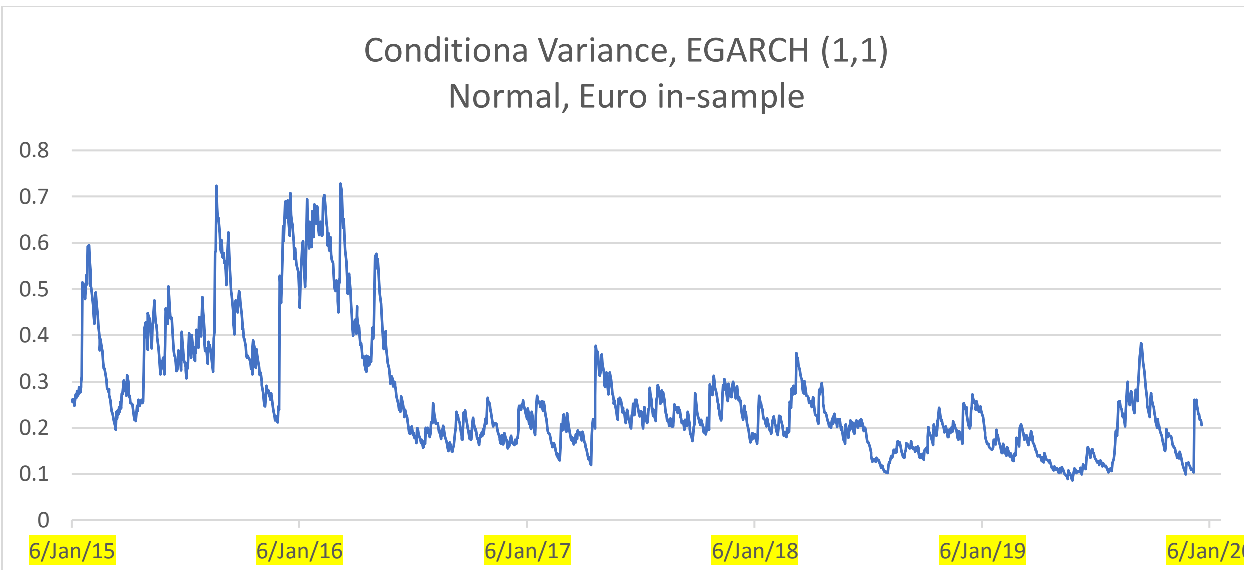

Conditional Variance, APGARCH (1,1)
Student's t, USD in-sample
0.6
0.5
0.4
0.3
0.2
0.1
0
06/01/15
06/01/16
06/01/17
06/01/18
06/01/19
06/01/20

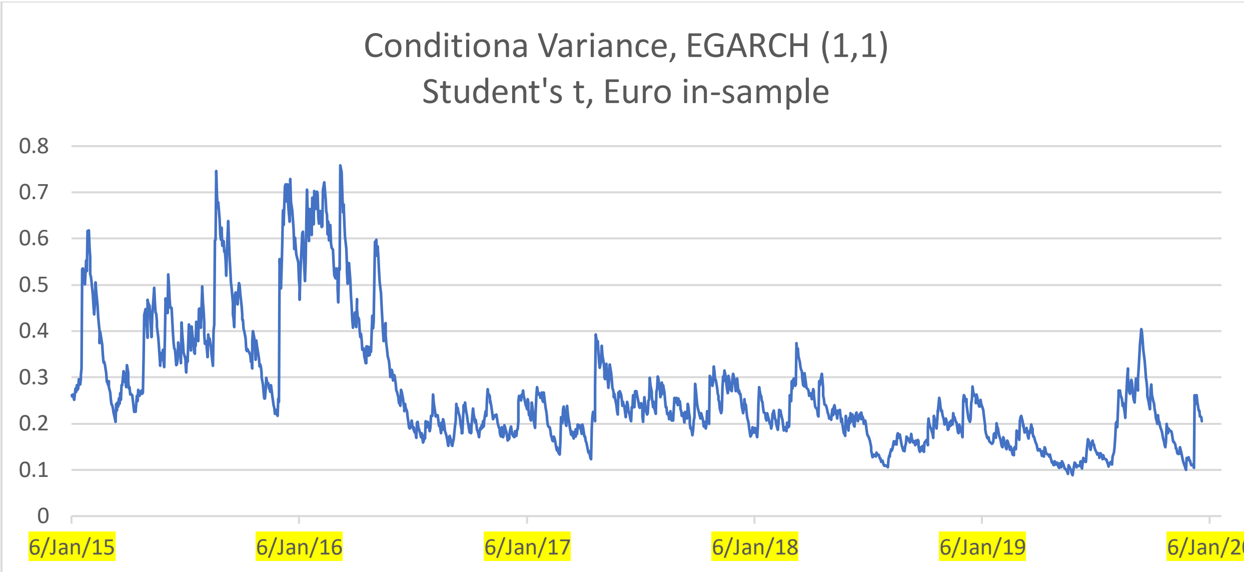

Conditional Variance, APGARCH (1,1)
GED, USD in-sample
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

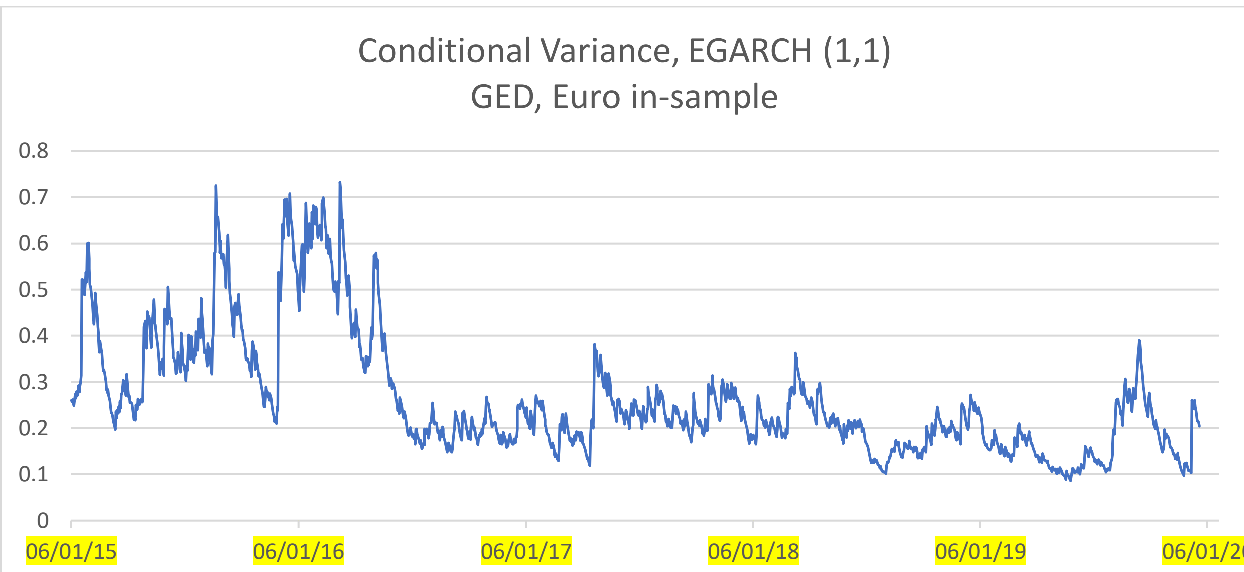

Conditional Variance, GARCH (1,1)
Normal, Euro in-sample
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
06/01/15
06/01/16
06/01/17
06/01/18
06/01/19

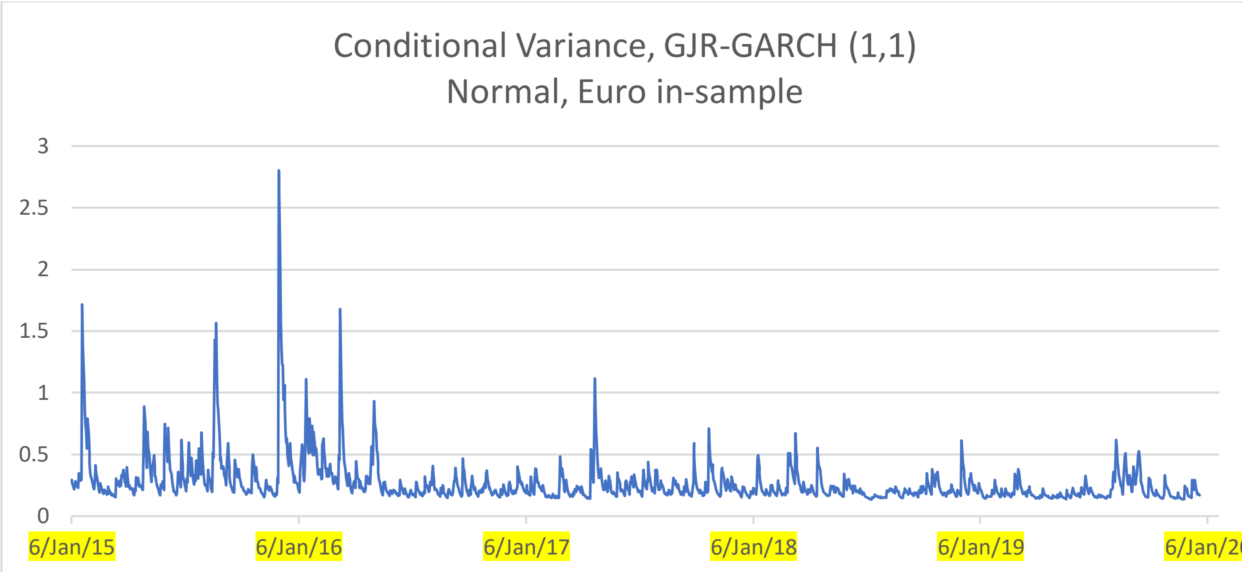

Conditional Variance, GARCH (1,1)
Student's t, Euro in-sample
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19

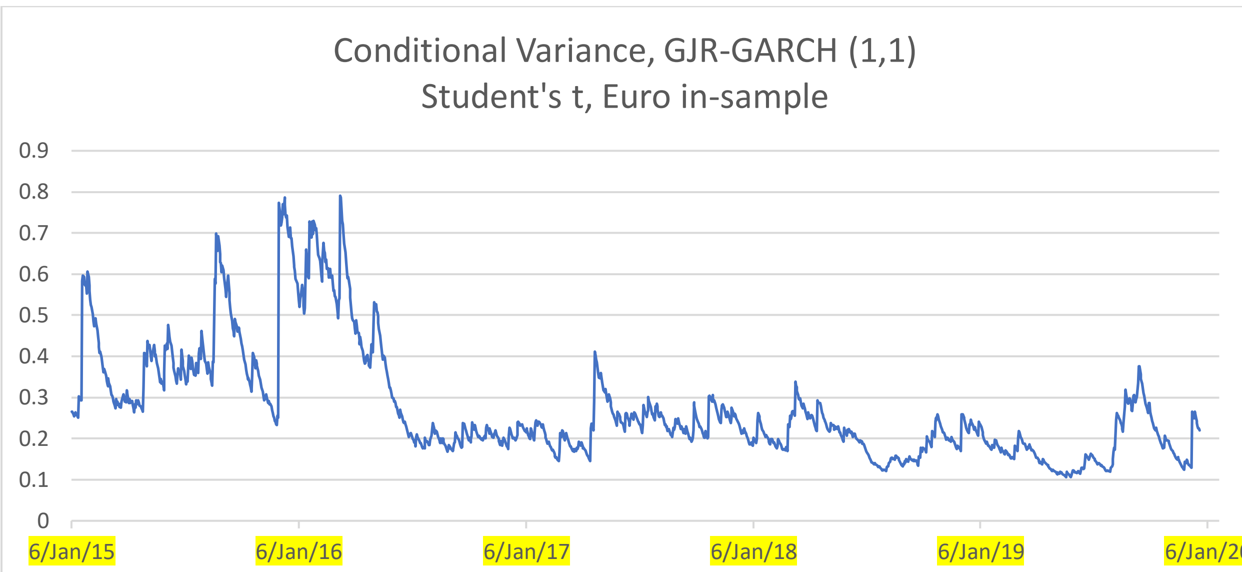

Conditional Variance, GARCH (1,1)
GED, Euro in-sample
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

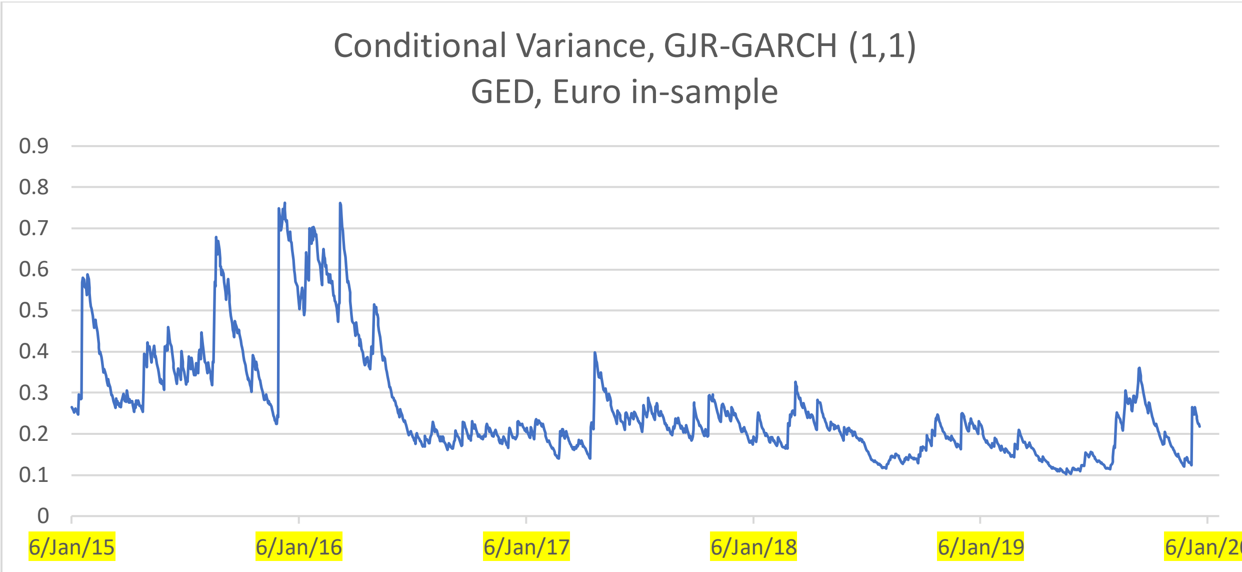

Conditiona Variance, EGARCH (1,1)
Normal, Euro in-sample
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

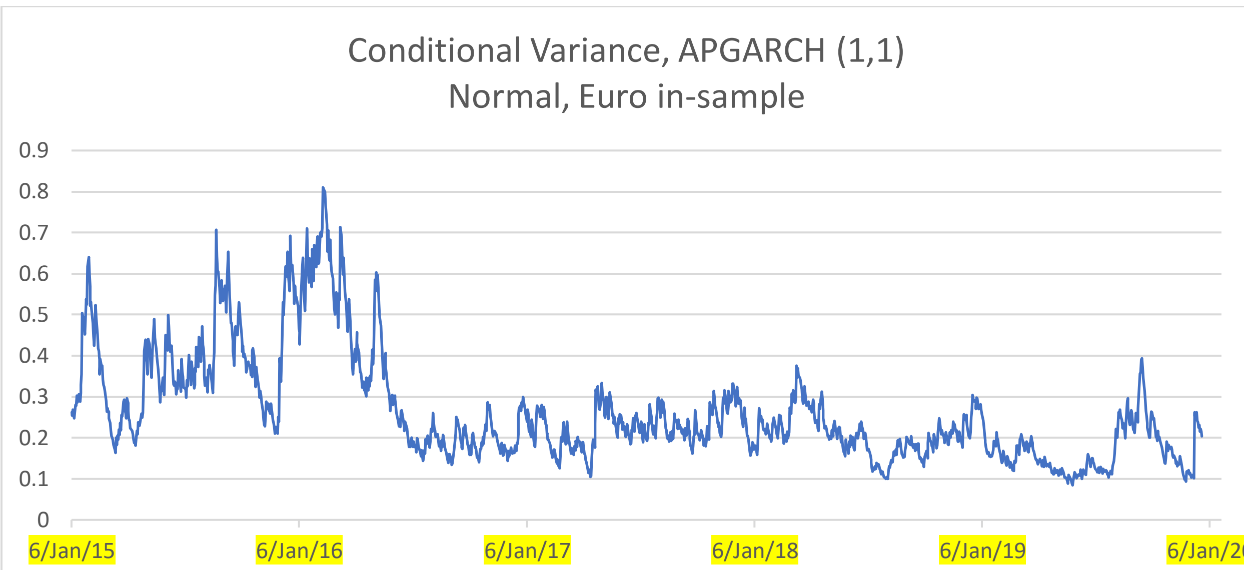

Conditiona Variance, EGARCH (1,1)
Student's t, Euro in-sample
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

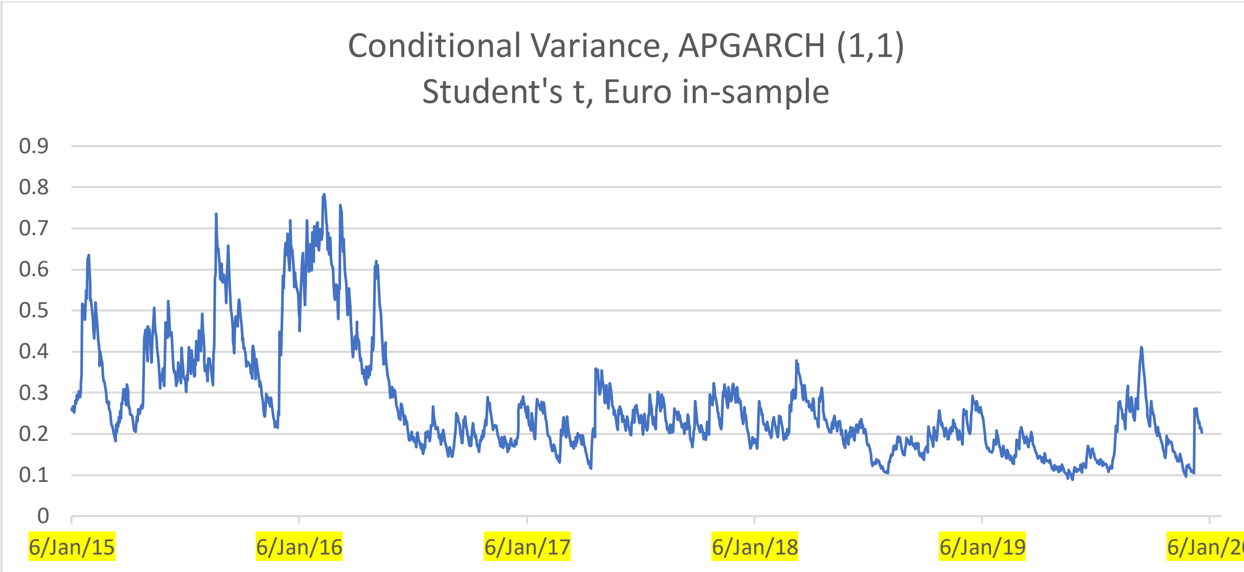

Conditional Variance, EGARCH (1,1)
GED, Euro in-sample
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
06/01/15
06/01/16
06/01/17
06/01/18
06/01/19
06/01/20

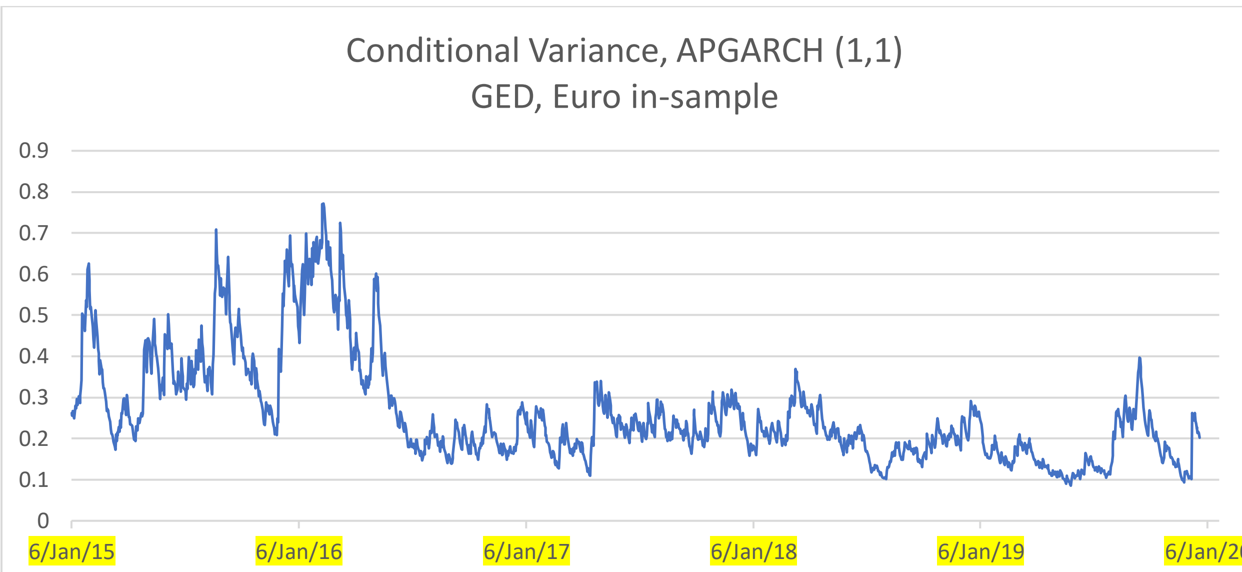

Conditional Variance, GJR-GARCH (1,1)
Normal, Euro in-sample
3
2.5
2
1.5
1
0.5
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

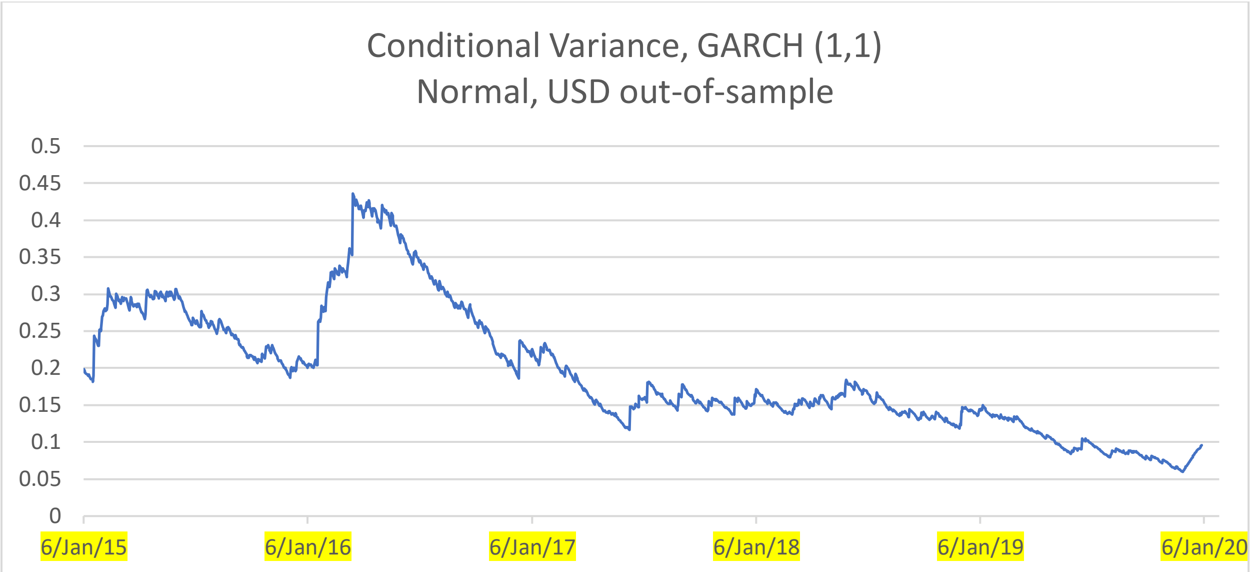

Conditional Variance, GJR-GARCH (1,1)
Student's t, Euro in-sample
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

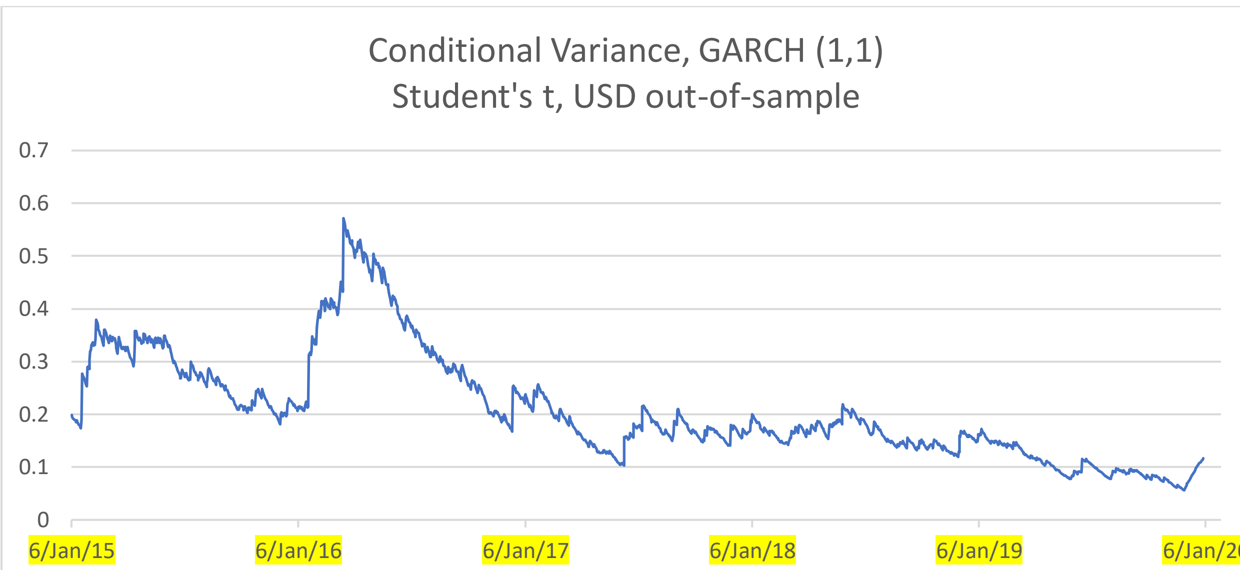

Conditional Variance, GJR-GARCH (1,1)
GED, Euro in-sample
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19

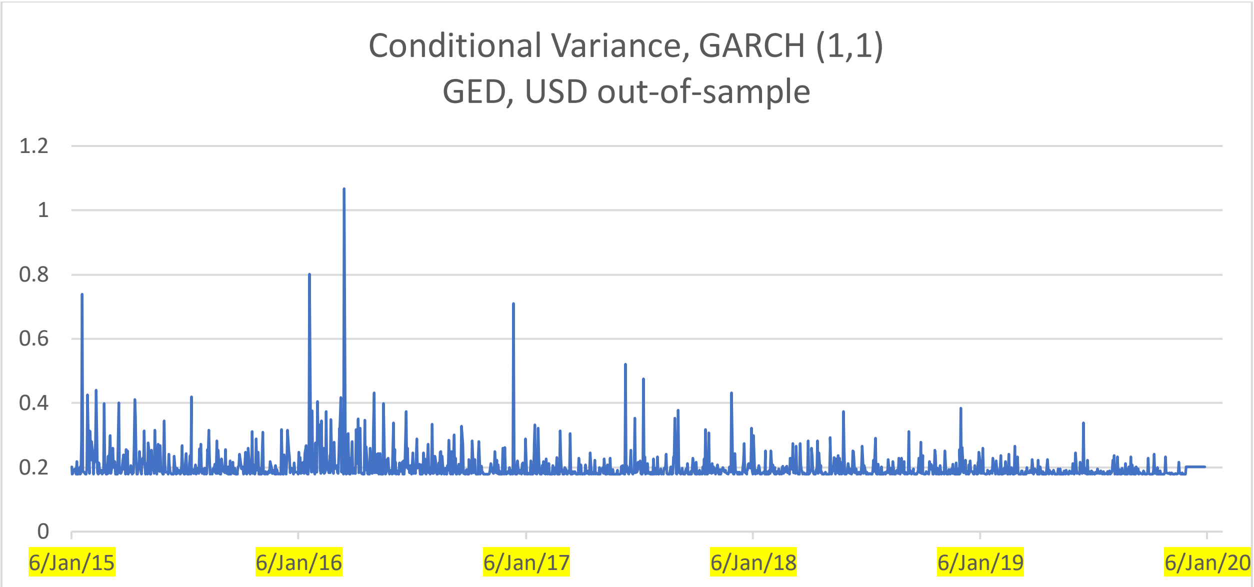

Conditional Variance, APGARCH (1,1)
Normal, Euro in-sample
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19


Conditional Variance, APGARCH (1,1)
Student's t, Euro in-sample
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

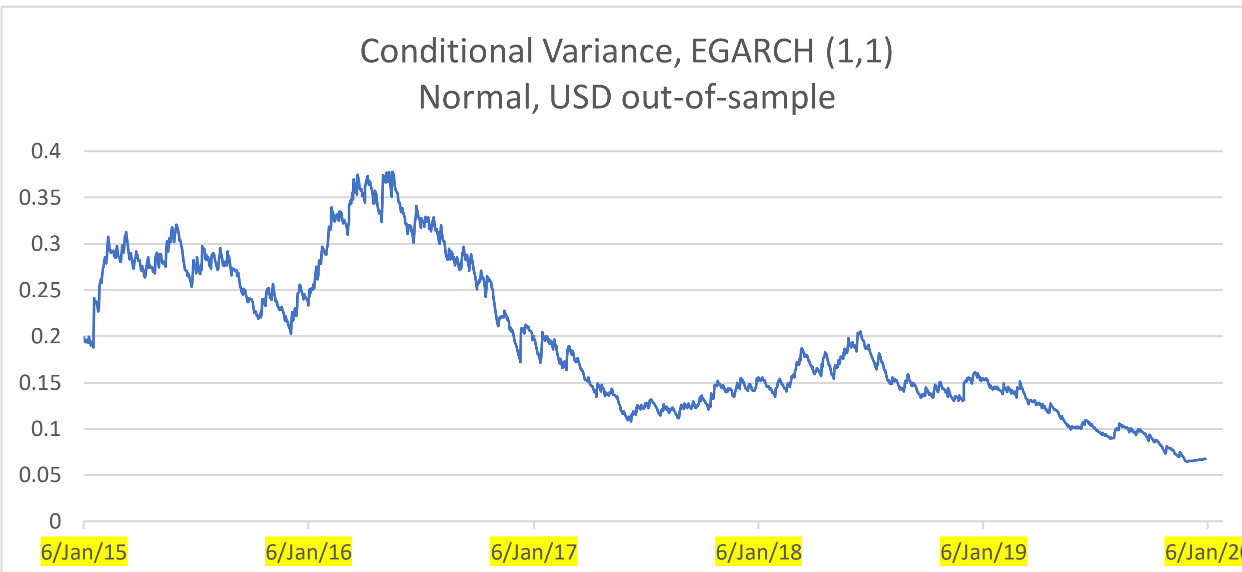

Conditional Variance, APGARCH (1,1)
GED, Euro in-sample
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20


Conditional Variance, GARCH (1,1)
Normal, USD out-of-sample
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

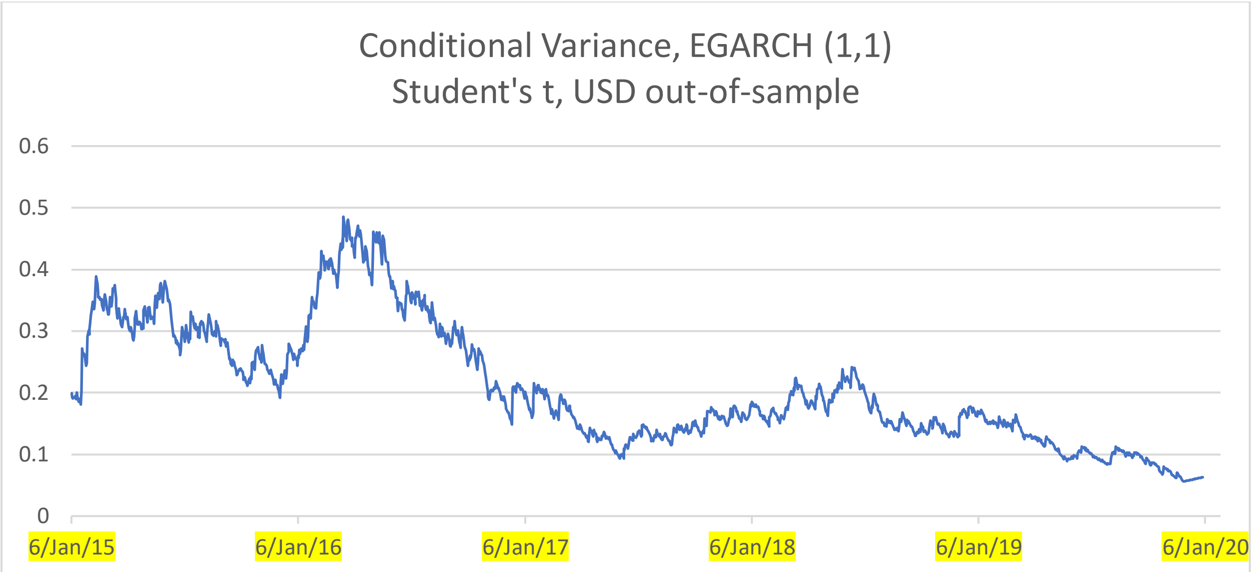

Conditional Variance, GARCH (1,1)
Student's t, USD out-of-sample
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

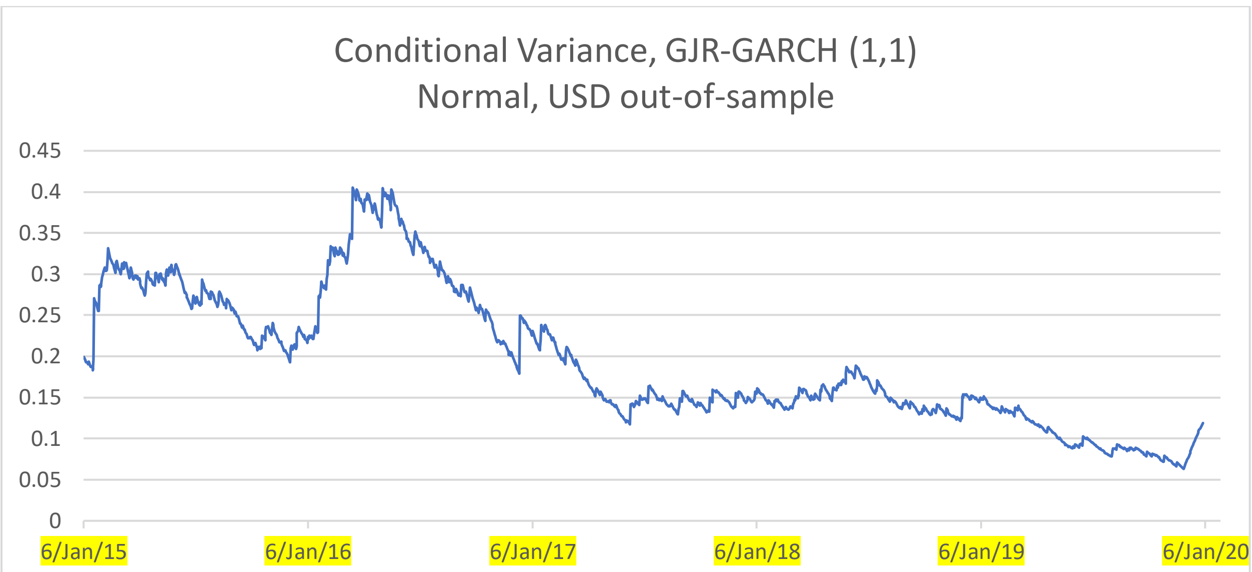


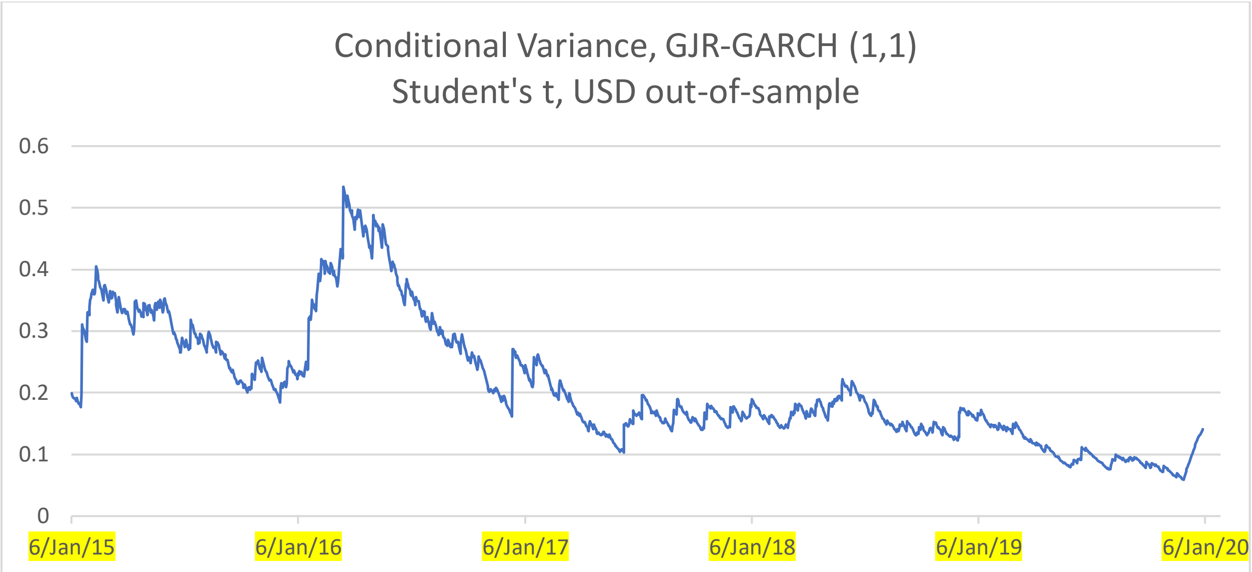
Conditional Variance, GARCH (1,1)
GED, USD out-of-sample
1.2
1
0.8
0.6
0.4
0.2
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

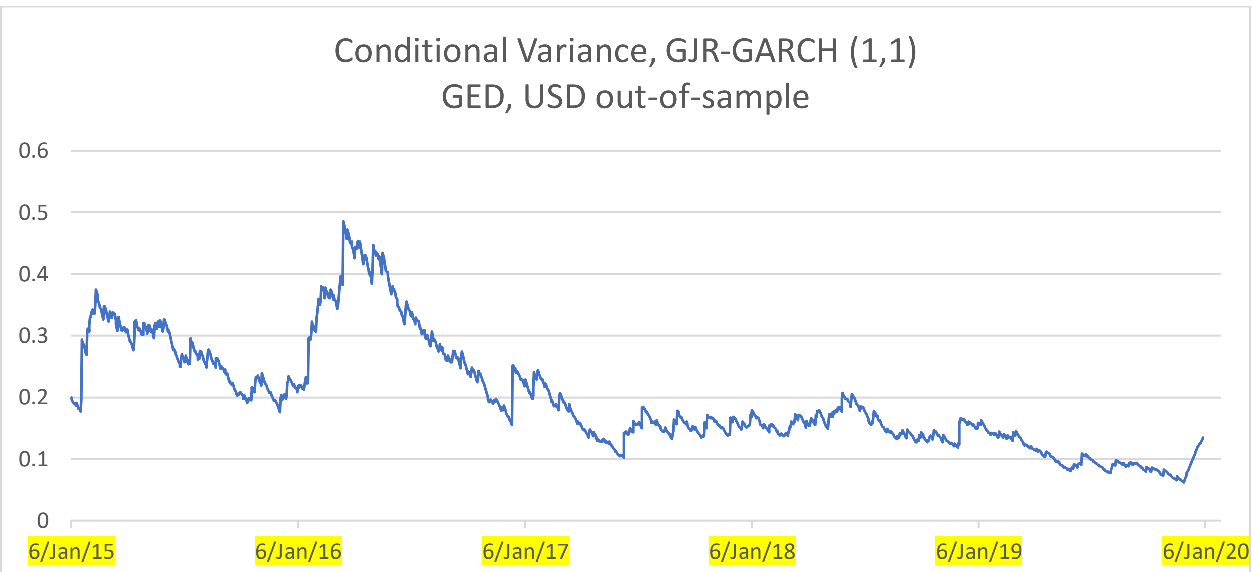


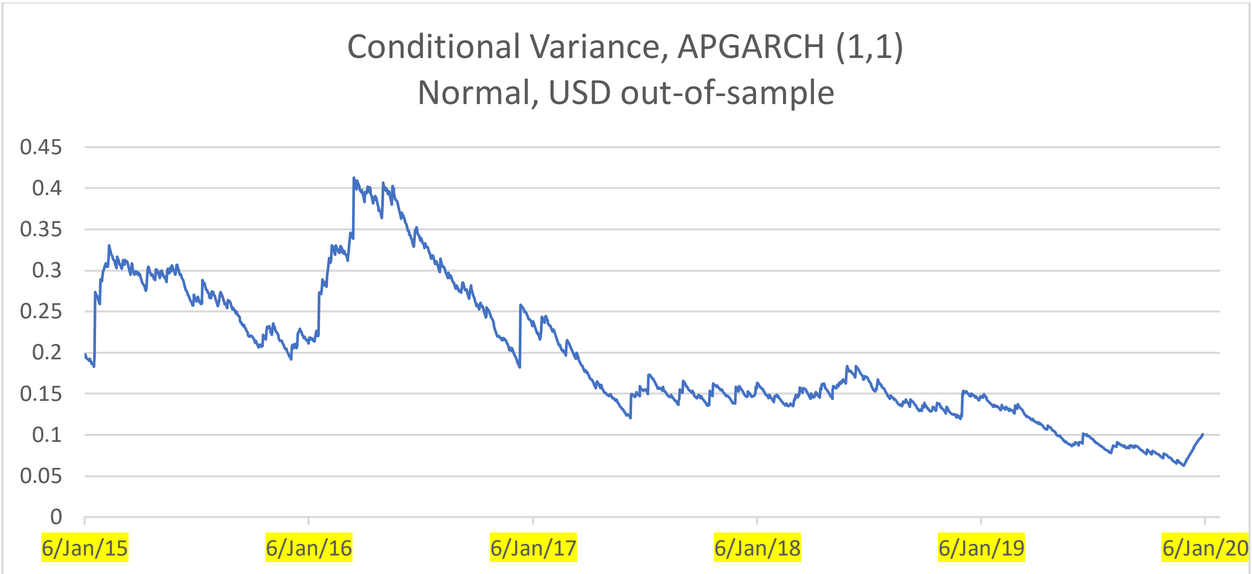
Conditional Variance, EGARCH (1,1)
Normal, USD out-of-sample
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20


Conditional Variance, EGARCH (1,1)
Student's t, USD out-of-sample
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

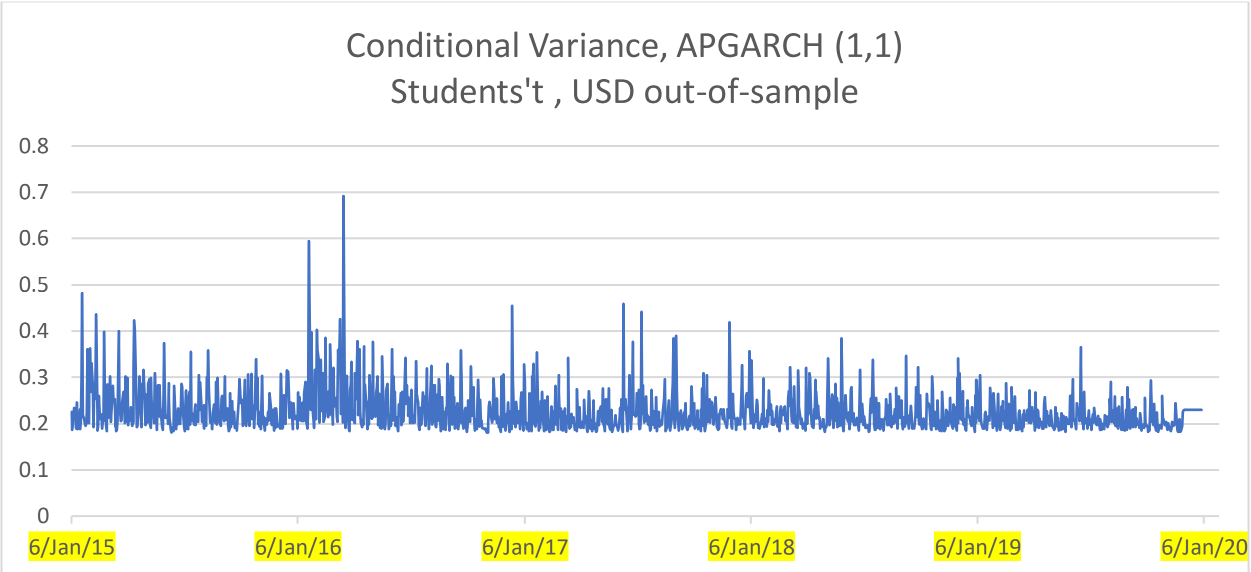

Conditional Variance, GJR-GARCH (1,1)
Normal, USD out-of-sample
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

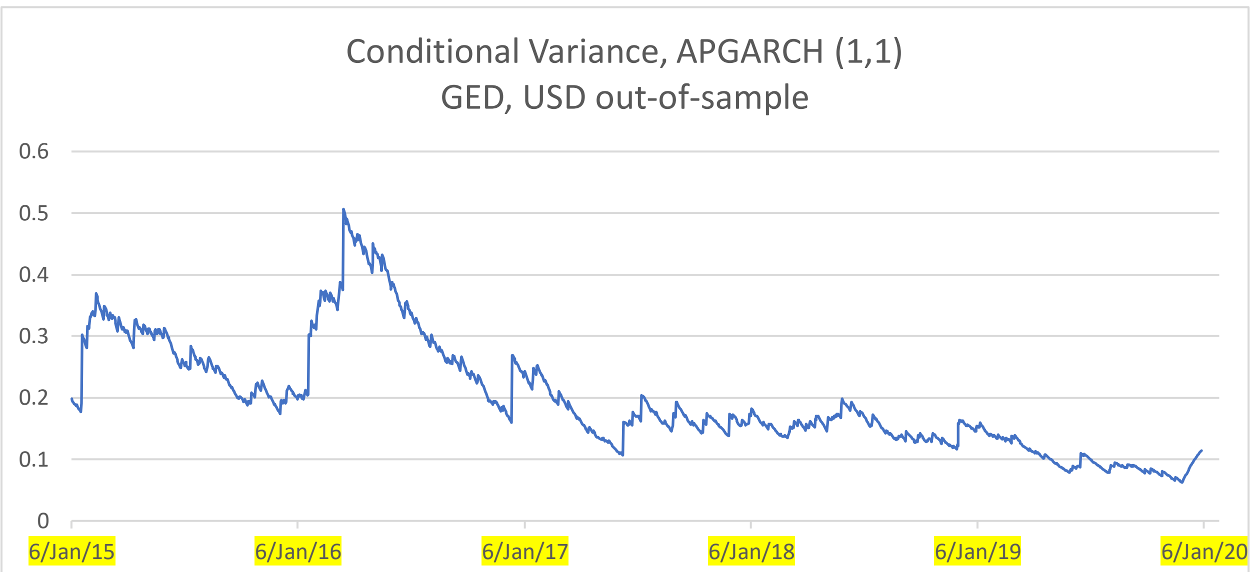

Conditional Variance, GJR-GARCH (1,1)
Student's t, USD out-of-sample
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

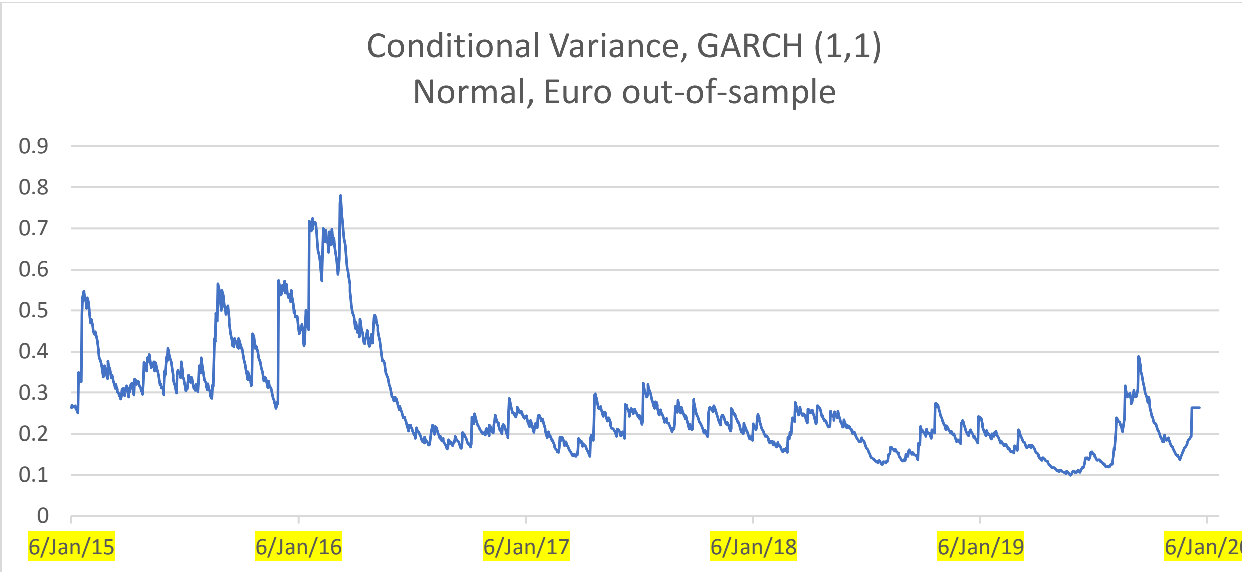

Conditional Variance, GJR-GARCH (1,1)
GED, USD out-of-sample
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

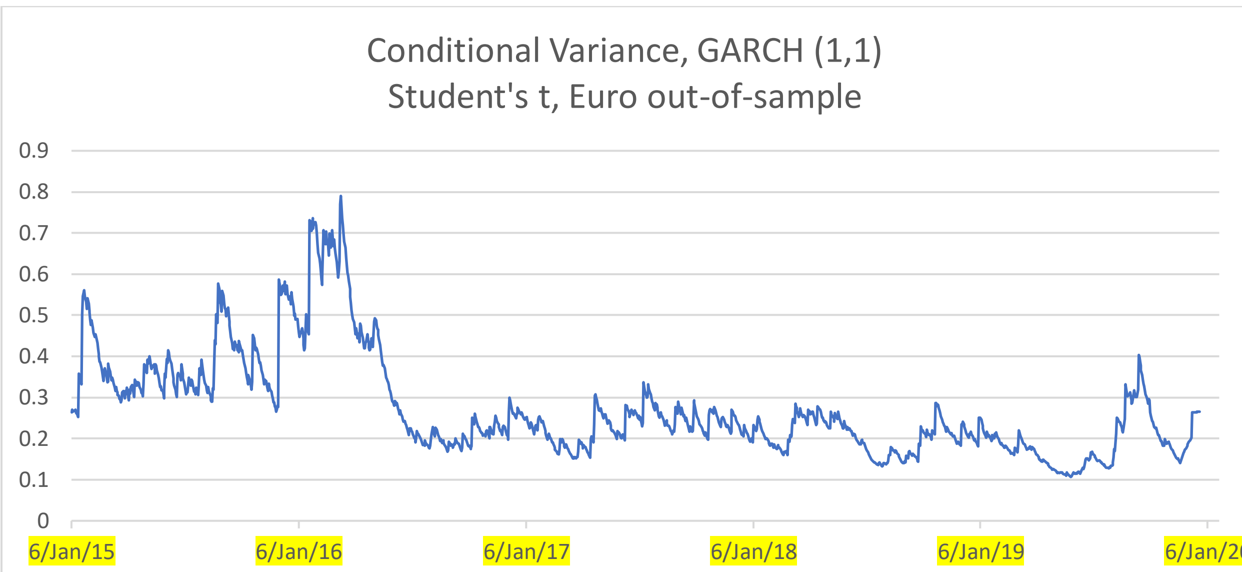


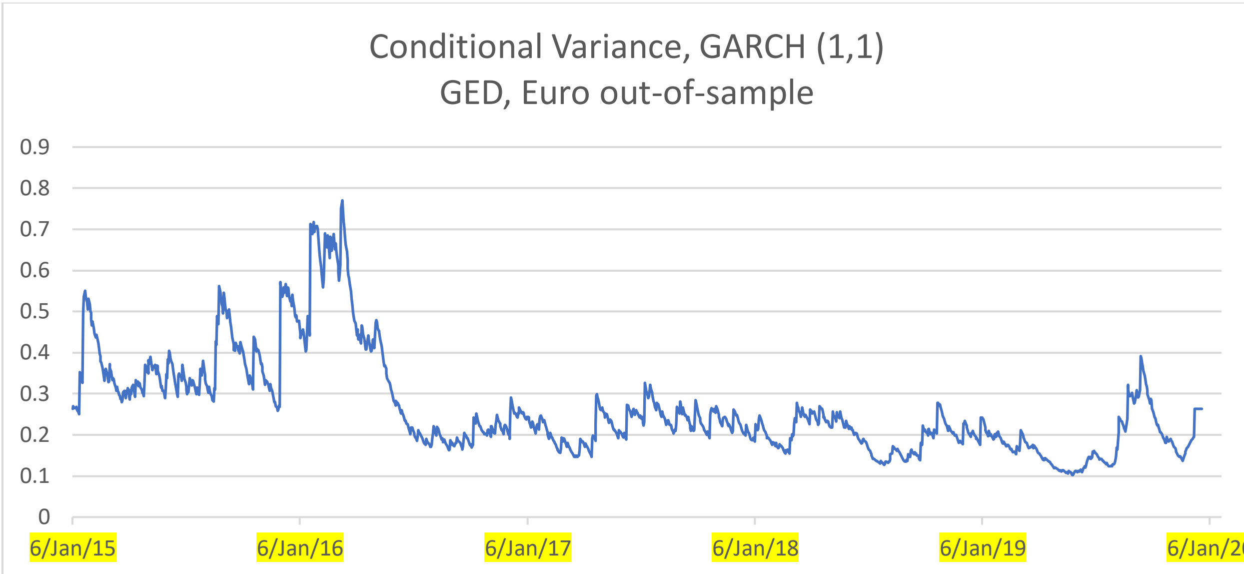
Conditional Variance, APGARCH (1,1)
Normal, USD out-of-sample
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

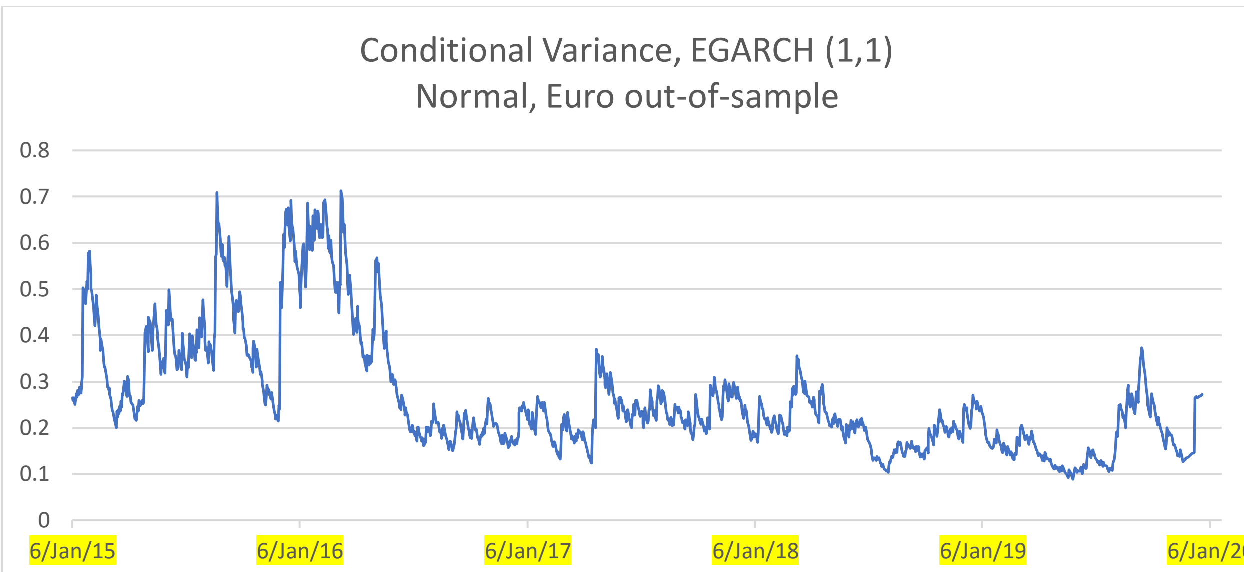

Conditional Variance, APGARCH (1,1)
Students't , USD out-of-sample
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

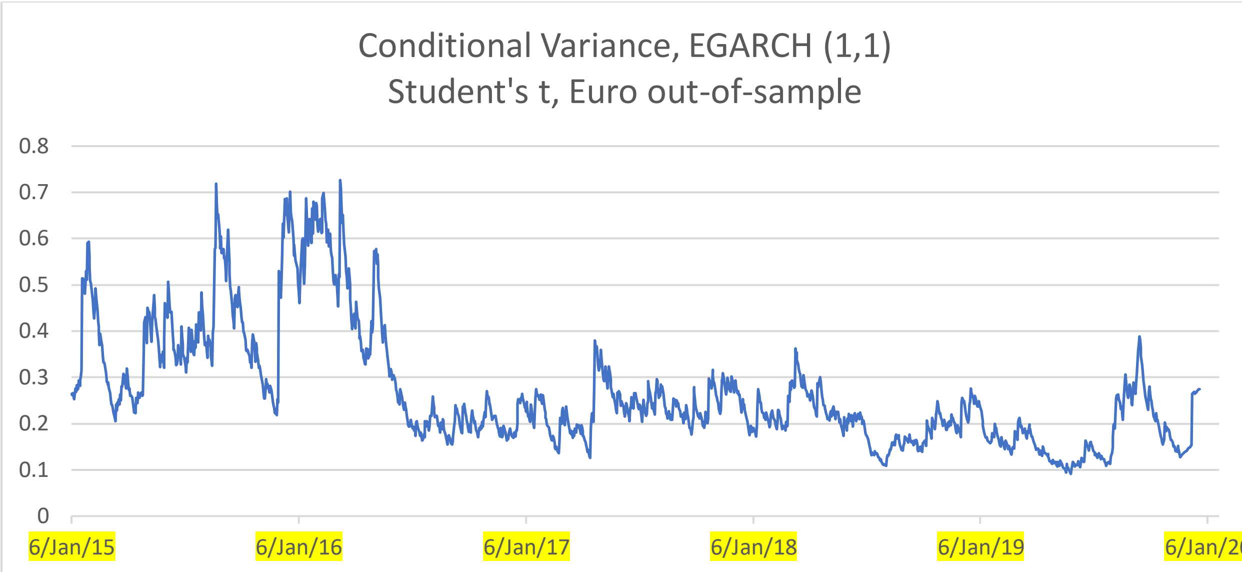

Conditional Variance, APGARCH (1,1)
GED, USD out-of-sample
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

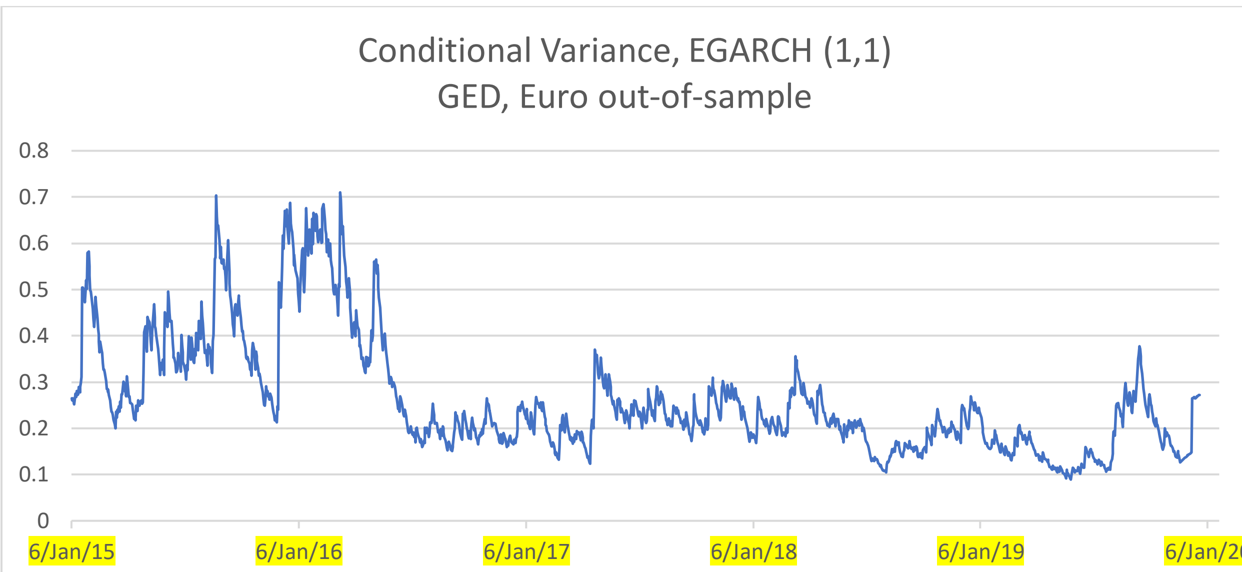

Conditional Variance, GARCH (1,1)
Normal, Euro out-of-sample
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

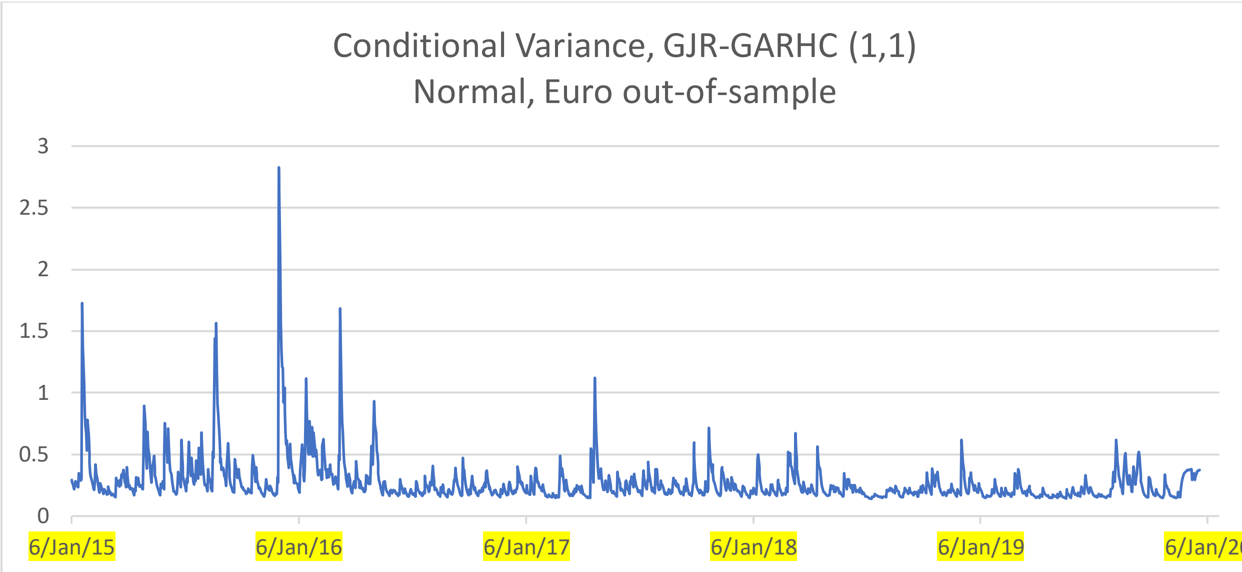

Conditional Variance, GARCH (1,1)
Student's t, Euro out-of-sample
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20

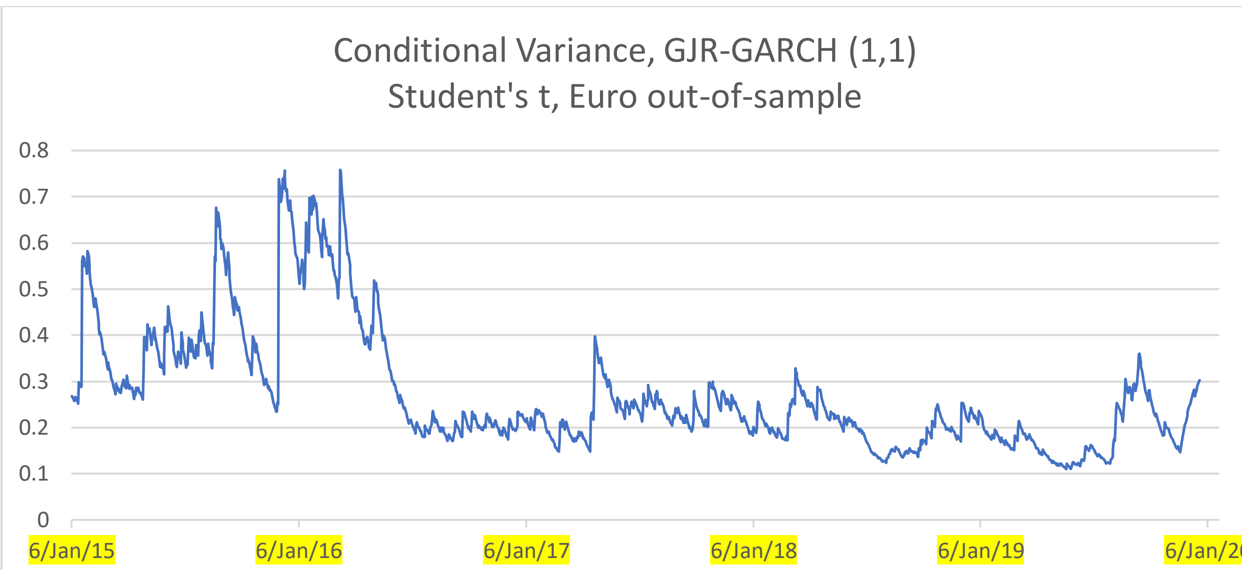

Conditional Variance, GARCH (1,1)
GED, Euro out-of-sample
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19

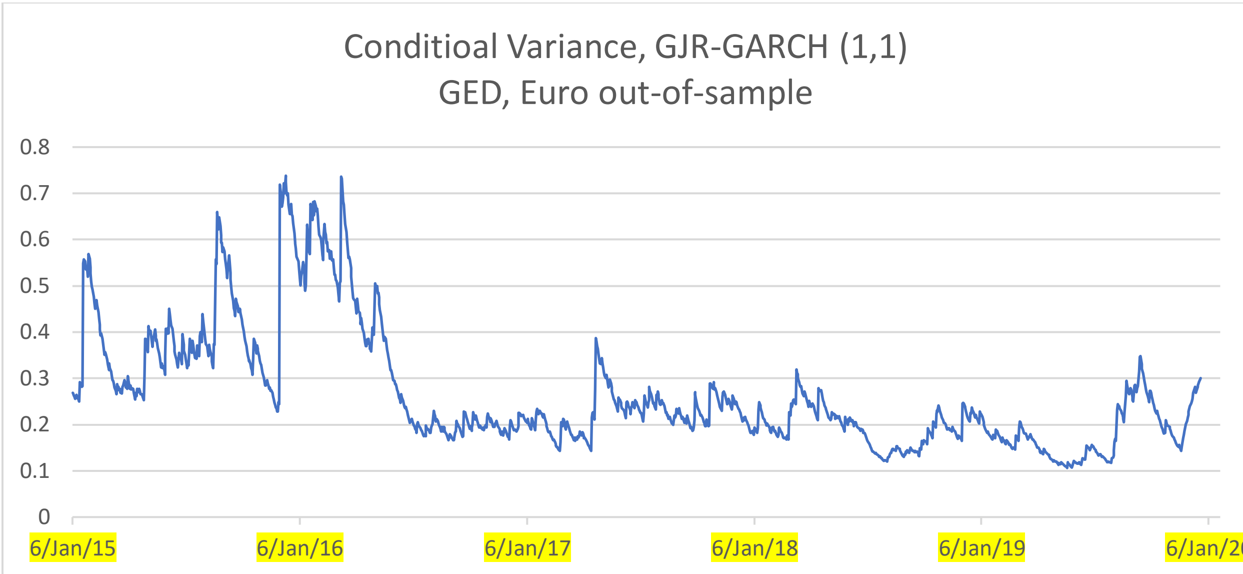

Conditional Variance, EGARCH (1,1)
Normal, Euro out-of-sample
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19

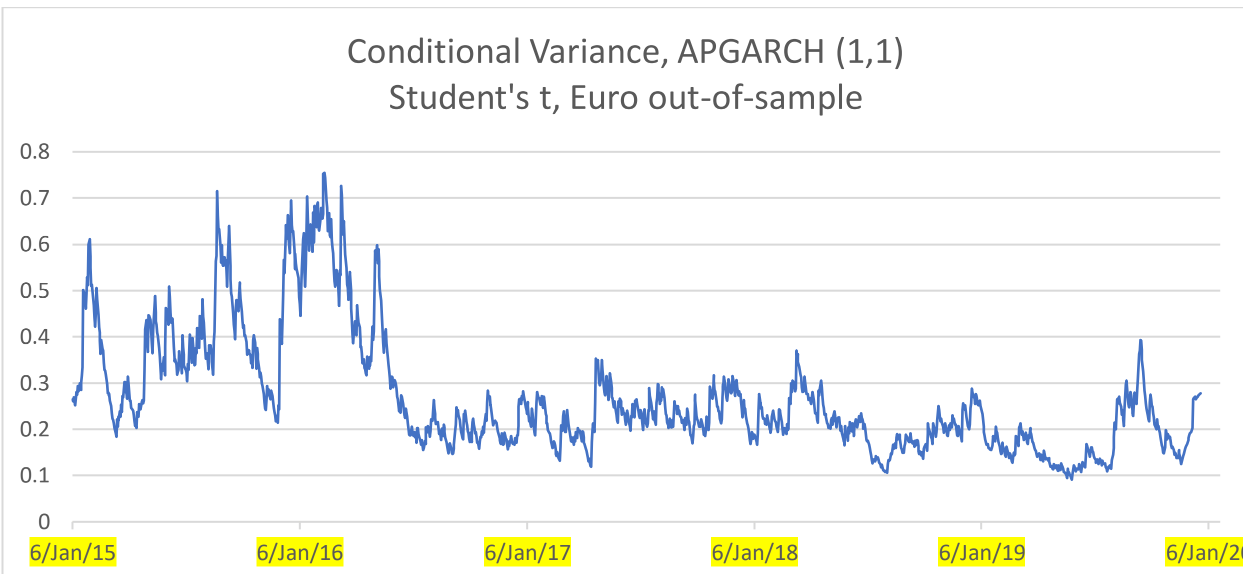

Conditional Variance, EGARCH (1,1)
Student's t, Euro out-of-sample
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19


Conditional Variance, EGARCH (1,1)
GED, Euro out-of-sample
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19


Conditional Variance, GJR-GARHC (1,1)
Normal, Euro out-of-sample
3
2.5
2
1.5
1
0.5
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20


Conditional Variance, GJR-GARCH (1,1)
Student's t, Euro out-of-sample
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20


Conditioal Variance, GJR-GARCH (1,1)
GED, Euro out-of-sample
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20


Conditional Variance, APGARCH (1,1)
Student's t, Euro out-of-sample
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
6/Jan/15
6/Jan/16
6/Jan/17
6/Jan/18
6/Jan/19
6/Jan/20