Course Code	MTH570		
Course Name	Numerical Solutions Differential Equations		
Credits	4		
Course Offered to	3rd/4th year UG; PG students		
Course Description	This course will be provide an overview of two standard numerical methods for partial differential equations (PDEs). The focus will be on essential theoretical analysis as well parabolic and hyperbolic partial differential equations. This will be followed by a short foray into linear system solvers and finite difference scheme for two-dimensional Poisson's (elliptic) problem. The second part of the course will deal with finite element methods exclusively for elliptic problems. The core ideas in functional analysis, variational formulation, error analysis, and computer implementation will be presented for the one-dimensional problem. This will be followed be a more practical treatment of two-dimensional problems. The last part will consist of an overview of the specialized topics of mixed and adaptive finite element methods.as computer implementation. The first part will be on finite difference methods. Key numerical schemes and underlying theory will be provided for one-dimensional		
Pre-requisites			
Pre-requisite (Mandatory)	**Pre-requisite (Desirable)**	**Pre-requisite(other)**	
Math I; Math III/Real Analysis I	Numerical Methods (MTH 270); Math IV	Python experience	
*Please insert more rows if required			
Post Conditions*(For suggestions on verbs please refer the second sheet)			
CO1	**CO2**	**CO3**	**CO4**
Students learn about basics of partial differential equations, some qualitative and quantitative aspects of their analytical and general solutions.	Students can derive and analyze finite difference methods for solving model problems of parabolic and hyperbolic partial differential equations.	Students can write computer code to solve model problems, interpret and visualize their solutions. They also learn to make appropriate choices for numerical linear algebraic methods.	Students understand basics of functional analysis, variational formulation and finite element spaces. They also learn stability analysis in the setting of Hilbert spaces.
Weekly Lecture Plan			
Week Number	**Lecture Topic**	**COs Met**	**Assignment/Labs/Tutorial**
Week 1	Introduction to partial differential equations (PDEs) including classification, initial- and boundary-value problems, boundary conditions and common PDEs; Python tutorial.	CO1	HW0
Week 2	Overview of one-dimensional parabolic PDEs (heat and convection-diffusion equations); introduction to finite differences; explicit and implicit schemes for one-dimensional parabolic equations;	CO1 + CO2	HW1

Week 3	Consistency, stability and Fourier analysis; maximum principle in parabolic PDEs;	CO2 + CO3	
Week 4	Overview of one-dimensional hyperbolic PDEs (advection equation); finite difference schemes (method of lines discretizations and Lax Wendroff schemes) for one-dimensional hyperbolic PDEs; Courant-Friedrichs-Lewy (CFL) condition	CO1 + CO2	HW2
Week 5	Lax equivalence theorem; von-Neumann analysis and stability condition	CO2	
Week 6	Order of accuracy of solution; dissipation and dispersion in finite difference schemes for advection equation	CO2 + CO3	HW3
Week 7	Overview of two-dimensional elliptic PDEs (Laplacian); Maximum principle for Laplacians; reentrant corner singularities	CO1	
Week 8	Interregnum: Direct and iterative methods for linear system solution; finite differences for two-dimensional Poisson's;	CO3	HW4
Week 9	Sobolev norms and spaces; weak derivatives; variational formulation; finite element method in one-dimensions and error estimates	CO4	
Week 10	Hilbert spaces; Riesz representation theorem; Lax-Milgram theorem	CO4	HW5
Week 11	Meshing; quadrature; two-dimensional finite element spaces	CO3 + CO4	
Week 12	Implementation of two-dimensional linear finite element for Poisson's; Use of FEniCS package for other elements	CO3	HW6
Week 13	Adaptive and mixed finite elements in one-dimension; FEniCS implementation	CO3	

*Please insert more rows if required

Weekly Lab Plan

Week Number	**Laboratory Exercise**	**COs Met**	**Platform (Hardware/Software)**

*Please insert more rows if required

Assessment Plan

Type of Evaluation	% Contribution in Grade
Assignments	50 (HW 0: 5%; HWs 1-6: each 7.5%)
Midsem	20
Endsem	30

Resource Material

Type	Title
Textbook	Partial Differential Equations with Numerical Methods by Stig Larsson and Vidar Thomée
Reference 1	Finite Difference Schemes and Partial Differential Equations (Second Edition) by John Strikwerda
Reference 2	Numerical Solution of Partial Differential Equations (Second Edition) by K. W. Morton and David Meyers
Reference 3	The Mathematical Theory of Finite Element Methods (Third Edition) by Susanne Brenner and Ridgway Scott