

Course Code	MTH599		
Course Name	Variational Calculus and their Applications		
Credits	4		
Course Offered to	PG		
Course Description	Variational Calculus is the simplest and the most direct means of unifying all branches under the discipline of Applied Mathematics. This course assumes very limited knowledge of vector calculus, ordinary differential equations and basic mechanics. Many new applications in applied mathematics, physics, chemistry, biology and engineering are included. This course will serve as a reference for advanced study and research in this subject as well as for its applications in the fields of neuroscience, signal processing, informatics and communications.		
Pre-requisites			
Pre-requisite (Mandatory)	Pre-requisite (Desirable)	Pre-requisite(Other)	
MTH-203	Basic knowledge in calculus, ordinary differential equations and partial differential equations		
Post Conditions*(For suggestions on verbs please refer the second sheet)			
CO1	CO2	CO3	
Students master the techniques of 1st/2nd variation and are able to solve equations involving functionals.	Students are able to model physical systems involving linear/non-linear problems in Variational Calculus.	Students apply the variational techniques to solve problems in relativistic mechanics	
Weekly Lecture Plan			
Week Number	Lecture Topic	COs Met	Assignment/Labs/Tutorial
1	First variation: Intro., weak variations, Eulerian equations, Legendre test; Applications: Catenary, optics, geodesics on a sphere	1,2	
2	Applications: Brachistochrone, minimal surfaces, fluid motion, Newton's solid of minimum resistance; Principle of least action, discontinuous solutions	1,2	Practice Problems (Set 1)
3	Second Variation: Intro., Jacobi's accessory equation, conjugate points (kinetic foci), property of conjugate points	1,2	Practice Problems (Set 2)
4	Analytical methods for conjugate points, conjugate points on catenary, parabolic trajectory, spherical geodesics, orbits under inverse square law.	1,2	Practice Problems (Set 3)
5	Generalization of 1st and 2nd variation: Intro., Maxima/minima of functionals of multiple variables, lemma on double integration, Application to other physical problems, theory of minimal surfaces	1,2	Practice Problems (Set 4)
6	Relative max/min for isoperimetric problems: Several examples of relative max/min, subsidiary equations of non-integral type, non-holonomic dynamical constraints, Isoperimetric problems using second variations	1,2	Practice Problems (Set 5)
7	Principle of least action: Intro., degrees of freedom, holonomic & non-holonomic systems, conservative/non-conservative systems, Hamilton's principle and proof	1,2	Practice Problems (Set 6)
8	Lagrange's equation of motion, energy equation for conservative force fields, special variation in external, geodesics on hyper-surfaces, Path of minimum time in stream-flow	1,2	Practice Problems (Set 7)
9	Hamilton's principle in Relativity theory: Physical basis, Michelson & Morley experiment, space-time continuum: Newtonian vs. relativistic concept, Hamilton's principle in relativity mechanics, mass & energy in relativity	2,3	Practice Problems (Set 8)
10	Applications to problems in elasticity: Illustration using Euler's equation, Rayleigh-Ritz method, Strum-Liouville functions, stress-strain relations, Saint-Venant torsion problem, Applications via Trefftz method, Galerkin method, torsion in beams	1,2,3	Practice Problems (Set 9)
11	Hilbert Integrals: Problems with variable end-points, determination of focal points via geometric, analytic methods, fields of extremals, method of Caratheodory, Bliss condition	1,2,3	Practice Problems (Set 10)
12	Strong variations: Weierstrassian function in simplified form, Weierstrassian theory for parametric form, Applications to geodesics on surfaces, special cases	1,2,3	Practice Problems (Set 11)
13	Some additional topics		Practice Problems (Set 12)
Weekly Lab Plan			
Week Number	Laboratory Exercise	COs Met	Platform (Hardware/Software)
*Please insert more rows if required			
Assessment Plan			
Type of Evaluation	% Contribution in Grade		
Quizzes/Assignments	30		
Midsem	30		
Endsem	40		
*Please insert more row for other type of Evaluation			
Resource Material			
Type	Title		
Textbook	1) Variational Optimization, Donald Smith 2)Calculus of variations, B. Van Brunt		
Reference	1) Quantum Variational Calculus, A. Malinowska 2) Variational Calculus in Science and Engineering, M. J. Forray		