设 $x=1+t$,

$$\begin{aligned}
\frac{\ln(1+x)}{x} &= \frac{\ln(2+t)}{1+t} = \frac{\ln 2+\ln(1+t/2)}{1+t}\\
&= \Big(\ln 2+\frac{t}{2}-\frac{t^2}{8}+o(t^3)\Big)\big(1-t+t^2+o(t^3)\big)\\
&= \ln 2+(\frac{1}{2}-\ln 2)t+(\ln 2-\frac{5}{8})t^2+o(t^3),\\
x\big(\ln(1+x)-\ln x\big) &= (1+t)(\ln 2+\ln(1+\frac{t}{2})-\ln(1+t))\\
&= (1+t)\big(\ln 2-\frac{t}{2}+\frac{3}{8}t^2+o(t^3)\big)\\
&= \ln 2+(\ln 2-\frac{1}{2})t-\frac{1}{8}t^2+o(t^3),\\
S=\frac{\ln(1+x)}{x}+x\ln(1+\frac{1}{x}) &= \ln 4+(\ln 2-\frac{3}{4})t^2+o(t^3),\\
(1+x)^{1/x}(1+\frac{1}{x})^x-4 &= e^S-4=4(\ln 2-\frac{3}{4})t^2+o(t^3),
\end{aligned}$$

故

$$\lim_{x\to 1}\frac{(1+x)^{1/x}(1+\frac{1}{x})^x-4}{(x-1)^2}=4\ln 2-3.$$