ECN430 Assignment1

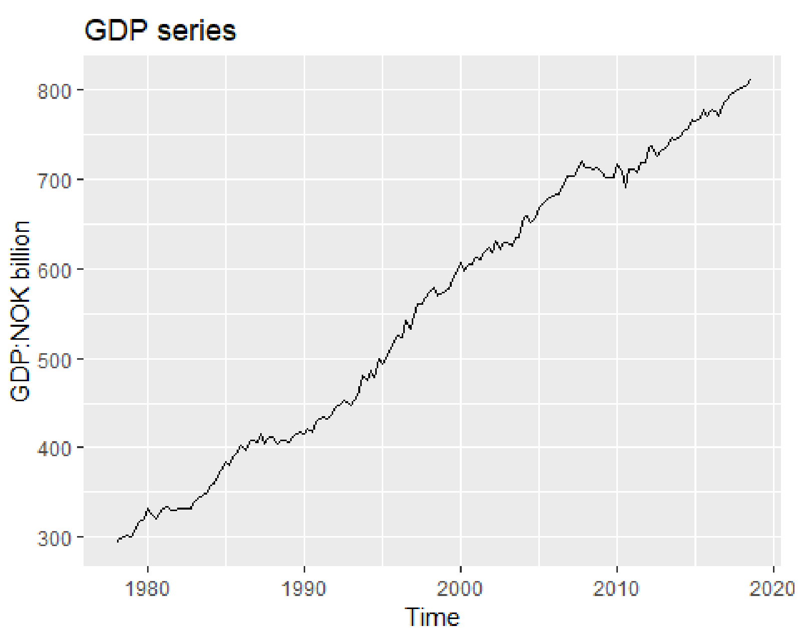
Group 1

Candidate numbers: 12 and 23

a) Plot and describe the series

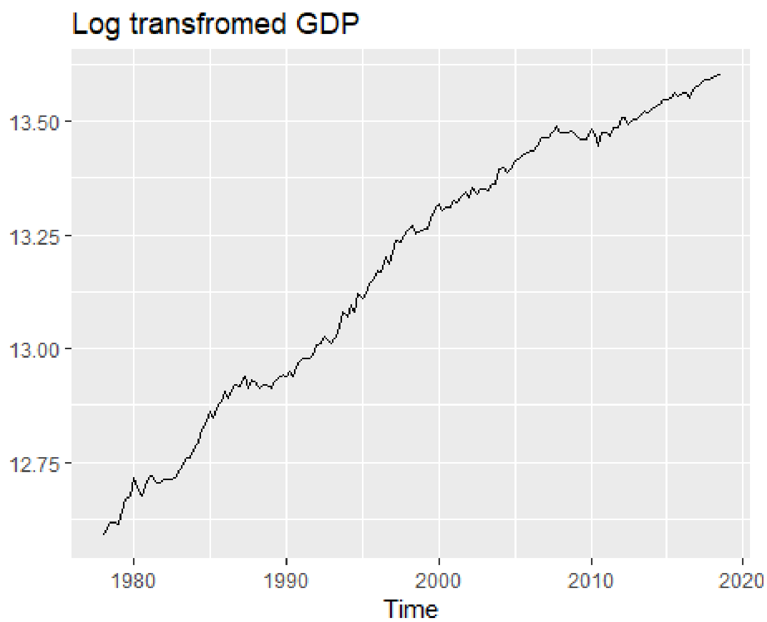
GDP series

The quarterly data series of GDP in Norway is plotted below. The figure shows a strong and steady increasing trend overtime. In terms of growth, generally the GDP grows steady over the horizon, except for some typical time periods. For example, two obvious economy recessions occurred in 1988 and 2008, the growth of GDP slowed down due to bank crisis in Norway and global financial crisis respectively. There are no significant quarterly effects indicated in the figure because we use seasonally adjusted data. Regarding the stationarity, this GDP series is obviously non-stationary, because it does not come back to a constant mean.



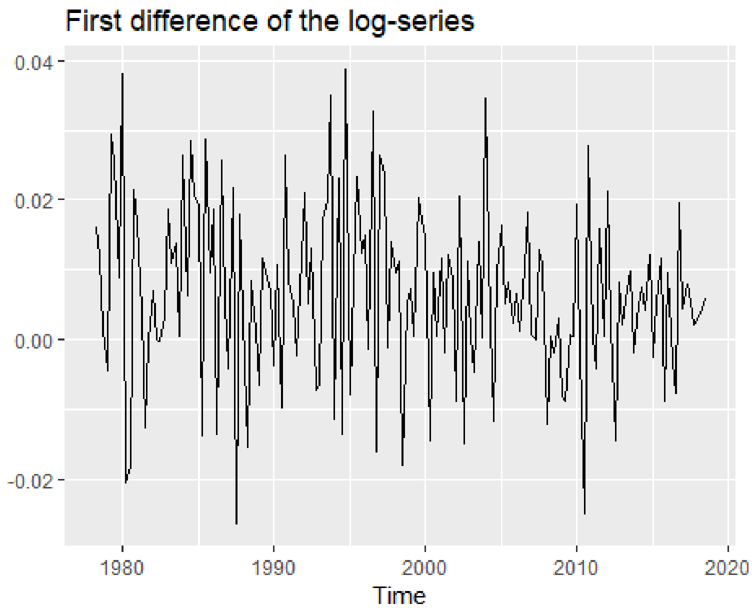
The log transformed GDP series

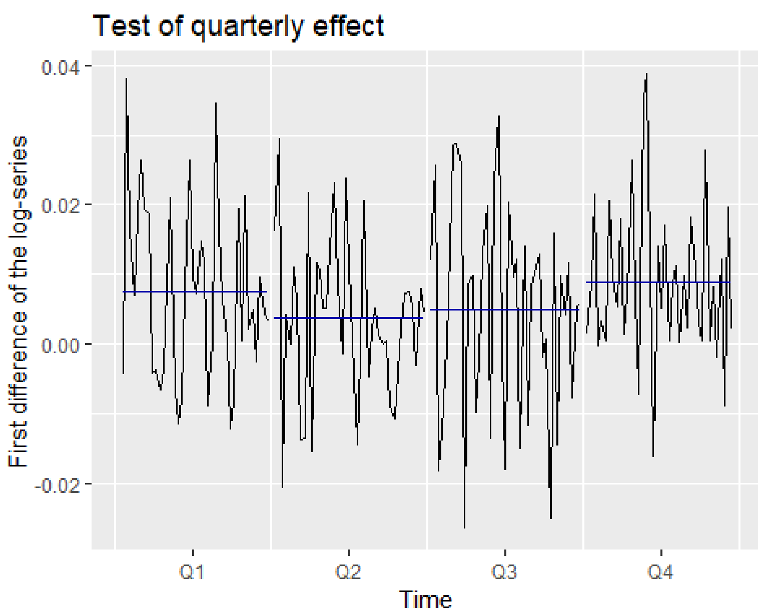
Overall, the quarterly data series of log transformed GDP shares similar properties with the GDP series, with a stable rising trend and growth. Also, the periods in 1988 and 2008 are exceptions, with fluctuant growth. However, the fluctuations in log transformed GDP series are less violent than those in previous GDP series, because log transformed data have less variance than the original data. The log transformed GDP series inheres no significant quarterly effects and is non-stationary as well.



The first difference of the log-series

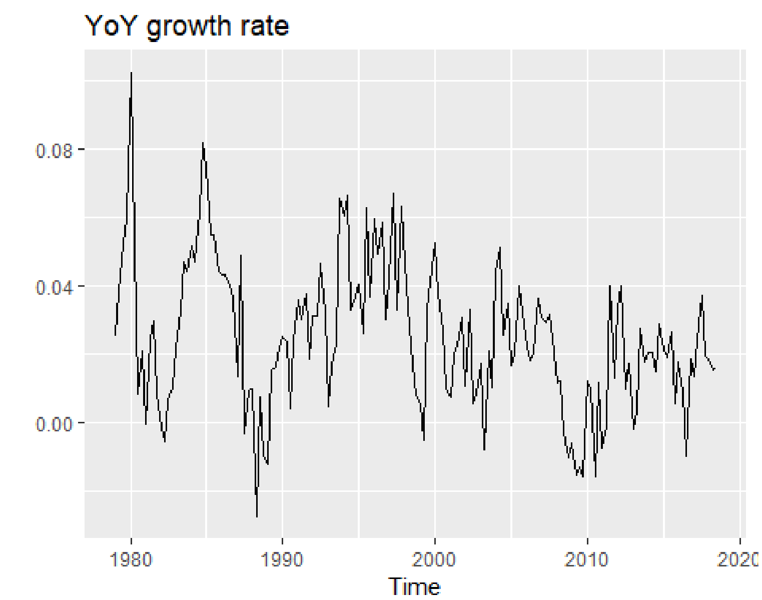
In the first difference of the log transformed GDP series, there are no trend or growth indicated in the graph, because differencing can stabilize the mean of a time series. About stationarity, because differencing makes series stable with a constant mean, we can see the first differenced log-GDP series always come back a constant mean. The variance also looks stable, so we regard the first differenced log-GDP series stationary. Regarding quarterly effects, we find it is difficult to judge from the plot. Hence, we plot each season as separate mini time series, and we can see the difference among quarters. The blue lines represent the mean of the observations within each season, and the four lines are at different levels. So, we can conclude that the first difference of the log-series has quarterly effect.

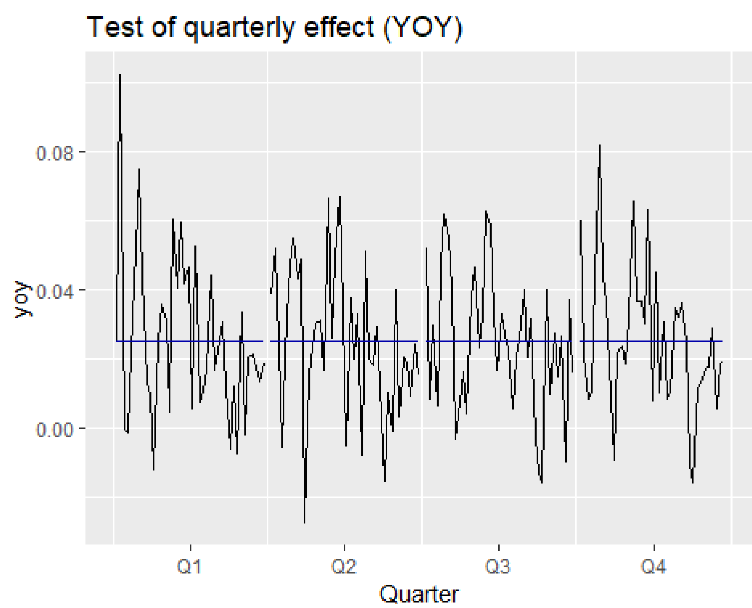




The YoY series

The year-on-year growth of GDP series is plotted below. Intuitively, there is no trend or growth indicated in the graph, because differencing stabilizes the time series. Accordingly, from this graph, we also see YOY series as stationary because it seems to have a constant mean and stable variance. In terms of quarterly effects, it is difficult to determine the time series plot. After plotting each season as separate mini time series, the mean of four seasons are the same. So, we can say there is no quarterly effect in YOY series.

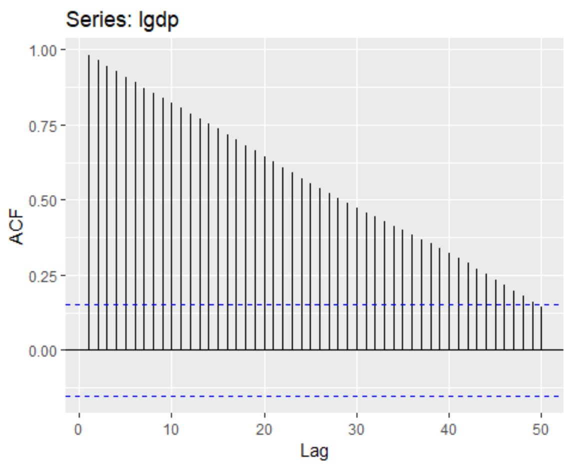


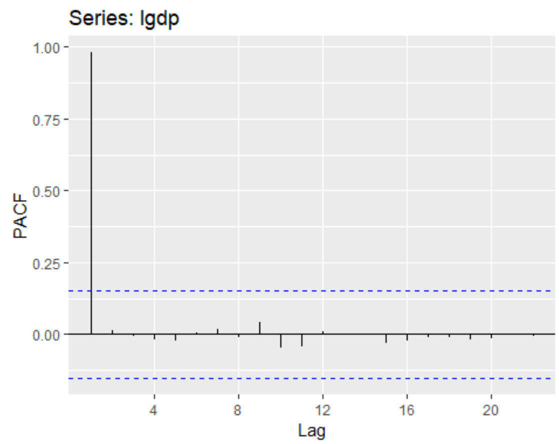


b) Use a correlogram to test the stationary

The log transformed GDP series

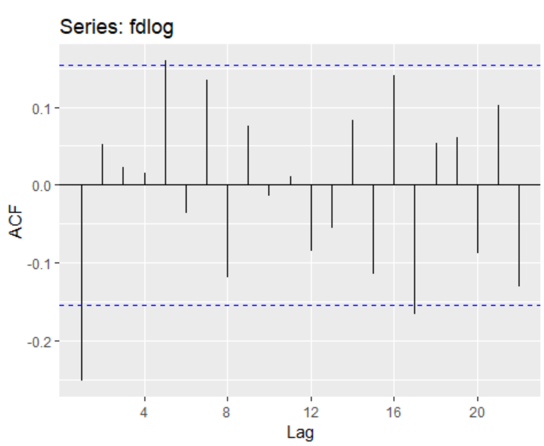
In part (a), we believed log transformed GDP series as nonstationary. In the ACF plot, the coefficients of lags remain significant until the 49th lag, indicating the shock impact decays really slowly. So, we can get the same inference that the log-transformed GDP series is nonstationary. We do not think it follows AR or MA model, as stationarity is a basic precondition.

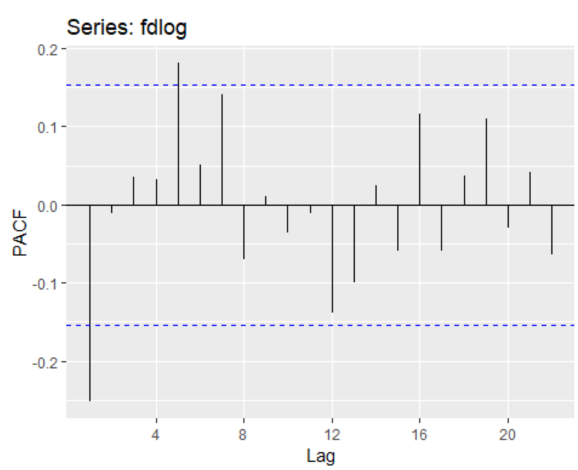




The first difference of the log-series

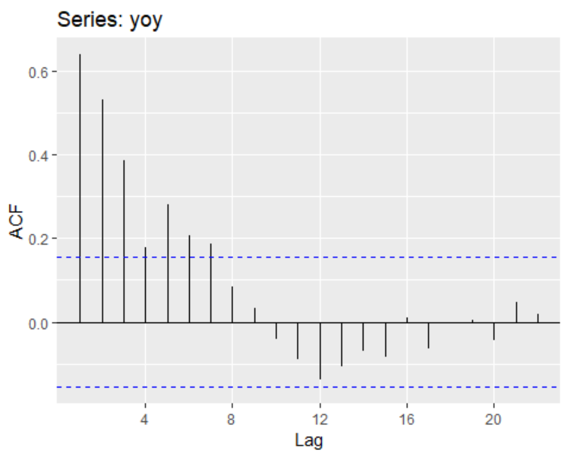
According to the correlogram, we think the first differenced log-transformed GDP series is stationary. In the ACF plot, there is a feature of MA(1) model, as its spike at first lag is significant, with the remaining decaying very quickly and suddenly. The subsequent spikes at lag 5 and lag 17 also seem significant, but we can ignore them as they are not in the first few lags. According to PACF plot, we can see this series also inheres feature of AR(1) model, because the partial autocorrelation function stop suddenly after the first lag. Hence this series should follow an ARMA model.

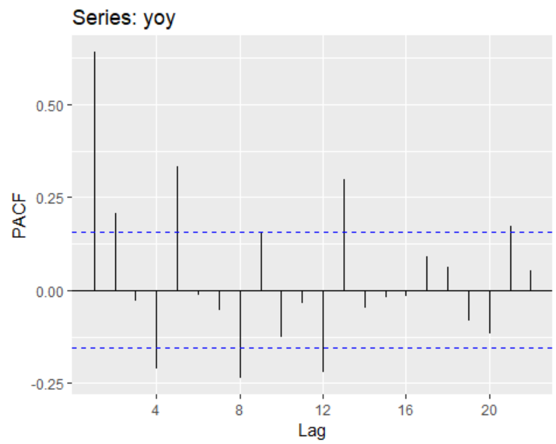




The YoY series

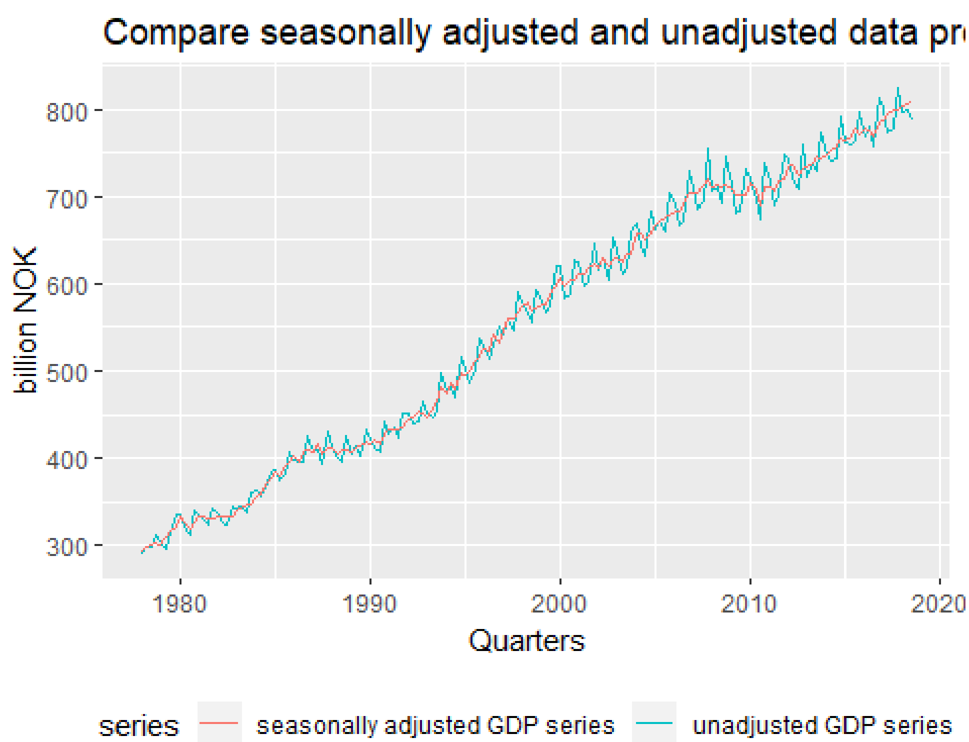
Practically, we use ACF to determine q in MA(q), and PACF to determine p in AR(p). We regard YoY series follows MA(3) model, because the first three spikes are significant in the ACF plot. We think the series also follows AR(2), as the spikes in the PACF plot stop suddenly after the first two lags. Therefore, we suggest that ARMA(2,3) model can appropriately fit the YoY series.





c) Take away seasonal effects

We should remove seasonal effects from the unadjusted data provided by SSB. As shown in the graph "Compare seasonally adjusted and unadjusted data provided by SSB", it is obvious that the unadjusted data inheres seasonal effects.



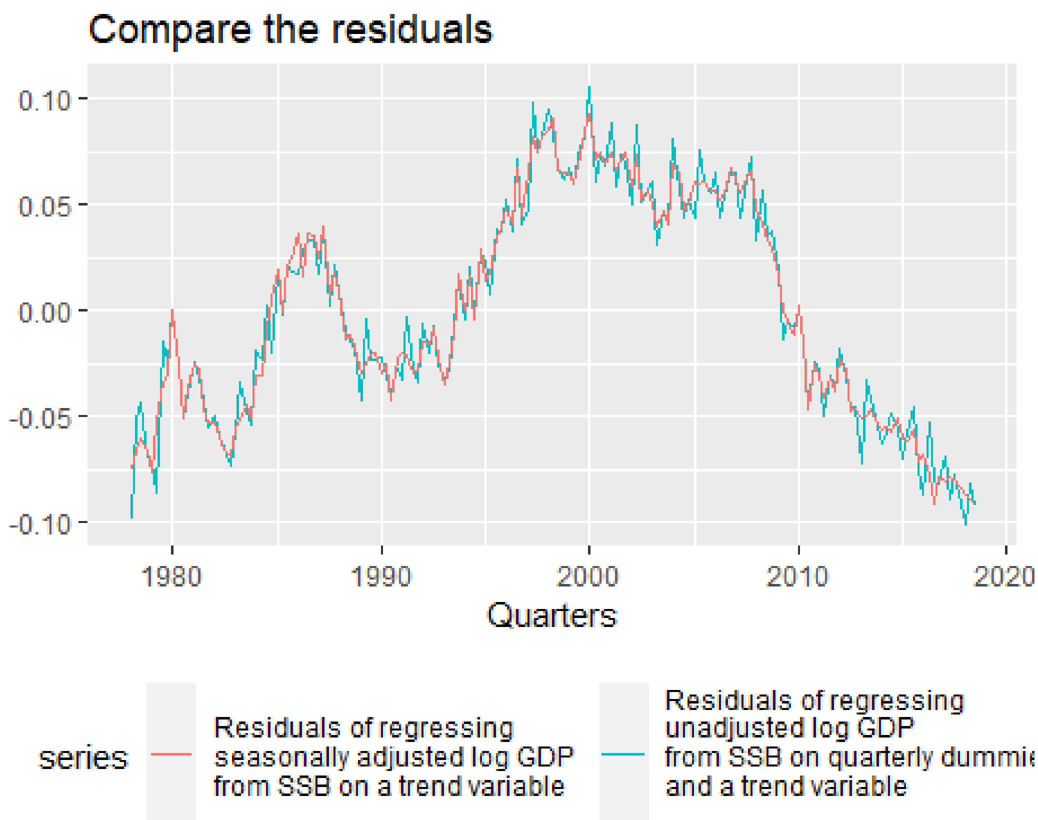
We include quarterly dummy variables and a potential variable "trend" into the regression model of the log transformed unadjusted GDP. As shown in the results below, the coefficients of three dummy variables are all significant. Therefore, we believe seasonal adjustments should be done before we further study the unadjusted GDP series.

```
Call:
tslm(formula = lugdp ~ trend + season)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.100941 -0.042334 -0.005627  0.045629  0.105581 

Coefficients:
              Estimate Std. Error  t value Pr(>|t|)    
(Intercept)  1.266e+01  1.088e-02 1164.196  < 2e-16 ***
trend        6.365e-03  8.777e-05   72.517  < 2e-16 ***
season2     -2.232e-02  1.164e-02   -1.917  0.05707 .  
season3     -3.325e-02  1.165e-02   -2.855  0.00488 ** 
season4      3.199e-02  1.172e-02    2.730  0.00705 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.05272 on 158 degrees of freedom
Multiple R-squared:  0.971,  Adjusted R-squared:  0.9703 
F-statistic:  1322 on 4 and 158 DF,  p-value: < 2.2e-16
```

According to the regression results, the coefficient of variable "trend" is also significant. So, we also detrend the log transformed seasonally adjusted GDP series with the results shown below.

```
Call:
tslm(formula = lgdp ~ trend)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.091353 -0.046183 -0.008594  0.049330  0.092498 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 1.266e+01  8.096e-03 1563.47   <2e-16 ***
trend       6.372e-03  8.563e-05   74.42   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.05144 on 161 degrees of freedom
Multiple R-squared:  0.9718, Adjusted R-squared:  0.9716 
F-statistic:  5538 on 1 and 161 DF,  p-value: < 2.2e-16
```

Then we compare the residuals of both regressions in the graph “Compare the residuals”, we can see that generally the lines of two residual series are overlapped, that is, we can get same answer with SSB data.



Moreover, we think the slight difference between two lines may due to different seasonal adjustment methods. To further insure our judgement, we test the correlation of two residuals, and they are around 99% correlated. So statistically we can believe the series we adjusted is identical to the adjusted series provided by SSB.

```
 Pearson's product-moment correlation

data:  reslugdp and reslgdp
t = 73.709, df = 161, p-value < 2.2e-16
alternative hypothesis: true correlation is not equal to 0
95 percent confidence interval:
 0.9802907 0.9893468
sample estimates:
      cor
0.9855047
```

d) Unit Root Test

We use the seasonally adjusted data from SSB so we believe that we have already controlled the seasonal effects before we use ADF test. In our R command argument, test type is set to "trend", with both an intercept and a trend added when doing tests. We choose AIC to decide about the lag-length used in the ADFs.

Our conclusion about stationarity of three series remain the same as in part (a), that is, the log transformed GDP series is nonstationary, while the first differenced log-transformed GDP series and YoY series are stationary. Also, the conclusions are identical using different testing methods. The detailed explanations are stated below and tests results are listed in the appendix.

Both the first differenced log-transformed GDP series and YoY series are stationary at the same confidence level, but we prefer YoY series, because in part (a) we find YoY series has no quarterly effects while there's slight quarterly effects in the first differenced log-transformed GDP series.

The log transformed GDP series

In the ADF test results for log transformed GDP series, the t-statistic is larger than the 10% critical value, failing to reject null hypothesis. Therefore, we accept the null hypothesis that the log transformed GDP is nonstationary.

In the KPSS test, the test statistic is greater than the 1% critical value, indicating that the null hypothesis is rejected. Thus, the log transformed GDP is non-stationary, which is the same as in the ADF test.

In the Elliot, Rothenberg and Stock test, the test statistic is much bigger than the 10% critical value, fail to reject null hypothesis, therefore, the log transformed GDP is nonstationary, which is the same as in the ADF test and KPSS test.

	test-statistic	Critical Values			Null hypothesis	Conclusion
		1%	5%	10%		
ADF	-0.9372	-3.99	-3.43	-3.13	Unit root	Unit root
KPSS	1.2545	0.739	0.463	0.347	stationary	Unit root
ERS	-0.597	-3.46	-2.93	-2.64	Unit root	Unit root

The first difference of the log-series

In the augmented Dickey-Fuller (ADF) test, the test statistic is smaller than the 1% critical value, therefore we reject null hypothesis with 99% confidence. Hence, the first differenced log-transformed GDP series is stationary.

In the KPSS test, the test statistic is smaller than the 5% critical value and fails to reject null hypothesis. Therefore, the first differenced log-transformed GDP series is stationary, which is the same as in the ADF test.

In the Elliot, Rothenberg and Stock test, the test statistic is smaller than the 1% critical value, therefore we reject null hypothesis at 99% confidence level. Hence the first differenced log-transformed GDP series is stationary, which is the same as in the ADF test and KPSS test.

	test-statistic	Critical values			Null hypothesis	Conclusion
		1%	5%	10%		
ADF	-10.4355	-3.99	-3.43	-3.13	Unit root	stationary
KPSS	0.3973	0.739	0.463	0.347	stationary	stationary
ERS	-4.2662	-3.46	-2.93	-2.64	Unit root	stationary

The YoY series

In the augmented Dickey-Fuller (ADF) test, the test statistic is smaller than the 1% critical value, therefore we reject null hypothesis with 99% confidence. So, the YoY series is stationary.

In the KPSS test, the test statistic is smaller than the 5% critical value and fails to reject null hypothesis. Thus, the YoY series is stationary, which is the same as in the ADF test.

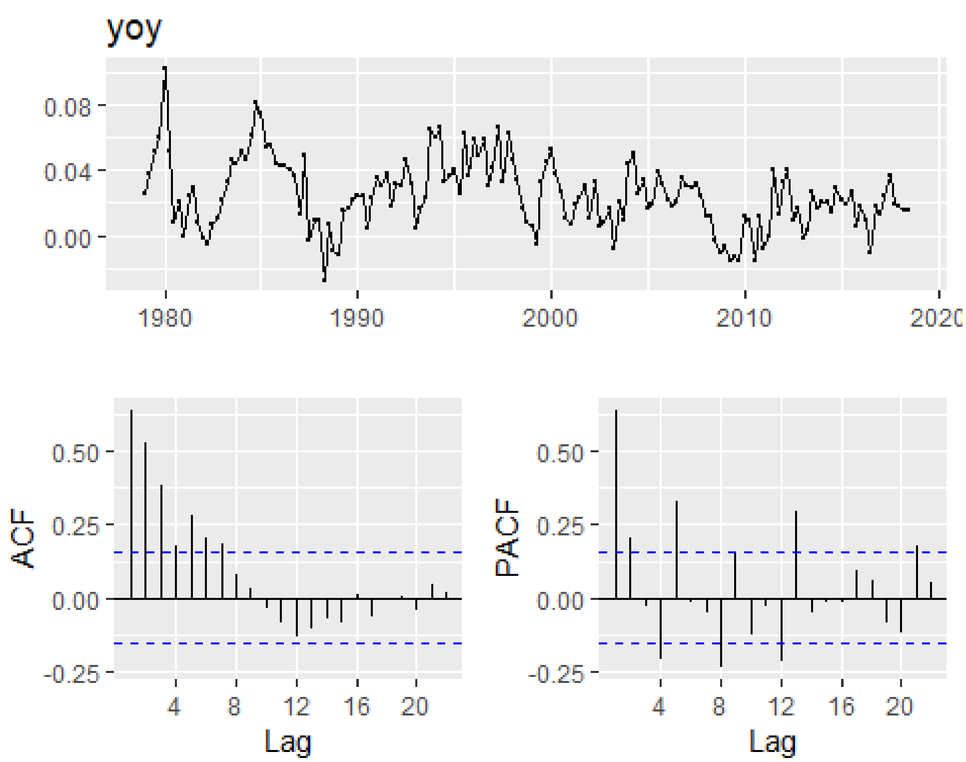
In the Elliot, Rothenberg and Stock test, the test statistic is smaller than the 1% critical value, therefore we reject null hypothesis with 99% confidence. Hence the YoY series is stationary, which is the same as in the ADF test and KPSS test.

	test-statistic	Critical values			Null hypothesis	Conclusion
		1%	5%	10%		
ADF	-4.5761	-3.99	-3.43	-3.13	Unit root	stationary
KPSS	0.4058	0.739	0.463	0.347	stationary	stationary
ERS	-3.8193	-3.46	-2.93	-2.64	Unit root	stationary

e) Partial autocorrelation function

Autocorrelation function (ACF) measures the relationship between y(t) and y(t-k) for different values of k. If y(t) and y(t-1) are correlated, then y(t-1) and y(t-2) must be correlated. However, we cannot determine whether y(t) and y(t-2) is correlated simply because they both correlated to y(t-1). Partial autocorrelation function (PACF) is a good method to overcome this problem. PACF can measure the relationship between y(t) and y(t-k) after removing the effects of lag 1, 2…, (k-1). In addition, practically, we use ACF to determine q in MA(q), and PACF to determine p in AR(p).

Since we prefer the YoY series, we make a correlogram of it. As stated in part (b), we regard YoY series follows an ARMA(2,3) model.



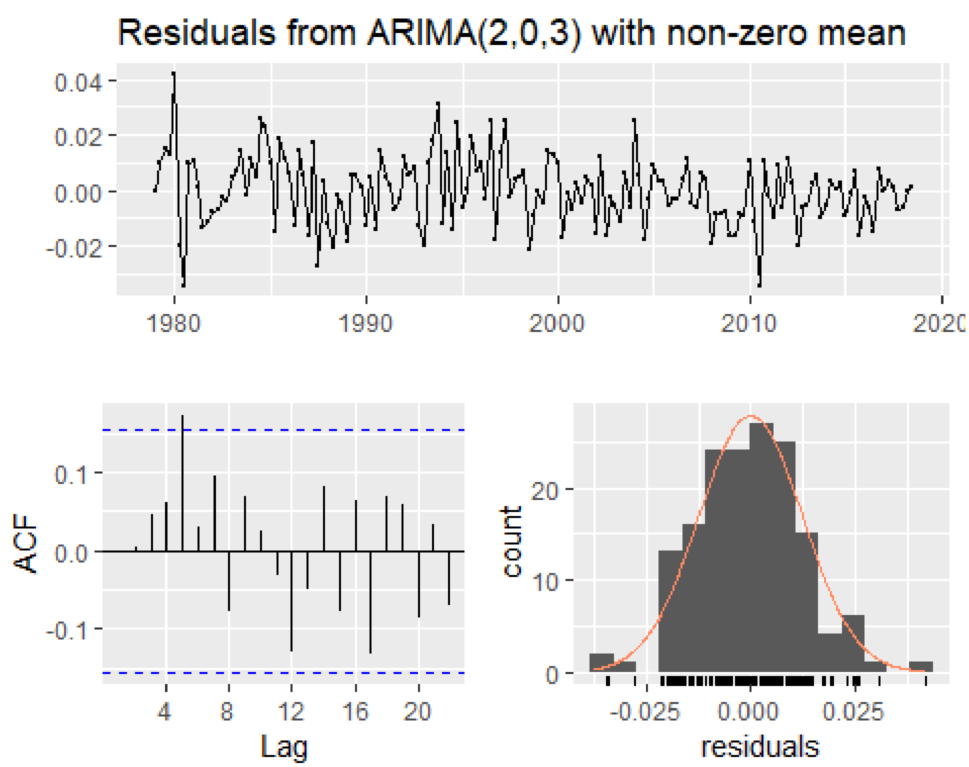
f) Fit an ARMA/ARIMA model

We fit an ARMA(2,3) model on the YoY series, along with variations including ARMA(2,1), ARMA(2,2) and ARMA(2,4). We prefer the ARMA(2,3) model because it has the smallest AICc value among four models. The detailed tests results are listed in the appendix.

AR(p)	MA(q)	AIC	AICc	BIC
2	1	-851.91	-851.51	-836.56
2	2	-885.42	-884.87	-867
2	3	-919.84	-919.1	-898.36
2	4	-919.54	-918.58	-894.99

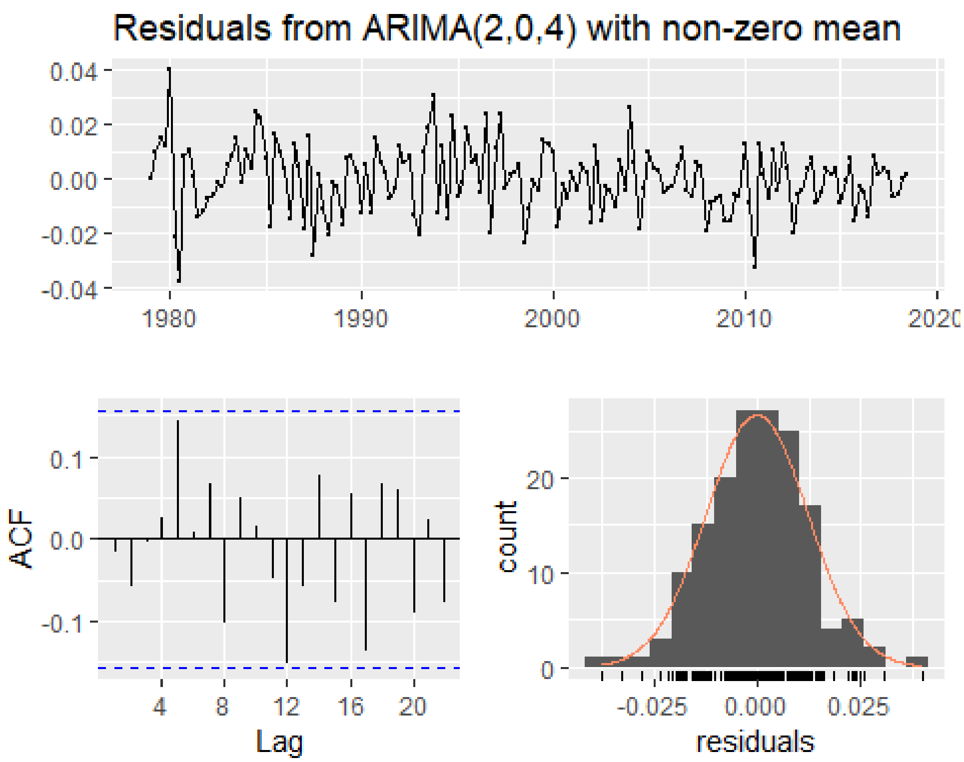
g) Test the residual

The ACF correlogram of the residuals of ARMA(2,3) model on YoY series shows that not all the spikes are within the bounds, which means the residuals may be correlated. Moreover, the p-value in Ljung-Box test is less than 0.05, also indicating the residual series does not follow Gaussian distribution of white noise.




```
  Ljung-Box test

data:  Residuals from ARIMA(2,0,3) with non-zero mean
Q* = 9.4177, df = 3, p-value = 0.02422

Model df: 6.   Total lags used: 9

       ar1         ar2         ma1         ma2         ma3   intercept

-0.25659044  0.01100460  0.99646193  0.99646966  0.99997406  0.02516158
```

So we go back to step (f) and look at ARMA(2,4) model, whose AICc is the second lowest. For ARMA(2,4), the ACF plot of the residuals shows that all the spikes are within the bounds, which means the residuals are uncorrelated. In addition, the residuals follow a normal distribution, and seem like white noise. In addition, in Ljung-Box test, p-value is larger than 0.05, which can also prove the residual series is stationary. Accordingly, we believe our preferred model ARMA(2,4) on YoY series as an appropriately fitted model.



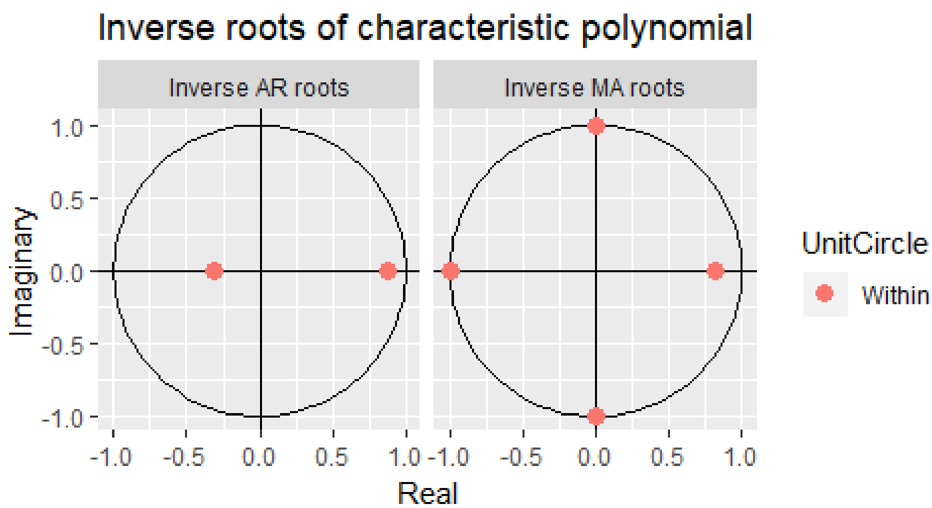

```
Ljung-Box test

data:  Residuals from ARIMA(2,0,4) with non-zero mean
Q* = 7.1602, df = 3, p-value = 0.06696

Model df: 7.   Total lags used: 10

       ar1        ar2        ma1        ma2        ma3        ma4  intercept
 0.5619339  0.2711374  0.1821107  0.1839008  0.1860937 -0.8156890  0.0253294
```

Characteristic equation and characteristic roots

The characteristic equation is followed: x^2 - 0.56x - 0.27 = 0. The characteristic roots from ARMA(2,4) are 0.8726424 and -0.3107085 respectively , with results shown as below.

```
    eigen() decomposition
$values
[1]  0.8726424 -0.3107085

$vectors
          [,1]       [,2]
[1,] 0.6574979 -0.2967160
[2,] 0.7534563  0.9549658
```

They are also demonstrated in the below unit circle graph. We only look at the AR(p) part because MA(q) is irrelevant for determining stationarity, so MA(q) part can be ignored as long as q is finite. In the AR(2) part in the graph, both the points representing characteristic roots are within the unit circle, indicating that the model ARMA(2,4) is stationary.



h) Compare models and interpret coefficients.

Compared to the ARMA(2,2) model on YoY series, our preferred model, ARMA(2,4) on YoY series, has smaller AICc value and BIC value. Hence, ARMA(2,4) on YoY series fits better than ARMA(2,2) on YoY series.

Regarding the interpretation of coefficients, for ARMA(2,4) on YoY series, every 1% increase on YoY growth of log-GDP in time period y(t-1) can lead to 56.19% increase on YoY growth of log-GDP in time period y(t), and similarly, every 1% increase on YoY growth of log-GDP in time period y(t-2) can lead to 27.11% increase on YoY growth of log-GDP in time period y(t).

```
Series: yoy
ARIMA(2,0,4) with non-zero mean

Coefficients:
         ar1     ar2     ma1     ma2     ma3      ma4    mean
      0.5619  0.2711  0.1821  0.1839  0.1861  -0.8157  0.0253
s.e.  0.1769  0.0809  0.1690  0.1685  0.1682   0.1682  0.0042

sigma^2 estimated as 0.0001579:  log likelihood=467.77
AIC=-919.54   AICc=-918.58   BIC=-894.99
```

For ARMA(2,2) on YoY series, every 1% increase on YoY growth of log-GDP in time period y(t-1) can lead to 48.15% increase on YoY growth of log-GDP in time period y(t), while every 1% increase on YoY growth of log-GDP in time period y(t-2) can lead to 24.74% decrease on YoY growth of log-GDP in time period y(t).

```
Series: yoy
ARIMA(2,0,2) with non-zero mean

Coefficients:
         ar1      ar2      ma1    ma2    mean
      0.4815  -0.2474  -0.0183  1.000  0.0252
s.e.  0.0819   0.0803   0.0268  0.028  0.0029

sigma^2 estimated as 0.0002026:  log likelihood=448.71
AIC=-885.42   AICc=-884.87   BIC=-867
```

i) Reestimate models until T=2007Q4

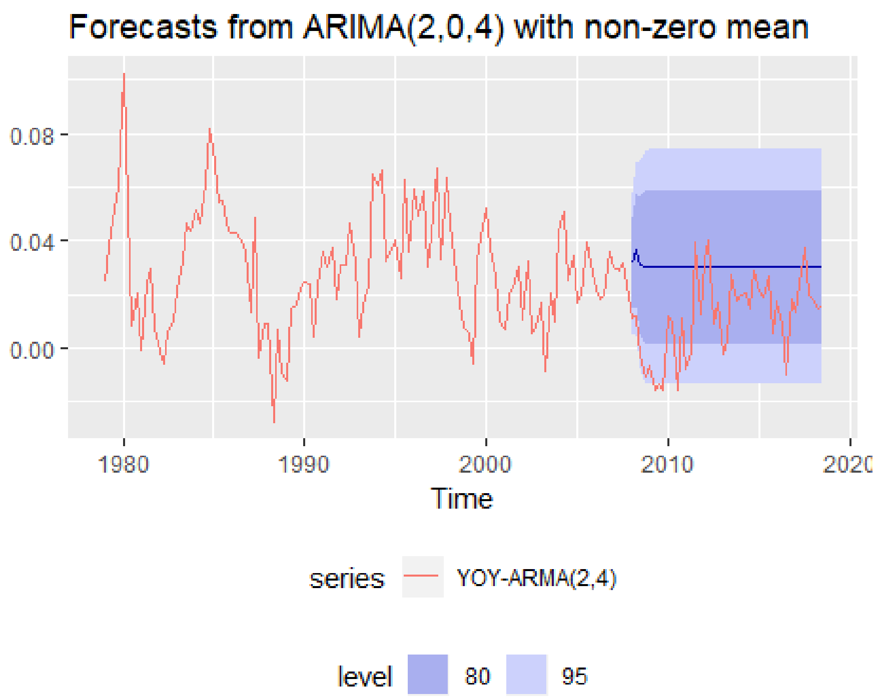
From time period 1978Q1 to 2007Q4, our preferred model still has smaller AICc value and BIC value, compared to the ARMA(2,2) model of YoY series. Hence, for the training data set, our preferred model fits better than ARMA(2,2) on YoY series.

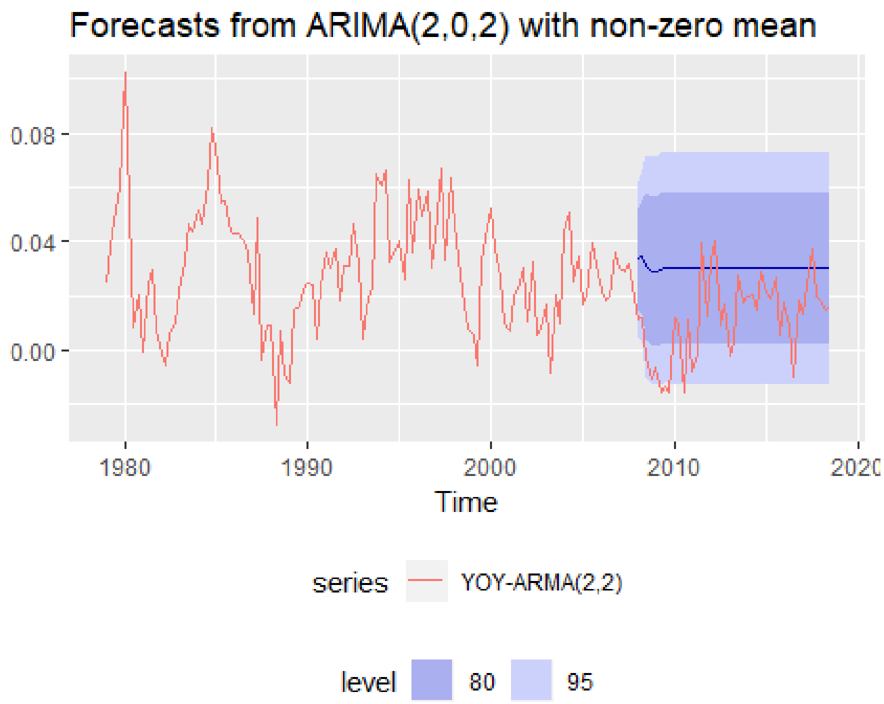
```
Series: yoy2
ARIMA(2,0,4) with non-zero mean

Coefficients:
          ar1      ar2     ma1     ma2     ma3     ma4    mean
      -0.5644  -0.0485  1.2691  1.2634  1.2831  0.2888  0.0302
s.e.   0.8318   0.2530  0.8273  0.8113  0.8130  0.8283  0.0038

sigma^2 estimated as 0.0001774:  log likelihood=334.39
AIC=-652.79   AICc=-651.44   BIC=-630.76

Series: yoy2
ARIMA(2,0,2) with non-zero mean
```

```
Coefficients:
         ar1      ar2      ma1     ma2    mean
      0.5224  -0.3285  -0.0571  1.0000  0.0301
s.e.  0.0999   0.0972   0.0448  0.0407  0.0032

sigma^2 estimated as 0.0002125:  log likelihood=324.53
AIC=-637.06   AICc=-636.29   BIC=-620.54
```

Generate a point forecast

We use the remaining data as a test set to make a forecast. As shown in the below two graphs, these point forecasts convey very little information because they lack precision. Thus, we focus on the forecast interval and find out the test sets of both models almost fall into the 95% confidence level, so generally speaking, both models have a good forecast.





j) Evaluate the bias and RMSE

According to the accuracy test results on both models, the Mean Error (ME) on test set of both models are -0.00008 and the Root Mean Squared Error (RMSE) on test set of both models are -0.02. As the bias and RMSE in both models are nearly identical, we should choose ARMA(2,2) rather than ARMA(2,4), as the former one is less complicated and easier to interpret.

What's more, the Mean Percentage Error (MPE) and Mean Absolute Percentage Error (MAPE) on test set of ARMA(2,2) model is slightly lower than those of ARMA(2,4) model. So, ARMA(2,4) may have a problem of overfitting, and is not an ideal model.

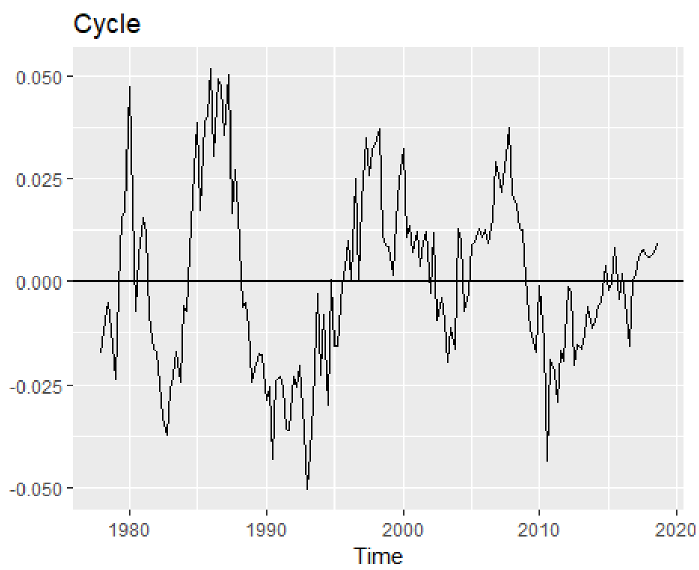
Above all, we changed our mind and prefer ARMA(2,2) model.

Model	Data	ME	RMSE	MAE	MPE	MAPE	MASE
ARMA (2,4)	Training set	-0.00008	0.01	0.01	-14.2	75.1	0.45
	Test set	-0.02	0.02	0.02	109.29	293.5	0.88
ARMA (2,2)	Training set	-0.00008	0.01	0.01	-10.9	78.4	0.48
	Test set	-0.02	0.02	0.02	108.99	292.3	0.88

The state of the Norwegian business cycle

We use HP filter to extract trend component and cycle component from the log transformed GDP data, and make plots of them. In the graph "HP filter", we can see that generally Norway GDP was growing at a steady trend over the horizon.

In the graph "Cycle", we can see at the beginning of the horizon, around 1978, the Norway economy is in a recession, which may due to the oil price shock in 1973 and 1974, causing a negative effect on the output growth for a prolonged period. Then followed by an expansion period until 1987, when bank crisis occurred in late 1980s, making the Norwegian business cycle enter a recession and subsequent rebound period throughout 1990s. The next fall is around 2007 and 2008, due to the financial crisis. 2011 seems to be the latest trough, and the later period until now is in a long rebound period during which happened several short term fluctuations due to the vulnerability of Norwegian Economy.



Appendix

ADF test for log transformed GDP series

```
Augmented Dickey-Fuller Test Unit Root Test


 Test regression trend


 Call:
 lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)

 Residuals:
       Min        1Q    Median        3Q       Max
 -0.035395 -0.007548  0.000452  0.007342  0.031201

 Coefficients:
               Estimate Std. Error t value Pr(>|t|)
 (Intercept)  2.424e-01  2.461e-01   0.985 0.326044
 z.lag.1     -1.823e-02  1.945e-02  -0.937 0.350076
 tt           6.959e-05  1.262e-04   0.552 0.582065
 z.diff.lag  -2.652e-01  7.720e-02  -3.435 0.000758 ***
 ---
 Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

 Residual standard error: 0.01236 on 157 degrees of freedom
 Multiple R-squared:  0.09699,    Adjusted R-squared:  0.07973
 F-statistic: 5.621 on 3 and 157 DF,  p-value: 0.001099
```

```
Value of test-statistic is: -0.9372 19.2136 2.9039

Critical values for test statistics:
      1pct  5pct 10pct
tau3 -3.99 -3.43 -3.13
phi2  6.22  4.75  4.07
phi3  8.43  6.49  5.47
```

KPSS test for log transformed GDP series

```
 KPSS Unit Root Test


Test is of type: mu with 13 lags.

Value of test-statistic is: 1.2545

Critical value for a significance level of:
                10pct  5pct 2.5pct  1pct
critical values 0.347 0.463  0.574 0.739
```

ERS test for log transformed GDP series

```
Elliot, Rothenberg and Stock Unit Root Test


Test of type DF-GLS
detrending of series with intercept and trend


Call:
lm(formula = dfgls.form, data = data.dfgls)

Residuals:
      Min        1Q    Median        3Q       Max
-0.035838 -0.007615 -0.001041  0.007553  0.032895

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
yd.lag       -0.01048    0.01756  -0.597  0.55137
yd.diff.lag1 -0.24837    0.08166  -3.041  0.00277 **
yd.diff.lag2  0.01340    0.08400   0.160  0.87348
yd.diff.lag3  0.06049    0.08392   0.721  0.47209
yd.diff.lag4  0.04131    0.08091   0.511  0.61039
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.01267 on 153 degrees of freedom
Multiple R-squared:  0.07016,    Adjusted R-squared:  0.03977
F-statistic: 2.309 on 5 and 153 DF,  p-value: 0.04688
```

```
 Value of test-statistic is: -0.597

Critical values of DF-GLS are:
                 1pct  5pct 10pct
critical values -3.46 -2.93 -2.64
```

ADF test for the first differenced log-transformed GDP series

```
Augmented Dickey-Fuller Test Unit Root Test

 Test regression trend

 Call:
 lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)

 Residuals:
       Min        1Q    Median        3Q       Max
 -0.034221 -0.008260  0.000219  0.006907  0.032210

 Coefficients:
               Estimate Std. Error t value Pr(>|t|)
 (Intercept)  1.218e-02  2.322e-03   5.246 4.99e-07 ***
 z.lag.1     -1.332e+00  1.277e-01 -10.435  < 2e-16 ***
 tt          -4.809e-05  2.180e-05  -2.206   0.0288 *
 z.diff.lag   4.372e-02  7.990e-02   0.547   0.5850
 ---
 Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

 Residual standard error: 0.01242 on 156 degrees of freedom
 Multiple R-squared:  0.639,  Adjusted R-squared:  0.632
 F-statistic: 92.04 on 3 and 156 DF,  p-value: < 2.2e-16


 Value of test-statistic is: -10.4355 36.3065 54.4561

 Critical values for test statistics:
       1pct  5pct 10pct
 tau3 -3.99 -3.43 -3.13
 phi2  6.22  4.75  4.07
 phi3  8.43  6.49  5.47
```

KPSS test for the first differenced log transformed GDP series

```
KPSS Unit Root Test


 Test is of type: mu with 13 lags.

 Value of test-statistic is: 0.3973
```

```
Critical value for a significance level of:
                10pct  5pct 2.5pct  1pct
critical values 0.347 0.463  0.574 0.739
```

ERS test for the first differenced log transformed GDP series

```
Elliot, Rothenberg and Stock Unit Root Test


 Test of type DF-GLS
 detrending of series with intercept and trend


 Call:
 lm(formula = dfgls.form, data = data.dfgls)

 Residuals:
       Min        1Q    Median        3Q       Max
 -0.044543 -0.007881 -0.000539  0.005934  0.030043

 Coefficients:
              Estimate Std. Error t value Pr(>|t|)
 yd.lag       -0.83589    0.19593  -4.266 3.48e-05 ***
 yd.diff.lag1 -0.40257    0.17682  -2.277   0.0242 *
 yd.diff.lag2 -0.37666    0.15441  -2.439   0.0159 *
 yd.diff.lag3 -0.30113    0.12371  -2.434   0.0161 *
 yd.diff.lag4 -0.20095    0.07825  -2.568   0.0112 *
 ---
 Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

 Residual standard error: 0.01252 on 152 degrees of freedom
 Multiple R-squared:  0.6356, Adjusted R-squared:  0.6236
 F-statistic: 53.03 on 5 and 152 DF,  p-value: < 2.2e-16


 Value of test-statistic is: -4.2662

 Critical values of DF-GLS are:
                  1pct  5pct 10pct
 critical values -3.46 -2.93 -2.64
```

ADF test for the YoY series

```
Augmented Dickey-Fuller Test Unit Root Test


 Test regression trend


 Call:
```

```
lm(formula = z.diff ~ z.lag.1 + 1 + tt + z.diff.lag)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.047999 -0.010267  0.000593  0.009096  0.051573 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.192e-02  3.725e-03   3.201  0.00166 ** 
z.lag.1     -3.249e-01  7.100e-02  -4.576 9.73e-06 ***
tt          -4.937e-05  3.051e-05  -1.618  0.10774    
z.diff.lag  -1.873e-01  7.925e-02  -2.364  0.01935 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.0162 on 153 degrees of freedom
Multiple R-squared:  0.2281, Adjusted R-squared:  0.213 
F-statistic: 15.07 on 3 and 153 DF,  p-value: 1.209e-08


Value of test-statistic is: -4.5761 6.9859 10.4704 

Critical values for test statistics: 
      1pct  5pct 10pct
tau3 -3.99 -3.43 -3.13
phi2  6.22  4.75  4.07
phi3  8.43  6.49  5.47
```

KPSS test for the YoY series

```
KPSS Unit Root Test 


 Test is of type: mu with 13 lags. 

 Value of test-statistic is: 0.4058 

 Critical value for a significance level of: 
                10pct  5pct 2.5pct  1pct
 critical values 0.347 0.463  0.574 0.739
```

ERS test for the YoY series

```
Elliot, Rothenberg and Stock Unit Root Test 


 Test of type DF-GLS 
 detrending of series with intercept and trend 
```

```
Call:
lm(formula = dfgls.form, data = data.dfgls)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.043747 -0.008266 -0.000090  0.008514  0.039335 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
yd.lag       -0.28735    0.07523  -3.819 0.000196 ***
yd.diff.lag1 -0.18169    0.08274  -2.196 0.029647 *  
yd.diff.lag2  0.04550    0.08241   0.552 0.581717    
yd.diff.lag3  0.04528    0.08109   0.558 0.577413    
yd.diff.lag4 -0.31774    0.07404  -4.291 3.18e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.01447 on 149 degrees of freedom
Multiple R-squared:  0.3757, Adjusted R-squared:  0.3547 
F-statistic: 17.93 on 5 and 149 DF,  p-value: 6.77e-14


Value of test-statistic is: -3.8193 

Critical values of DF-GLS are:
                 1pct  5pct 10pct
critical values -3.46 -2.93 -2.64
```

Fitting ARIMA models

```
Series: yoy 
ARIMA(2,0,1) with non-zero mean 
Coefficients:
         ar1    ar2     ma1    mean
      0.4666  0.230  0.0400  0.0248
s.e.  0.2125  0.147  0.2084  0.0043
sigma^2 estimated as 0.0002647:  log likelihood=430.95
AIC=-851.91   AICc=-851.51   BIC=-836.56



Series: yoy 
ARIMA(2,0,2) with non-zero mean 
Coefficients:
         ar1      ar2      ma1    ma2    mean
      0.4815  -0.2474  -0.0183  1.000  0.0252
s.e.  0.0819   0.0803   0.0268  0.028  0.0029
sigma^2 estimated as 0.0002026:  log likelihood=448.71
AIC=-885.42   AICc=-884.87   BIC=-867
```

```
Series: yoy
ARIMA(2,0,3) with non-zero mean
Coefficients:
          ar1     ar2     ma1     ma2     ma3    mean
      -0.2566  0.0110  0.9965  0.9965  1.0000  0.0252
s.e.   0.0832  0.0807  0.0318  0.0362  0.0309  0.0031
sigma^2 estimated as 0.0001587:  log likelihood=466.92
AIC=-919.84   AICc=-919.1   BIC=-898.36



Series: yoy
ARIMA(2,0,4) with non-zero mean
Coefficients:
         ar1     ar2     ma1     ma2     ma3      ma4    mean
      0.5619  0.2711  0.1821  0.1839  0.1861  -0.8157  0.0253
s.e.  0.1769  0.0809  0.1690  0.1685  0.1682   0.1682  0.0042
sigma^2 estimated as 0.0001579:  log likelihood=467.77
AIC=-919.54   AICc=-918.58   BIC=-894.99
```

Evaluate the bias and RMSE for the ARMA(2,4) model

```
                   ME                   RMSE         MAE          MPE
Training set       -8.006013e-05        0.01291237   0.01031028   -14.2480
Test set           -1.878305e-02        0.02396166   0.02016975   109.2899
                   MAPE                 MASE         ACF1         Theil's U
Training set       75.06919             0.4514958    -0.002072612 NA
Test set           293.50348            0.8832502    0.440345684  0.7212641
```

Evaluate the bias and RMSE for the ARMA(2,2) model

```
                   ME                   RMSE         MAE          MPE
Training set       -7.555701e-05        0.01426062   0.01103361   -10.88523
Test set           -1.866987e-02        0.02384286   0.02007043   108.98994
                   MAPE                 MASE         ACF1         Theil's U
Training set       78.3798              0.4831710    0.05416782          NA
Test set           292.3178             0.8789008    0.43420097   0.7152266
```

Reference

Applied time series for macroeconomics, Hilde C. Bjørnland, Leif Anders Thorsrud, 2nd ed.

Forecasting: Principles and Practice, Rob J. Hyndman and George Athanasopoulos, 2nd edition