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Project/ Thesis Title

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Dedicated to my parents, Mr. A

And

Mrs. B

Abstract

gsjhdgsjdgwdfjegu

Key words: Dynamic Networks, Periodic Behaviors, Supergraph,

Acknowledgment

In this very special moment, first and foremost I would like to express my heartiest gratitude to the almighty God for allowing me to accomplish this MS study successfully. I am really thankful for the enormous blessings that the Almighty has bestowed upon me not only during my study period but also throughout my life.

In achieving the gigantic goal, I have gone through the interactions with and help from other people, and would like to extend my deepest appreciation to those who have contributed to this dissertation itself in an essential way.

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Author Name

October, 2020

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Chapter 1

Introduction

This chapter presents an overview of the periodic behavior mining in social networking research including the goal, and potential applications in this area. Then, different existing methods are discussed briefly to figure out the advantages as well as disadvantages of each particular technique. Based on the discussion, motivations behind the proposed method are explained clearly in this thesis. The organization of this thesis is presented at the end of this chapter.

1.1 Overview

The goal of periodic behavior miming is to identify regularly occurring behavior in dynamic networks. In general, a dynamic network is a powerful model for representing time varying systems where interactions among entities change from time to time. Dynamic network analysis has gained the attention of the computer science community because of its different applications such as human societies and behavior analysis, wild animal communities' behavior matching, sensor networks activities prediction and the study of uses of mobile cell.

Dynamic network dataset size increases very rapidly. Not all the information in networks conveys interesting meaning. The definition of interesting depends on the context. Data mining is the studies of extracting interesting information from data. The items those are frequent in database must hold some valuable meaning such as market basket analysis [2], which indicates items purchase behavior of consumer. Structured data mining has attracted researchers attention in the last few years. Graphs are probably one of the best representations of structured data and relevant

research field is extremely vast. Here an overview of the most interesting problems and their solution techniques are discussed briefly.

In the middle of 1990s, the first studies on graph mining methods [3, 4] proposed discovering graph representation technique for structures data. In [5], an inductive logic programming based technique has been proposed that extracts frequent subgraph from graph data. An efficient method for frequent subgraph mining is proposed by Nijssen and Kok in [6].

Since the 2000s, the data mining research community showed great effort in periodic pattern mining. Their research interest is distributed in several domains [7–9] like as transactional datasets, daily traffic patterns, stock data, meteorological data and web logs data. Periodic patterns in graph database is also incredibly interesting and information has been shown in [1, 10] for human periodic behavior analysis in social network and celebrity selection. Therefore, periodic patterns mining is one of the important tasks in data mining.

Ozden et al. [11] introduced discovering periodic pattern association rule that show regular cyclic pattern over time, while Bettini et al. [12] presented a technique to find temporal patterns in time sequence. Partial periodic pattern mining is another very interesting research since Han et al. [13] proposed partial time series behavior mining methods. Yang et al. [14] proposed a asynchronous periodic pattern mining model that mine all patterns whose periods cover a range within a subsequence and whose occurrences may be shifted due to disturbance in time series data. Huang and Chang proposed another asynchronous method in [15]. In this their method, minimum number of repetitions required for valid segment and valid sequence.

The introduction of a probabilistic model in periodic pattern mining is another topic of research. In this technique, the value of a pattern is continuous and monotonical that probability decreases with occurrence. In [16] Yang et al. introduced an efficient algorithm, InfoMiner, to mine surprising patterns and associated subsequences based on the information gain.

Graph mining is incredibly interesting research in current world. Dynamic networks represent a sequence of graphs over time. Changing of graphs from time to time mention entities behaviors change in dynamic networks. Graph vertexes represent the members of population and interaction between two members at particular time is represented by establishing an edge between two vertexes.

The population in dynamic networks can be diverse in nature: humans [17, 18], animals [19], networked computers [20]. Social network [6] analysis is the best-known example in dynamic networks analysis. Among the analysis of dynamic networks, finding periodic pattern is most interesting and conveys very meaningful information yet often-infrequent pattern.

In this perspective, Lahiri and Berger-Wolf in [1, 10] introduced a new mining problem to find periodic patterns in dynamic social networks. Periodic patterns occur regularly in networks that change over time. These patterns do not exist in all time intervals, that's why they mine all periodic subgraphs that occur in a minimum number of times. They deal with the concept of closed subgraph mining that has been widely used in frequent pattern mining [21]. Occam's Razor principle is followed for parsimony closed pattern mining. $PSEMiner$ algorithm is proposed for finding periodic subgraph in dynamic networks which is polynomial unlike many related subgraph and itemset mining problem. Here, they create pattern tree that maintains all parsimonious subgraphs shown up to timestep t, and it tracks subgraphs that is periodic or might be periodic at some point in future. In timestep t, the graph G_t is read, and the pattern tree is traversed and common subgraph between G_t and tree node is updated with the new information including modifying, adding and deleting tree node. Each node indicates unique subgraph. This process creates a lot of unessential tree node which is time consuming.

Apostolico et al. proposed the speedup method $ListMiner$ for periodic pattern subgraph mining in [22]. Proposed method identified patterns based on projected timesteps graph that solve unused tree node problem. Exact number of tree nodes is created that must be essential for pattern mining. This method maintains a list

of structure. At timestep t, the graph G_t is read, and the projection $\pi_{p,m}$ list is updated adding new list nodes, where p is period and $m = t \bmod p$. This approach is much faster than previous approach because it creates less number of list nodes. However, the number of list nodes is not a small number and many nodes store some common information redundantly that is memory consuming. The number of list depends on maximum period P_{max} because the process mine all periodic patterns up to P_{max} and the number of list nodes in the list depends on entities set and total timesteps that is also a time-consuming process.

1.2 Motivation

From the above analysis, it can be seen that periodic pattern mining in social networks plays a significant role. Although, $PSEMiner$ and $ListMiner$ mine periodic patterns correctly, these process store every timesteps graph $< G_1, G_2, ..., G_t >$ in memory and finding periodic pattern from G_t it computes common subgraph patterns among $< G_1, G_2, ..., G_{t-1} >$ those patterns mining is time consuming. Dynamic networks is a continuous process and it varies time to time. If some graph G_t occurs one time and will never happen in future, existing processes maintain that graph also. Therefore, our motivation of this work is to propose a method for periodic behaviors mining that overcomes the limitations of existing works. My proposed technique helps to mine periodic behaviors that need storing of dynamic networks entities (vertexes and edges) only one time and need common entities computation once for graph G_t. This process is time and memory efficient.

1.3 Problem Statement

Dynamic network is a model of structure that changes over time. This structure is generally modeled as real-world phenomena where a set of uniquely identifiable entities are interacting with each other over time. In global organization the interaction between entities can be a complex network system over time [23]. In this thesis we

deal with the detection of a specific types of interaction behavior, called periodically reoccurring behavior in networks that has been changed change dynamically. Our main goal is to mine parsimonious periodic behavior even if it persists only for a short period and infrequent. More clearly it can say, given a dynamic network (DG) and a minimum support threshold ($\sigma \geq 2$), the periodic subgraph mining problem is to mine all parsimonious periodic subgraph embeddings in DG that satisfy the minimum support. The periodic subgraph embedding (PSE) for a subgraph F is $S(t, p, s)$ that means subgraph F occurs at first t times, continues until s times where occurs interval is p. We define $S(t, p, s) = \{t, t + p, ..., t + p(s - 1)\}$, where $t \geq 0$ and $p, s \geq 1$. A subgraph F may have several periodic supports set. Not all of those periodic supports are parsimonious. Let subgraph F occurred two periodic set $P_1 = S(t_1, p_1, s_1)$ and $P_2 = S(t_2, p_2, s_2)$. We say that P_1 subsumes/contains P_2 if and only if $S(t_2, p_2, s_2) \not\subseteq S(t_1, p_1, s_1)$. That result P_1 is parsimonious periodic pattern for subgraph F but P_2 is not parsimonious.

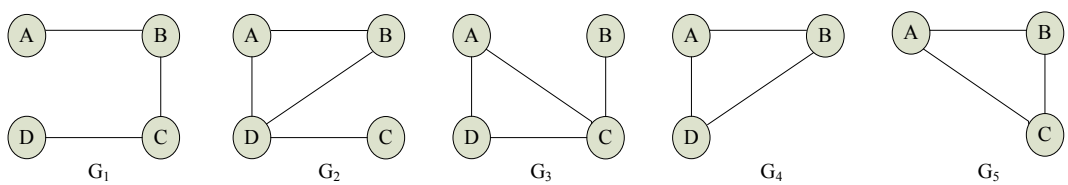


Figure 1.1: Dynamic graph structure

For example, iteration between A and B occur $< 1, 2, 4, 5 >$ timesteps graph. Suppose minimum $\sigma \geq 2$ then we find periodic patterns $P_1 = \{1, 1, 2\}$, $P_2 = \{1, 3, 2\}$, $P_3 = \{1, 4, 2\}$, $P_4 = \{2, 2, 2\}$, $P_5 = \{2, 3, 4\}$ and $P_5 = \{4, 1, 2\}$. On the other hand, interaction between C and D occur $< 1, 2, 3 >$ timesteps. We find periodic patterns $P_6 = \{1, 1, 2\}$, $P_7 = \{1, 2, 2\}$, $P_8 = \{2, 1, 2\}$ and $P_9 = \{1, 1, 3\}$. Periodic patterns P_6, P_7, P_8 subsumed in P_9. This purpose C and D interaction contains only P_9 periodic patters that is parsimonious periodic pattern. Our goal is finding all parsimonious periodic patterns in dynamic networks.

1.4 Contributions

The main contributions of this dissertation are described as follows: We introduce the problem of finding regularly occurred periodic patterns in dynamic networks. The design and development of $SPBMiner$, an online algorithm that improves the worst case time performance of the algorithms [1, 10] by at least σ factor proportional to the total number of timestamps in dynamic networks. It shows σ times better performance than algorithm [22]. Proposed miner method space complexity is independent on dynamic networks timestamps. It's space complexity is $(\sigma^2 ln(T))$ times less than [1, 10] and σ times less than [22].

We propose a supergraph based technique that stores all dynamic networks interaction entities (vertexes and edges) only one times. Each entity maintains a data structure, which stores an occurred time set and a list of periodic descriptor sets. When one entity holds interactions in networks, its descriptor and timestep update are performed with modifying, deleting and adding descriptors. Each timestep supergraph conveys individual periodic entities. We combine those periodic entities based on starting position, period, and support and find periodic patterns. Then we mine parsimonious periodic patterns. Extensive performance analyses show that our proposed method is significantly efficient and effective for periodic behaviors mining in dynamic networks when networks density is medium or high.

1.5 Organization of the Thesis

The dissertation is organized as follows:

- • **Chapter 1 Introduction.** In this chapter an introduction to the periodic patterns mining researches is presented. The definition, importance and existing approaches are clearly introduced. After that, the dissertation focuses the contribution.
- • **Chapter 2 Related Work.** This chapter first shows the state of the art methods of the periodic patterns mining research. Then describe two

existing periodic pattern mining works $PSEMiner$ and $ListMiner$ in dynamic networks. The limitations of these methods are clearly addressed, as these are the focuses of this dissertation.

- • **Chapter 3 SPBMiner.** We present our proposed technique for mining periodic behaviors in dynamic networks.
- • **Chapter 4 Experiments Analysis.** In this chapter, it has been shown the effectiveness and efficiency of our proposed method.
- • **Chapter 5 Conclusion and Future Work.** Finally, this chapter concludes the dissertation indicating the limitations and future works.

Chapter 2

Related Works

In data mining research, Periodic patterns mining appears in different perspective. In this chapter, periodic patterns mining relevant literatures in unstructured and structured databases have been reviewed. The main focus of this thesis is mining periodic patterns in dynamic networks these are the structured data representation model. At first, some periodic patterns mining research in unstructured database are described briefly. Then we discuss structured periodic patterns mining problem related works those are most relevant our work. The two proposed algorithms $PSEMiner$ and $ListMiner$ are explained in details. Finally, we conclude this chapter mentioning some limitations of existing works and make clear our motivation.

2.1 Related Works on Unstructured Databases

Most of the proposed periodic patterns mining techniques deal with unstructured data such sequential and transactional databases. This research area is not our main goal; as a result, we explain some proposed techniques in briefly. In the most general model, given a sequence of symbol set $S = \{x_1, x_2, ..., x_T\}$, where each symbol x_i represents a universal set L. A pattern sequence $P = \{y_1, y_2, ..., y_p\}$, which period is p and each $y_i \subseteq L \bigcup \{*\}$. The $'*'$ character indicates matching any symbol. This pattern-mining problem mine all such patterns from the input sequence data those satisfy minimum support.

Han et al. first introduced partial periodic pattern mining algorithm in time-series databases [13]. Mining interesting properties of partial patterns such as Apriori property [24] and the max-subpattern hit set property have been maintained in

this algorithm. Ma and Hellerstein [25] proposed a similar, Apriori inspired method containing two level wise algorithms in unknown periods value. They also proposed novel approach based on chi-square test for defining periodicity. Yang et al. [14] introduced a new asynchronous type of periodic pattern mining algorithm that mine all patterns within coverage rage of data sequence and maximum number of disruption allowed. Huang et al. [15] proposed another asynchronous method, which valid segment and valid sequence is measured by minimum number of repetitions of patterns.

In periodic pattern mining problem, the introduction of a probabilistic model is another interesting topic of research. In this technique, a pattern value is continuous and monotonically, which probability decreases with number of occurrences. In [16] Yang et al. established an efficient algorithm named InfoMiner that mine surprising patterns and associated subsequences based on the information gain. Yin et al. [26] proposed probability based the latent periodic topic analysis in text database. Latent periodic topic analysis find a latent topic space to fit the data corpus as well as detect whether a topic in periodic or not.

Mining frequent patterns [24, 27–29] from transactional database has been actively and widely studied in data mining. Tanbeer et al. [30] introduced a novel concept of mining periodic frequent patterns in transactional database. Those patterns are frequent and appear at a regular interval by given user in the database is periodic frequent pattern. Periodic pattern mining is interesting in others data mining research area. Sumithi and Sathiyabama [31] proposed an efficient method that discovers the hidden periodic patterns from a spatio-temporal database. They show that if there are any periodic pattern could unveil important information to data analyst as well as facilitate data management substantially.

2.2 Related Works on Structured Databases

Periodic structured data mining is especially interesting research in current world. A dynamic network is an extraordinarily powerful mathematical representation for

time-varying systems those consist of many interactions among entities in structured data model. It is most useful when the environment extremely complex and behavior is continuously changing over time. It is supplementary influential representation than canonical social network that allows one to map explicitly system structure changes [23].

Dynamic networks represent a sequence of graphs over time. Graph vertexes set characterize the members of population and interactions among members at some particular time are represented by establishing edges among vertexes. Dynamic networks population can be diverse nature: humans [17, 18], animals [19], networked computers [20]. Social network [6] analysis is the best-known example in dynamic networks analysis. Monthly e-mail, yearly family reunions, monthly banking information reports and familiar face of stranger at the morning coffee shop are all periodic a significant that are easily missing to the study in the collection of public interactions data [10]. Among the analyses of dynamic networks, finding periodic patterns is most interesting and conveys very meaningful information yet often-infrequent pattern. There are two potential applications. First one, these periodic patterns represent stable interaction patterns that can be of qualitative interest in and of themselves. For example, ecologists study animals movements and social patterns are found by tracking devices [32]. Periodic subgraphs are corresponding seasonal association or mating patterns that are hidden in mass quantities of arbitrary animals movements [19]. The second important application is periodic behavior can be predict behavior by virtue of repeating regularly. For example, mining predictable interactions from sensors logs can be used in different types of mobile and ubiquitous applications [33]. Yan et al. [34] proposed regular behavior miner algorithm that mine maximal frequent subgraph in dynamic networks.

In this perspective mining periodic patterns in dynamic networks, Lahiri and Berger-Wolf in [1, 10] proposed $PSEMiner$ algorithm. This algorithm mine periodic patterns in networks those change over time and occur in a minimum number of times. They deal with the concept of closed subgraph mining that has been

widely used in frequent pattern mining [21]. Occams Razor principle is followed for parsimony closed pattern mining. Finding periodic subgraph in dynamic networks is polynomial unlike many related subgraph and itemset mining problem. In this process, pattern tree structure is created that maintains all periodic subgraphs seen up to timestep t, and it tracks subgraphs that is periodic or might be periodic at some point in future. At timestep t, the graph G_t is read, the entire pattern tree is traversed and common subgraph between G_t and tree node is updated with the new information including modifying, adding and deleting tree nodes. In this process, a number of common subgraphs are created that is useless. Each node indicates unique subgraph. There is another method $ListMiner$ [22] that speedup periodic patterns mining. This method finds patterns based on projected timesteps graph that solve unused tree node problem. Exact number of tree nodes are created those must be essential for pattern mining. This method maintains a list structure. At timestep t, the graph G_t is read, and the projection $\pi_{p,m}$ list is updated adding new list nodes, where p is period and $m = t \bmod p$. This approach is T faster than previous approach because it creates less number of list nodes. Our main motivation is above two methods $PSEMiner$ and $ListMiner$. The following subsections we describe both techniques in detail.

2.2.1 PSEMiner

2.2.1.1 Preliminaries

The concept of periodic subgraph mining in dynamic networks has been firstly proposed by this algorithm in [1, 10]. The algorithm makes a single pass over the data and capable of accommodating perfect closed subgraph patterns periodicity. Dynamic networks are a representation of interaction between a set of unique entities, which change time to time. Let $V \in N$ represent the set of entities. Interactions between entities may be directed or undirected and are supposed to have been recorded over a period of T isolated timesteps. We use natural quantizations specific to each of our dataset such as one day per timesteps. The only essential is that a timestep

represent a meaningful amount of real time where the periodicity of mined subgraphs will be set of chosen timesteps. The purpose of this study is known the periodic interactions between elements, whose belong to dynamic network.

1. DEFINITION. *Graph: A graph G = (V,E) is a simple interactions set E among entities set V. The interactions E and entities V in G represent with unique label set* $R \in \boldsymbol{N}$.

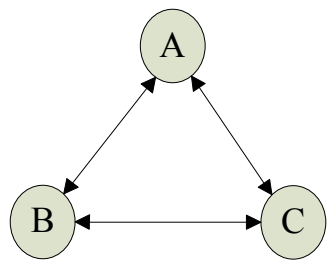


Figure 2.1: Graph

Figure 2.1 shows the graph that can be represent by integer set $R = \{1, 2, 3, 4, 5, 6\}$ where vertex set $\{A, B, C\}$ labeled by $\{1, 2, 3\}$ and edge set $\{A - B, A - C, B - C\}$ labeled by $\{4, 5, 6\}$ respectively. These unique labeled representations show that two graphs similarity measurement is very easy. Suppose given two graph G_1 and G_2 with unique vertexes and edges labels, testing whether G_1 is subgraph of G_2 or vice versa is defined by the corresponding R_1 and R_2 representation are subset of each other.

The maximum common subgraph (MCS) between two graphs find by unique label vertex and edge set intersection between two corresponding representation set. Figure **??** shows the maximum common subgraph (MCS) calculation process between two graphs using unique labeled vertex and edge set.

2. DEFINITION. *Dynamic Networks: A dynamic network* $DN =< G_1, G_2, ..., G_T >$ *is a time series graphs, where graph* $G_t = (V_t, E_t)$ *represent interactions set* E_t *among unique vertex set* $V_t \subseteq V$ *at timestep t.*

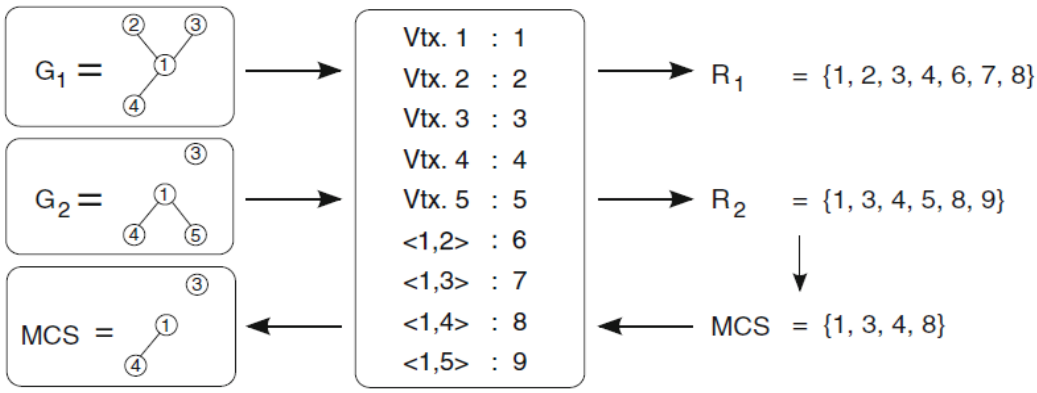


Figure 2.2: The unique set representation between graphs that demonstrates the computation of the MCS of two graphs [1].

Figure 2.3 shows the example of a dynamic network with five timestep. Definition **??** reduces the high computational complexity of many algorithmic tasks on graphs database. Since vertexes and edges are represented by unique entity, each timestep t, vertex set V_t in graph G_t are unique that reduce computational complexity for certain hard graph mining problems, such as maximum common subgraph and subgraph isomorphism testing [35, 36].

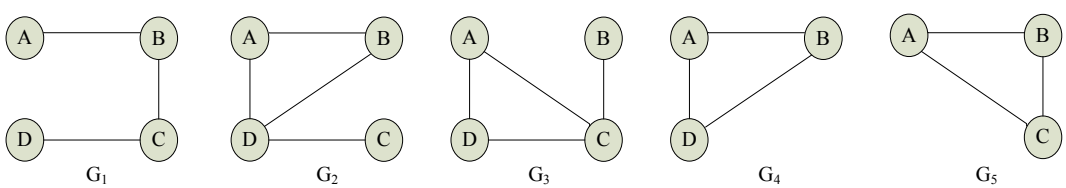


Figure 2.3: An example of a dynamic graph structure with 5 consecutive timesteps

3. DEFINITION. *Periodic Graph: A graph* $G = (V_g \subseteq V\ , E \subseteq V \times V)$ *in dynamic network* $DN =< G_1, G_2, ..., G_T >$ *is a periodic graph with period p, if G is a subgraph of* $< G_x, G_{x+p}, ..., G_{x+np} >$, *where* $0 \leq x \leq p$ *and* $n \geq \sigma$ *(min support).*

In figure 2.3 the graph with vertex $\{A, B, D\}$ and their connected edge $\{A-B, A-D, B-D\}$ is periodic with period 2 because it occurs at timestep 2 and 4. Vertex set $\{C, D\}$ and their connected edge is periodic with period 1, it

appears in timestep 1, 2 and 3. If minimum support is 2 then it is period also at time $\{1, 2\}$ and $\{2, 3\}$. In large dynamic network, the number of periodic patterns could be very large. Reducing these kinds of redundant, closed periodic subgraphs are mined from networks. The periodic graph does not appear each timestep, if it appears minimum times σ in dynamic network with period p it is periodic. The number of times graph occurs periodically in networks called periodic support set.

4. DEFINITION. *Periodic Support Set: Given a dynamic networks DN of T timesteps and any subgraph F = (V, E). The periodic support set $S_p(F)$ of F in DN is the set of all timesteps t, which start at t_i timestep and repeating every p period interval where F is a subgraph of DN, which denote F subgraph of G_t. The representation of support set $S_P(F) = (t_i, p, s) = \{t_i, t_i + p, ..., t - i + p(s - 1)\}$, such that $\forall t_i$ ($t_i \in S_p(F) \leftrightarrow F \subseteq G_{t_i}$) and neither $G_{t_i - p}$ nor $G_{t_i + ps}$ contains F as subgraph.*

F is **frequent periodic pattern** if its support exceeds a user defined minimum support threshold value $\sigma \leq T$.

Definition 4 is the basic formulation of well known frequent pattern mining problem that satisfy the downward closure property. In downward closure property, every sub pattern of a frequent periodic pattern F is also frequent.

5. DEFINITION. *Closed Subgraph : For any subgraph F = (V, E) in a dynamic networks DN of T timesteps is closed if it is maximal for its support set.*

There are a difference between frequent closed subgraph support set and periodic closed subgraph support set. A single subgraph F can have multiple periodic pattern support set to allow dis-join and overlapping periodic behavior. Thus, we require extraction of all periodic subgraph embeddings, rather than just the periodic subgraphs. According [1], the definition of periodic subgraph embedding is follows.

6. DEFINITION. *Periodic Subgraph Embedding (PSE): Given a dynamic network DN and a arbitrary subgraph F. The periodic subgraph embedding (PSE) is a pair of $< F, S_p(F) >$, where F is closed subgraph over a periodic support set $S_p(F)$ with $|S_p(F)| > \sigma$ and $S_p(F)$ is temporally maximum for F.*

Mining frequent periodic closed behavior patterns is a well-designed solution to the redundancy of general frequent pattern mining problem, since it captures all the information of the more general formulation produce small number of output results except any loss of information. However, there are periodic closed patterns that periodicity contains by other periodicity, This kinds of patterns also redundant. To avoid this kinds of redundancy parsimonious pattern is proposed.

7. DEFINITION. *Parsimonious PSE: A PSE that is not subsumed by any other PSE is Parsimonious PSE.*

1. PROPERTY. *Subsumption : Two periodic subgraphs F_1 and F_2, their support set $S_p(F_1) = (t_1, p_1, s_1)$ and $S_p(F_2) = (t_2, p_2.s_2)$. F_1 support set $< F_1, S_p(F_1) >$ contains or subsumed F_2 support set $< F_2, S_p(F_2) >$ if and only if the following condition hold.*

i. $F_2 \subseteq F_1$

ii. $t_2 \geq t_1$

iii. $p_2 \ mod \ p_1 = 0$ *and* $p_1 < p_2$

iv. $t_2 + p_2(s_2 - 1) \leq t_1 + p_1(s_1 - 1)$

v. $(t_2 - t_1) = p.k$ *for some integer* $k > 0$.

For example, a subgraph F of period 1 with adequate support 10, then it also periodic at period 2 and 3 if minimum threshold is 3. In this case, PSE that period 2 and 3 are not Parsimonious PSE.

8. DEFINITION. *Periodic Subgraph Mining Problem: Given a dynamic network DN and a minimum support threshold $\sigma \geq 2$, the Periodic Subgraph Mining Problem mine all parsimonious periodic subgraph embeddings in DN those satisfy the minimum support.*

Since real world networks, there are some event that are not exactly periodic, present a definition of what constitutes near periodicity.

9. DEFINITION. *Noisy Subgraph: A noisy subgraph exhibits jitter in its period if its period is near-constant rather than constant. If a jitter value of $J \geq 0$, the extend support of subgraph F as follows: $S(F) =< t : F \subseteq G_t >$ and $\forall i : |t_{i+1} - t_i| \leq p \pm J$.*

2.2.1.2 Complexity Analysis of Periodic Subgraph Mining

This section discusses the proof (taken from [1, 10]) time complexity of periodic subgraph mining is *polynomial* unlike many related subgraph mining problems. To this purpose method, the unique labeling of vertexes and edges plays significant role. The proof shows the PSE mining problem is solvable in polynomial time that is contrast to the more general frequent subgraph-mining problem, which is $NP-hard$ for enumeration and $\#P-complete$ for counting periodic subgraph, even though unique vertex labels are considered [37, 38]. The concept of projection of a discrete time series data in [39] is used to find the maximum number of PSEs in dynamic networks.

10. DEFINITION. *Projection: Given a dynamic network DN, a projection* $\pi_{p,m} =< G_m, G_{m+p}, G_{m+2p}, ..., G_{m+sp} >$ *is a subsequence of graphs among DN, where p is the period of projection and* $0 \leq m < p$ *is the phase offset.*

It should be clear from the definition of periodicity and projection that the subgraph of every graph in the projection is a periodic subgraph if s is greater than or equal minimum support σ that means periodic support set is $\geq \sigma$.

1. PROPOSITION. *Let F be the maximal common subgraph (MCS) of any* $s \geq \sigma$ *consecutive positions of any projection* $\pi_{p,m}$. *If* $F \neq \phi$, *then it is a periodic subgraph and the s consecutive timesteps form projection* $\pi_{p,m}$ *are part of a PSE for F.*

Proof: Suppose the MCS of F of any $s \geq \sigma$ consecutive positions is not empty, that implies F is maximal over a support set of at least σ periodic timesteps. Subgraph F might or might not be temporally maximal. However, on the other case, the s timesteps are part of some valid periodic support set of size at least σ. According to definition 6 it satisfy the condition of periodic subgraph and thus F is a periodic subgraph.

1. COROLLARY. *In the worst case, computational complexity of mining periodic subgraph embeddings in a dynamic network, the MCS of every* $s \geq \sigma$ *consecutive posi-*

tions is not empty then contains a unique PSE.

Proof: If every periodic subgraph subset is $s \geq \sigma$ timesteps in the dynamic network contains a unique maximal common subgraph, then they all need to be enumerated by any mining algorithm. It has been showen that it is possible using an explicit edifice. At different edge in each $s \geq \sigma$ consecutive positions of every projection to ensure each edge is part of a unique periodic subgraph embedding. Let edge e be created in this way with support set (SP) in some $\pi_{p,m}$. Considering only SP , we know that it is temporally maximal for the edge e because e does not exist in any other timesteps. Moreover, the MCS of SP is non-empty because it contains at least the edge e. Thus, each edge is part of a unique PSE whose support set is SP . Since a different edge was placed in every $s \geq \sigma$ consecutive positions of every projection, the number of $PSEs$ is equal to the number of edges created. No additional $PSEs$ can be created since every permissible support set, i.e. with support greater than σ is already part of a unique PSE. Therefore, the described structure is a worst case instance for its size.

The next step unambiguously computes the upper bound on the total number of $PSEs$ in the worst-case network instances. According to corollary 1, we only need to count the number $s \geq \sigma$ consecutive positions of every projection to derive this bound. In order to do this, we first state the bounds on several other parameters.

2. Proposition. *In a dynamic network with T timesteps, the maximum period of any periodic subgraph with support at least is $P = \lfloor (T-1)/(\sigma-1) \rfloor$.*

Proof: For a given period p, the subgraph is $F \subseteq G_1$. In the other $T-1$ timesteps, for every periodic embedding subgraph $F \subseteq G_j$ in $\sigma - 1$ consecutive timesteps $< T_{1+p}, T_{1+2p}, ..., T_{1+p(\sigma-1)} >$. The last index $1+p(\sigma-1) \leq T$, because T is the index of the last timestep. From this inequality it has been derived that $p \leq (T-1)/(\sigma-1)$.

3. Proposition. *In a dynamic network with T timesteps, the length of any projection is $|\Pi_{p,m}| = \lceil (T-m)/p \rceil$.*

Proof: Since $\pi_{p,m} =< G_m, G_{m+p}, G_{m+2p}, ... >$, the projection starts after m timesteps, and so there are $T-m$ timesteps remaining. Since indexes of two following timesteps differ by p positions that the length of profection $\pi_{p,m}$ is $\lceil (T-m)/p|$.

Given the above expressions, the exact bound for mining number of closed PSE can be obtained by construction [1].

1. THEOREM. *In a dynamic network with T timesteps, there are at most $O(T^2 ln(\frac{T}{\sigma}))$ closed PSEs at minimum support σ.*

Proof: From Corollary 1, the maximum number of PSEs possible in a dynamic network at minimum support σ is equal to the number of $s \geq \sigma$ length windows over all possible projections of the network. For a given projection $\pi_{p,m}$ and value of s, it is clear that the number of length-s windows over the projection is $|\pi_{p,m}| - s + 1$. Thus, for a given value of s, the number of length-s windows over all projections can be obtained by substituting the expressions from propositions **??** and **??**:

$$\sum_{p=1}^{\lfloor \frac{T-1}{s-1} \rfloor} \sum_{m=0}^{p-1} \left(\left\lceil \frac{T-m}{p} \right\rceil - s + 1 \right)$$

The expression mention the maximum period of a pattern from proposition 2, where σ was replaced by s since we only want projections which contain at least one length-s window for any s. The outer summation is over all possible periods that find from 2. The inner summation is over all possible phase offset values m for a given period p. Finally, the term inside the summation is the number of $length - s$ windows in any projection, where $|\pi_{p,m}|$ has been substituted from Proposition refprojectionlength. We now sum this expression over all possible values of s, which run from σ to T, and relax the floor and ceiling expressions for an asymptotic closed form approximation.

$$\sum_{s=\sigma}^{T} \sum_{p=1}^{\lfloor \frac{T-1}{s-1} \rfloor} \sum_{m=0}^{p-1} \left(\left\lceil \frac{T-m}{p} \right\rceil - s + 1 \right) \tag{2.1}$$

$$\approx \sum_{s=\sigma}^{T} \sum_{p=1}^{\lfloor \frac{T-1}{s-1} \rfloor} \sum_{m=0}^{p-1} \left(\left\lceil \frac{T-m+p}{p} \right\rceil - s + 1 \right) \tag{2.2}$$

From the above formula 2.2, we find the simplifies expression $O(T^2.H(\frac{T-1}{\sigma-1}))$, where $H(n) = \sum_{k=1}^{n} \frac{1}{k}$ is approximated by $ln(n)$. Thus, the number of closed PSEs at minimum support σ is $O(T^2 ln(\frac{T}{\sigma}))$.

2. THEOREM. *Periodic Subgraph Mining in dynamic networks is in P.*

Proof: Suppose an algorithm that outputs the maximum common subgraph of every σ length window of every projection. Since the vertexes and edges are uniquely labeled that the common can be found in time $O(V + E)$ [35]. In the worst case, the $O(T^2 ln(T/\sigma))$ periodic patterns computing time $\Theta((V + E)T^2 ln(T/\sigma))$ that guaranteed to every closed periodic subgraph is mined. Thus, the mining problem is in P.

2.2.1.3 Basic Description of PSEMiner Method

The main algorithm has been used a special data structure called pattern tree. This structure maintains information about embedding subgraphs seen up to timestep t and tracks subgraphs that is periodic or might become periodic at some point in future. At timestep t, the graph G_t is read and the pattern tree is updated with modification, addition and deletion tree nodes information. Each tree node represents one unique subgraph. The most important parameter of this algorithm is the maximum period P_{max}. When the P_{max} is restricted, the algorithm perform as an online algorithm, retaining only the parts of dataset in memory that periodicity be calculated. However, in many applications, this information is irrelevant such as sensor data streaming. In this case, unrestricted maximum period value must be set and requires large computational burden and the entire dataset hold on in memory.

Data Structures: The algorithm maintains five primary data structures to track PSEs.

Pattern tree: The tree structure characterizes a subgraph relationship among periodic subgraphs. It maintains all PSEs up to timestep t and tracks all periodic

subgraphs those are periodic or might become periodic in future.

Tree node: The node of pattern tree is tree node. Each tree node contains a different subgraphs and a list of descriptors. Descriptors modification, addition and deletion are the primary operations on the tree node. Except root node, every tree node must observe; all descendants of a tree node are associated with proper subgraphs of F, but not all subgraphs of F are necessarily its descendants in the tree.

Descriptor: A descriptor D is one kind of data structure that represents periodic support set (t_i, p, s) for tree node, where starting time t_i, period p and support s. It is unique for subgraphs F. The last time of descriptor define $t_j = t_i + p(s-1)$ and the expected time $t_e = t_j + p$. At time step t, a descriptor is alive if $t_e \geq t$ otherwise descriptor is not alive and must be flashed out from the pattern tree and if it satisfy minimum support than write subgraph as PSE. A descriptor where $t_i = t_j$ is a special case called anchor descriptor it does not represent periodic support set and it always live unless P_{max} is defined and $t - t_i > P_{max}$ that means it in no longer needed.

Subgraph hash map: The random access of tree node that is associating with subgraphs is required finding subgraphs position directly. For this purpose, subgraph hash map function exists for graphs since the set representation R has a global ordering by $R \subset N$.

Timeline list: The timeline list is an optional component that links tree nodes to the future timesteps at which they are expected to appear.

Description of Pattern Tree Update: The update process of pattern tree is the core part of this periodic subgraphs mining methods. Initially tree node of the pattern tree is empty. At each timestep t, graph G_t traverses the pattern tree in a breadth-first search (BFS) to update tree node with the new information contained

in G_t. This leave out every tree node N with subgraph F which $MCS(F, G_t) = \Phi$. The algorithm first search $MCS(F, G_t)$ is in hash table if the node exists in pattern tree then update it otherwise a new node is created as a child of N. Moreover, entire graph G_t exists in pattern tree ensure by adding a new child of root as an anchor node. At each time t, graph G_t traverse the tree, one of the following three conditions holds at each treenode N with subgraph F. Let $C = MCS(F, G_t)$ be the maximal common subgraph of G_t and F.

Update descriptors: If F subset of G_t, that means F has appeared entirely at timestep t. Suppose D is a descriptor in N and $t_e = t_j + p$ is the next expected time.

(a) If $t_e = t$, then time step t is added to D to ensure temporal maximality.

(b) If $t_e < t$, then D is no longer live. It is written to the output stream if its support is greater than or equal to σ , and removed from the tree.

(c) If $t_e > t$, then the expected time has not been processed yet, so nothing is happened.

(d) If D is an anchor descriptor then timestep t might be second occurrence of F, a new descriptor D' is created with period $p' = t - t_i$ and phase offset $m' = (t_i - 1) \bmod p'$. If N does not contain a descriptor with the same period and phase offset of D' then add D' as a descriptor at N.

For every child node N' with subgraph F' of N that $F' \subset F \subseteq G_t$. So the process updates all descriptor of the N' without calculating MCS, that save computation time.

Propagate descriptors: Let C is not empty the above condition does not hold then $C \subset F$ is present at timestep t. If a treenode for C does not already exist in the tree, determined using the subgraph hash map, it is created as a child of N with subgraph F. If D be any descriptor at N and $t_e = t$, then D represents a PSE

which subgraph C must inherit and continue. The treenode for C receives a copy of D, if a live descriptor of the same period and phase offset does not already exist. The subgraph F with descriptor D is written to the output stream if the support of D is greater than or equal to σ, and then D is removed from treenode N.

Dead Subtree: If C is empty, then there are no common subgraph between G_t and F and no descriptor at N are directly affected by the observation of G_t. The common subgraph calculations for all descendant of N are avoided.

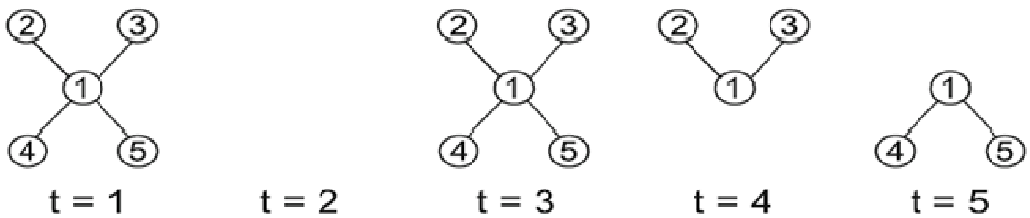


Figure 2.4: An example of a dynamic network [1].

Figure 2.5 shows the structure of pattern tree during the execution of the method at each timestep on the dynamic network from Figure 2.4. For simplicity, we have described a especially basic version of the technique. The main aspect of this method is that it outputs all $PSEs$, which superset are all $PPSEs$. Non-parsimonious $PSEs$ can be post-processed out of the output.

2.2.1.4 Extension to the PSEMiner Technique

Mining Parsimonious PSEs: The most significant improvement of this technique is to mine only parsimonious PSEs in dynamic networks. Mining parsimonious PSEs from PSEMiner an indicator bit is set to each descriptor to indicate subsumption. The indication bit is cleared when the descriptor is created. When any descriptor D from tree node N with subgraph F is flushed, its subsumed bit is checked if it is cleared, the D is compared to all other live descriptors at N. If D is subsumed by another descriptor, it is not written to the output. On the other

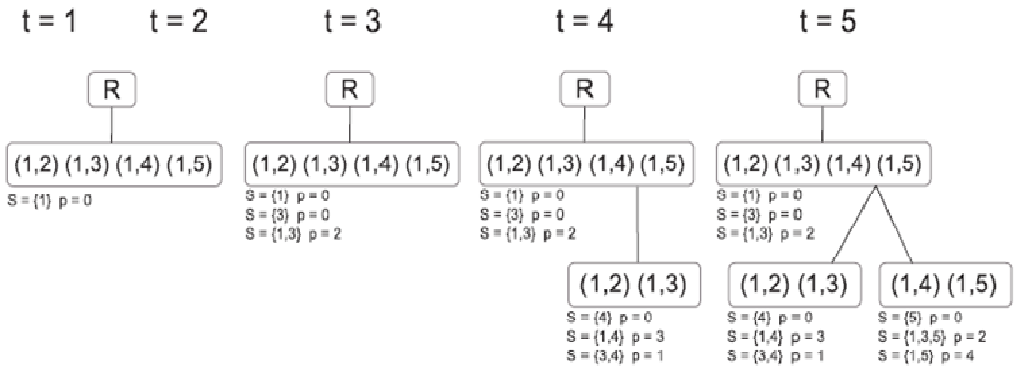


Figure 2.5: The pattern tree at each timestep for the dynamic network shown in figure 2.4, considering only edges for brevity [1].

case, if D subsumes some other descriptors D', the subsumed bit for D' is set. If the support of D' increases in the future, its subsumed bit is cleared. However, if the indicator bit is set then the descriptor is not written in output as parsimonious PSE.

Including Smoothing: In view of the fact that real-world dynamic networks are unlikely to contain perfectly periodic subgraphs, Lahiri and Berger-Wolf [1, 10] used smoothing as a mechanism for accommodating imperfect periodicity subgrpahs. Given a user-defined smoothing parameter $S \geq 1$, the dynamic network $DN =< G_1, G_2, ..., G_T >$ is mapped in a new network G', in which each element $G_{i'} = G_i \bigcup ... \bigcup G_i + S$. However, the following two circumstances handle the elimination of artifacts introduced by the smoothing process.

1. The minimum period P_{min} is set to S.

2. PSEs of the same subgraph those share the same period, and those differ in their starting positions by at most $S - 1$ timesteps, are merged together. In other words, only the PSE with the highest support is retained.

By introducing this smoothing mechanism, they allow a window of timesteps within which the order of events does not matter. No smoothing is performed at $S = 1$.

Sorted descriptor list : Descriptor list at each node can be stored sorted by the next expected timestep. So, for a given time step, only those descriptors are read that expected time lass than or equal to current time. This process reduces the number of descriptors that need to be examined during each tree update, at the computational cost of having to sort the list of descriptors after each update. If the number of descriptors per tree node is not very large, the computational overhead is minimal in practice.

Lazy tree updates: Generally, the algorithm spends most of the running time for calculating intersections of integer sets. However, the MCS of two graphs is calculated in time linear in the number of vertices and edges, the size of the graphs results in a relatively expensive intersection computation. The sparsity of the network generally results in a relatively small number of tree nodes, which means many such intersections between large sets must be performed. Thus, to improve the practical efficiency of the algorithm, we can delay calculating intersections until it is essential.

Using a timeline to trim the tree: The timeline is a technique that associates each future timestep with a list of tree nodes those have at least one descriptor expected at that timestep. It can be dynamically updated unimportant cost once per tree node and stored in space linear in the number of tree nodes. After the tree update at timestep t, all tree nodes are still associated with timestep t are guaranteed not to have been visited during the tree update, and have at least one descriptor that is no longer periodic. These tree nodes can be visited and the invalid descriptors removed. Thus, at the end of each tree updates operation, the tree node only contains descriptors those are lived at the next timestep. This technique ensures that the pattern tree contains a minimal number of descriptors and tree nodes at any given timestep.

2.2.1.5 Space and time complexity

Theorem 1 shows that the maximum number of PSEs are $O(T^2 ln(P_{max}))$ where T is the total timestep and P_{max} is the maximum period. For every time step t, the tree is completely traversed. Thus, the worst-case time complexity of the algorithm involves traversing each descriptor in the tree once for each timestep and calculating the MCS at each treenode. The MCS of two graphs can be calculated in time $O(V+E)$. This yields a total time complexity of $O((V+E)T^3 ln(P_{max}))$. The worst-case space complexity of our algorithm is $O((V+E+P_{max}^2)T^2 ln(P_{max})$ when P_{max} is specified. If P_{max} is unrestricted then the time complexity is $O((V+E)T^3 ln(T/\sigma))$ and space complexity is $O((V+E))T^2 ln(T/\sigma) + T^3 lnT$.

2.2.2 ListMiner

The $ListMimer$ algorithm in [22, 40] mine periodic pattern in dynamic networks that improves the worst-case time complexity of $PSEMiner$ by a factor T. In the $PSEMiner$, it can be observed that at each time step t, the graph G_t, traverses every node of pattern tree. At each node N the following options occur: (i) the maximal common subgraph $C = MCS(N, G_t)$ is computed (ii) the corresponding node N_1 with subgraph C is searched in the pattern tree, and (iii) each descriptor at node N_1 is then checked for consistency of periodicity in t. However, the descriptor is updated, otherwise either it is deleted or no action takes place. If no action is taken, the time consuming MCS computation is useless.

The main idea of the $ListMiner$ algorithm is reduce maximal common subgraph computation. For this inspiration, only timesteps in which the graphs are intersected has been considered that contain a periodic subgraph. Consider a fixed period p, every timestep t belongs to a single projection $\pi_{p,m}$, where m=(t mod p). Thus, every projection is considered separately, because graphs belong to different projections cannot be periodic with the same period p.

More correctly, for a fixed period p, the T timesteps are partitioned into p projections. For example, if $p = 1$, there is a single projection contains all the

timesteps graphs. If $p = 2$, all graphs are partitioned two projections such as $\pi_{2,1} =< G_1, G_3, G_5, G_7, ... >$ and $\pi_{2,0} =< G_2, G_4, G_6, G_8, ... >$, etc. However, a list is created for every projection. This list contains the intersection between every possible sequence of consecutive graphs. For example if the projection is $\pi_{2,1}$ the list is composed by $G_1, G_3, ..., G_1 \bigcap G_3, G_3 \bigcap G_5, ..., G_1 \bigcap G_3 \bigcap G_5$ and so on. In this way every possible PSE is generated. By iterating the process for all possible choices of period p, all PSE will be found. In the following subsections, these concepts are formalized and the discussion of the technique is given.

2.2.2.1 Preliminaries

11. DEFINITION. *Projection $\pi_{p,m} =< G'_1, G'_2, ..., G'_x >$ is given, where $G'_j = G_{pj+m}$ and $x = \lceil (T - m)/p \rceil$, we call run $S_{i,j}$ every subsequence of consecutive graphs from $\pi_{p,m}$. For two fixed indexes i and j, $1 \leq i \leq j \leq x$ we have $S_{i,j} =< G'_i, ..., G'_j >$.*

4. PROPOSITION. *Every timestep t, graph G_t for period p belongs to a single projection $\pi_{p,m}$ where m=t mod p.*

Proof: From definition, $G_t \in \pi_{p,m}$ if t=qp+m for some q. It is known that for every $t \in Z$ there is a unique remainder $m \in N$ such that t=qp+m where $p, q \in Z$ and $p > 0$. Since m is unique, G_t belongs only to $\pi_{p,m}$.

5. PROPOSITION. *For a given period p, there are exactly p projections.*

Proof: The previous proposition the number of all possible values of remainders m is p.

In the projection $\pi_{p,m}$, the maximal common subgraph for consecutive positions $s \geq \sigma$ is not empty, is a periodic subgraph, and the s consecutive timesteps are part of a PSE. Therefore the method considered every period p, every projection $\pi_{p,m}$ and computing the MCS among all graphs of every $S_{i,j}$ of length at least σ, and saving only subgraphs which are temporally maximal. Using the following property the MCS of every run can be calculated in time $V + E$.

2. PROPERTY. *Given a dynamic graph stream* $< G_i, G_{i+1}, ..., G_x >$ *where* $1 \leq i \leq x \leq t$*, the maximal common subgraph (MCS) of this run is:*
MCS of $< G_i, G_{i+1}, ..., G_x >$ = *MCS of* $<$ *MCS of* $< G_i, G_{i+1}, ..., G_{x-1} >, G_x >$.

Proof: Using the associative property of intersection between sets this property can be proved. Since from definition 11 every graph can be considered as a set of natural numbers, the intersection of $< G_i, G_{i+1}, ..., G_x >$ is equal to the intersection between $< (G_i, G_{i+1}, ..., G_{x-1}), G_x >$. Using this theorem, the MCS of a given run $S_{i,j}$, can be obtained calculating the MCS between $S_{i,j-1}$ and the j-graph of run $S_{i,j}$. The time needed for such intersection is $V + E$.

2.2.2.2 Data Structures

Mining periodic subgraphs in dynamic networks $ListMiner$ algorithm uses three primary data structures: lists, listnodes and a bidimensional array that contains every list. To mine only parsimonious subgraphs another data structure hash map is necessary.

Every list contains some listnodes that is associated to a specific projection $\pi_{p,m}$. Every listnode describes a run $S_{i,j}$ of the projection and used to describe a single temporally maximal PSE.

List: Every projection $\pi_{p,m}$, is associated with a specific list, where $1 \leq p \leq P_{max}$, $0 \leq m < p$. Every node of the list contains the MCS among all graphs of projection $\pi_{p,m}$. Specifically, each time step t, every list L contains listnodes in reverse order where $S_{t,t}$ is the first node and $S_{1,t}$ is the last node. The MCS of all runs $S_{x,t}$ is temporally maximal, where $1 \leq x \leq t$. Therefore, each node N in the list has a graph that is properly contained in the graph of its predecessor. This property allows efficient traversal of every list by the mining algorithm, and also allows the list to be built and manipulated quickly.

Listnode: A single projection $\pi_{p,m}$ is represent by a list L, every listnode describes a single PSE that contains:

Start index: This is the first timestep index of the PSE;

End index: This is the last timestep index of the PSE

Graph G: It is the graph, which represent the PSE. The MCS subgraph between all graphs from timestep T_{start} to timestep T_{end} is mentioned by graph G which indexes differ by period p.

Support: It is the number of elements of the support set. It is calculating by following way: $Support = (T_{end} - T_{start})/p$. This data structure is equivalent to the descriptor used in Berger Wolf 's algorithm [1].

Bidimensional array: The bidimensional array A is used to store all lists. The list associated to the projection $\pi_{p,m}$ is stored at position $A[m][p]$. Using A allows to perform list lookup in constant time.

Subgraph hash map: For mining parsimonious PSE this data structure is used. The key of hash map is a graph id that is calculated by MCS among graphs. For every key, the associated object is a list of descriptors. Descriptor is a triple $< p, s, e >$ where p is the period, s is the first timestep and e is the last timestep of the PSE. This information is added to the hash map when a listnode is flushed out from the list.

Since two PSE could have the same graph, the object associated to every key is a list of descriptors. Before writing in output a PSE P with graph G, G is used as a key to access to the hash map. If it exists in the hash map, the corresponding list is traversed, and for every descriptor D the algorithm controls if D subsumes P. Otherwise it is printed in output and its descriptor is added to the list. If it does not exist in the hash map then P is the first PSE with graph equal to G. This means that all other $PSEs$ with graph equal to G will have a period greater than the period of P. Therefore, P can be safely printed in output because it cannot be subsumed.

2.2.2.3 Parameter

The algorithm is like as $PSEMiner$ algorithm that is a single-pass, polynomial time and space algorithm for mining all closed PSE in a dynamic network. It does not require any parameters, but it optionally accepts the following: (i) Minimum support threshold $\sigma \geq 2$ and (ii) Maximum period P_{max} (default: unrestricted).

2.2.2.4 Description of the ListMiner Technique

The algorithm starts creating an empty bidimensional array A. At timestep t, graph G_t is read and stored the list in the bidimensional array in position $A[p][m]$ where $m = t \bmod p$. Begin with, a new listnode $N = (G_t, t, t, 1)$ is added at the head of the list because it could be the first element of a future PSE. Thereafter, the function update is called that calculates the MCS between the graph in each listnode and G_t. Whenever a PSE of a subgraph node is detected; it is checked for subsumption, and eventually printed in output. When all timesteps have been elaborated, some listnodes could remain in the lists because the next expected time of those graphs could be equal or greater than T. However, the algorithm must further check others PSE subsume the PSE. If it is not subsumed and the support of every PSE is greater than, or equal to σ then it must report it in output. For more clearness, the description of update stage and subsumed stage are given below.

Update Procedure: The update procedure is the core part of this ListMiner algorithm. For every timestep t all the lists associated to projections $\pi_{p,m}$, where $p \leq min(t, P_{max})$ and $m = t \bmod p$, are updated with the new information contained in G_t. The update process of list L is started by adding a head listnode for G_t. This listnode is built as follows: the graph is set to G_t , start and end indexes are set to t because t is the first and the last index of the run, and the support is set equal to 1. During the traversal of the list, one of the following three conditions holds at each listnode N with graph F. Let $C = F \bigcap G_t$ be the MCS of G_t and F.

1. If $F \subseteq G_t$ that means subgraph $C = F$ entirety appeared in G_t. Therefore

the MCS is F, and the listnode is updated in the following way:

- Graph and start index are unaffected
- End index is set to t because the last timestep where the $MCS(C)$ occurs is t.
- Support is incremented by one unit because there is another timestep (t) where C appears.

Given that all successors of a node N with subgraph $F' \subset F \subseteq G_t$, then F' is a also subgraph of G_t. However, the algorithm updates all successors of node N in the same way without calculating the MCS, thus saving computational time.

2. If C is empty, that means $MCS(G_t, F) = \phi$ have no common subgraph. List node N with subgraph F and all its successors are eliminated from the rest of the list and, if their supports satisfy the minimum support, flushed out from the list and store in output as PSE.

3. If C is not empty and $F \not\subseteq G_t$, a subgraph C of F is present at timestep t. In this case the algorithm first check if the listnode parameters describe a subgraph that is frequent and not subsumed. If it is so, it is printed in output. Then the algorithm updates the listnode N in the following way:

- Graph is set to C.
- Start index is unaffected.
- End index is set to t because C appears at time t.
- Support is equal to support(N)+1.

The next listnode in the list is then considered.

Moreover, whenever the update involves not just the start/end indexes, but also the graph variable. If they are equal, the previous node is deleted, since it would represent the same graph within a smaller periodic interval, therefore it would not respect the condition of temporal maximality.

Subsumed Procedure: This procedure mine parsimonious PSE. In order to do this procedure uses a subgraph hash map H. For a given PSE, P with graph F the procedure checks if a list associated to F exists in H. If not, then P is not subsumed because it is the first PSE with graph F and printed in output and stored in the hash map. Otherwise, for every descriptor of the list the algorithm verifies its existence in other descriptors that respects parsimonious conditions If there is periodicity P that is subsumed and it is not printed in output, otherwise P is memorized in the hash map and flushed in output.

2.2.2.5 Time and space complexity

In the above discussion it has been observed that there are exactly p projections for a given period p and the length of every projection is $|\pi_{p,m}| = \lceil (T-m)/p \rceil$.

Since the algorithm creates a new listnode for every element of the projection the maximum number of listnode is the length of the projection. For every timestep t and for every list, in the worst case the algorithm calculates the MCS for every node of the list and Gt.

Therefore the number of MCS is:

$$\sum_{p=\sigma}^{P_{max}} \sum_{m=o}^{p} \sum_{j=0}^{\lceil (T-m)/p \rceil} (j) \tag{2.3}$$

The summations are: for every period p, for every projection with period p the algorithm creates a list. The number of elements in the list is increased by one at every step. Therefore, also the number of MCS to calculate is increased by one at every step, from 0 to the maximum length of the projection.

Since the MCS can be computed in $O(V+E)$ time, the total complexity of the basic algorithm (without subsumption) is $O((V+E)T^2 ln(P_{max})$. Since P_{max} is unrestricted in the worst case, its maximum value is $O(T/\sigma)$. Therefore the complexity time in the worst case is $O((V+E)T^2 ln(T/\sigma)$ that is smaller by a factor T than $PSEMiner$ [1, 10].

For every period p there are p projections with $O(T/p)$ elements. Therefore the number of listnodes for every period is $O(T)$. Every listnode contains an associated graph. Therefore the total space complexity is $O(P_{max}(V+E)T)$. In the worst case P_{max} is unrestricted ($P_{max} = O(T/\sigma)$), so the space complexity is $O((V+E)T^2/\sigma)$.

2.3 Limitation of Existing Works

From the previous sections, it has been shown for mining periodic patterns each graph G_t needs a large number of MCS computation. If dynamic networks density were medium or high, its computation cost would be very high. However, one efficient method that reduces number of MCS computation and mine periodic patterns efficiently in dynamic networks is very essential.

Chapter 3

Supergraph Based Periodic Behaviors Mining (SPBMiner)

This chapter presents the main contribution of the thesis: the design and development of $SPBMiner$, a supergraph based periodic behaviors mining technique that improves the worst-case time and space complexity of $PSEMiner$ and $ListMiner$ algorithms. From the previous chapter, it has been observed that in $PSEMiner$ that at each time step t, graph G_t must traverses every node of the pattern tree that build by graphs $< G_1, G_2, ...G_{t-1} >$. The exhaustive visit of pattern tree performed at each timestep is inefficient. It also generates useless common subgraphs when tree node expected time is less than current time t. An alternative approach $ListMiner$ has been proposed where the graphs in dynamic networks are partitioned based on period p. This process creates unique list for all possible period p, and phase $m = t \bmod p$. It traverses only list nodes rather than the trees from the graph and the common subgraph is created when it is necessary. It is shown in chapter 2 that at timestep t, $ListMiner$ stores same graph G_t at p times for each period p. The approach is extremely memory consuming because dynamic networks are generally large. Both of these two techniques store entire subgraphs differently though the numbers of interactions are same. If one interaction is changed, the whole graph should be restored. This redundant information storing is exceedingly memory consuming.

The key idea of the method proposed in this thesis is to reduce the number of common subgraph computation. It needs only one MCS calculation at each timestep graph G_t. On the other hand, all common and uncommon behav-

iors/subgraphs entities among dynamic networks are stored only once that requires less memory and capable of avoiding redundant information storage. More precisely, we propose supergraph based period behavior mining algorithm called $SPBMiner$. Supergraph is a graph stores information of all graphs with the common patterns of the graphs being stored only once. In the next sections we will describe the methodology of $SPBMiner$. At each time t, find periodicity of all entities periodicity individually and identify periodic entities. After selecting periodic entities, those entities are combined and periodic behaviors/subgraphs are recognized. The following sections formalize these concepts and detailed descriptions of the algorithms are presented.

3.1 Preliminaries

Dynamic networks are a representation of interactions among a set of unique populations that change from time to time. Let $V \in N$ represent the set of populations. Interactions among populations may be directed or undirected and are supposed to have been recorded over a period of T isolated timestamps. We use natural quantization, specific to each of our dataset such as one day/ one hour per timesteps. The only essential is that a timestep represents a meaningful amount of real time where the periodicities of mined behavior subgraphs will be set of chosen timestep.

12. DEFINITION. *Dynamic Networks: A dynamic network* $DN =< G_1, G_2, ..., G_T >$ *is time series graphs, where* $G_t = (V_t, E_t)$ *is a simple interactions* E_t *among populations* $V_t \in V$ *at timestep* t*. Interaction and populations denoted as follows. These interactions and populations are called entities in the rest of this thesis.*

(i) $l(v_i)$ *is the unique label of population* $v_i \in V_t$*.*

(ii) Interaction between v_i *and* v_j *represent as* $(v_i, v_j) \in E_t$ *where* $l(v_i) < l(v_j)$*.*

Figure 2.3 shows the example of a dynamic network with five timesteps. Definition 12 reduces the high computational complexity of many algorithmic tasks on graphs database. Since a vertex represents a unique population, each timestep t, vertex

set V_t in graph G_t are unique that reduce computational complexity for certain hard graph mining problems, such as maximum common subgraph and subgraph isomorphism testing [35, 36].

13. DEFINITION. *Supergraph: Supergraph is a graph database that stores all the graphs into one graph with the common subgraphs of the graphs being stored only once.*

Figure 3.1 shows the supergraph SG that compacts two graph G_1 and G_2 where common subgraph is stored only once.

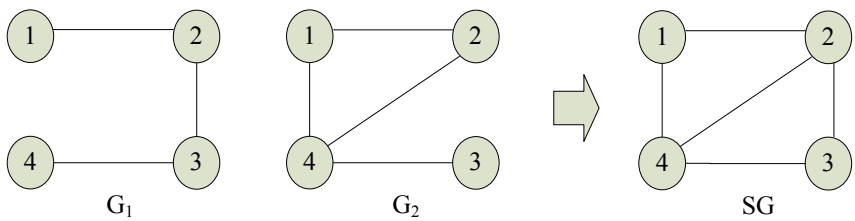


Figure 3.1: Compact Supergraph.

3. PROPERTY. *Graph Representation: For a graph G = (V, E) with unique vertex labels, the set representation $\Re$ for G is formed by vertex and edge represent as two unique vertex labels in $\Re$ where $\Re$ is natural number.*

Since each vertex is uniquely expressed by its label, each edge is also expressed by the interaction between two unique vertexes. This allows each vertex to be labeled as a unique integer, even across different graph over the same vertex set. Two graphs will be same if their vertex label sets are same and its corresponding edge means connected vertexes label sets are same. Figure 3.2 shows two graphs G_1 and G_2 where vertexes sets are same but edges sets are different that's why they are not same. Connectivity information is remain unchanged in this representation. Each vertex is connected with other vertexes as connected edges.

4. PROPERTY. *Subgraph Testing: The measurement of subgraph testing whether G_1 is a subgraph of G_2 or vice versa can be done by checking unique vertex label repre-*

G_1			G_2	
Vertex Label	Connected Edge (Vertex Label)		Vertex Label	Connected Edge (Vertex Label)
1			1	
2	1		2	1
3	2		3	
4	3		4	1, 2, 3

Figure 3.2: Graph representation with unique vertex label and corresponding connected vertex label

sentation sets and their corresponding connected edge edge label of G_1 is subset of G_2 or vice versa.

5. PROPERTY. *Maximum Common Subgraph (MCS): The MCS between two graphs is defined by common vertex label and their corresponding common connected vertex label. It may be connected or disconnected subgraph. We use intersection operation $\bigcap$ to represent the common subgraph of two graphs.*

From figure 3.2 we find the maximum common subgraph contains all unique vertex labels $< 1, 2, 3, 4 >$ and two common connected edge $(1, 2)$ and $(3, 4)$. Following figure 3.3 shows the maximum common subgraph representation and its structure.

Maximum Common Subgraph ($G_1 \cap G_2$)	
Vertex Label	Connected Edge (Vertex Label)
1	
2	1
3	
4	3

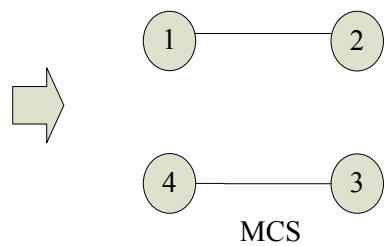


Figure 3.3: Maximum common subgraph between G_1 and G_2

6. PROPERTY. *Hashing: Since the unique vertex labels set represent by $\Re$ has a global ordering by feature of $\Re \in N$, A graph can be hashed like a vertex set. Each vertex connected with other vertexes that also represent by $\Re \in N$, another hashed can be used for denote connected edges within graph hashed.*

For periodic patterns, time indication is particularly important, because periodic patterns depend on period and starting time. A pattern occurring number is also essential for counting support.

14. DEFINITION. *Periodic Support Set: Given a dynamic network DN of T timesteps and any subgraph F = (V, E). The periodic support set $S_p(F)$ of F in DN is the set of all timesteps t, which start at t_i timestep and repeating every p timesteps where F is a subgraph of DN, which denote F subgraph of G_{t_i}. The representation of support set $S_P(F) = (t_i, p, s) = \{t_i, t_i + p, ..., t_i + p(s-1)\}$, such that $\forall t_i (t_i \in S_p(F) \leftrightarrow F \subseteq G_{t_i})$ and neither $G_{t_i - p}$ nor $G_{t_i + ps}$ contains F as subgraph.*

F is a **frequent periodic pattern** if its support exceeds a user defined minimum support threshold value σ.

Definition 14 is the formulation that is derived from the well known frequent pattern mining problem that satisfy the downward closure property. In downward closure property, every sub pattern of a frequent periodic pattern F is also frequent.

15. DEFINITION. *Closed Subgraph: For any subgraph F = (V, E) in a dynamic network DN of T timesteps is closed if it is maximal for its support set. In other words, while F support is maintaining, no vertex or edge can be added of F.*

There are a difference between frequent closed subgraph support set and periodic closed subgraph support set. A single subgraph F can have multiple periodic pattern support set to allow disjoin and overlapping periodic behavior. Thus, we require extraction of all periodic subgraph embeddings, rather than just the periodic subgraphs. The basic definition of periodic subgraph embeddings (PSEs) is given at definition 6 there is the possibility that a periodic subgraph embedding carries information in other periodic subgraph embeddings. Suppose a periodic graph's period is p, it is also periodic at 2p and for every multiple of p, which depends on only the threshold value. If they are frequent, they will be the output as periodic embedding and redundancy problem is occurred.

Mining parsimonious periodic subgraphs embedding is a well-designed solution to the redundancy of general periodic patterns. Since it captures all the information and produces small number of output results that is defined in the previous chapter at definition 7. It maintains subsumption property in property1. For example, a subgraph F of period 1 with adequate support 10, then it is also periodic at period 2 and 3 if minimum threshold is 3. In this case, PSE at period 2 and 3 are not Parsimonious PSE.

3.2 Supergraph based Behaviors Mining

The existing works $PSEMiner$ and $ListMiner$ find all $PSEs$ based on tree and list structure. In these cases, tree node and list node represent periodic pattern that stores common entities redundantly. These two processes need to convey all timesteps graph $< G_1, G_2, ..., G_{T-1} >$ for mining periodic patterns at T timesteps graph G_T. Dynamic networks are online process, total timestamps increase time to time that decrease the mining performance because it needs to check all previous graph information or their common subgraphs. To solve this kind of problem we propose supergraph-based method that stores common entities of all graphs only once and stores the periodic information of the entities called descriptors. For mining periodic patterns, it needs only one comparison between current graph and previous supergraph.

We now introduce $SPBMiner$, our proposed algorithm for mining all periodic subgraphs in dynamic networks. At first, we start by describing the most basic form of the algorithm, which mines periodic subgraphs. Then prune non-closed and non-parsimonious periodic subgraphs and mine parsimonious periodic subgraphs. After that, we mention some simple optimization of the basic algorithm that improves the performance. The basis architecture of $SPBMiner$ has been shown in figure **??**.

$SPBMiner$ works on following idea: at each timestep dynamic network is read, we maintain a supergraph embeddings all networks entities seen up to timestep t. This supergraph maintains two kinds of data structure for each entity. One is time-

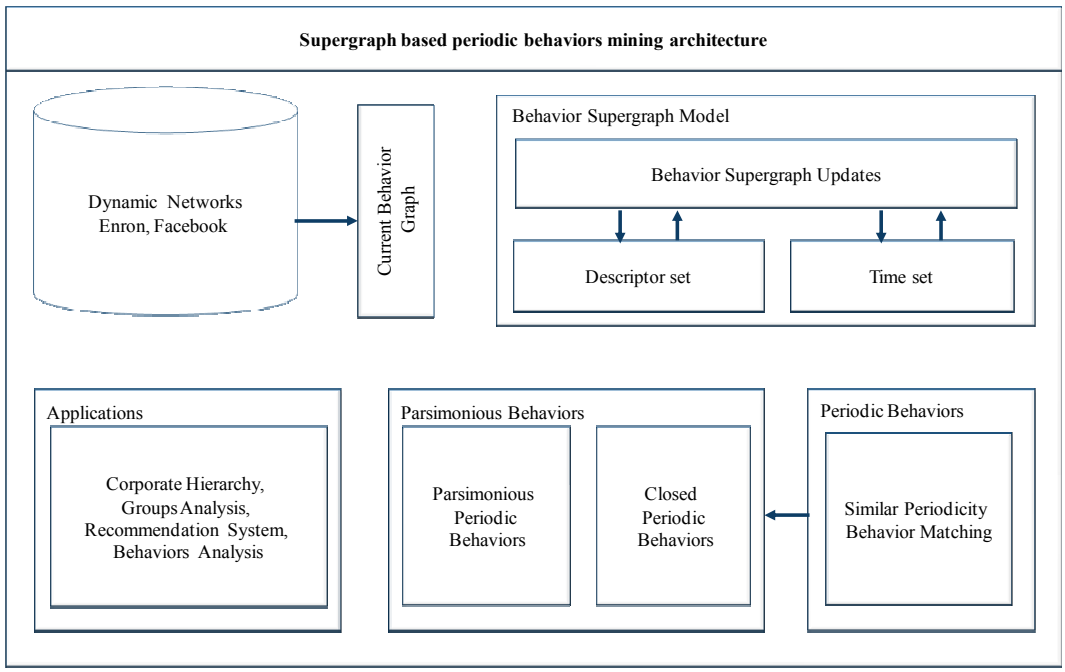


Figure 3.4: SPBMiner Architecture

set, which stores active time of entity. Other one is descriptor list, which represents entity periodicity. Once entities cease to be periodic, they are flashed from the supergraph and insert to periodic hash table as a periodic entity. Periodic hash table is one kind of data structure that stores entities base on period and starting time. If a group of entities flashed out same time and their period and support are same then store together and build periodic subgraph. After mining periodic subgraphs we use another kind of hash that stores subgraph as key and corresponding hash table keys represent descriptors information save as value. Each descriptor is checked when it is saved in value list. If its supergraph with the same descriptors does not exist, it is closed. If it is not subsumed by other descriptors, it is considered as parsimonious descriptor and subgraph is the parsimonious subgraph for the descriptor. The algorithm parameters, data structures and proof of correctness are described in the following sections.

3.2.1 Parameters

Our algorithm is online periodic subgraph mining single pass and space algorithm for mining all parsimonious periodic subgraph embeddings in a dynamic network. It needs two parameters as follows.

i. Minimum support threshold $\sigma \geq 2$ (default 3)

ii. Maximum period P_{max} (default 40).

The online algorithm bound the maximum period of supergraph entities. When entities create new descriptor, it generates maximum P_{max} number of descriptor and the minimum support threshold is used to indicate flushed entity is periodic or not.

3.2.2 Data Structure

As the algorithm looks into the stream graphs, it maintains three kinds of data structures: timeset, descriptor set and periodic subgraph hash table.

3.2.2.1 Descriptor

16. DEFINITION. *Descriptor: A descriptor D is one kind of data structure that represents periodic support set* (t_i, p, s) *for any entities, where starting time* t_i*, period* p *and support* s*. Descriptor D for an entity E is live if it's expected time* $(t_e = t_i + ps) >$ *current time* t_c *or if* $t_e = t_c$ *and entity E is present at* G_t*. A descriptor that is not live considered as a periodic entity if it support satisfies minimum threshold value.*

In supergraph, each entity contains a set of descriptor that is periodic or be periodic in future timesteps. Generating future periodic descriptors, each entity needs to store occurred time in previous timesteps.

3.2.2.2 Time Set

17. DEFINITION. *Time Set (TS): Time set is one kind of data structure that stores active time of the entity.*

The length of entity time set depends on maximum periodic length. Our proposed method is online periodic patterns mining in that case we consider P_{max} as maximum period. The timeset maintains the following lemma.

1. LEMMA. *The maximum size of TS for any entity is* P_{max}.

Proof: Periodic entity repeats every p period time intervals. Descriptor indicates entity periodicity information and support. If one entity appears in current graph, it updates descriptors of common supergraph entity. If descriptor expected time is equal to current time, then update these descriptors information. On the other hand, current entity generates a set of descriptors that would be periodic next time. In this time previous appeared time of entity is needed and these time is stored in timeset (TS). For new descriptor, maximum period is P_{max} because if it appeared in previous and live it already exists in descriptors. However, entity TS is maximum P_{max} stores all periodic time exclusive of missing any information.

3.2.2.3 Periodic subgraph hash table

Supergraph flushed out periodic support set for each entity. Our main goal is finding periodic subgraphs. In support of that principle, we need to add flushed entities descriptor to generate subgraphs. Hash table is especially efficient for this variety of structure. We have used starting position, period and support combined as a key value of hash table and have stored corresponding entities into patterns.

3.3 Description of the algorithm

Now we describe the update of supergraph information that is the core part of our process for periodic patterns mining. Initially the supergraph SG is empty. At timestep t, graph G_t is read. The common entities between SG and G_t are updated into SG. Entity updates timeset and descriptor set including addition, deletion and modification. Descriptors are flushed at deletion step. If descriptors support are greater than minimum threshold value, the entity is periodic. The process at

time t is completed ensuring all uncommon entities in current graph G_t includes in supergraph.

The entire process partitions three parts. We describe each part in the following subsections in details.

3.3.1 Update Common Entities in Supergraph

The supergraph entity information: timeset and descriptor sets are updated when entity is active. An entity appears in graph G_t is active otherwise inactive. Each graph G_t, the supergraph should be updated with common and uncommon entities. Let $F = SG \bigcap G_t$ be the maximal common subgraph of supergraph SG and G_t.

3.3.1.1 Timeset Update

Entity timeset presents active time steps of the entity. Each graph G_t, time set presents only single current timestep t. However, supergraph SG compact a set of time series graphs that result it should be stored entity active timesteps. Those time steps are needed to generate entity periodicity those are not periodic at this moment but would be periodic in future. $SPBMiner$ stores only maximum periodic number of timesteps instead of all time because entity appears reiteration at each periodic time interval. If timeset size is maximum, which is defined in Lemma 1, then delete timesteps using first in first out (FIFO) method.

3.3.1.2 Descriptors Creation

When entity $E \in F$, is active at current time t, it can creates some new periodic descriptors these are not currently periodic. Using entity timeset, it creates future periodic descriptors. Descriptor D period is the difference between timeset time and current time.

2. LEMMA. *Each entity E, the new created descriptor at time t is maximum* P_{max}.

Proof: New descriptor indicates entity periodicity that would be periodic at some point in future. In our method, we define maximum period P_{max}. Thus,

entity can generate descriptor that period is 1 to P_{max}. So, the maximum number of descriptor at any time step is maximum P_{max}.

Suppose an entity timeset TS [1, 2, 3, 4] and current time is 5. It can create four descriptors. These descriptors periodic support sets are like as (1,4,2), (2,3,2), (2,2,2) and (4,1,2) where first element is starting position, second one is period and last one is support value.

3.3.1.3 Descriptors Update

If entity $E \in F$, i.e. entity E is active at time step t, Let D is a descriptor of $E \in SG$ and $te = t_i + ps$ be the next expected time for D. If $te = t$, then D has appeared where it was expected and created a new descriptor $D' = D$. Time t is added to D' support to ensure temporal maximality. If the support of D is greater than or equal to σ then remove from supergraph entity E and stores in the periodic hash table. The following property is maintained at updates stage.

3. LEMMA. *For entity E, the maximum number of updated descriptors at time t is* $min(t, P_{max})$.

Proof: Descriptor D update occurs when expected time is equivalent to current time. Each period p, there is exacts p number of descriptors that should be updated at time t. These descriptors are defined by period and phase such as p mod m where $0 \leq m < p$. If period $p = 4$ then descriptors period and phase pairs (1,0), (2,0), (3,1) and (4,0) should be updated. The maximum value of period p is P_{max}. Therefore, the number of descriptors at any time t will be $min(t, P_{max})$.

3.3.1.4 Descriptors Deletion

Suppose entity E has descriptor D that has expected time $te < t$, then D has not appeared when expected and is no longer live. If it's support $\geq \sigma$, it will be stored in periodic hash table as a periodic entity and will remove D from $E \in SG$.

4. LEMMA. *At time step t, average* $2 * P_{max}$ *number of descriptors have been deleted from entity E.*

Proof: Lemma 2 and Lemma 3 show that at time t, maximum P_{max} number of descriptors have been created and updated. These means maximum $2 * P_{max}$ number of descriptors are added at each timestep. The number of deleted descriptors will be same as added descriptors in entire timesteps. However, the numbers of deleted descriptors are not fixed and there is no upper bound because at last step a large number of descriptors be alive. In this issue, we can define the average number of descriptors deletion at each time step and it will be same as creating and updating number $2 * P_{max}$.

Each time step t, entity E creates new descriptors, updates descriptors and finally deletes descriptors that may be periodic or not. Therefore, the number of descriptors at entity E maintain following property define in lemma 5.

5. LEMMA. *The maximum number of descriptor of any entity at time t is* P_{max}^2.

Proof: The number of descriptors of entity depends on size of entity time set. Lemma 2 proved that every time step entity creates P_{max} future behavior descriptors including period 1....P_{max}. If same periodic descriptors exist, then update those descriptors by creating new descriptors and delete old descriptors. Each periodic descriptor can be started at any position of phase m, where $0 \leq m < P_{max}$. Thus, the maximum number of descriptors of entity is P_{max}^2.

3.3.1.5 Entities Deletions

An entity appears in G_t is active otherwise inactive. Sorting of long time inactive entity in graph is inefficient because it needs memory and more computation time for mining common subgraph between supergraph and G_t.

6. LEMMA. *Inactive entity survive maximum* P_{max} *time in supergraph.*

Algorithm 1: SPBMiner $(\{BG_1, BG_2, ...BG_T\}, \sigma)$

Data: $BG_1, BG_2, ..., BG_T$: Dynamic behavior graphs at timestep $1, 2, ..., T$; σ : min_sup;

Result: Parsimonious Periodic Behaviors Sets

```
1  BSG ← φ          /* Behavior supergraph is empty          */;
2  for t = 1 to T do
3      for ∀ E ∈ BSG do
4          if E ∈ BG_t then
5              Update_Entity(BSG_E, t, σ, com);     /* Update common entity
                */;
6          else
7              Update_Entity(BSG_E, t, σ, uncom);       /* Update uncommon
                entity                                */;
8              if t − E^{|ST|−1} > P_max then
9                  Remove(BSG, E);      /* Remove dead entity      */;
10             end
11         end
12     end
13     for ∀ E ∉ BSG and E ∈ BG_t do
14         Add_Entity(BSG, E); /* Add uncommon entity          */;
15     end
16 end
17 MineLastTimestep_Behaviors(BSG, σ); /* Mining periodic behaviors
    at last timesteps                                    */;
```

Proof: For each entity, maximum P_{max} periodic descriptors are contained. When one periodic expected time is passed, those periodic descriptors are removed. After P_{max} time there is no descriptor in entity if entity does not become active within this period. When entity descriptor set is empty upto P_{max} time, we said this entity is dead. Then we can delete those entities from supergraph because there is no chance to be periodic within bounded period if it appears in future it will create new periodic descriptors.

Suppose entity E, appears in 1 and 6 timesteps and $P_{max} = 3$ then we can delete entity at time step 5 because it already reached dead entity. Next time if it appears, it will be considered as new entity and stores newly.

3.3.2 SPBMiner algorithm

Algorithm 1 shows the proposed $SPBMiner$ algorithm. The main algorithm performs from line 2 to line 17. Initially supergraph is empty. At each time step t, supergraph is updated based on current graph entities that are mentioned at line 5. However, there are entities in supergraph that are not appeared in current graph at all timesteps. These entities update occur at line 7. If any entity remains inactive in consecutive P_{max} times, we can remove it by line 9. There are some entities in current graph those do not exist in supergraph. Those entities become compact with supergraph at line 14. Finally, we mine periodic entity by line 17 those are live at last time step.

3.3.2.1 Entity Updates algorithm

The core part of the $SPBMiner$ algorithm is Update_Entity() procedure. This procedure tracks entities periodicity and support information that is significantly important for mining periodic behaviors in dynamic networks. Algorithm 2 shows the entity updates algorithm. Entity E is updated including descriptor set and timeset update, deletion and creation. Descriptor operations are performed by line 1- 14. When descriptor expected time is equal to current time, it creates new descriptors,

and updates its information and flashed out old descriptors with corresponding entity by line 3-8. If descriptors expected time are less than current time then flashed out descriptors and corresponding entity. These flashed entity would be periodic entity if it support is greater than or equal to σ (minimum support).

The other data structure of an entity is timeset (TS) that stores last P_{max} times when the entity was active. At timestep t, entity may be periodic or will be periodic at some point in future. For generating future periodic descriptor set, the algorithm creates new descriptors at line 20. If descriptors do not exists in entity then add by line 22 and finally add current time t in timeset TS at line 26. This algorithm returns update entity to main $SPBMiner$ algorithm.

Algorithm 3 shows flashed procedure for descriptor D with entity E. If the support of D is greater than minimum support σ then consider as a periodic entity. All those entities based on it start timestep, period and support have been combined. For this purpose, I create periodic hash table key combining starting position, period and support in line 1. If entity first creates periodic descriptor set into hash table, otherwise add to corresponding hash key value by line 6. If hash key contains then find out hash value and add current entity and store again based on same hash key mentioned by line 4.

3.3.2.2 Mining Periodic Behaviors

Periodic hash table values indicate periodic behaviors. However, these kinds of behaviors are neither closed nor parsimonious. When mining closed and parsimonious periodic behaviors, we have to maintain two basic lemmas those are defined by lemma 7 and lemma 8.

7. LEMMA. *Let periodic behaviors F support set S(F) = (m, p, s) then F is closed behaviors if there are no* $S(F') = (m, p, s')$ *where* $s' > s$ *and* $F \subseteq F$.

Proof: According to definition 5, closed periodic behaviors are those subgraphs which have no superset with same support or same graph with larger support. Then straightforward we can find out closed periodic subgraphs from periodic hash table.

Algorithm 2: Update_Entity(BSG_E, t, σ)

Data: BSG_E: Behavior supergraph entity E; t: current timestep; σ : min_sup;

Result: Update Behavior Supergraph Entity BSG_E

```
for ∀ D ∈ BSG_E^D do
    /*  Descriptors update in entity E  */;
    if D.te == t then
        D' = D;
        D'.sup = D.sup + 1;
        D'.te = D'.last + D'.period;
        Insert_Descriptor(BSG_E^D, D'); /* Insert Update Descriptor */;
        Flashed(D, E, σ);  /*  Delete Descriptor  */;
    else
        if D.te < t then
            Flashed(D, E, σ);  /*  Delete Descriptor  */;
        end
    end
end
for ∀ t' ∈ BSG_E^TS do
    /*  Create new descriptors  */;
    if t − t' > P_max then
        Remove(BSG_E^TS, t');  /*  Delete time t'  */;
    else
        D = new Descriptor(t', t, t − t', 2);
        if D ∉ BSG_E^D then
            Add_Descriptor(BSG_E^D, D);  /*  Add new descriptor  */;
        end
    end
end
Add_Time(BSG_E^TS, t);  /*  Add time t to Time Set  */;
return BSG_E  /*  Return update supergraph entity  */;
```

Algorithm 3: Flashed(D, E, σ)

Data: D: Descriptor; E: graph entity; σ:min_sup

Result: Insert into Periodic Hash

```
1 hashkey ← CreateHashKey(D.period, D.phase, D.support); /* Create
    hashkey                                              */;
2 if D.support ≥ σ then
3     if Periodic Table contains Hashkey then
4         Periodic_Subgraph ←
            Find_Value(Priodic_Patterns, hashkey) ⋃ E;
          Set_Value(Priodic_Patterns, hashkey, Periodc_Subgrpah, D.start);
5     else
6         Set_Value(Priodic_Patterns, hashkey, E, D.start);
7     end
8 end
```

Suppose subgraph $F = (A, B), (C, D)$, $S(F) = (1, 1, 3)$ is closed because position $(1, 1, 6)$ subgraph $F' \not\supseteq F$. On the other hand $F = (A, B), (B, C)$, $S(F) = (2, 2, 2)$ is not closed because position $(2, 2, 3)$ subgraph $F' \subseteq F$.

Using this lemma, I can prune non-closed periodic subgraphs that reduce redundant information. Another kind of redundant periodicity subgraphs exist in periodic hash table subsumed by others is defined in property 1. Mining parsimonious periodic patterns by the following lemma puts strongly effective influence.

8. LEMMA. *Let periodic subgraph F support set S(F) = (m, p, s) then F is subsumed by* $S(F') = (\beta * \lfloor m/p \rfloor, \beta, s')$ *if* $F \subseteq F'$, $p \bmod \beta == 0$ *and* $s' \geq p * s$.

Proof: I can limit myselves to the discovery of all periodic subgraphs of period 1. If subgraph F is periodic at (m,p.s) support set. It also periodic at $(\lfloor m/p \rfloor, 1, s')$ of projection of graph, for all $1 < p \leq P_{max}$ and $0 \leq m < p$. Then it also is periodic at divisor of p. If $\beta \bmod p == 0$ then β is divisor and $(\beta * \lfloor m/p \rfloor, \beta, s')$ is periodic support with graph F'. If $F' > F$ and $s' \geq p * s$ then F is subsumed by F' .

Suppose subgraph $F = (A, B), (C, D)$, $S(F) = (2, 2, 2)$ that means it occurs 2,4,6 timesteps. It may be subsumed by $(1, 1, s')$ where $s' \geq 2 * 2$. That reason $F' = (A, B), (C, D) = F$ $S(F') = (1, 1, 6)$ subsumed F which shows in table 3.1. F' also subsumed subgraph at position (2,2,3). So F is subsumes by F' and F is not parsimonious.

Table 3.1: Closed and Parsimonious Periodic subgraphs characteristics

Hash Key	Pattern	Closed	Parsimonious
1_1_6	(A,B),(C,D)	Yes	Yes
1_1_3	(A,B),(B,C)	Yes	Yes
2_2_2	(A,B),(C,D)	No	No
2_2_3	(A,B),(C,D)	Yes	No

Algorithm 4 shows the parsimonious periodic patterns mining procedure. Each hash key mentions the periodicity of pattern F that contains periodic descriptor as a hash value. The pattern F checks if a pattern F' is associated with large support and contains periodicity information with F' periodicity. It checks closed and parsimonious patterns. Finally, from the algorithm we can mine parsimonious periodic patterns.

3.3.3 Example

Suppose the dynamic network in figure 3.5 is the input. We explain our $SPBMiner$ algorithm in details systematically. It maintains supergraph update including descriptor and TS operations at each time. New periodic patterns are found and insert into hash table then mine parsimonious periodic patterns from periodic patterns hash table. In this example we consider $\sigma = 2$.

At time step 1, graph G_1 is considered supergraph because initially supergraph is empty and the representation of supergraph is shown in table 3.2. In this time

Algorithm 4: Parsimonious Periodic Pattern (*Periodic_Patterns*)

Data: BSG_E: *Periodic_Patterns*: All periodic patterns;

Result: Parsimonious Periodic Patterns

1 $Iterator \leftarrow Periodic\_Patterns.begin()$;
2 **while** $Itegator < Periodic\_Patterns.end()$ **do**
3 Hash key ← Iterator.Value();
4 Descriptor D ← Iterator.Value ();
5 $Find(phase, period, support) \leftarrow D$;
6 Hash newKey = new Key$(phase, period, sup')$; $where(sup' \geq support)$
7 **if** $Periodic\_Patterns(newKey)\&(F' = Value.Periodic\_Pattenrs(newkey)) \supseteq F$ **then**
8 Delete.Periodic_Patterns(newKey); `/* Delete nonclosed patterns */`;
9 **end**
10 Hash newKey = new Key$(\beta * \lfloor m/p \rfloor, \beta, sup')$; where $(sup' \geq support)\&p \bmod \beta == 0$
11 **if** $Periodic\_Patterns(newKey)\&(F' = Value.Periodic\_Pattenrs(newkey)) \supseteq F$ **then**
12 Delete.Periodic_Patterns(newKey); `/* Delete non parsimonious patterns */`;
13 **end**
14 **end**
15 **return** *Periodic_Patterns* ;

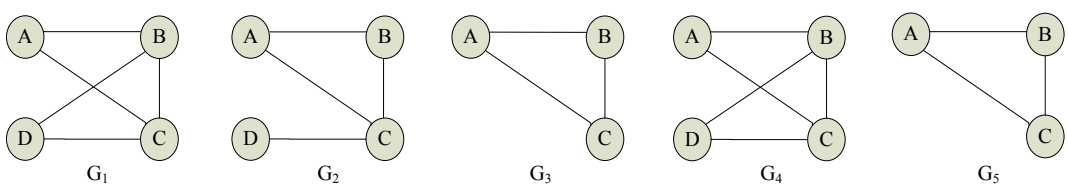


Figure 3.5: Dynamic Networks

periodic patterns hash table is empty.

Table 3.2: Supergraph representation at timestep 1

Super Graph					Periodic Patterns	
V_1	V_2	Timeset	Descriptors		Keys	Patterns
B	A	1	(1,1,1)			
C	A	1	(1,1,1)			
	B	1	(1,1,1)			
D	B	1	(1,1,1)			
	C	1	(1,1,1)			

At time step 2, graph G_2 is read and updates the supergraph. It updates time set and descriptors. Except interaction $(B - D)$ all interactions are appeared. The common interactions with supergraph creates new descriptors $(1, 1, 2)$ based on existing descriptors $(1, 1, 1)$. Existing descriptors $(1, 1, 1)$ are flashed from supergraph. It should not be added in periodic patterns because its support is less than σ. No new interaction appear so no need to add interactions to supergraph. The current supergraph structure representation is reported in table 3.3.

Table 3.3: Supergraph representation at timestep 2

Super Graph					Periodic Patterns	
V_1	V_2	Timeset	Descriptors		Keys	Patterns
B	A	1,2	(1,1,2)			
C	A	1,2	(1,1,2)			
	B	1,2	(1,1,2)			
D	B	1				
	C	1,2	(1,1,2)			

Graph G_3 is read and updates the supergraph at time step 3. It also updates time set and descriptors like as same way those are described at time step 2. Interactions $(A - B), (A - C)$ and $(B - C)$ appear at graph G_3. The existing descriptors $(1, 1, 2)$

expected time is current time. So it creates duplicate descriptor and updates it as $(1, 1, 3)$. It also creates $(1, 2, 2)$ and $(2, 1, 2)$ descriptors for future periodicity. All descriptors, which expected time is equal to current time or less than current time flashed out from supergraph with interactions duplicate entity. If it satisfies minimum support threshold value then inserts into periodic patterns hash table where hash key is defined as starting position, period and support. That's why descriptors $(1, 1, 2)$ is flashed out and corresponding interactions is stored in periodic hash table, which hash key is $1 - 1 - 2$. Table 3.4 shows the current supergraph presentation and periodic pattern hash table.

Table 3.4: Supergraph representation at timestep 3

Super Graph					Periodic Patterns	
V_1	V_2	Timeset	Descriptors		Keys	Patterns
B	A	1,2,3	(1,1,3)(1,2,2),(2,1,2)		1-1-2	(A-B),(A-C),(B-C)(C-D)
C	A	1,2,3	(1,1,3)(1,2,2),(2,1,2)			
	B	1,2,3	(1,1,2)(1,2,3),(2,1,2)			
D	B	1				
	C	1,2				

At time step 4, graph G_4 is read and updates the supergraph. All interactions in supergraph are active that why it updates or creates new descriptors based on timeset times. Interactions $(A - B), (A - C)$ and $(B - C)$ update and create new descriptors for future time. On the other hand $(B - D)$ and $(C - D)$ interactions are empty descriptors those way they create new descriptors. The descriptors, which expected time is equal or less then current time should be flashed out and create periodic patterns. Because of this flashed out descriptors $(1, 1, 3)$ and $(2, 1, 2)$ create two periodic patterns. The structure of supergraph and periodic patterns hash table after time 4 are shown in table 3.5.

Graph G_5 is read and updates the supergraph at time step 5. Interactions $(A - B), (A - C)$ and $(B - C)$ are active that why they update descriptors based

Table 3.5: Supergraph representation at timestep 4

Super Graph					Periodic Patterns	
V_1	V_2	Timeset	Descriptors		Keys	Patterns
B	A	1,2,3,4	(1,1,4)(1,2,2),(2,1,3),		1-1-2	(A-B),(A-C),(B-C)(C-D)
			(1,3,2)(3,2,2),(3,1,2)		1-1-3	(A-B),(A-C),(B-C)
C	A	1,2,3,4	(1,1,4)(1,2,2),(2,1,3),		2-1-2	(A-B),(A-C),(B-C)
			(1,3,2)(3,2,2),(3,1,2)			
	B	1,2,3,4	(1,1,4)(1,2,2),(2,1,3),			
			(1,3,2)(3,2,2),(3,1,2)			
D	B	1,4	(1,3,2)			
	C	1,2,4	(1,3,2),(2,2,2)			

on existing descriptors and also create new descriptors, which would be periodic in future and flashed out those descriptors which are not live. The table 3.6 shows the current supergraph and periodic patterns.

However, graph G_5 is the last graph in our dynamic network. Supergraph should be traversed and flushed out interactions and store those periodic patterns, which satisfy minimum support. Then finally, we find all periodic patterns like table 3.7.

The periodic patterns in periodic hash table all patterns are neither closed nor parsimonious. Mining parsimonious periodic patterns, we should check two kinds of property. First, one is closed pattern mining and second one is parsimonious pattern mining that means others do not subsume its periodicity. If any pattern satisfies two properties, we said that it is parsimonious periodic pattern. At first, we prune non-closed patterns. After removing non-closed patterns, those patterns may be parsimonious or not. There are a number of patterns that are subsumed by other patterns mention in table 3.8. After pruning non-parsimonious patterns we find parsimonious patterns these are the output of our proposed $SPBMiner$ algorithm shows in table 3.9.

Table 3.6: Supergraph representation at timestep 5

Super Graph					Periodic Patterns	
V_1	V_2	Timeset	Descriptors		Keys	Patterns
B	A	1,2,3,4,5	(1,1,5)(1,2,3),(2,1,4),		1-1-2	(A-B),(A-C),(B-C)(C-D)
			(1,3,2)(2,2,2),(3,1,2),		1-1-3	(A-B),(A-C),(B-C)
			(1,4,2)(2,3,2),(3,2,2),(4,1,2)		1-1-3	(A-B),(A-C),(B-C)
C	A	1,2,3,4,5	(1,1,5)(1,2,3),(2,1,4),		2-1-2	(A-B),(A-C),(B-C)
			(1,3,2)(2,2,2),(3,1,2),		1-1-4	(A-B),(A-C),(B-C)
			(1,4,2)(2,3,2),(3,2,2),(4,1,2)		1-2-2	(A-B),(A-C),(B-C)
	B	1,2,3,4	(1,1,5)(1,2,3),(2,1,4),		2-1-3	(A-B),(A-C),(B-C)
			(1,3,2)(2,2,2),(3,1,2),		3-1-2	(A-B),(A-C),(B-C)
			(1,4,2)(2,3,2),(3,2,2),(4,1,2)			
D	B	1,4	(1,3,2)			
	C	1,2,4	(1,3,2),(2,2,2)			

Table 3.7: Periodic Patterns

Keys	Patterns	Keys	Patterns
1-1-2	(A-B),(A-C),(B-C),(C,D)	2-1-4	(A-B),(A-C),(B-C)
1-1-3	(A-B),(A-C),(B-C)	1-3-2	(A-B),(A-C),(B-C),(B-D),(C-D)
2-1-2	(A-B),(A-C),(B-C)	2-2-2	(A-B),(A-C),(B-C),(C-D)
1-1-4	(A-B),(A-C),(B-C)	3-1-3	(A-B),(A-C),(B-C)
1-2-2	(A-B),(A-C),(B-C)	1-4-2	(A-B),(A-C),(B-C)
2-1-3	(A-B),(A-C),(B-C)	2-3-2	(A-B),(A-C),(B-C)
3-1-2	(A-B),(A-C),(B-C)	3-2-2	(A-B),(A-C),(B-C)
1-1-5	(A-B),(A-C),(B-C)	4-1-2	(A-B),(A-C),(B-C)
1-2-3	(A-B),(A-C),(B-C)		

Table 3.8: Pruning non closed and non parsimonious periodic patterns

Patterns	Keys	Closed	Reason	Parsimonious	Reason
(A-B),(A-C),(B-C),(C,D)	1-1-2	Yes		Yes	
	2-2-2	Yes		Yes	
(A-B),(A-C),(B-C)	1-1-3	No	1-1-4		
	2-1-2	No	2-1-3		
	1-1-4	No	1-1-5		
	1-2-2	No	1-2-3		
	2-1-3	No	2-1-4		
	3-1-2	No	3-1-3		
	1-1-5	Yes		Yes	
	1-2-3	Yes		No	1-1-5
	2-1-4	Yes		No	1-1-5
	3-1-3	Yes		No	1-1-5
	1-4-2	Yes		No	1-1-5
	2-3-2	Yes		No	1-1-5
	3-2-2	Yes		No	1-1-5
	4-1-2	Yes		No	1-1-5
(A-B),(A-C),(B-C),(B-D),(C-D)	1-3-2	Yes		Yes	

Table 3.9: Parsimonious Periodic Patterns

Patterns	Keys	Start	Period	Phase	Support
(A-B),(A-C),(B-C),(C,D)	1-1-2	1	1	0	2
	2-2-2	2	2	0	2
(A-B),(A-C),(B-C)	1-1-5	1	1	0	5
(A-B),(A-C),(B-C),(B-D),(C-D)	1-3-2	1	3	1	2

3.3.4 Description of the implementation

The algorithm is implemented in C++. In the next section, most important elements of the program are presented.

Graph: The graph is represented by integer vector where interactions present by integer value and it is unique.

Super Graph: Super graph like as graph. Each interaction has descriptor set an timeset which indicate entity periodicity information and active time respectively.

Timeset (TS): It is integer array that size is maximum period and value is time.

Descriptor Set: It is the descriptor object vector in entity. It describes the entity periodicity information like as period, phase, support, start and last occurred times.

Periodic Hash Table: The periodic has table is implemented by google dense hash map [41]. The key of hash table is the combination of starting position, period and support (key = start-period-support).

Maximal common subgraph: For every timestep G_t we have to calculate the MCS between supergraph SG_{t-1} and G_t. For MCS computation common interactions are checked. Parsimonious Periodic Patterns: In this step pattern is the key and corresponding descriptors are value of the hash map.

3.4 Time and Space Complexity

3.4.1 Time Complexity

Supergraph maintaining is the main part of our proposed algorithm. Suppose V is the vertex number of our supergraph. Then total entities are $V * (V-1)/2$ if supergraph is strongly connected. However, worst case super graph entities is $O(V^2)$. Lemma 5 proved that every entity has maximum $(P_{max})^2$ descriptors. Updating these descriptors, need $(P_{max})^2$ time and updating timeset (TS) needs P_{max} time. So each timestep requires $O(V^2)((P_{max})^2 + P_{max}) = O(V^2(P_{max})^2)$ times. This yields the total time complexity is $O((V^2)T(P_{max})^2)$ when P_{max} is specified. If P_{max} is not specified than the time complexity is $O(V^2T(T/\sigma)^2)$. In practice, however,

the vertex number is smaller and descriptor set is sorted. Then the computational cost is usually smaller then the worst case, as we will demonstrate.

3.4.2 Space Complexity

For every timestep t, supergraph has maximum V^2 entity and each entities contains TS and descriptor sets. According to lemma 1 and 5, the size of time set and descriptor set is P_{max} and $(P_{max})^2$ respectively. Therefore the total space complexity is $O((V^2)(P_{max})^2)$ that is total time independent. In the worst case P_{max} is unrestricted $P_{max} = T/\sigma$, so the space complexity is $O(V^2(T/\sigma)^2)$.

3.5 Summary

In this chapter, we have introduced an attractive periodic patterns mining technique $SPBMiner$, which mine periodic patterns as well as closed and parsimonious periodic patterns from dynamic networks over the user given minimum support. The efficiency of the finding such patterns is shown in the complexity analysis section which is better than the existing $PSEMiner$ and $ListMiner$ methods.

Chapter 4

Experimental Evaluation

In this chapter, the performances and the characteristics of $SPBMienr$ are compared with two existing algorithms $PSEMiner$ and $ListMiner$. I used three real world dynamic social networks in the experimental result analysis. Artificial datasets are also created to better understanding of every algorithm. I clearly mention the differences, weak and strong points in my algorithm.

For this comparison, these three algorithms are implemented in $C++$. I implemented $SPBMienr$ and $ListMiner$ and used $PSEMiner$ source code available in [42]. The experiments are run on 3.3 GHz Intel Core i5 with 4GB RAM, in windows 7. These algorithms use google dense/sparse hash library [41]. In all experiments, the reported computation time is the sum of the user and CPU time. Memory usage is the maximum resident set size reported by $C++$ memory uses function.

4.1 Datasets Description

Three real dynamic social networks datasets and six artificial datasets are used to evaluate our $SPBMiner$ algorithm.

4.1.1 Real Datasets

Dynamic networks are collected from different sources and covering a range of interaction dynamics. These networks are described below.

Enron e-mails: The Enron e-mail corpus is a publicly accessible record of e-mails commutation among employees of the Enron Corporation [43]. Senders and list of recipients timesteps were extracted from message headers for each e-mail on file. It has been considered a day as the quantization timestep that means if at least one e-mail was sent between two individuals employees on a particular day we can said that interaction is presented.

Facebook Wall Post: This facebook dataset [44] gathered wall post information about 90,269 users September 26, 2006 to January 22, 2009. In total, there are observed 838,092 wall posts, for an average of 13.9 wall posts per user. This covers communication between 188,892 distinct pairs of users. One-day quantization is measured as unique timestep.

Reality Mining: Cell phones with nearness tracking technology were distributed to 100 students at the Massachusetts Institute of Technology over the course of an academic year [33]. The timestep quantization is chosen as 1 day.

4.1.2 Artificial Data

Artificial datasets are used to better understand the performances of these algorithms. The intention of these set of experiments is to illuminate why and when our algorithm outperforms the others. The following datasets are created using GraphGen a synthetic graph generator [45].

Experiment 1.1: This dataset creates random graph with 50 different interactions among 10000 populations at each timestep and the total timestep is 1000. The interactions represent a sequence of 50 different random numbers among 1 to 10000.

Experiment 1.2: This dataset creates random graph with 100 different interactions among 10000 populations at each timestep and the total timestep is 1000.

The interactions represent a sequence of 100 different random numbers among 1 to 10000.

Experiment 1.3: This dataset creates random graph with 200 different interactions among 10000 populations at each timestep and the total timestep is 1000. The interactions represent a sequence of 200 different random numbers among 1 to 10000.

Experiment 1.4: This dataset creates random graph with 400 different interactions among 10000 populations at each timestep and the total timestep is 1000. The interactions represent a sequence of 400 different random numbers between 1 and 10000.

Experiment 1.5: This dataset generates graph with 800 various interactions among 10000 populations at each timestep and the total timestep is 1000. The interactions represent a sequence of 800 different random numbers 1 to 10000.

Experiment 1.6: This dataset builds random graph with 1000 different interactions among 10000 populations at each timestep and the total timestep is 1000. The interactions represent a sequence of 1000 different random numbers 1 to 10000. The table **??** shows the datasets in details.

The variation of artificial data parameters for the artificial networks expresses diversity characteristic of networks including low density (Experiment 1.1 and 1.2), medium density (Experiment 1.3 and 1.4) and high density (Experiment 1.5 and 1.6). The experiments analyses will show the density of networks is a parameter that has a significant influence on our $SPBMiner$ algorithm.

4.2 Experimental Time Analysis

This section shows the execution times comparison analysis among our proposed method and other two existing works. The execution times of three techniques are

Table 4.1: Parameters of various datasets

Dataset	Timestep	Vertexes	Edges	Avg. Active Edge Density	P_{max}
Enron	2588	82614	330452	0.0015	40
Reality mining	544	100	4900	0.025	40
Facebook	1563	46951	193337	0.002	40
Experiment 1.1	1000	150	10000	0.005	40
Experiment 1.2	1000	150	10000	0.01	40
Experiment 1.3	1000	150	10000	0.02	40
Experiment 1.4	1000	150	10000	0.04	40
Experiment 1.5	1000	150	10000	0.08	40
Experiment 1.6	1000	150	10000	0.10	40

performed where minimum support $\sigma = 3$ and mining patterns are parsimonious.

The reality mining dataset is high density network. The number of vertexes is low (100) and the number of timesteps is medium (544). The $SPBMiner$ algorithm generates periodic descriptors for each entity (interactions between two vertexes) that result low number of vertexes interactions mining needs short time and less memory. Similarly, experiment 1.5 creates a sequence of 400 casual numbers among 1 to 10000 and experiment 1.5 creates 800 numbers among 1 to 10000. These networks are also dense. Therefore, these dense networks are changed by time to time defined by different graph structures. The probability of common subgraph computation between two graphs are high that reason $ListMiner$ and $PSEMiner$ needs more MCS computation.

The figure **??** shows that $SPBMiner$ is faster than two existing works in reality mining datasets. It's vertex number is low and it is dense that way it creates more common sub-behaviors' set in $ListMiner$ and $PSEMiner$. On the other hand, Facebook dataset is medium density and its time is slightly high. Although in this case my algorithm performs better than $PSEMiner$ but it is not good as $ListMiner$ because it mine common patterns between 387 entities where our

proposed process traverse large than 387 entities at each timestamps that why need little bit large computation time. In Enron dataset due to sparse data my propose method $SPBMiner$ shows low performance than $PSEMiner$ and $ListMiner$. The variation of network density is the main cause that explained below using artificial datasets.

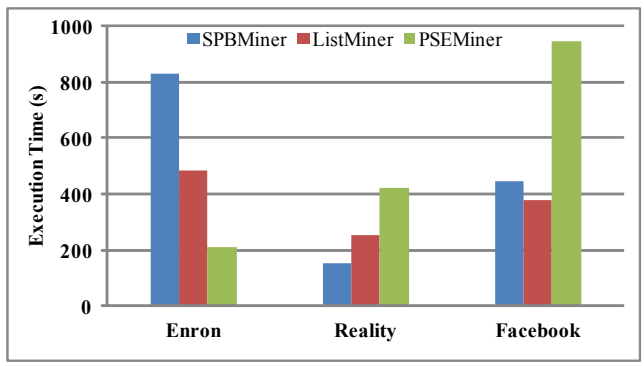


Figure 4.1: Execution times comparison on real datasets.

In the high-density context, $PSEMiner$ is much slower than $SPBMiner$. Because PSEMiner builds periodic pattern tree using graphs and current graph traverse the entire tree node and find common subgraph. Since the dense graph, it need more time to compute MCS between graph and tree nodes. The execution times comparison among three methods in the figure 4.2 shows that at experiment 1.1, $PSEMiner$ is faster than $ListMiner$ and $SPBMiner$. The $SPBMiner$ execution time analyses have been shown in experiment 1.3 $ListMiner$ is minimum 1.5 times slower and $PSEMiner$ is 2 times slower than $SPBMiner$. Another density dataset experiment 1.4 shows our proposed $SPBMiner$ is around 1.5 times faster than $ListMimer$ and around four times faster than $PSEMiner$. In the medium density context, analysis of experiment 1.3 reports that all these three methods execution times are almost same. Although, the theoretical time bound complexity of $SPBMiner$ is better than ListMiner and PSEMiner, experiment 1.1 and 1.2 reports $PSEMiner$ is faster than $ListMiner$ and $SPBMiner$ because of sparse dataset. In this case, MCS computation is less and faster. According to above analyses, we

can say that *SPBMiner* is time efficient when network density is medium or high.

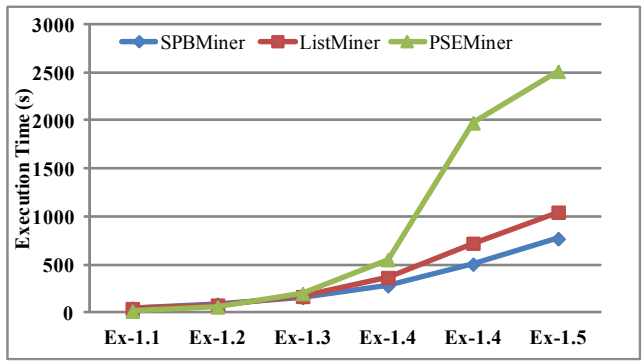


Figure 4.2: Execution times comparison on artificial datasets.

4.3 Experimental Space Analysis

The analysis of the memory requirement of *SPBMiner*, *ListMiner* and *PSEMiner* are presented in this section. Figure 4.3 shows the results comparison of the memory usage of these algorithms with σ =3.

SPBMiner use less memory in facebook dataset because it density is not so high and it create less number of periodic behaviors than Enron datasets and periodic behaviors length is small that way it needs less memory. In Reality dataset requires large memory because for each entity descriptors set generate P^2_{max} descriptors maximum that why it needs large memory. Enron dataset has large number of entities that way it requires little bit large memory. In conclusion we said that our *SPBMiner* methods is memory efficient in medium density networks.

SPBMiner uses less memory than *ListMiner* and *PSEMiner* in experiment 1.1 , 1.2 and 1.3 in figure 4.4. The others dataset *SPBMiner* is slightly higher than *ListMiner*.

This behavior can be justified by theoretical analysis of the space complexity. The space complexity of *PSEMiner* is $((V + E)N + P^2_{max} + G)$ where N is the number of nodes in the tree, G is the number of descriptor, and V, E are the number of vertexes and edges, respectively. The space complexity of *ListMiner* is always

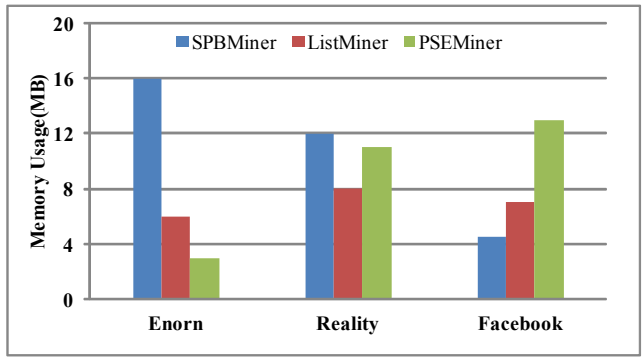


Figure 4.3: Memory usage on real datasets.

$((V + E)TP_{max})$. Since the most part of the memory is used to store graphs, the dominant term of the space complexity expression in $PSEMiner$ is (V+E)N. The space complexity of $SPBMiner$ is $((V + E)P^2_{max}$. In dataset experiment 1.1, 1.2, 1.3, 1.4 numbers of interactions (vertexes and edges) is 50, 100,200 and 400. P_{max} is 40 so memory complexity of our process is less than existing methods $ListMiner$ and $PSEMiner$. In experiment 1.5, entities are 800 and $P_{max} = 40$ that way $ListMiner$ requires 800*1000*40 that is less then requires theoretical memory in $SPBMiner$, which is $10000 * (40)^2$. Thus $ListMiner$ is memory efficient. If time step would be too large that case $SPBMiner$ would be memory efficient. Therefore, when the number of entities density is more and time steps are low or density is high and total timesteps are also high than the space complexity of $SPBMiner$ is better then $PSEMiner$ and $ListMiner$.

4.4 Analysis of Dense Networks Effects

In previous section, we report the $SPBMiner$ algorithm efficiency depend on network density. In section we will explain how density is affected our approach. Analyzing density effects, from figure 4.2 experiment 1.3 network density is 2% that time all these algorithms execution time is almost same. But experiment 1.4 network density is 4%, total number of interactions 10000 and 400 is active at each timesteps. In this time our $SPBMiner$ is 23% faster than $ListMiner$ and 49%

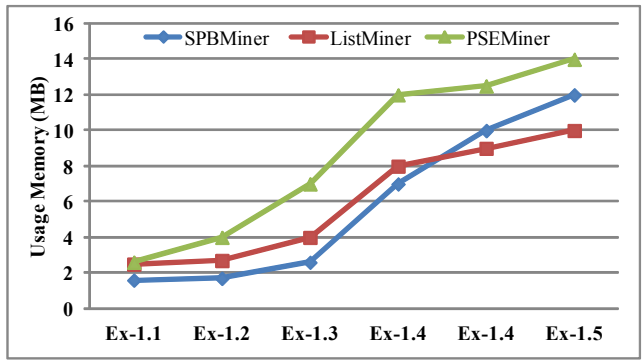


Figure 4.4: Memory usage on real datasets.

faster than $PSEMiner$. Experiment 1.5 is dense data that case $SPBMiner$ is 30% faster than $ListMiner$ and 75% faster than $PSEMiner$. All these experiment networks continue 1000 timesteps and considered minimum threshold $sigma$= 3 and P_{max} = 40. Finally, we say that our approach is exceptionally faster than existing works in medium and dense datasets.

4.5 Analysis of Parsimonious Periodic Patterns

In this section, the analysis of parsimonious periodic patterns mined by the $SPBMimer$ algorithm is reported. The analysis is performed in two tracks. First, one is number of periodic patterns vs support and second one is number of periodic patterns vs period.

Experiments analyses showed that the highest number of parsimonious periodic patterns and the highest values of support occur on high-density networks.

4.5.1 Parsimonious Periodic Pattern Division and Support Values

Figure 4.5 shows the number of parsimonious periodic patterns (y axis) for each real dataset based on support value (x-axis). It has been shown that dense dataset reality mining produce a large number of parsimonious periodic patterns more than 10000 though its total timestep is only 544. With the increases of support its patterns

number decrease rapidly. Facebook dataset produce large number of data at support 3,4,5 that represent most of the patterns are within support 3 to 5 and then it reduce very rapidly and at the end support 19 it produces only 7 patterns. On the other hand, Enron dataset produce large dataset also and reduces the periodic pattern in a sequentially after support 6. All these experiment run based on period P_{max} that has been mentioned in table 4.1.

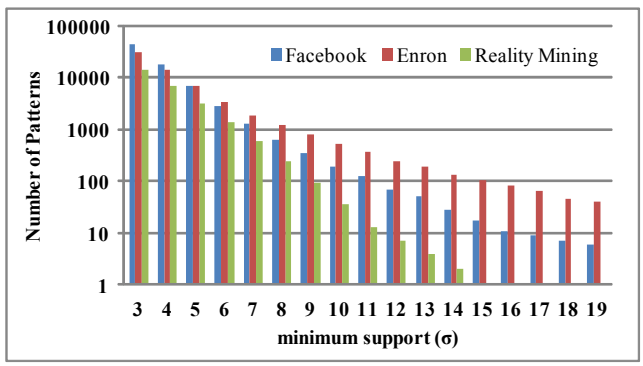


Figure 4.5: Parsimonious periodic patterns vs minimum support

4.5.2 Parsimonious Periodic Pattern Division and Period Values

Figure 4.6 shows the number of parsimonious periodic patterns (y axis) for each artificial dataset based on different period value (x-axis). It has been shown that Enron dataset produces a large number of parsimonious periodic patterns and generating patterns are decreases with the increases of period. It clearly shows that at period 7, 14, 21, 28 and 35 produces large number of periodic patterns that indicate at each week they have a particular day that day their email communication be high. It may be their weekly report of or performance analysis email. On the other facebook dataset number of patterns are not so very period to period though there are some differences between them . All these patterns are mined based on minimum support $\sigma = 3$.

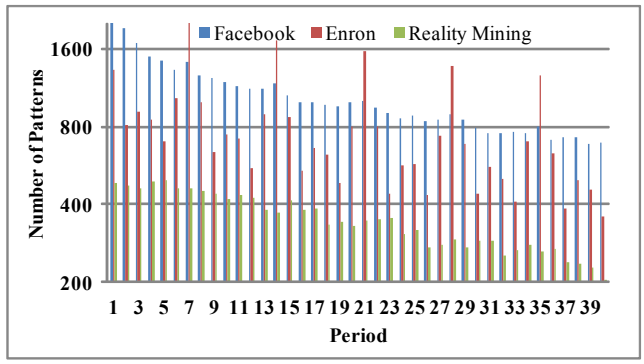


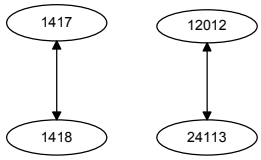
Figure 4.6: Parsimonious periodic patterns vs period value

4.5.3 Knowledge Discovery

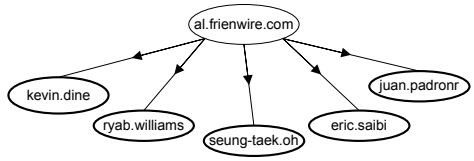
In the figure 4.7 (a) and (b) have been shown that facebook dataset contains pair to pair communication that are weekly and continue up to 7 weeks and other one is daily and 2 days interval wall post communication continue up to 10 times. Figure 4.7(c) represents interesting communications where one email users send emails continuously 84 days. From those kinds of relationship, we said that they are very strongly connected in the Enron Corporation. In figure 4.7(d) shows one email users send email each week interval and it continue around 4 month. From this kinds or relationship, I suppose that they are works one projects for 4 month and their communication is strong for these period.

4.5.4 Scalability Analysis

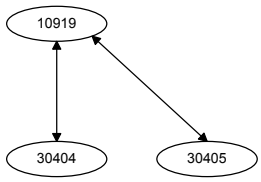
Execution time depends on minimum support and maximum period value. With the increases of maximum period, the execution time increase. On the other hand, with increase of minimum support the execution time reduces. Figure 4.8(a) shows that the minimum support scalability where $P_{max} = 50$. And figure 4.8(b) shows that the scalability analysis of our proposed SPBMiner as well as ListMiner and PSEMiner. Our propose SPBMiner is more scalable than two existing works because when P_{max} increase it create a large number of nodes and it more comparison that result it needs large time. In this experiment I use artificial data experiment 1.4 and



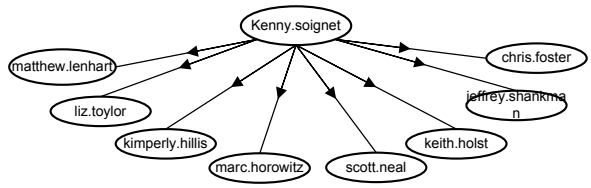
(a) Facebook: period 7 support 7



(c) Enron: period 1 support 84



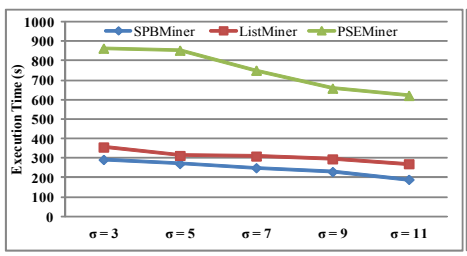
(b) Facebook: period 1 & 2 support 10



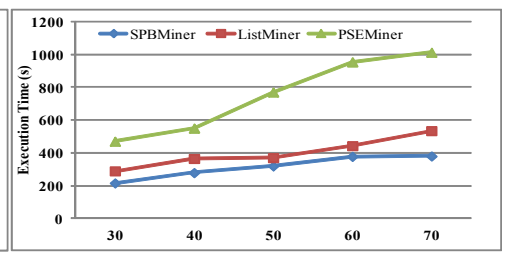
(d) Enron: period 7 support 31

Figure 4.7: Finding inherent patterns from facebook and Enron dataset

minimum support = 3.



(a) Execution time vs minimum support



(b) Execution time vs maximum period

Figure 4.8: Scalability test for experiment 1.4 dataset

4.6 Summary

In this chapter, we have shown the effectiveness and efficiency of our proposed *SPBMiner*. Experimental space and time analysis show that our method are significantly efficient for medium and dense dynamic networks and outperform the existing algorithm in both execution time and memory usage. We also mine some interesting periodic patterns form real datasets that represent very informative information.

Chapter 5

Conclusions and Future Work

In this chapter we summarize the research works presented in this dissertation and make final concluding remarks with few directions for future works.

5.1 Summary of the Dissertation

The main contribution of this dissertation is the design and development of $SPBMiner$, an efficient algorithm for solving the periodic behaviors mining problem in dynamic networks. The time complexity of $SPBMiner$ is $((V + E)TP_{max}^2)$, where V is the size of the population of networks, E is the set of interactions among populations, T is the number of observations (timesteps) and P_{max} is the maximum period. Our proposed $SPBMiner$ improves the worst case time complexity of $ListMiner$ by $/sigma$ and $PSEMiner$ by $(/sigma^2 ln(T))$ times .

The theoretical analyses of our proposed method support experimental analyses. The performances and the behaviors of $SPBMiner$, $ListMiner$ and $PSEMiner$ were compared using two real-world dynamic social networks and several artificial datasets. The experiments have been shown the performances of the algorithms depend on networks density. In the high-density networks, $SPBMiner$ is faster than $ListMiner$ and $PSEMiner$. Contrarily in a low-density context $PSEMiner$ and $ListMiner$ is faster than $SPBMiner$. However, in the worst case dataset analysis confirms that $SPBMiner$ is actually more efficient than $ListMiner$ and $PSEMiner$. Moreover, in real scenarios, where the maximum period P_{max} is restricted, $SPBMiner$ took few seconds to execute and uses less than 15 MB of memory.

Finally, qualitative analyses of parsimonious patterns show the periodicities interactions among students and corporate executives. The daily and weekly behaviors of interactions among people are shown in the experiments. Additionally, the mined parsimonious patterns revealed the hidden characteristics of the interactions among the population. In particular, the patterns characteristic of college students are shown peer-to-peer and corporate relationship are mostly hierarchical.

5.2 Future Research Directions

With the capabilities of the proposed method, we plan to investigate and explore the following related problems, extensions:

• We would like to investigate whether other efficient algorithms would lead to better discovery of periodic patterns because our model does not maintain effectiveness and efficiency in offline networks when period is unrestricted.

• Designing sequential periodic behaviors mining technique is significantly interesting in data mining and knowledge discovery. We would like to develop efficient method for mining sequential periodic patterns in dynamic networks.

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Appendix A

List of Publications

International Journal Papers

1. **Sajal Halder**, Yongkoo Han, A. M. Jehad Sarkar and Young-Koo Lee. *An Entertainment Recommendation System using the Dynamics of User Behavior over Time.* Decision in process in the Journal of Systems and Software.

2. Md. Rezaul Karim, **Sajal Halder** , Byeong-Soo Jeong, and Ho-Jin Choi. *Efficient Mining Frequently Correlated, Associated-correlated and Independent Patterns Synchronously by Removing Null Transactions.* Human Centric Technology and Service in Smart Space, pages 93-103, 2012.

3. **Sajal Halder**, A. M. Jehad Sarkar and Young-Koo Lee. *A synthetic trajectory-based moving objects generator.* Under review in International Journal of Artificial Intelligence Tools.

4. **Sajal Halder**, Md. Mostofa Kamal Rasel, Yongkoo Han, and Young-Koo Lee. *Mining Spatiotemporal Moving Objects Swarm.* Under review in Kyung Hee University Journal..

International Conference Papers

5. Sajal Halder, Yongkoo Han and Young-Koo Lee. *Discovering Periodic Patterns using Supergraph in Dynamic Networks.* Accepted in 5th International Conference on Data Mining and Intelligent Information Technology Applications (ICMIA),Jun 18-20, South Korea, 2013.

6. Sajal Halder, A. M. Jehad Sarkar and Young-Koo Lee. *Movie Recommendation System Based on Movie Swarm.* Second International Conference on Cloud and Green Computing (CGC), China, Nov 1-3, 2012.

7. Sajal Halder, Md. Samiullah, A. M. Jehad Sarkar and Young-Koo Lee. *MovieSwarm: Information Mining technique for Movie Recommendation System.* In the 7th International Conference on Electrical and Computer Engineering (ICECE), Bangladesh, Dec 20-22, 2012.

Thesis/Project Works

8. Sajal Halder, Uzzal Kumar Dutta, Uttam Kumer Biswas and Asish Kumar Biswas "Classification of Multiple Protein Sequences by means of Irredundant Patterns", B.Sc. Final Year Project, Department of Computer Science and Engineering (CSE), University of Dhaka (DU), Bangladesh, February, 2011.