Hypergraph colourings

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Theorem (Wanless and Wood 2022+, arXiv:2008.00775)

If there exist a real $\beta \geqslant 1$ and an integer $c \geqslant 1$ such that for every $v \in V(G)$,

$$c \geqslant \beta + \sum_{k \geqslant 0} \frac{E_k(v)}{\beta^k},$$

then the number of c-colouring of G is at least $\beta^{|V(G)|}$.

For $v \in e \in E(G)$, let S be the minimum-sized subset of $e \setminus \{v\}$ that determines the bad event B_e and

$$E_k(v) = \left| \left\{ (v,e) : v \in e \in E(G), k = |e| - 1 - |S| \right\} \right|.$$

Example

For proper colouring of an r-uniform hypergraph G. For each $e \in E(G)$, bad event is

$$B_e = \{e \text{ is monochromatic}\}.$$

If $w \in (e \setminus \{v\})$, then $\{w\}$ determines B_e, implying that $E_{r-2}(v) \leqslant \Delta$.

Wanless-Wood condition

$$c \geqslant \beta + \sum_{k \geqslant 0} \frac{E_k(v)}{\beta^k},$$

Theorem (Alves, Procacci, Sanchis 2021, arXiv:1509.04638)

If

$$k \geqslant \inf_{x>0} \left(\frac{1}{x} \left(1 + \sum_{j \geqslant 0} \left(\max_v E_j(v) \right) x^j \right) \right),$$

then we can avoid all bad events via entropy-compression in an expected number of steps linear in $|V(G)|$.

Questions

- To simplify the entropy-compression proof via Wanless and Wood's induction.
- To produce exponentially many colourings via entropy-compression.

Thanks for your attention!