When rhs is not constant, but f:

$$\min_{\alpha,\beta,\gamma,\omega}: \quad F := \|\hat{d} - d\|^2 = \hat{d}^2 - 2\hat{d}^T d + d^2$$

s.t. $K(\alpha,\beta,\gamma,\omega)\, d = G$. $K_1 = \alpha A + \beta B + \gamma C + \omega D$, $G_1 = \alpha g_1 + \beta g_2 + \gamma g_3 + \omega g_4$.

$d = K_1^{-1} G_1$,

$$\begin{bmatrix} 1 & & \\ & 1 & \\ & & \boxed{K_1} \end{bmatrix} d = \begin{bmatrix} 0 \\ 0 \\ \boxed{G_1} \\ 0 \end{bmatrix}$$

$$\frac{\partial F}{\partial \alpha} = -2\hat{d}^T \frac{\partial d}{\partial \alpha} + 2 d^T \frac{\partial d}{\partial \alpha}$$

$$\frac{\partial d}{\partial \alpha} = \begin{pmatrix} 0 \\ 0 \\ \frac{\partial d_1}{\partial \alpha} \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ -K_1^{-1} A K_1^{-1} G_1 + K_1^{-1} g_1 \\ 0 \end{pmatrix}, \quad \frac{\partial d}{\partial \beta} = \begin{pmatrix} 0 \\ 0 \\ -K_1^{-1} B K_1^{-1} G_1 + K_1^{-1} g_2 \\ 0 \end{pmatrix}$$

$$\frac{\partial d}{\partial \gamma} = \begin{pmatrix} 0 \\ 0 \\ -K_1^{-1} C K_1^{-1} G_1 + K_1^{-1} g_3 \\ 0 \end{pmatrix}, \quad \frac{\partial d}{\partial \omega} = \begin{pmatrix} 0 \\ 0 \\ -K_1^{-1} D K_1^{-1} G_1 + K_1^{-1} g_4 \\ 0 \end{pmatrix}$$

$$\frac{\partial^2 d}{\partial \alpha^2} = \begin{pmatrix} 0 \\ 0 \\ 2K_1^{-1} A K_1^{-1} A K_1^{-1} G_1 - 2K_1^{-1} A K_1^{-1} g_1 \\ 0 \end{pmatrix}$$

$\frac{\partial^2 d}{\partial \beta^2}$, $\frac{\partial^2 d}{\partial \gamma^2}$, $\frac{\partial^2 d}{\partial \omega^2}$ is similar.

$$\frac{\partial^2 d}{\partial \alpha \partial \beta} = + K_1^{-1} B K_1^{-1} A K_1^{-1} G_1 + K_1^{-1} A K_1^{-1} B K_1^{-1} G_1 - K_1^{-1} A K_1^{-1} g_2 - K_1^{-1} B K_1^{-1} g_1$$

Let $\alpha, \beta, \gamma, \omega$ be x_i, $i = 1,2,3,4$, $H_{x_i} = \begin{cases} A, & 1 \\ B, & 2 \\ C, & 3 \\ D, & 4 \end{cases}$ $h_{x_i} = \begin{cases} g_1, & 1 \\ g_2, & 2 \\ g_3, & 3 \\ g_4, & 4 \end{cases}$

$$\frac{\partial^2 d}{\partial x_i \partial x_j} = K_1^{-1} H_{x_i} K_1^{-1} H_{x_j} G_1 + K_1^{-1} H_{x_j} K_1^{-1} H_{x_i} G_1 - K_1^{-1} H_{x_i} K_1^{-1} g_j - K_1^{-1} H_{x_j} K_1^{-1} g_i$$