Circuit Theory and Electronics Fundamentals 2020/2021

Integrated Masters in Aerospace Engineering, Técnico, University of Lisbon

First Laboratory Report

March 25, 2021

Rui Rodrigues, 95844
Tiago Silva, 95850
Tomás Ribeiro, 95854

Contents

1 Introduction

The objective of this laboratory assignment is to study a circuit containing a voltage source V_A, a current-controlled voltage source V_C, a current source I_D and a voltage-controlled current source I_B connected to different fixed value resistors R_1, R_2, R_3, R_4, R_5, R_6 and R_7. The circuit can be seen in Figure 1.

In Section 2, a theoretical introduction is made in order to contextualize all the main principles that sustain our analysis of the circuit. This circuit is carefully analysed according to two of the most efficient ways to solve a circuit: the Mesh Current Method and the Node Voltage Method, both presented in Section 3. In Section 4, the circuit is analysed by simulation and the results are compared to the theoretical results obtained in Section 3. The conclusions of this study are outlined in the final part of the report, in Section 5.

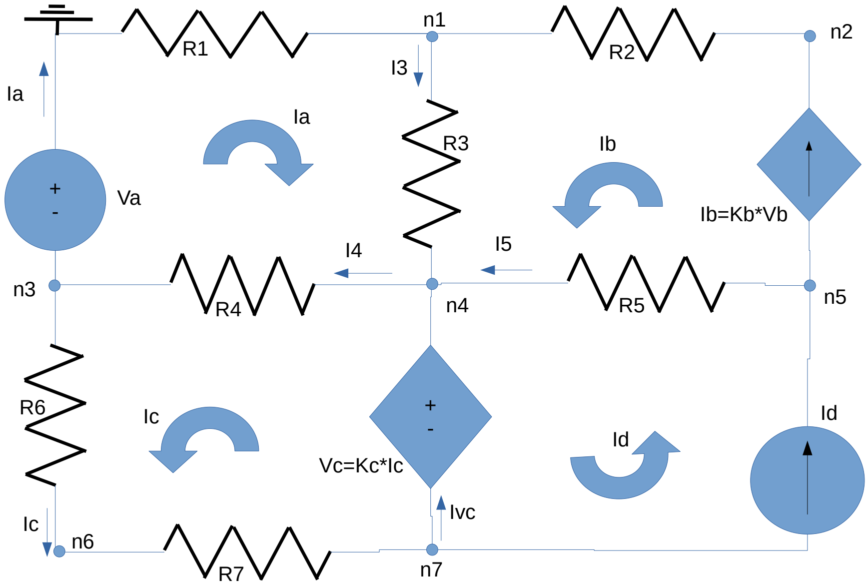

Figure 1: First laboratory circuit.

2 Theoretical Introduction

The Mesh Current Method is a well-organized method for solving a circuit and is based on Kirchhoff's Voltage Law (KVL). To apply this method, we need to define what mesh current is. When we use the term mesh current, we are referring to an imagined current flowing around a loop. To aplly this first step of this method, we first need to identify and distinguish a loop from a mesh. A loop corresponds to any closed path around the circuit and, to trace it, we start at any component terminal and trace a path through connected elements until we get back to the starting point. A loop is allowed to go through an element just one time. That leads us to the definition of a restricted kind of loop, a mesh, which contains no other loops.

The implementation of the Mesh Current Method to analyse the circuit was done following the common sequence of steps, summarized below.

- Identify the meshes.
- Assign a current variable to each mesh, using a consistent direction (clockwise or counterclockwise).
- Write Kirchhoff's Voltage Law equations around each mesh.
- Solve the resulting system of equations for all mesh currents.
- Solve for any element currents and voltages you want using Ohm's Law.

The Node Voltage Method is another way to analyze a circuit. This method is based on Kirchhoff's Current Law (KCL). To apply this method, we need to define what node voltage is. When we use the term node voltage, we are referring to the potential difference between two nodes of a circuit. We select one of the nodes in our circuit to be the reference node and, therefore, all the other node voltages are measured with respect to the referenced one. This reference node is called the ground node and, as it gets the ground symbol in Figure 1, corresponds to the node between resistor R_1 and voltage source V_A. The potential of the ground node is defined to be $0V$ and the potentials of all the other nodes are measured relative to ground.

The implementation of the Node Voltage Method to analyse the circuit was done following the common sequence of steps, summarized below.

- Assign a reference node.
- Assign node voltage names to the remaining nodes.
- Solve the easy nodes first, the ones with a voltage source connected to the reference node.
- Write Kirchhoff's Current Law for each node.
- Solve the resulting system of equations for all node voltages.
- Solve for any currents you want to know using Ohm's Law.

3 Theoretical Analysis

This circuit is carefully analysed according to two of the most efficient ways to solve a circuit: the Mesh Current Method, in Subsection 3.1, and the Node Voltage Method, in Subsection 3.2.

3.1 Mesh Current Method Analysis

By analysing the circuit, identifying the meshes and assigning currents that work as variables, we can start to write a system of equations with KVL that help us solve the circuit:

$$\begin{cases} MeshA : R_1 \times I_a + (I_a + I_b) \times R_3 + (I_a + I_c) \times R_4 = V_a \\ MeshC : (I_a + I_c) \times R_4 + R_6 \times I_c + R_7 \times I_c - K_c \times I_c = 0 \end{cases}$$

From here, we can rewrite the system into an equivalent, adding a few equations to make it a system of 7 equations with 7 variables:

$$\begin{cases} MeshA : (R_1 + R_3 + R_4) \times I_a + R_3 \times I_b + R_4 \times I_c = V_a \\ MeshC : R_4 \times I_a + (R_4 + R_6 + R_7 - K_c) \times I_c = 0 \\ I_a - I_4 + I_c = 0 \\ \frac{1}{1 - K_b \times R_3} \times I_a - I_3 = 0 \\ I_c - I_{vc} = I_d \\ I_b + I_5 = I_d \\ -I_a - I_b + I_3 = 0 \end{cases}$$

From here, we can write a matrix equation that can be solved in GNU Octave:

$$\begin{bmatrix} R_1 + R_3 + R_4 & R_3 & 0 & 0 & 0 & R_4 & 0 \\ R_4 & 0 & 0 & 0 & 0 & R_4 + R_6 + R_7 - K_c & 0 \\ 1 & 0 & 0 & -1 & 0 & 1 & 0 \\ \frac{1}{1 - K_b \times R_3} & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_3 \\ I_4 \\ I_5 \\ I_c \\ I_{vc} \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ 0 \\ 0 \\ I_d \\ I_d \\ 0 \end{bmatrix}$$

The table below presents the solution to the system of equations from the Mesh Current Method:

Mesh Analysis Currents	
Mesh Current Ia	2.35432076795e-04
Mesh Current Ib	-2.46391641626e-04
Mesh Current Ic	9.14823521600e-04
Mesh Current Id	1.03087754949e-03
Branch Current Ia	2.35432076795e-04
Branch Current Ib	-2.46391641626e-04
Branch Current I3	-1.09595648304e-05
Branch Current I4	1.15025559840e-03
Branch Current I5	1.27726919112e-03
Branch Current Ic	9.14823521600e-04
Branch Current Ivc	-1.16054027890e-04
Branch Current Id	1.03087754949e-03

3.2 Node Voltage Method Analysis

We start to apply the Node Voltage Method by starting to identify the nodes of the circuit. In this case, our circuit has 7 nodes and each one has a node voltage designated V_1, V_2, V_3, V_4, V_5, V_6 and V_7, according to the related node. Then, we apply the KCL to each one of the nodes.

$$\begin{cases} Node1: I_1 + I_2 = I_3 \\ Node2: I_B = I_2 \\ Node3: V_3 = -V_A \\ Node4: V_4 - V_7 = V_C \\ Node5: I_B + I_5 = I_D \\ Node6: I_6 = I_7 \\ Node7: V_4 - V_7 = V_C \end{cases}$$

After that, we rewrite it into an equivalent system defining the currents in terms of node voltages.

$$\begin{cases} Node1: \frac{0-V_1}{R_1} + \frac{V_2-V_1}{R_2} = \frac{V_1-V_4}{R_3} \\ Node2: I_B = \frac{V_2-V_1}{R_2} \\ Node3: V_3 = -V_A \\ Node4: V_4 - V_7 = V_C \\ Node5: I_B + \frac{V_5-V_4}{R_5} = I_D \\ Node6: \frac{V_3-V_6}{R_6} = \frac{V_6-V_7}{R_7} \\ Node7: V_4 - V_7 = V_C \end{cases}$$

Now, we have a system of 6 equations (note that applying KCL to node 4 and node 7 generate the same exact equation). Between these two nodes, the circuit presents a voltage source V_A. Therefore, to obtain another equation to add to the system, we need to consider a supernode that includes both node 4 and node 7 and, after that, apply KCL to the supernode we just created.

We obtain another equation: $I_3 + I_5 + I_C = I_4 + I_D$.

Once again, we define the currents in terms of node voltages and obtain an equivalent equation: $\frac{V_1-V_4}{R_3}+\frac{V_5-V_4}{R_5}+I_C=\frac{V_4-V_3}{R_4}+I_D$.

At this point, we have a system with 7 equations. However, besides the node voltages V_1, V_2, V_3, V_4, V_5, V_6 and V_7, we still have two more variables to determine its value, I_B and I_C. So, we need to find two more equations to complete our system. We have to define the missing value currents in terms of node voltages and get the two extra equations.

$$\begin{cases} I_B=\frac{V_1-V_4}{K_B} \\ I_C=\frac{V_3-V_6}{R_6} \end{cases}$$

This takes us to our final system of equations, with 9 equations to find the values of 9 variables.

$$\begin{cases} V_4-V_7-K_CI_C=0 \\ -V_3=V_A \\ -\frac{1}{R_5}V_4+\frac{1}{R_5}V_5+I_B=I_D \\ -\frac{1}{R_2}V_1+\frac{1}{R_2}V_2-I_B=0 \\ \frac{1}{R_6}V_3+(-\frac{1}{R_6}-\frac{1}{R_7})V_6+\frac{1}{R_7}V_7=0 \\ (-\frac{1}{R_1}-\frac{1}{R_2}-\frac{1}{R_3})V_1+\frac{1}{R_2}V_2+\frac{1}{R_3}V_4=0 \\ K_BV_1-K_BV_4-I_B=0 \\ \frac{1}{R_6}V_3-\frac{1}{R_6}V_6-I_C=0 \\ \frac{1}{R_3}V_1+\frac{1}{R_4}V_3+(-\frac{1}{R_3}-\frac{1}{R_4}-\frac{1}{R_5})V_4+\frac{1}{R_5}V_5+I_C=I_D \end{cases}$$

We can transform this system of equations and put it into a matrix form, ready to be solved in GNU Octave.

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & -K_C \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\frac{1}{R_5} & \frac{1}{R_5} & 0 & 0 & 1 & 0 \\ -\frac{1}{R_2} & \frac{1}{R_2} & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & \frac{1}{R_6} & 0 & 0 & -\frac{1}{R_6}-\frac{1}{R_7} & \frac{1}{R_7} & 0 & 0 \\ -\frac{1}{R_1}-\frac{1}{R_2}-\frac{1}{R_3} & \frac{1}{R_2} & 0 & \frac{1}{R_3} & 0 & 0 & 0 & 0 & 0 \\ K_B & 0 & 0 & -K_B & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & \frac{1}{R_6} & 0 & 0 & -\frac{1}{R_6} & 0 & 0 & -1 \\ \frac{1}{R_3} & 0 & \frac{1}{R_4} & -\frac{1}{R_3}-\frac{1}{R_4}-\frac{1}{R_5} & \frac{1}{R_5} & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ I_B \\ I_C \end{bmatrix} = \begin{bmatrix} 0 \\ V_A \\ I_D \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ I_D \end{bmatrix}$$

These are the final values obtained with the application of the Node Voltage Method:

Nodal Analysis Voltages	
Node Voltage 1	-2.45701846174e-01
Node Voltage 2	-7.41098638311e-01
Node Voltage 3	-5.02924600001e+00
Node Voltage 4	-2.11713274972e-01
Node Voltage 5	3.74034647031e+00
Node Voltage 6	-6.88395862436e+00
Node Voltage 7	-7.80086877032e+00

4 Simulation Analysis

4.1 Operating Point Analysis

Node/Component	Value [A or V]
i(vaa)	-2.35432e-04
i(vab)	9.148235e-04
i(hc)	1.160540e-04
@gb[i]	-2.46392e-04
@idd[current]	1.030878e-03
@r1[i]	2.354321e-04
@r2[i]	-2.46392e-04
@r3[i]	-1.09596e-05
@r4[i]	1.150256e-03
@r5[i]	1.277269e-03
@r6[i]	9.148235e-04
@r7[i]	9.148235e-04
n1	-2.45702e-01
n2	-7.41099e-01
n3	-5.02925e+00
n4	-2.11713e-01
n5	3.740346e+00
n6	-6.88396e+00
n7	-7.80087e+00
na	-5.02925e+00

Table 1: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volts.

Ngspice is a circuit-simulation program that makes it possible to have an accurate representation of how the circuit would behave if it was actually assembled. Table 1 shows the simulated operating point results for the circuit under analysis.

As it can be seen, the table shows the values of the voltage in all nodes, the currents in all of the branches and also the currents in independent voltage sources. When looking at the simulation results there is one very interesting detail which is very important: Ngspice operates with the idea that the positive current flows from the positive pole to the negative pole in all components, sources included. This explains why, for example, in the the very same branch where resistor R_1 and voltage source V_A are, the current given by ngspice in the whole branch ($@r1[i]$) is the symetric of the one given specifically in the voltage source V_A. The same logic applies to I_{Vc}.

Another important detail about this table is the node N_A and the voltage source $V_{AB} = 0V$, which are absent from the circuit's picture. This is because the Current-Controlled Voltage Source is dependent on current I_C, but Ngspice requires a voltage source where this current flows through, which made necessary the use of an auxiliary voltage source in series with R_6 and, therefore, an auxiliary node.

4.2 Relative Error Analysis

Error Table (in %)	
Current I_A	9.85620510589e-06
Current I_B	1.45448812660e-04
Current I_3	3.20901927999e-04
Current I_4	3.49143686654e-05
Current I_5	1.49628435100e-05
Current I_C	2.36111667431e-06
Current I_{Vc}	2.40318729941e-05
Node 1	6.26066681442e-05
Node 2	4.88043571119e-05
Node 3	7.95345230513e-05
Node 4	1.29879657939e-04
Node 5	1.25739394366e-05
Node 6	1.99831968215e-05
Node 7	1.57633694998e-05

When the simulation results given by Ngspice are compared to the theoretical ones obtained in Section 3, it is possible to highlight the fact that these are, in reality, extremely close to each other. As it can be seen, the highest error in percentual value is in the order of 10^{-4}, which is negligible. Such result can be explained by the fact that this is a very simple circuit with very simple components, thus not having a lot of chances to differ greatly.

For reasons explained in Subsection 4.1, the value considered for I_A was the one given by the resistor R_1 and the value considered for I_{Vc} was the symetric of the one given by Ngspice.

5 Conclusion

In this first laboratory assignment, all the major goals of the project were achieved. We concluded with success our first interaction with a new software (Ubuntu), with a simulation platform (Ngspice), with a computational language program (GNU Octave) and with a text report editor (LaTeX). The analysis of the circuit was also finished with success through simulation and theoretical interpretation.

The main objective of the report was completed with the study of a circuit containing a voltage source V_A, a current-controlled voltage source V_C, a current source I_D and a voltage-controlled current source I_B connected to different fixed value resistors R_1, R_2, R_3, R_4, R_5, R_6 and R_7. Mesh currents and node voltage were analised both theoretically, using the Octave maths tool, and by circuit simulation, using the Ngspice tool. The simulation results matched the theoretical results precisely. This accuracy was confirmed by the matemathical calculation of relative errors, which were proved to be really small. The reason for this perfect match is the fact that this is a straightforward circuit containing only simple and linear components, so the theoretical and simulation models cannot differ. For more complex components, the theoretical and simulation models could differ. However, this is not the case of the analysis of this report, where the results are obtained sucesfully and with notorious precision.