Circuit Theory and Electronics Fundamentals 2020/2021

Integrated Masters in Aerospace Engineering, Técnico, University of Lisbon

Third Laboratory Report

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Rui Rodrigues, 95844
Tiago Silva, 95850
Tomás Ribeiro, 95854

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1 Introduction

The objective of this laboratory assignment is to study and implement an Audio Amplifier circuit while, at the same time, trying to maximize the figure of merit M. The Audio Amplifier circuit can be separated into two stages: a Gain Stage, which can be simply described as the point in the system where the signal passes through an amplifier, and an Output stage. The role of the Output Stage is to provide power gain. It is also important to note that an Output Stage should have high input impedance and low output impedance. The circuit can be seen in Figure 1.

The implementation of the Audio Amplifier circuit was made according to the theoretical or simulation principles we were based on, using the equations presented in the class slides as the main material source to be able to do this fourth laboratory assignment.

In Section 2, a theoretical introduction is made in order to contextualize all the main principles that sustain our construction and analysis of the circuit. A theoretical Audio Amplifier circuit is built and carefully analysed in Section **??**, where the results are obtained in GNU Octave. Also, in Section 4, the Audio Amplifier circuit is analysed by simulation through the use of NGSpice to simulate the real electric circuit behaviour. The results of the simulation of Section 4 are then compared to the theoretical results obtained in Section **??** and the comparative results are expressed in Section 5. The figure of merit, calculated according to the components used to build the simulation circuit, can also be found in Section 5. The conclusions of this study are outlined in the final part of the report, in Section 6.

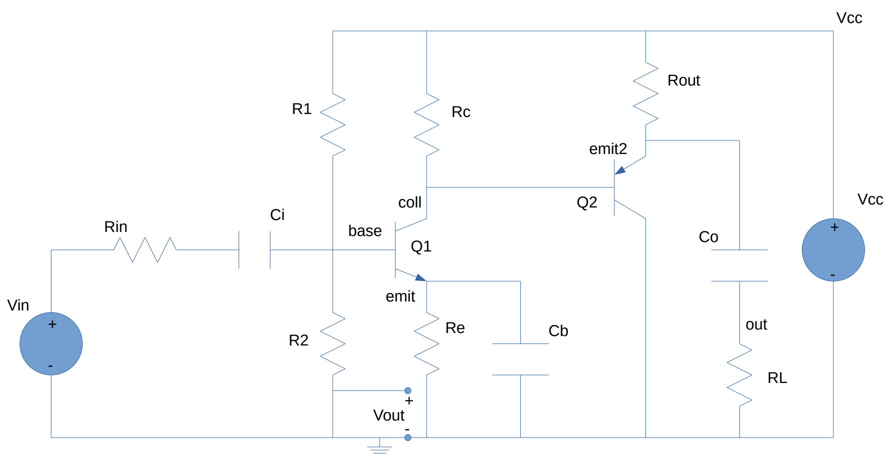

Figure 1: Fourth laboratory circuit.

2 Theoretical Introduction

An audio amplifier is an electronic amplifier designed to take input as the low strenght audio signals and generate the output signal that consists of the high strenght value. Audio amplifiers are found in all manners of sound systems including sound reinforcement, public address, home audio systems and musical instrument amplifiers. It is the final electronic stage in a typical audio playback chain before the signal is sent to the speakers.

The preceding stages in such a chain are low power audio amplifiers which perform tasks like pre-amplification of the signal (this is usually associated with record turntable signals, microphones signals and electric instrument signals, for example), equalization (adjusting the balance between frequency components within an electronic signal), tone controls, mixing different input signals or adding electronic effects such as reverb. The inputs can also consist of a several number of audio sources like record players, CD players, digital audio players or cassette players. Most of the audio power amplifiers require these low-level inputs, which are line level. Amplification of the signal produced is necessary and must be amplified before they can be processed in either analog or digital circuits.

The main goal of audio amplifiers is to reproduce input audio signals at sound-producing output levels, with desired volume and power levels - faithfully, efficiently and at low distortion. Audio frequencies ranges from about 20 Hz to 20 kHz, so the amplifier must have good frequency response over this range. Power capabilities vary widely depending on the application, from the miliwatts in headphones, to a few watts in TV or PC audio, to ten of watts for ”mini” home stereos and automotive audio, to hundred of watts and beyond for more powerful home and commercial sound systems - and to fill theaters or auditoriums with sound.

A straightforward analog implementation of an audio amplifier uses transistors in linear mode in order to create an output voltage that is a scaled copy of the input voltage. The forward voltage gain is usually high (at least 40 dB). If the forward gain is part of a feedback loop, the overall loop gain will also be high. Feedback is often used because high loop gain improves performance - supressing distortion caused by nonlinearities in the forward path and reducing power supply noise by increasing the power-supply rejection (PSR).

The Audio Amplifier circuit presented above consists of two stages: a Gain Stage that, as the name suggests, its main goal is to maximize the voltage gain; and an Output Stage, which is the stage that is connected to the speaker. The Output Stage gives further improvement to the power gain and transfers this power to the speaker with minimum loss.

3 Theoretical Analysis

This theoretical analysis has, as its main purpose, showing how this circuit would behave in theory. This section is divided in two given that this circuit, as explained in the theoretical introduction, is mainly composed of two different stages - the gain stage and the output stage. However, there will also be a third subsection which will include the frequency response analysis for the whole circuit.

3.1 The Gain Stage

As already explained in the very first section of this report, the final objective of this lab assignment is to make an amplifier. Given so, this stage will have, as its main goal, to get a voltage gain as high as possible. In this subsection, the OP analysis of circuit will be made (DC current) and the gain, plus the input and output impedances, will be calculated.

3.1.1 OP Analysis

The first subject being analyzed is how does the circuit behave when the current is continuous (DC). This part is crucial when it comes to the study of the rest of the topics, such as the impedances, given that the incremental parameters of the transistors will be calculated based on the values obtained in the OP analysis.

$$\begin{cases} R_{B1} = \\ R_{B2} = \\ R_B = \frac{R_{B1}R_{B2}}{R_{B1}+R_{B2}} \\ V_{BEON} = \approx 0.7V \\ I_E = (1+\beta_F)I_B \\ V_E = R_E I_E \\ V_C = R_C I_C \\ V_O = V_{CC} - V_C \end{cases}$$

3.1.2 Voltage Gain

In order to compute the voltage gain, the values of transistor's incremental parameters are required, given by the expressions below:

$$g_m = \frac{I_C}{V_T} \tag{1}$$

$$r_\pi = \frac{\beta_F}{g_m} \tag{2}$$

$$r_o \approx \frac{V_A}{I_C} \tag{3}$$

$$\frac{v_o}{v_i} = -g_m(R_C||r_o)\frac{r_\pi||R_{B1}||R_{B2}}{R_S + r_\pi||R_{B1}||R_{B2}}v_S \tag{4}$$

In which v_S is the voltage supplied by the source.

3.1.3 Input and Output Impedances

Yet another very important property of this stage to be studied are the impendances, both the input and output ones. The relevance of the impedance, specially the output one, is that it gives an indication about the type of components that can be connected to it. Thus, through deductions made in the theoretical class, the final expressions that give the required values are the following:

$$Z_I = R_{B1}||R_{B2}||r_\pi \tag{5}$$

$$Z_O = R_C||r_o \tag{6}$$

And so, the final results for this stage were obtained:

Gain Stage Computations	
Data	Values
Voltage Gain	-202.010493
Input Impedance	523.575168
Output Impedance	730.963158

Analyzing the table above with detail, there are two specific values that are of great interest to this report: the voltage gain and the output impedance. When coming to the voltage gain, this stage had as its major function, to obtain a very high value, to amplify the sound as much as possible. It can be said that this objective was fulfilled. Looking now at the output impedance, a value of 730.96Ω was computed. This value creates a problem: this stage has an astonishingly high output impedance value to be connected to an 8Ω speaker. Given so, there is only one way to keep both the present Gain Stage and the 8Ω speaker, which is by adding an intermediate stage which will lower the output impedance while maintaining the voltage gain. This intermediate stage is the output stage and its computations are going to be presented right next.

3.2 The Output Stage

As it can be seen from the output impedance obtained in the previous stage, it has a very high value to be connected to the 8Ω of the speaker in the end of the circuit. Therefore, the reasoning of this stage is precisely to solve that issue: to force the output impedance of the amplifier to have a reasonable value to be connected to an 8Ω speaker.

3.2.1 OP Analysis

Just like it was done for the Gain Stage, the operating point analysis for the output stage is also required. The fact that, such as the Gain Stage, this present stage also has a transistor, makes this analysis specially important given that the transistor's incremental parameters depend on the values obtained in this subsection. To do it, the following equations are going to be used.

$$
\begin{cases}
V_{CC} = 12V \\
V_{BEON} = \approx 0.7V \\
R_E = \omega \\
R_E I_E + V_{BEON} + V_I - V_C C = 0 \\
V_O = V_{CC} - R_E I_E \\
V_O = V_I + V_{BEON}
\end{cases}
$$

3.2.2 Voltage Gain

As it was previously explained, the main justification for the existence of this Output Stage is to make the output impedance compatible with the 8Ω speaker. Given so, the voltage gain of this stage is expected to be 1, because it should simply transport the voltage gain of the previous stage to the already mentioned speaker.

It will be more convenient to work with admitances, therefore:

$$g_\pi = \frac{1}{r_\pi} \tag{7}$$

$$g_E = \frac{1}{R_E} \tag{8}$$

$$g_o = \frac{1}{r_o} \tag{9}$$

By using the Kirchhoff Current Law (KCL), the expression for the voltage gain of this output can be deduced:

$$\frac{v_o}{v_i} = \frac{g_m}{g_\pi + g_E + g_o + g_m} \tag{10}$$

3.2.3 Input and Output Impedances

Of all the computations already made for this stage, the impedances are certainly the most important ones. As it was already mentioned, the output impedance of this stage is supposed to be compatible with the 8Ω speaker so, by using the expressions deducted in class, the formulas that follow next were obtained:

$$Z_I = \frac{g_\pi + g_E + g_o + g_m}{g_\pi(g_\pi + g_E + g_o)} \tag{11}$$

$$Z_O = \frac{1}{g_\pi + g_E + g_o + g_m} \tag{12}$$

This way, the final results for this stage were obatined:

Output Stage Computations	
Data	Values
Voltage Gain	0.990747
Input Impedance	78634.698490
Output Impedance	3.171596

From looking at the table the results are, overall, very good. Starting by the voltage gain, as mentioned in the theoretical explanation about this stage, it should have a value of 1, which is very well approximated by the 0.99 obtained. Besides that, when it comes to the impedances, the attention should go to the output one, given that it is the one that is going to connect to the speaker. And so, for this impedance, we get an amazing value of 3.17Ω, which is more than perfect to connect to the 8Ω speaker.

3.3 Frequency Response Analysis

Something that is of great interest to analyze an amplifier is how its voltage gain will vary accordingly with the frequency. As a consequence of that, this subsection will look forward to study the frequency response of the whole circuit, varying the frequency from an initial value of $10Hz$ to a final value of $10MHz$. It is important to add that, in this final topic of the theoretical analysis, a total number of 10 points per decade will be considered.

When varying the frequency, at some point, the voltage gain will have a constant value and thus have a graphical representation similar to a an upland. This way, there will be two frequency, commonly referred to as the cut-off frequencies, that will delimit the already mentioned upland, which will correspond to a gain approximately $3dB$ lower than the constant voltage gain. However, the graphical representation of the frequency response in this report will be somewhat different from the what was just described. In this report, the higher cut-off frequency will be considered as infinite and, when it comes to the lower cut-off frequency, even though it will be calculated, the shape of the graph line for points lower than it will not. The reason why the choice to ignore the graph of the points located between $10Hz$ and the lower cut-off frequency was made is simply because of the difficulty of its computation, added to the fact that they are of no interest to the amplifier. This will result in a graphic made of a single constant function that will vary from the lower cut-off frequency to $100MHz$.

Similarly to what was done in the previous subsections, the formula already deduced in the theoretical classes to compute the value of the constant voltage gain value mentioned in the previous paragraph is presented:

$$\frac{v_o(f)}{v_i(f)} = \frac{g_B + g_{m2}}{g_B + g_{e2} + g_{o2} + g_{m2}} \times Av_{GainStage} = = \frac{g_B + g_{m2}}{g_B + g_{e2} + g_{o2} + g_{m2}} \frac{(r_\pi||R_{B1}||R_{B2})}{(R_S + r_\pi||R_{B1}||R_{B2})} v_S(-g_m(R_C||r_o) \tag{13}$$

Just as a side note, the variables included in the previous formula, such as g_{e2}, g_{o2} and g_{m2} are relative to the output stage while the ones that do not have the "2" belong to the Gain Stage.

Yet another formula of extreme importance to this topic is the one that will calculate the value of the lower cut-off frequency:

$$f_{Lower\ Cut-Off} = \frac{1}{min[Z_{I1}C_I, Z_{O2}C_O, 1/g_{m1}C_E]2/pi} \tag{14}$$

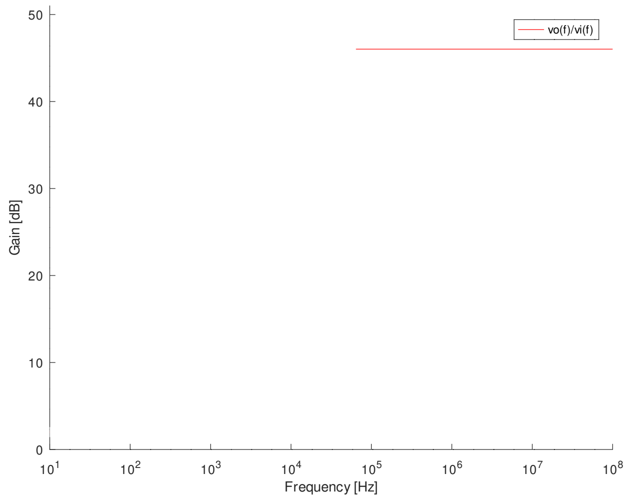

Figure 2: The voltage gain

4 Simulation Analysis

In this section, we can find the results of the topics required in the simulation analysis. The numeric results or graphics are presented alongside a short explanation of the interpretation of the problem. All of the results were obtained using NGSpice and the section is divided in three different subsections.

4.1 Output Voltage Gain

The output stage of the common collector amplifier allows the circuit to be compatible with the speaker impedance limits. It is possible to analyse the output voltage of this second part of the amplifier circuit relating it with the variation of the frequency. The NGSpice analysis results in the plot shown below.

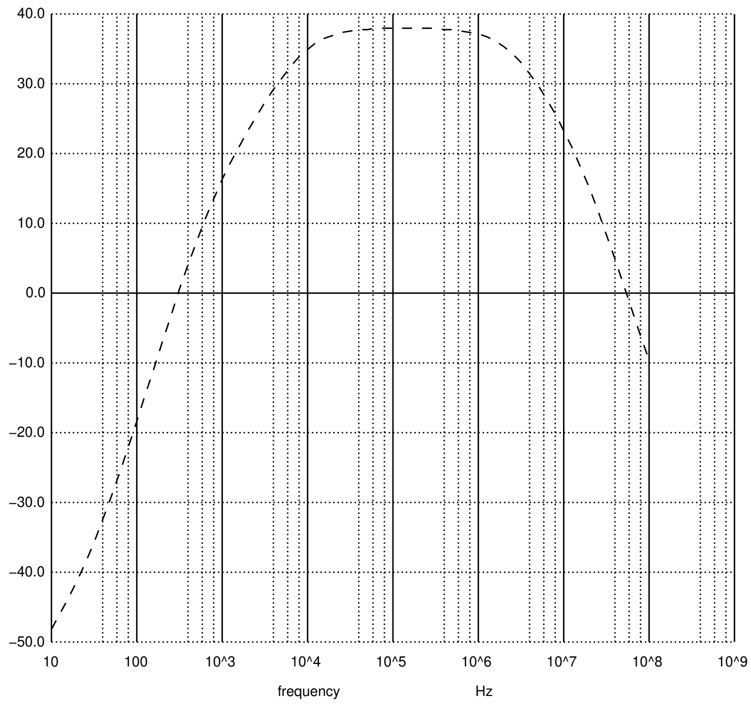

Figure 3: The Output Voltage of the Output Stage of a Common Collector Amplifier.

where f is expressed in Hertz (Hz) along the x-axis
and $v_{dB_{out}}$, the output voltage of the amplifier, is expressed in in Volts (V) using a decibel scale along y-axis.

The interpretation of the graphic allows us to see that the output voltage tends to become constant when closer to its maximum values. In fact, it is possible to compute the output voltage gain as an approximation of the maximum output voltage level.

NGSpice Formula	Output Voltage Gain (V)
vdb(out)[40]	3.793151e+01

4.2 Bandwidth and Cut-Off Frequencies

Also from the intepretation of the plot presented in the previous usbsection, we can calculate the cut-off frequencies. In this case, since we are using an electric circuit simulation tool like NGSpice, we are able to compute values for two existing cut-off frequencies. They are calculated as 3dB cut off frequencies, which means that their correspondent output voltage is the same and it is 3dB lower than the maximum output voltage level. From the lower and upper cut-off frequencies it is possible to calculate the bandwidth.

Variable	Frequency Value (Hz)
f1	1.004524e+04
f2	2.189576e+06
f2-f1	2.179531e+06

where f_1 is the lower cut-off frequency,
af_1 is the upper cut-off frequency and
$f_2 - f_1$ is the bandwith (the difference between the upper and lower cut-off frequencies).

4.3 Input and Output Impedances

The impedances of the common collector amplifier are really important values to obtain when trying to interpretate the viability of the amplifier circuit. And that's why it is really important to calculate the input impedance and the output impedance for both the gain and output stages of the amplifier. After all the values are computed, the circuit can be approved as a common colector amplifier if all the impedances are within the limits allowed by the driver (input audio source) and the load (speaker).

The following values were obtained using different NGSpice circuits built specifically to calculate each one of the impedances. Alternate voltage sources with different parameters were used in order to aplly the Ohm's law with impedances and compute the impedance for each case studied. In both output stage's impedances and also in the output impedance of the gain stage, an auxiliary capacitor was applyed to the circuits in series with the voltage source in order to cancel the DC variations of the circuit and get a straight analysis exclusively from sinusoidal components.

NGSpice Formula	Gain Stage Input Impedance (Ω): a, b from $Z_{in} = a + bj$
v(in2)[40]/(v(in)[40]-v(in2)[40])*100/1000	6.077977e-01,-1.08849e-01

NGSpice Formula	Gain Stage Output Impedance (Ω)

NGSpice Formula	Output Stage Input Impedance (Ω)

NGSpice Formula	Output Stage Output Impedance (Ω)

5 Relative Error and Graphic Analysis

Comparison of various data between Octave and Ngspice			
DATA	Octave	NGSpice	Percentual Relative Error
Output Voltage Gain	0.990747	37.93151	3728.576821
Input Impedance	78634.698490	65771.74	16.35786585
Output Impedance	3.171596	-3.17595	200.137281

In the table above, we have the comparison between Octave and NGSpice of the Output Voltage Gain, the Input Impedance and the Output Impedance. The percentual error is also included to make the interpretation easier. As we can see, the percentual relative errors presented are very high and thus cannot be ignored. A possible explanation for these discrepancies is that NGSpice uses a very complex transistor model, similarly to what happened in the previous laboratory assignment with the diode model, which does not have fixed parameters, and so it becames very hard for Octave to match the results.

5.1 Figure of Merit

Figure of Merit	
Merit Figure	85.094462

To end our report, we present the figure of merit. It is important to remind that this figure is obtained upon the devices and values used in Ngspice. During our work, we have performed several incremental modifications to improve the merit figure. It was really important for us to understand the influence of the coupling capacitors on the bandwidth, the purpose of the bypass capacitor in relation with the voltage gain and the effect of resistors and capacitors.

The variation of these parameters allowed us to achieve an improved figure of merit, while trying to maximize the voltage gain and the bandwith and to minimize the cost of the circuit' components and to achieve the lowest lower cut-off frequency as posible. The values applied both in NGSpice and Octave proved to be our best options to make the figure of merit as high as possible.

6 Conclusion

In this fourth laboratory assignment, all the major goals of the project were achieved. We concluded with success a further interaction with a new software (Ubuntu), with a simulation software (Ngspice), with a computational language program (GNU Octave) and with a text report editor (LaTeX). The construction and analysis of the circuit was also finished with success through simulation and theoretical interpretation, which allowed a good comparative analysis between the differences presented in the behaviour of a simulated and theoretical electric circuit.

Even though the simulation results didn´t match the theoretical results precisely, by the analysis of the percentual relative errors, we can say, in fact, this assignment was very helpful in the sense that we were able to build an Audio Amplifier circuit which allowed us to get a better understanding of how this type of circuits work.