Circuit Theory and Electronics Fundamentals 2020/2021

Integrated Masters in Aerospace Engineering, Técnico, University of Lisbon

First Laboratory Report

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Contents

1 Introduction

The objective of this laboratory assignment is to study an RC circuit containing a voltage source V A , a current-controlled voltage source V C , a voltage-controlled current source I B and a capacitor C connected to different fixed value resistors R 1 , R 2 , R 3 , R 4 , R 5 , R 6 and R 7 , through a sinusoidal analysis using the following equation: SEE EQUATION PROBLEM The circuit can be seen in Figure 1. In Section 2, a theoretical introduction is made in order to contextualize all the main principles that sustain our analysis of the circuit. This circuit is carefully analysed following the equation already presented that describes our RC circuit. In Section 4, the circuit is analysed by simulation and the results are compared to the theoretical results obtained in Section 3. The conclusions of this study are outlined in the final part of the report, in Section 5.

FIGURA ERRADA.

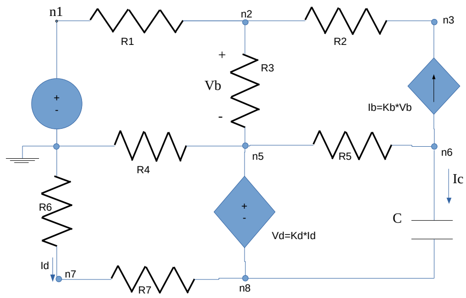

Figure 1: Second laboratory circuit.

2 Theoretical Introduction

The Node Voltage Method is another way to analyze a circuit. This method is based on Kirchhoff's Current Law (KCL). To apply this method, we need to define what node voltage is. When we use the term node voltage, we are referring to the potential difference between two nodes of a circuit. We select one of the nodes in our circuit to be the reference node and, therefore, all the other node voltages are measured with respect to the referenced one. This reference node is called the ground node and, as it gets the ground symbol in Figure 1, corresponds to the node between resistor R_1 and voltage source V_A. The potential of the ground node is defined to be $0V$ and the potentials of all the other nodes are measured relative to ground.

The implementation of the Node Voltage Method to analyse the circuit was done following the common sequence of steps, summarized below.

- Assign a reference node.
- Assign node voltage names to the remaining nodes.
- Solve the easy nodes first, the ones with a voltage source connected to the reference node.
- Write Kirchhoff's Current Law for each node.

- Solve the resulting system of equations for all node voltages.
- Solve for any currents you want to know using Ohm's Law.

A n RC circuit (also known as an RC filter or RC network) stands for a resistor-capacitor circuit. An RC circuit is defined as an electrical circuit composed of the passive circuit components of a resistor (R) and capacitor (C), driven by a voltage source or current source. Due to the presence of a resistor in the ideal form of the circuit, an RC circuit will consume energy, akin to an RL circuit or RLC circuit. This is unlike the ideal form of an LC circuit, which will consume no energy due to the absence of a resistor. Although this is only in the ideal form of the circuit, and in practice, even an LC circuit will consume some energy because of the non-zero resistance of the components and connecting wires.

Circuits manipulate electrical signals. Signals convey energy, information or both (vital functions) and these signals can be decomposed into series or integrals of basic signals. Through this, we can use the equation presented to make a transient analysis of the circuit.

3 Theoretical Analysis

INTRO HERE

3.1 Theoretical - Topic I

We start to apply the Node Voltage Method by starting to identify the nodes of the circuit. In this case, our circuit has 8 nodes and each one has a node voltage designated V_1, V_2, V_3, V_4, V_5, V_6, V_7 and V_8, according to the related node. Then, we apply the KCL to each one of the nodes.

$$\begin{cases} Node1: V_1 = V_S \\ Node2: I_1 + I_2 = I_3 \\ Node3: I_B = I_2 \\ Node4: V_4 = 0 \\ Node5: V_5 - V_8 = V_D \\ Node6: I_C + I_B + I_5 = 0 \\ Node7: I_D = I_7 \\ Node8: V_5 - V_8 = V_D \end{cases}$$

After that, we rewrite it into an equivalent system defining the currents in terms of node voltages.

$$\begin{cases} Node1: V_1 = V_S \\ Node2: \frac{V_1 - V_2}{R_1} + \frac{V_3 - V_2}{R_2} = \frac{V_2 - V_5}{R_3} \\ Node3: I_B = \frac{V_3 - V_2}{R_2} \\ Node4: V_4 = 0 \\ Node5: V_5 - V_8 = K_D * I_D \\ Node6: I_C + I_B + \frac{V_6 - V_5}{R_5} = 0 \\ Node7: I_D = \frac{V_7 - V_8}{R_7} \\ Node8: V_5 - V_8 = V_D \end{cases}$$

Now, we have a system of 6 equations (note that applying KCL to node 5 and node 8 generate the same exact equation). Between these two nodes, the circuit presents a voltage source V_D. Therefore, to obtain another equation to add to the system, we need to consider a supernode that includes both node 5 and node 8 and, after that, apply KCL to the supernode we just created.

We obtain another equation: $I_3 + I_5 + I_C + I_D = I_4$.

Once again, we define the currents in terms of node voltages and obtain an equivalent equation: $\frac{V_2 - V_5}{R_3} + \frac{V_6 - V_5}{R_5} + I_C + I_D = \frac{V_5}{R_4}$.

At this point, we have a system with 7 equations. However, besides the node voltages V_1, V_2, V_3, V_5, V_6, V_7 and V_8, we still have two more variables to determine its value, I_B and I_D (note that $I_C = 0V$ when $t < 0$). So, we need to find two more equations to complete our system. We have to define the missing value currents in terms of node voltages and get the two extra equations.

$$\begin{cases} I_B = (V_2 - V_5) * K_B \\ I_D = \frac{0 - V_7}{R_6} \end{cases}$$

This takes us to our final system of equations, with 10 equations to find the values of 10 variables.

$$\begin{cases} V_1 = V_S \\ \frac{1}{R_1}V_1 + (-\frac{1}{R_1} - \frac{1}{R_2} - \frac{1}{R_3})V_2 + \frac{1}{R_2}V_3 + \frac{1}{R_3}V_5 = 0 I_B - \frac{1}{R_2}V_3 + \frac{1}{R_2}V_2 = 0 \\ V_5 - V_8 - K_D * I_D = 0 \\ I_B + \frac{1}{R_5}V_6 - \frac{1}{R_5}V_5 = 0 \\ I_D - \frac{1}{R_7}V_7 + \frac{1}{R_7}V_8 = 0 \\ I_D + \frac{V_2 - V_5}{R_3} + \frac{V_6 - V_5}{R_5} - \frac{V_5}{R_4} = 0 \\ I_D + \frac{1}{R_3}V_2 + (-\frac{1}{R_3} - \frac{1}{R_4} - \frac{1}{R_5})V_5 + \frac{1}{R_5}V_6 = 0 \\ I_B - -K_B * V_2 + K_B * V_5 = 0 \\ I_D + \frac{V_7}{R_6} = 0 \end{cases}$$

We can transform this system of equations and put it into a matrix form, ready to be solved in GNU Octave.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{R_1} & -\frac{1}{R_1} - \frac{1}{R_2} - \frac{1}{R_3} & \frac{1}{R_2} & \frac{1}{R_3} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{R_2} & -\frac{1}{R_2} & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & -K_D \\ 0 & 0 & 0 & -\frac{1}{R_5} & \frac{1}{R_5} & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{R_7} & -\frac{1}{R_7} & 0 & 0 & -1 \\ 0 & -K_B & 0 & K_B & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{R_3} & 0 & -\frac{1}{R_3} - \frac{1}{R_4} - \frac{1}{R_5} & \frac{1}{R_5} & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{R_6} & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ I_B \\ I_C \\ I_D \end{bmatrix} = \begin{bmatrix} V_S \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

These are the final values obtained with the application of the Node Voltage Method:

Nodal Analysis Voltages [in Volts]	
Node Voltage 1	5.02924600001e+00
Node Voltage 2	4.78354415384e+00
Node Voltage 3	4.28814736170e+00
Node Voltage 5	4.81753272504e+00
Node Voltage 6	5.57990489781e+00
Node Voltage 7	-1.85471262435e+00
Node Voltage 8	-2.77162277031e+00

3.2 Theoretical - Topic II

$$\begin{cases} V_s = 0V \\ V_x = V(6) - V(8) \end{cases}$$

Nodal Analysis Voltages [in Volts]	
Node Voltage 1	0.00000000000e+00
Node Voltage 2	-4.32466740589e-16
Node Voltage 3	-1.30442858919e-15
Node Voltage 5	-3.72642498997e-16
Node Voltage 6	8.35152766812e+00
Node Voltage 7	3.35827209436e-16
Node Voltage 8	4.33186198764e-16
Req, Equivalent Resistor	3094.147869
Time Constant	0.003190

where $R_{eq} = \frac{V_x}{I_x}$ is expressed in Ohms (Ω).
EXPLAIN WHY WE NEED TO DO THAT! Ix se calhar adicionamos uma coluna na tabela?

3.3 Theoretical - Topic III

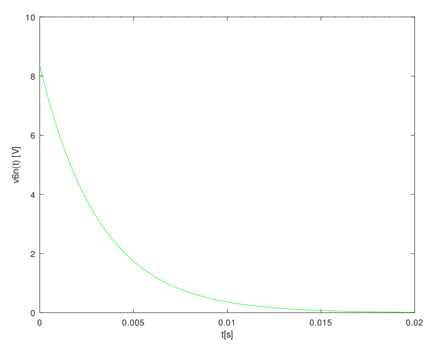

Figure 2: The natural solution, $V6_n(t)$, during time interval [0, 20] ms.

where t is expressed in seconds (s) along the x-axis and
$V6_n(t)$, the natural solution, is expressed in Volts (V) along the y-axis.

3.4 Theoretical - Topic IV

$$\begin{cases} V_s = 1 \\ f = 1kHz = 1000Hz \\ t = 20ms = 0.020s \\ C --> Z_C = \frac{1}{\omega C} \end{cases}$$

$v_s = sin(\omega t) = cos(\frac{\pi}{2} - \omega t) = e^{i(\frac{\pi}{2} - \omega t)}$

SEE HERE WHAT'S MISSING Nodal Analysis Voltages [in Volts]	
Phasor of Node 1	6.12323399574e-17+i1.57079632679e+00
Phasor of Node 2	5.72250611431e-17+i1.57079632679e+00
Phasor of Node 3	4.91453779795e-17+i1.57079632679e+00
Phasor of Node 5	5.77793983751e-17+i1.57079632679e+00
Phasor of Node 6	8.26523333807e-02+i-1.42082340747e+00
Phasor of Node 7	-2.34568735257e-17+i-1.57079632679e+00
Phasor of Node 8	-3.46204860767e-17+i-1.57079632679e+00

3.5 Theoretical - Topic V

$$\{ f = 1kHz = 1000Hz$$

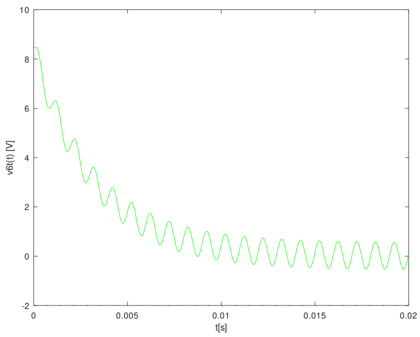

Figure 3: The final total solution, $V6(t)$, and $v_s(t)$ during time interval [-5, 20] ms.

where t is expressed in seconds (s) along the x-axis and
$V6(t)$, the total solution, and v_s are expressed in Volts (V) along the y-axis.

3.6 Theoretical - Topic VI

$$\begin{cases} f = \in [0.1, 1]MHz \\ v_c(f) = v_6(f) - v_8(f) \end{cases}$$

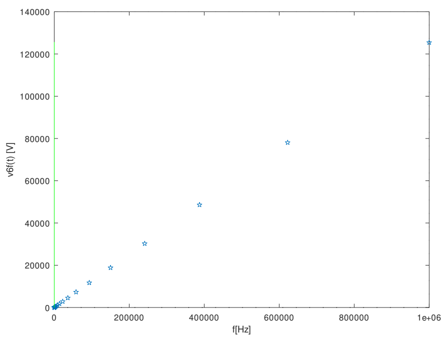

Figure 4: Magnitude of $v_s(f)$, $v_c(f)$ and $v_6(f)$ during frequency interval [0.1 , 1] MHz.

where f is expressed in Hertz (Hz) along the x-axis and the magnitude of $v_s(f)$, $v_c(f)$ and $v_6(f)$ is expressed with a logscale decibel (dB) along the y-axis.

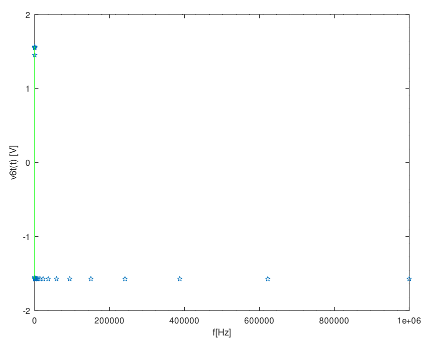

Figure 5: Phase $v_s(f)$, $v_c(f)$ and $v_6(f)$ during frequency interval [0.1 , 1] MHz.

where f is expressed in Hertz (Hz) along the x-axis and the phase of $v_s(f)$, $v_c(f)$ and $v_6(f)$ is expressed in degrees along the y-axis.

4 Simulation Analysis

4.1 Simulation - Topic I

Node/Component	Value [A or V]
@c1[i]	0.000000e+00
@gb[i]	-2.46392e-04
@r1[i]	2.354321e-04
@r2[i]	-2.46392e-04
@r3[i]	-1.09596e-05
@r4[i]	1.150256e-03
@r5[i]	2.463916e-04
@r6[i]	9.148235e-04
@r7[i]	9.148235e-04
n1	5.029246e+00
n2	4.783544e+00
n3	4.288147e+00
n5	4.817533e+00
n6	5.579905e+00
n7	-1.85471e+00
n8	-2.77162e+00
na	0.000000e+00

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

4.2 Simulation - Topic II

Node/Component	Value [A or V]
@gb[i]	0.000000e+00
@r1[i]	0.000000e+00
@r2[i]	0.000000e+00
@r3[i]	0.000000e+00
@r4[i]	0.000000e+00
@r5[i]	2.699137e-03
@r6[i]	0.000000e+00
@r7[i]	0.000000e+00
n2	0.000000e+00
n3	0.000000e+00
n5	0.000000e+00
n6	8.351528e+00
n7	0.000000e+00
n8	0.000000e+00
na	0.000000e+00

XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX

4.3 Simulation - Topic III

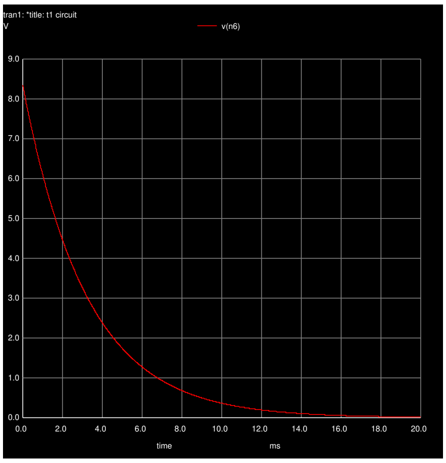

Figure 6: LALALAALALALALALALAL.

4.4 Simulation - Topic IV

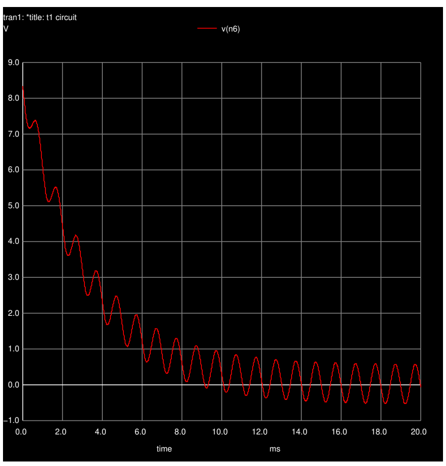

Figure 7: LALALAALALALALALALAL.

4.5 Simulation - Topic V

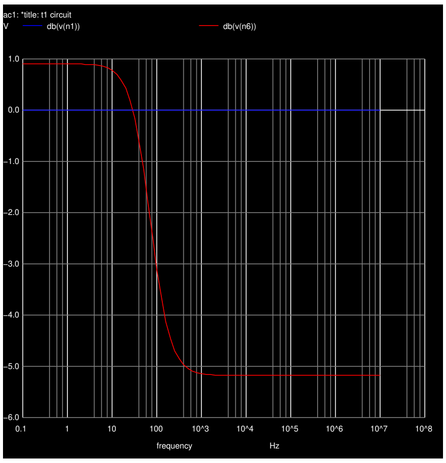

Figure 8: LALALAALALALALALALAL.

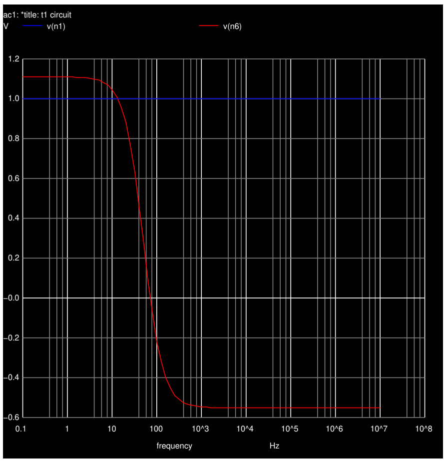

Figure 9: LALALAALALALALALALAL.

5 Relative Error Analysis

5.1 Topic I

ANALISE DO ERRO COM 4 COLUNAS			
Node 1	5.029246e+00	5.02924600001e+00	1.98836979289e-10
Node 2	4.783544e+00	4.78354415384e+00	3.21593851440e-06
Node 3	4.288147e+00	4.28814736170e+00	8.43484611997e-06
Node 4	4.817533e+00	4.81753272504e+00	5.70752959068e-06
Node 5	5.579905e+00	5.57990489781e+00	1.83143454860e-06
Node 6	-1.85471e+00	-1.85471262435e+00	1.41496768946e-04
Node 7	-2.77162e+00	-2.77162277031e+00	9.99527366038e-05

6 Conclusion

In this first laboratory assignment, all the major goals of the project were achieved. We concluded with success our first interaction with a new software (Ubuntu), with a simulation platform (Ngspice), with a computational language program (GNU Octave) and with a text report editor (LaTeX). The analysis of the circuit was also finished with success through simulation and theoretical interpretation.

The main objective of the report was completed with the study of a circuit containing a voltage source V_A, a current-controlled voltage source V_C, a current source I_D and a voltage-controlled current source I_B connected to different fixed value resistors R_1, R_2, R_3, R_4, R_5, R_6 and R_7. Mesh currents and node voltage were analised both theoretically, using the Octave maths tool, and by circuit simulation, using the Ngspice tool. The simulation results matched the theoretical results precisely. This accuracy was confirmed by the matemathical calculation of relative errors, which were proved to be really small. The reason for this perfect match is the fact that this is a straightforward circuit containing only simple and linear components, so the theoretical and simulation models cannot differ. For more complex components, the theoretical and simulation models could differ. However, this is not the case of the analysis of this report, where the results are obtained sucesfully and with notorious precision.