TÉCNICO LISBOA

Circuit Theory and Electronics Fundamentals 2020/2021

Integrated Masters in Aerospace Engineering, Técnico, University of Lisbon

Third Laboratory Report

May 5th, 2021

Rui Rodrigues, 95844
Tiago Silva, 95850
Tomás Ribeiro, 95854

Contents

1 Introduction

The very fundamental concept that characterizes an amplifier is its ability to convert a low strength input signal into one of higher strength. In our daily lives, there are multiple applications of this type of devices. What perhaps is the most common form we can find it is in audio components, more specifically, in speakers, which the sound enters with a lower level and exits with a higher one.

Besides the already mentioned use of amplifiers in speakers, the truth is that there are many other interesting possible applications. Among them, we have multiple examples such as alarms, sensors and protections systems.

In Section 2, a theoretical introduction is made in order to contextualize all the main principles that sustain our construction and analysis of the circuit. A theoretical Audio Amplifier circuit is built and carefully analysed in Section 3, where the results are obtained in GNU Octave. Also, in Section 4, the Audio Amplifier circuit is analysed by simulation through the use of NGSpice to simulate the real electric circuit behaviour. The results of the simulation of Section 4 are then compared to the theoretical results obtained in Section 3 and the comparative results are expressed in Section 5. The figure of merit, calculated according to the components used to build the simulation circuit, can also be found in Section 5. The conclusions of this study are outlined in the final part of the report, in Section 6.

2 Theoretical Introduction

When it comes to this amplifier, it is exactly of the same type of the first one mentioned in the introduction, it is the circuit of a speaker. As such, the objective of this lab assignment is to build a circuit that has an input source of $10mV$ and an output speaker of 8Ω.

In very broad terms, this circuit is going to be divided in two: a gain stage that will amplify the input sound signal as much as possible and an output stage, which will make the amplifier's output impedance compatible with impedance of the speaker.

3 Theoretical Analysis

This theoretical analysis has, as its main purpose, showing how this circuit would behave in theory. This section is divided in two given that this circuit, as explained in the theoretical introduction, is mainly composed of two different stages - the gain stage and the output stage. However, there will also be a third subsection which will include the frequency response analysis for the whole circuit.

3.1 The Gain Stage

As already explained in the very first section of this report, the final objective of this lab assignment is to make an amplifier. Given so, this stage will have, as its main goal, to get a voltage gain as high as possible. In this subsection, the OP analysis of circuit will be made (DC current) and the gain, plus the input and output impedances, will be calculated.

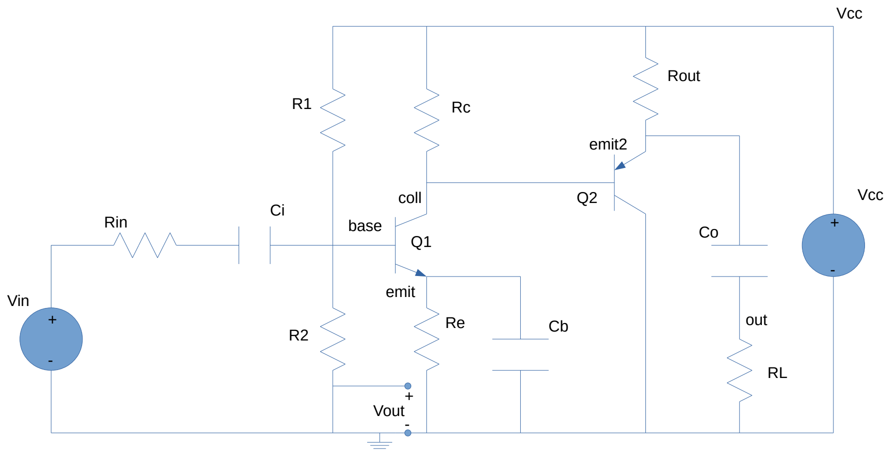


Figure 1: Fourth laboratory circuit.

3.1.1 OP Analysis

The first subject being analyzed is how does the circuit behave when the current is continuous (DC). This part is crucial when it comes to the study of the rest of the topics, such as the impedances, given that the incremental parameters of the transistors will be calculated based on the values obtained in the OP analysis.

$$\begin{cases} R_{B1} = \\ R_{B2} = \\ R_B = \frac{R_{B1}R_{B2}}{R_{B1}+R_{B2}} \\ V_{BEON} = \approx 0.7V \\ I_E = (1+\beta_F)I_B \\ V_E = R_E I_E \\ V_C = R_C I_C \\ V_O = V_{CC} - V_C \end{cases}$$

3.1.2 Voltage Gain

In order to compute the voltage gain, the values of transistor's incremental parameters are required, given by the expressions below:

$$g_m = \frac{I_C}{V_T} \tag{1}$$

$$r_\pi = \frac{\beta_F}{g_m} \tag{2}$$

$$r_o \approx \frac{V_A}{I_C} \tag{3}$$

$$\frac{v_o}{v_i} = -g_m(R_C||r_o)\frac{r_\pi||R_{B1}||R_{B2}}{R_S + r_\pi||R_{B1}||R_{B2}}v_S \quad (4)$$

In which v_S is the voltage supplied by the source.

3.1.3 Input and Output Impedances

Yet another very important property of this stage to be studied are the impendances, both the input and output ones. The relevance of the impedance, specially the output one, is that it gives an indication about the type of components that can be connected to it. Thus, through deductions made in the theoretical class, the final expressions that give the required values are the following:

$$Z_I = R_{B1}||R_{B2}||r_\pi \quad (5)$$

$$Z_O = R_C||r_o \quad (6)$$

And so, the final results for this stage were obtained:

Gain Stage Computations	
Data	Values
Voltage Gain	-202.010493
Input Impedance	523.575168
Output Impedance	730.963158

Analyzing the table above with detail, there are two specific values that are of great interest to this report: the voltage gain and the output impedance. When coming to the voltage gain, this stage had as its major function, to obtain a very high value, to amplify the sound as much as possible. It can be said that this objective was fulfilled. Looking now at the output impedance, a value of 730.96Ω was computed. This value creates a problem: this stage has an astonishingly high output impedance value to be connected to an 8Ω speaker. Given so, there is only one way to keep both the present Gain Stage and the 8Ω speaker, which is by adding an intermediate stage which will lower the output impedance while maintaining the voltage gain. This intermediate stage is the output stage and its computations are going to be presented right next.

3.2 The Output Stage

As it can be seen from the output impedance obtained in the previous stage, it has a very high value to be connected to the 8Ω of the speaker in the end of the circuit. Therefore, the reasoning of this stage is precisely to solve that issue: to force the output impedance of the amplifier to have a reasonable value to be connected to an 8Ω speaker.

3.2.1 OP Analysis

Just like it was done for the Gain Stage, the operating point analysis for the output stage is also required. The fact that, such as the Gain Stage, this present stage also has a transistor, makes this analysis specially important given that the transistor's incremental parameters depend on the values obtained in this subsection. To do it, the following equations are going to be used.

$$\begin{cases} V_{CC} = 12V \\ V_{BEON} = \approx 0.7V \\ R_E = \omega \\ R_E I_E + V_{BEON} + V_I - V_C C = 0 \\ V_O = V_{CC} - R_E I_E \\ V_O = V_I + V_{BEON} \end{cases}$$

3.2.2 Voltage Gain

As it was previously explained, the main justification for the existence of this Output Stage is to make the output impedance compatible with the 8Ω speaker. Given so, the voltage gain of this stage is expected to be 1, because it should simply transport the voltage gain of the previous stage to the already mentioned speaker.

It will be more convenient to work with admitances, therefore:

$$g_\pi = \frac{1}{r_\pi} \tag{7}$$

$$g_E = \frac{1}{R_E} \tag{8}$$

$$g_o = \frac{1}{r_o} \tag{9}$$

By using the Kirchhoff Current Law (KCL), the expression for the voltage gain of this output can be deduced:

$$\frac{v_o}{v_i} = \frac{g_m}{g_\pi + g_E + g_o + g_m} \tag{10}$$

3.2.3 Input and Output Impedances

Of all the computations already made for this stage, the impedances are certainly the most important ones. As it was already mentioned, the output impedance of this stage is supposed to be compatible with the 8Ω speaker so, by using the expressions deducted in class, the formulas that follow next were obtained:

$$Z_I = \frac{g_\pi + g_E + g_o + g_m}{g_\pi(g_\pi + g_E + g_o)} \tag{11}$$

$$Z_O = \frac{1}{g_\pi + g_E + g_o + g_m} \tag{12}$$

This way, the final results for this stage were obatined:

Output Stage Computations	
Data	Values
Voltage Gain	0.990747
Input Impedance	78634.698490
Output Impedance	3.171596

From looking at the table the results are, overall, very good. Starting by the voltage gain, as mentioned in the theoretical explanation about this stage, it should have a value of 1, which is very well approximated by the 0.99 obtained. Besides that, when it comes to the impedances, the attention should go to the output one, given that it is the one that is going to connect to the speaker. And so, for this impedance, we get an amazing value of 3.17Ω, which is more than perfect to connect to the 8Ω speaker.

3.3 Frequency Response Analysis

Something that is of great interest to analyze an amplifier is how its voltage gain will vary accordingly with the frequency. As a consequence of that, this subsection will look forward to study the frequency response of the whole circuit, varying the frequency from an initial value of $10Hz$ to a final value of $10MHz$. It is important to add that, in this final topic of the theoretical analysis, a total number of 10 points per decade will be considered.

When varying the frequency, at some point, the voltage gain will have a constant value and thus have a graphical representation similar to a an upland. This way, there will be two frequency, commonly referred to as the cut-off frequencies, that will delimit the already mentioned upland, which will correspond to a gain approximately $3dB$ lower than the constant voltage gain. However, the graphical representation of the frequency response in this report will be somewhat different from the what was just described. In this report, the higher cut-off frequency will be considered as infinite and, when it comes to the lower cut-off frequency, even though it will be calculated, the shape of the graph line for points lower than it will not. The reason why the choice to ignore the graph of the points located between $10Hz$ and the lower cut-off frequency was made is simply because of the difficulty of its computation, added to the fact that they are of no interest to the amplifier. This will result in a graphic made of a single constant function that will vary from the lower cut-off frequency to $100MHz$.

Similarly to what was done in the previous subsections, the formula already deduced in the theoretical classes to compute the value of the constant voltage gain value mentioned in the previous paragraph is presented:

$$\frac{v_o(f)}{v_i(f)} = \frac{g_B + g_{m2}}{g_B + g_{e2} + g_{o2} + g_{m2}} \times Av_{GainStage} = = \frac{g_B + g_{m2}}{g_B + g_{e2} + g_{o2} + g_{m2}} \frac{(r_\pi || R_{B1} || R_{B2})}{(R_S + r_\pi || R_{B1} || R_{B2})} v_S(-g_m(R_C || r_o) \tag{13}$$

Just as a side note, the variables included in the previous formula, such as g_{e2}, g_{o2} and g_{m2} are relative to the output stage while the ones that do not have the ”2” belong to the Gain Stage.

Yet another formula of extreme importance to this topic is the one that will calculate the value of the lower cut-off frequency:

$$f_{Lower\ Cut-Off} = \frac{1}{min[Z_{I1}C_I, Z_{O2}C_O, 1/g_{m1}C_E]2/pi} \tag{14}$$

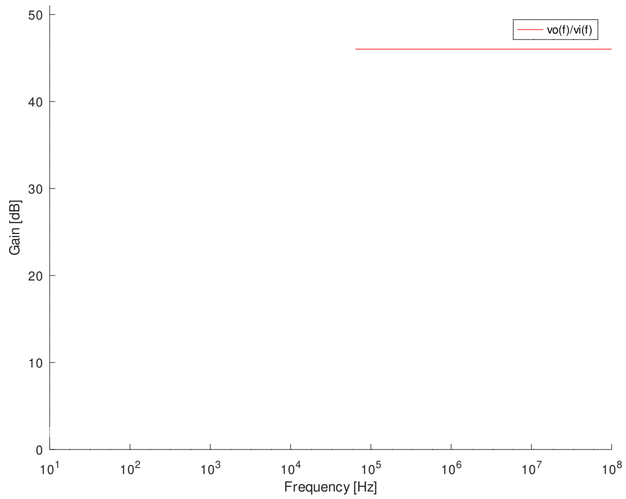


Figure 2: The voltage gain

Data	Values
Total Voltage Gain	-200.038143
Total Voltage Gain (in dB)	46.022256

4 Simulation Analysis

In this section, we can find the results of the topics required in the simulation analysis. The numeric results or graphics are presented alongside a short explanation of the interpretation of the problem. All of the results were obtained using NGSpice and the section is divided in three different subsections.

4.1 Output Voltage Gain

The output stage of the common collector amplifier allows the circuit to be compatible with the speaker impedance limits. It is possible to analyse the output voltage of this second part of the amplifier circuit relating it with the variation of the frequency. The NGSpice analysis results in the plot shown below.

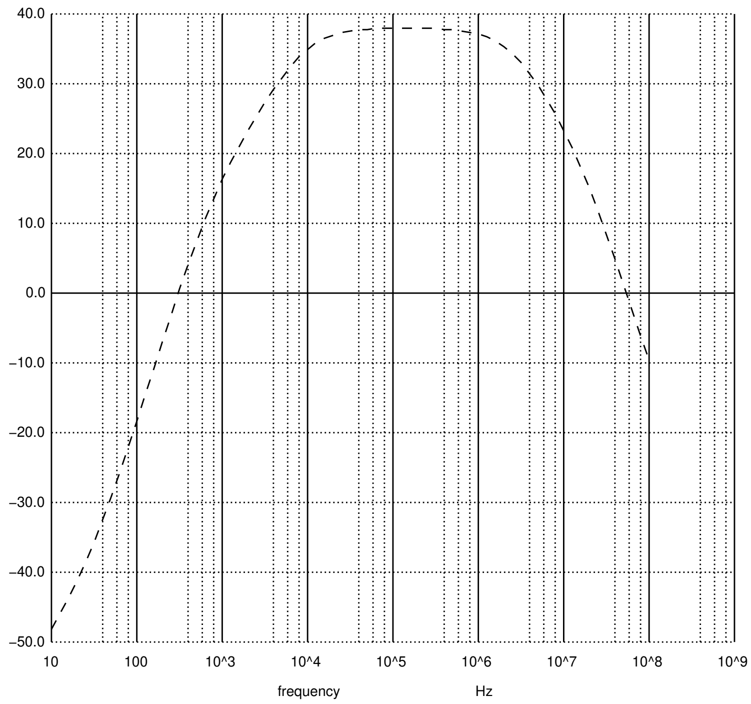


Figure 3: The Output Voltage of the Output Stage of a Common Collector Amplifier.

where f is expressed in Hertz (Hz) along the x-axis
and $v_{dB_{out}}$, the output voltage of the amplifier, is expressed in in Volts (V) using a decibel scale along y-axis.

The interpretation of the graphic allows us to see that the output voltage tends to become constant when closer to its maximum values. In fact, it is possible to compute the output voltage gain as an approximation of the maximum output voltage level.

NGSpice Formula	Output Voltage Gain (V)
vdb(out)[40]	3.793151e+01

4.2 Bandwidth and Cut-Off Frequencies

Also from the intepretation of the plot presented in the previous usbsection, we can calculate the cut-off frequencies. In this case, since we are using an electric circuit simulation tool like NGSpice, we are able to compute values for two existing cut-off frequencies. They are calculated as 3dB cut off frequencies, which means that their correspondent output voltage is the same and it is 3dB lower than the maximum output voltage level. From the lower and upper cut-off frequencies it is possible to calculate the bandwidth.

Variable	Frequency Value (Hz)
f1	1.004524e+04
f2	2.189576e+06
f2-f1	2.179531e+06

where f_1 is the lower cut-off frequency,
af_1 is the upper cut-off frequency and
$f_2 - f_1$ is the bandwith (the difference between the upper and lower cut-off frequencies).

4.3 Input and Output Impedances

The impedances of the common collector amplifier are really important values to obtain when trying to interpretate the viability of the amplifier circuit. And that's why it is really important to calculate the input impedance and the output impedance for both the gain and output stages of the amplifier. After all the values are computed, the circuit can be approved as a common colector amplifier if all the impedances are within the limits allowed by the driver (input audio source) and the load (speaker).

The following values were obtained using different NGSpice circuits built specifically to calculate each one of the impedances. Alternate voltage sources with different parameters were used in order to aplly the Ohm's law with impedances and compute the impedance for each case studied. In both output stage's impedances and also in the output impedance of the gain stage, an auxiliary capacitor was applyed to the circuits in series with the voltage source in order to cancel the DC variations of the circuit and get a straight analysis exclusively from sinusoidal components.

NGSpice Formula	Gain Stage Input Impedance (Ω): a, b from $Z_{in} = a + bj$
-v(in)[40]/i(vaux)[40]	5.745327e+02

NGSpice Formula	Gain Stage Output Impedance (Ω)
-v(coll)[40]/i(vaux)[40]	7.999195e+02

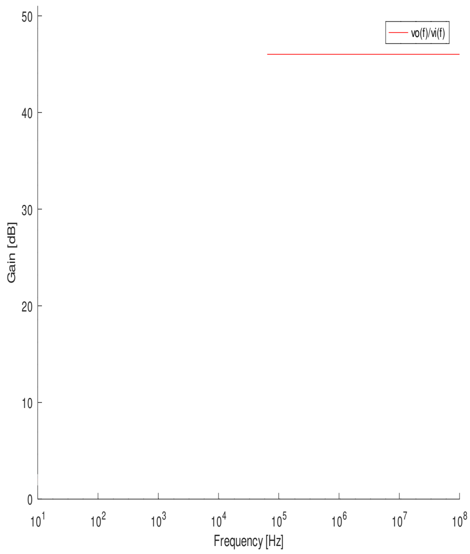
NGSpice Formula	Output Stage Input Impedance (Ω)
v(aux)[40]/i(vaux)[40]	6.577174e+04

NGSpice Formula	Output Stage Output Impedance (Ω)
-v(aux)[40]/i(vin)[40]	3.175952e+00

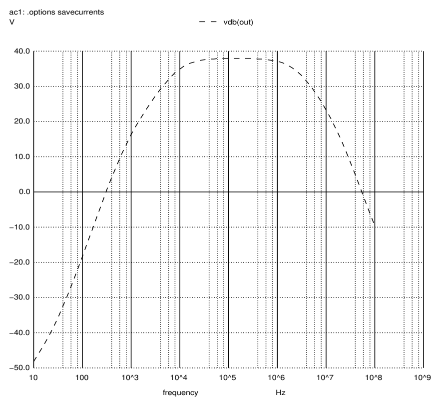
5 Relative Error and Graphic Analysis

5.1 Frequency Response Analysis

A frequency response analysis was performed using both NGSpice for simulation interpretation and GNU Octave for theorethical analysis. The graphics ploted by each one of the tools are not as similar as they would be without the several approximations made during our analysis. The causes for the absence of values for a range of frequencies in the GNU Octave graphic are already explained in the respective section. Therefore, it is important to note that the values for the maximum output gain voltage levels are really similar between the theorethical and simulation analysis. Besides that, also the lower cut-off frequencies are really close between the two analysis, making both graphics close to each other when looking to the right side of the x-axis, verifying to a good approximation to high frequency values.



(a) The Output Voltage of the Output Stage of a Common Collector Amplifier, obtained using GNU Octave.



(b) The Output Voltage of the Output Stage of a Common Collector Amplifier, obtained using Ngspice.

5.2 Input and Output Impedances Analysis

		NGSpice Value	Octave Value	Relative Error	Percentual Error [%]
Gain Stage	Input Impedance	574.5327	523.575168	0.08869387591	8.869387591
	Output Impedance	799.9195	730.963158	0.08620410179	8.620410179
Output Stage	Input Impedance	65771.74	78634.69849	0.1955696852	19.55696852
	Output Impedance	3.17595	3.171596	0.001370928384	0.1370928384
	Voltage Gain	37.93	46.02225	0.2133469549	21.33469549

In the table above, we have the comparison between Octave and NGSpice values for the of the Output Voltage Gain and all the input and output impedances of the two stages of the common collector amplifier. Both the relative error and percentual error are included to make the interpretation easier.

As we can see, some of the percentual errors are very high and thus cannot be ignored. In fact, some of the parameters defined for NGSpice circuit's components can influence in some way the impedances results being measured by those circuits. Besides that, all the formulas involved in the input and output impedances calculation process were really complex and could have minor approximations that have a final result of major differnece between the theorethical values and the simulation measured ones. A possible general explanation for these discrepancies is also that NGSpice uses a very complex transistor model, similarly to what happened in the previous laboratory assignment with the diode model, which does not have fixed parameters, and so it becames very hard for Octave to match the results.

5.3 Figure of Merit

To end our report, we present the figure of merit. It is important to remind that this figure is obtained upon the devices and values used in Ngspice. During our work, we have performed several incremental modifications to improve the merit figure. It was really important for us to understand the influence of the coupling capacitors on the bandwidth, the purpose of the bypass capacitor in relation with the voltage gain and the effect of resistors and capacitors.

The variation of these parameters allowed us to achieve an improved figure of merit, while trying to maximize the voltage gain and the bandwith and to minimize the cost of the circuit' components and to achieve the lowest lower cut-off frequency as posible. The values applied both in NGSpice and Octave proved to be our best options to make the figure of merit as high as possible.

6 Conclusion

In this fourth laboratory assignment, all the major goals of the project were achieved. We concluded with success a further interaction with a new software (Ubuntu), with a simulation software (Ngspice), with a computational language program (GNU Octave) and with a text report editor (LaTeX). The construction and analysis of the circuit was also finished with success through simulation and theoretical interpretation, which allowed a good comparative analysis between the differences presented in the behaviour of a simulated and theoretical electric circuit.

In this laboratory assignment, we were asked to build and simulate an Audio Amplifier circuit. By creating and simulating this circuit, composed of two stages: a Gain Stage and an Output Stage, we can safely say it was succesfuly implemented, based on the analysis of the percentual relative errors presented on Section **??**. As we can see, the simulation results matched the theoretical results with little deviation. Besides, the comparative analysis of the graphics ploted by NGSpice and Octave and presented side by side proved the similitude of the results obtained throughout this assignment.

Ultimately, we can finish this report by acknowledging the fact that this laboratory assignemnt proved to be really sucessful and very helpful in the sense that we were able to build an Audio Amplifier circuit which allowed us to get a better understanding of how this type of circuits work.