PHY224 Fitting Exercise 3 (Histograms): Carbon Dioxide Abundance And Surface Temperature For The Last 1000 Years

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Also note: Any constructive feedback or recommendation about formatting or otherwise for future improvement(s), is greatly appreciated.

In this exercise, we are plotting histograms and performing a log model fitting of the data provided by the NOAA. This data included 1000 years of temperature changes and their corresponding CO_2 levels.

1 The Log Model

Using the model provided in the PDF, we adopted the format: $\Delta T = alog(C) + b$. Also from the PDF, since after the industrial revolution the change in temperature can not keep up with the increase in CO_2 level, therefore we will be performing the curve fitting using data from before 1700. Our findings were:

$$A = 8.6 \pm 1.1$$

$$B = -48.7 \pm 5.9$$

Therefore, our model is:

$$\Delta T = 8.6log(CO_2) - 48.7 \text{ in unit of } C^o \tag{1}$$

If we also include the uncertainties, we would have:

$$\Delta T = (8.6 \pm 1.1)log(CO_2) + (-48.7 \pm 5.9) \text{ in unit of } C^o \tag{2}$$

This model has a reduced chi-squared value of: $\chi_r^2 = 0.593$. While this is much less than the ones we found from EX1, 2. From the PDF we also learned that a value less than 1 doesn't indicate that this is a good fit. Therefore, the actual data might need to be modeled by something more complex. Perhaps a combination of the models used in EX1 and EX2 along with this logarithmic model.

1.1 Overlappings

We checked if the periods have overlapping temperature and values between the pre- and post-industrial periods (separated at year = 1769) by looking at the difference between the mean values relative to their spread. See the plot at the last section for more detail.
We found that pre-industrial mean temperature change is approximately: $-0.53C^o$ with a standard deviation of $0.13C^o$. And the CO_2 mean of 280.02 ppm with a standard deviation of 1.82 ppm.
For post-industrial period, we found a mean temperature change of approximately: $-0.40C^o$ with a standard deviation of $0.21C^o$. The CO_2 mean level is 298.13 ppm with a standard deviation of 19.36 ppm.

Using this data and looking at the overlap by comparing the difference and the spread (see python for the exact code) and found that:

Yes, the two periods have overlapping temperature values and CO_2

2 Filtering

We repeated the above steps, however this time we first observed when the CO_2 level has a visible upward trend. See last section, with the subsection named "Graph of CO_2 level over years" for more details. We observed that just after year = 1900 the level of CO_2 started to increase sharply, and deduced that 1930 seemed like a nice year given that we don't know the exact nature of fossil fuel adoption.
This time, we found that pre-industrial mean temperature change is approximately: $-0.53C^o$ with a standard deviation of $0.13C^o$. And the CO_2 mean of 281.35 ppm with a standard deviation of 4.64.
For post-industrial, we found a mean temperature change of approximately: $-0.12C^o$ with a standard deviation of $0.06C^o$. The CO_2 mean level is 323.61 ppm with a standard deviation of 15.11 ppm.

Using this data and looking at the overlap by comparing the difference and the spread (see python for the exact code) and found that:

No, the two periods no longer have overlapping temperature values and CO_2

3 Multivariate Data

Having computed the log model's curve fitting, our resulting model is $f(CO_2) = \Delta T = 8.6log(CO_2) - 48.7$. To see the temperature difference as a result of doubling the CO_2 level, we can perform the following math:

$$f(2 \times CO_2) - f(CO_2) = 8.6log(2 \times CO_2) - 48.7 - (8.6log(CO_2) - 48.7)$$

$$\rightarrow 8.6log(2 \times CO_2) - 8.6log(CO_2) = 8.6(log(2 \times CO_2) - log(CO_2))$$

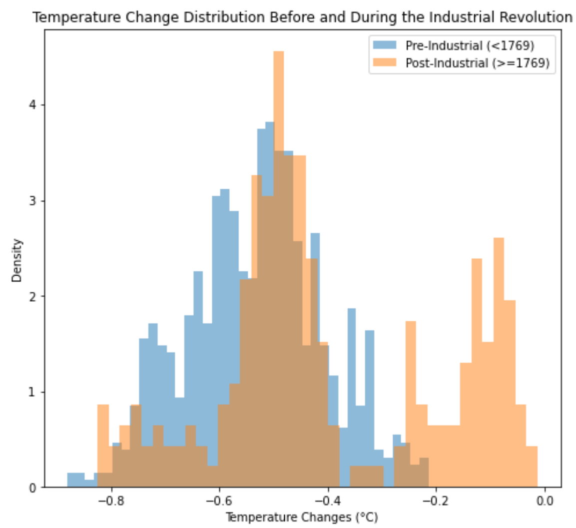
$$\rightarrow 8.6log\left(\frac{2 \times CO_2}{CO_2}\right) = 8.6log(2)$$

Therefore, the temperature increased as the result of doubling the CO_2 level is:

$$8.6log(2) \text{ in unit of } C^o$$

4 Plots

4.1 Histograms comparing pre- and post- industrial period at year = 1769



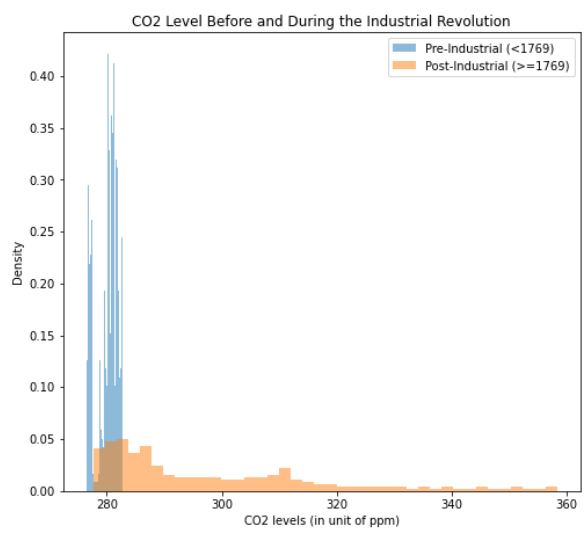


Figure 1: Histogram of Temperature Changes and CO_2 Levels separated into pre- and post- industrial periods at year = 1769

4.2 Graph of CO_2 level over years

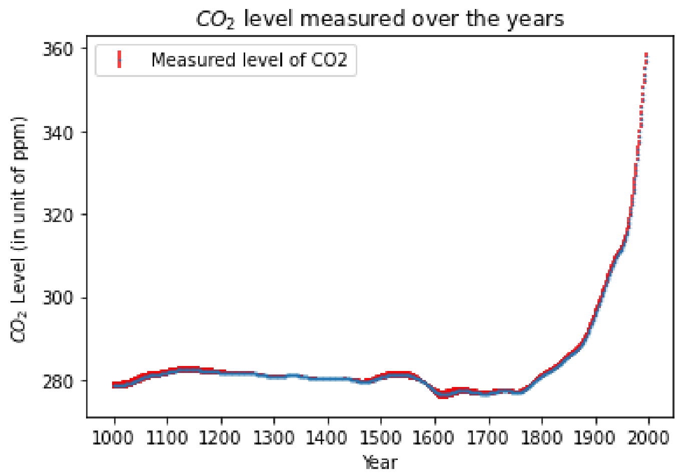


Figure 2: Measured data of CO_2 over the years

4.3 Histograms comparing pre- and post- full industrial period at year = 1930

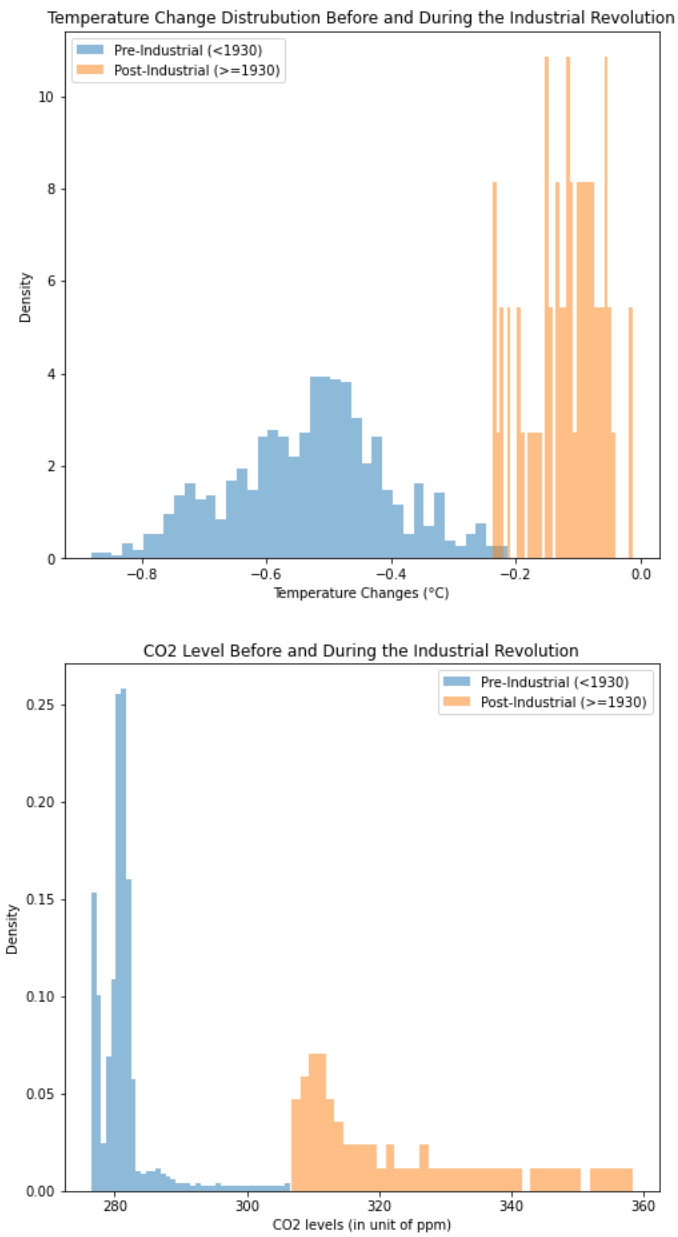


Figure 3: Histogram of Temperature Changes and CO_2 Levels separated into pre- and post- industrial periods at year = 1930

4.4 Fitted log model from data before 1700 (for more accurate prediction)

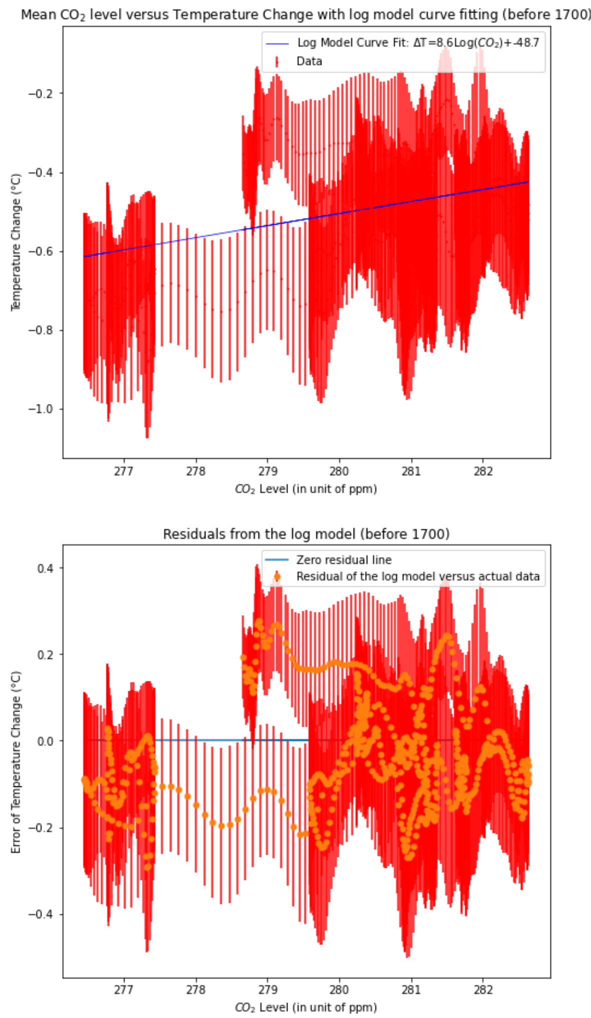


Figure 4: Fitted Log model and it's residual plots. With the function $\Delta T = (8.6 \pm 1.1)log(CO_2) + (-48.7 \pm 5.9)$ in unit of C^o