PHY224 Oscilloscope Capacitor Exercise
Time Dependent Measurements - the RC Circuit

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Also note: Any constructive feedback or recommendation about formatting or otherwise for future improvement(s), is greatly appreciated.

1 Introduction

In this report, we will explore the time dependent measurements of the capacitor in our RC circuit (the time constant τ). More specifically, we will take measurements of the time-varying signals generated via charging and discharging of our chosen capacitor and resistor. Then use our prediction model to fit this measured data. Since real world phenomenons involve a plethora of complex interactions, we will see how well and how close our predictions are in agreement/disagreement with the theoretical value.

1.1 Important Notations

1. Resistance R measured in unit of ohms (Ω).
2. Time t measured in standard unit of seconds (s), however we might also use other variants of seconds (such as μs). These will be explicitly shown, especially during unit conversions.
3. Capacitance C measured in standard unit of Farads (F) or also during our experiment in nanoFarads (nF). They will also be explicitly differentiated especially during unit conversions.
4. Voltage V measured in standard unit of volts (V), we also use unit of milliVolts (mV). They will be shown.
5. Reduced chi-squared values χ_r^2, unitless. Used to determine the accuracy of our prediction models below.

1.2 Equations used

1. Time Constant: $\tau = RC$ in unit of seconds (s)
2. Charging Prediction Model: $V = V_0 e^{-\frac{t}{RC}}$ in unit of volts (V)
3. Discharging Prediction Model: $V = V_0 e^{-\frac{t-t_0}{RC}}$ in unit of volts (V) with a time offset (t_0) to accommodate the exported data.
4. Reduced chi-squared value: $\chi_r^2 = \frac{1}{N-m} \sum_{i=1}^{N} \left(\frac{y_i - f(x_i)}{u(x_i)} \right)^2$ where N is the number of data points, m is the number of model parameters, x_i is the specific time (independent variable), y_i is the corresponding measured data point, $f(x_i)$ is the corresponding predicted value, and $u(x_i$ is the uncertainty associated with that data point.
5. Oscilloscope Uncertainty (vertical): $\pm$[DC vertical gain (3% full scale) + 0.25% full scale]. We are ignoring the vertical offset accuracy as our device was setup with $0V$ offset.
6. Oscilloscope Uncertainty (horizontal for time): $\pm$[0.0016$\times$screen width + 200ps]. We are ignoring the additional timebase accuracy as it was not taught in class nor mentioned in the exercise instruction.

2 Methods

The methods involved with this experiment can be broken down into the following sections:

2.1 Set Ups

Equipments: DSOX1202G (Oscilloscope), UofT LCR Breakout Box, Wires, USB Drive, Computer
Capacitor Spec: $C - 22nF$
Resistor Spec: $R - 470\Omega$
Wave Form: Square waves, with noise reduction (mode AVG) via the Acquire option and number of avg = 128.
Generator Frequency and Offsets: $10.000kHz$ with $0.00V$ offset.
Measurement Functions Added: Rise Time, Fall Time
Math Function: $f(CH1 - CH2)$ in unit of volts.

2.2 Data gathering and processing methods

1. Method 1: Using the oscilloscope's method, we can directly measure the 10-90 rise and fall time.
2. Method 2: Using the turning knob for tracing the curve(s) on the oscilloscope, manually log down the necessary data, such as the peak, voltage, etc. Separate them appropiately based on charging versus discharging behaviours. Then use Python to fit the collected data.
3. Method 3: Exporting the oscilloscope data, separate them based on charging and discharging behaviours, and then using Python to fit the data.
Note: Both method 2 and 3 require the use of the prediction models we described in the above section.

Fitting method:
Based on the above prediction models, they are then applied to our $curve\_fit$ function from the Python library *scipy.optimize* based on the aforementioned measurement methods. We are specifically interested in fitting the parameters R, C. For which we can then use to obtain the time constant for comparison with the theoretical value in the result and conclusion sections of this report.

2.3 Circuit Schematics

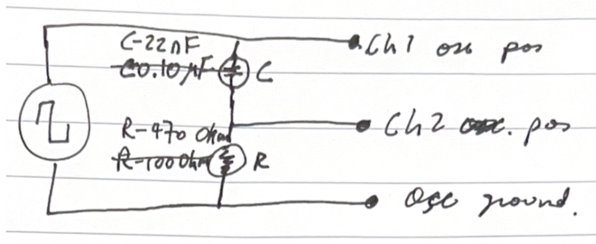

Figure 1: Screenshot of the circuit schematics from the lab notebook. Along with the specs and positioning of the resistor, capacitor, source and channels for the system.

3 Results and Plots

In this section we will discuss results of our data analysis and curve fitting to the models, the reduced chi-squared values, and the time constant τ that is present in the chosen capacitor and resistor.

3.1 Theoretical Value

Given that in theory, the time constant is expressed as: $\tau = RC$. Where our capacitor is $22nF = 2.2 \times 10^{-8}F$, and resistor is 470Ω.
Then our theoretical time constant is: $\tau = RC = 470\Omega * 2.2 \times 10^{-8}F = 1.034 \times 10^{-5}s$

$$\tau = 1.034 \times 10^{-5}s$$

3.2 Measured Value and Comparison

3.2.1 Method 1: Rise - Fall Time Method

Given that in our oscilloscope setting, our rise-fall time was set to 10-90. Our measured times are:
Rise time: $24.06\mu s = 2.406 \times 10^{-5}s$
Fall time: $25.50\mu s = 2.55 \times 10^{-5}s$

We can see the ratio between the measured rise time and theoretical value is:

$$\frac{2.406 \times 10^{-5}s}{1.034 \times 10^{-5}s} \approx 2.327$$

Similarly for the ratio between the measured fall time and theoretical value is:

$$\frac{2.55 \times 10^{-5}s}{1.034 \times 10^{-5}s} \approx 2.466$$

We also might be interested that in theory, the 10-90 rise time should be $\approx 2.2\tau$[1]. Since our rise time ratio is 2.327, we can claim that this is fairly accurate. However it is important to note that, given a real life situation, the wires that connect between the source and the capacitor and the resistor, are not ideal (i.e. they have non zero electric resistance). Therefore, the system's overall resistance is larger than that of the resistor alone. In which case the term R in $\tau = RC$ has a larger magnitude. Thus we can expect a slightly longer rise time constant than the theoretical value. Similar logic can be applied to the fall time.

[1] Valley, George E. Jr.; Wallman, Henry (1948), Vacuum Tube Amplifiers, MIT Radiation Laboratory Series, vol. 18, New York: McGraw-Hill., PG.73

3.2.2 Method 2: Manual data gathering

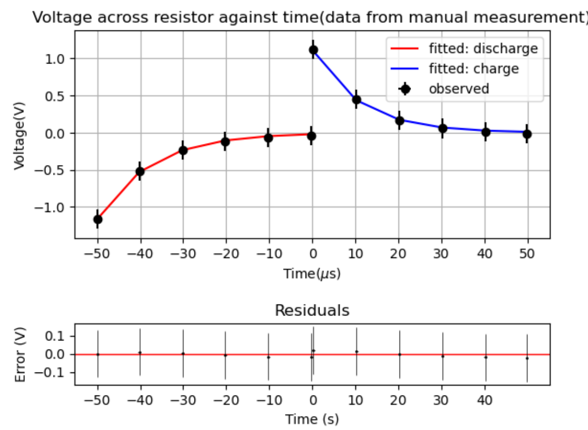

Figure 2: Plots of the manually measured data using oscilloscope cursor and the residuals, fitted to the prediction model.

We took manual measurements of the charging and discharging curves on the oscilloscope using the cursor knob. The range encapsulates a full period, with a negative and positive portion. The negative portion (concave down) represents the voltage across the resistor as the capacitor releases charge, and the positive portion (concave up) is the voltage when the capacitor stores charge. We applied the method of curve fitting to our prediction model listed above. Where we found:

1. Discharge (concave down): $R \approx -0.4996 \pm 9 \times 10^6 \Omega, C \approx 2.4727 \pm 4 \times 10^8 nF$
2. Charge up (concave up): $R \approx 3.4053 \pm 4 \times 10^{15} \Omega, C \approx 3.1767 \pm 4 \times 10^{15} nF$

Our fitted models then are:

$$V_{discharge} = V_0 e^{-\frac{t-t_0}{(-0.4996\Omega)(2.4727nF)}} \text{ in unit of volts } (V)$$

$$V_{charge} = V_0 e^{-\frac{t}{(3.4053\Omega)(3.1767nF)}} \text{ in unit of volts } (V)$$

With a reduced-chi squared value of $\chi_r^2 \approx 0.014$.

Note 1: Our large uncertainty values could be attributed to the fact that our fitted model is related by an exponential curve, creating a large variance as the input grows. Correcting for this change seems beyond the scope of this exercise, so it will be left as is.
Note 2: from class we learned that a value of $\chi_r^2 < 1$ implies that the data is being overfitted. This indeed makes sense, as we have a small dataset with a prominent curve. This makes it more susceptible to incorporating noise and

fitting too closely to the data points.

Measured Time Constant:

1. Via discharge: $abs(-4.4996\Omega) \times 2.4727nF = 4.4996\Omega \times 2.4727 \times 10^{-6}F \approx 1.113 \times 10^{-5}s$
2. Via charge: $3.4053\Omega \times 3.1767nF = 3.4053\Omega \times 3.1767 \times 10^{-6}F \approx 1.082 \times 10^{-5}s$.

When comparing these values with our theoretical time constant of $\tau = 1.034 \times 10^{-5}s$. We can see that they are roughly within a reasonable range to our theoretical expectation. Similar to the measured rise-fall time, the slightly large magnitude might correspond to the imperfections within the system (such as larger overall resistance), and should expect that in real life, things won't behave under ideal models.

3.2.3 Method 3: Exported data analysis

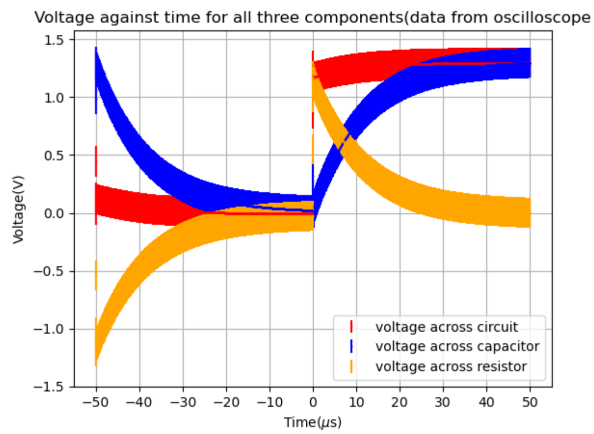

Figure 3: This is the plot of the raw exported data from the oscilloscope, with uncertainty error bars.

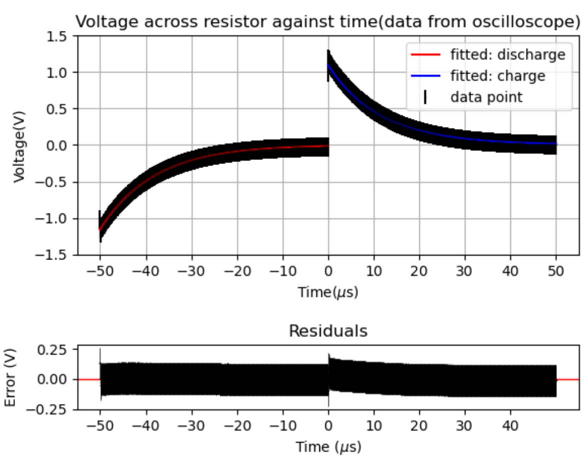

Figure 4: Plots of the exported measurement data and the residuals, fitted to the prediction model.

We exported the data directly from the oscilloscope onto a USB as per instruction in the exercise PDF. Furthermore, we calculated the uncertainties for the voltage reading based on the instrument manual. In the Python script, we performed the curve fitting on the exported data file named *scope_04.csv*.
Note: given that our models' parameters are both R, C, then we can directly obtain the values of resistance and capacitance with their respective uncertainties. They are:

1. Discharge: $R \approx 1.5287 \pm 2 \times 10^{13}\Omega, C \approx 7.7707 \pm 8 \times 10^{13} nF$
2. Charge: $R \approx 4.5904 \pm 5 \times 10^{14}\Omega, C \approx 2.5279 \pm 3 \times 10^{14} nF$

Our fitted models then are:

$$V_{discharge} = V_0 e^{-\frac{t-t_0}{(1.5287\Omega)(7.7707nF)}} \text{ in unit of volts } (V)$$

$$V_{charge} = V_0 e^{-\frac{t}{(4.5904\Omega)(2.5279nF)}} \text{ in unit of volts } (V)$$

With a reduced-chi squared value of: $\chi_r^2 \approx 0.009$.

Note: Though the timestep (Δt) is extremely small, giving a larger dataset compared to the manual data, the reduced chi-squared value is smaller than that of our manual data. This is explained by the difference in the data points and the model points values being very near to 0. This implies that we are fitting to every data point. However, this is acceptable since we are dealing with very small magnitudes of time in which voltage variations are minuscule.

Measured Time constant:

1. Via discharge: $1.5287\Omega \times 7.7707nF = 1.5287\Omega \times 7.7707 \times 10^{-6}F \approx 1.188 \times 10^{-5}s$
2. Via charge: $4.5904\Omega \times 2.5279nF = 4.5904\Omega \times 2.5279 \times 10^{-6}F \approx 1.160 \times 10^{-5}s$

When comparing these with our theoretical value time constant values of $\tau = 1.034 \times 10^{-5}s$, we can see that they are roughly similar, but with slightly larger magnitude. This makes sense, as there are many imperfections within our system such as deviation of resistance and capacitance from the manufacturer's intended values, imperfect wires, generated heat, and many others.

4 Conclusion

This experiment analyzed the time-dependent behavior of an RC circuit, comparing measured and theoretical time constants. With a theoretical value of $\tau = 1.034 \times 10^{-5} s$, our measured values closely aligned: through method 1, our rise and fall time were 2.327τ and 2.466τ respectively. Method 2 yields $1.113 \times 10^{-5} s$ for discharging, $1.082 \times 10^{-5} s$ for charging. Finally, method 3 gives $1.188 \times 10^{-5} s$ for discharging, $1.160 \times 10^{-5} s$ for charging

The slight deviations are attributed to non-ideal components and additional circuit resistance. Unfortunately, the uncertainties found from the parameters are unusually large which could be explained by the nature of the model function. Dealing with this requires further analysis beyond the reach of this exercise. Both models yielded near-zero reduced chi-squared values which indicates overfitting, but this is justified by the sizes of the datasets and the timescale of the experiment.