PHY224 Circuit Exercise 3
Diode

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Also note: Any constructive feedback or recommendation about formatting or otherwise for future improvement(s), is greatly appreciated.

1 Introduction

In this experiment, we will be observing the behaviour of the diode provided in class via positive and negative voltage readings. Through curve fitting the models discussed in the following section, we will analyze whether the diode follows the Shockley equation, estimate the leakage current and thermal voltage.

1.1 Important Notations

Notations and other information to know:
1. Current I measured in the standard unit of amps (A). We will also use other scale of this unit such as milli-amps (mA) to obtain datapoints with adequate accuracy.
2. Saturation current I_s uses the same units as current I, this is the equivalent in magnitude to the reverse current (i.e. $I \to -I_s$).
3. Voltage V measured in the standard unit of volts (V).
4. Thermal voltage V_T uses the same units as voltage V, specific to the Diode.
5. Ideality factor η, unitless. This number is based on the Diode material and manifacturing process, usually ranges from 1 to 2.
6. Reduced chi-squared values χ_r^2, unitless. Used to determine the accuracy of our prediction models below.

1.2 Equations/Models used

1. Prediction Exponential Model: $I = ae^{bV}, \forall(a,b) \in \mathbb{R}^2$ in unit of milli-amps (mA).
2. Prediction Shockley Model: $I = I_s\left(e^{aV} - 1\right), \forall a \in \mathbb{R}$. For $a = \frac{1}{\eta V_T}$, this is equivalent to: $I = I_s\left(e^{\frac{V}{\eta V_T}} - 1\right)$ in unit of milli-amps (mA).
3. Reduced chi-squared value: $\chi_r^2 = \frac{1}{N-m}\sum_{i=1}^{N}\left(\frac{y_i - f(x_i)}{u(x_i)}\right)^2$ where N is the number of data points, m is the number of model parameters, x_i is the specific time (independent variable), y_i is the corresponding measured data point, $f(x_i)$ is the corresponding predicted value, and $u(x_i)$ is the uncertainty associated with that data point.
4. Data Uncertainty calculations[1]

2 Methods

The methods involved with this experiment can be broken down into the following sections:

2.1 Circuit Schematics

[1]See Appendix for the screenshots of the Multimeter and power supply manufacturer electrical specs used to calculate the uncertainty of the measured data.

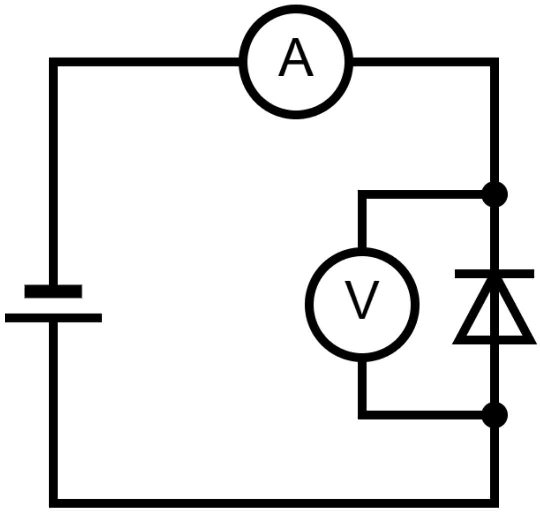

Figure 1: Initial circuit diagram of the setup, where the Diode has a 4 point connection (Positive Voltage Reading).

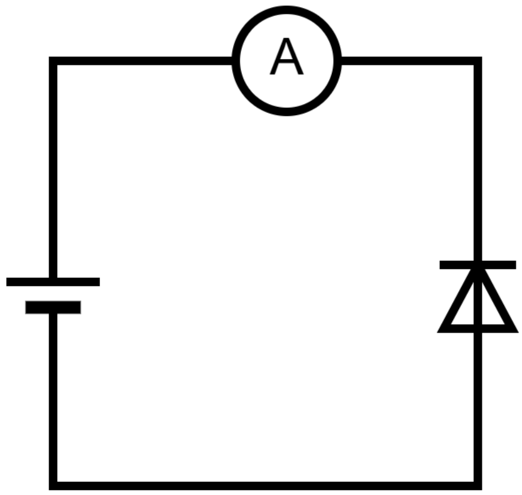

Figure 2: Circuit diagram of the setup, where the Diode has a 2 point connection (Negative Voltage Reading).

2.2 Set Ups

Equipments: 2x Keysight U1272A (multimeters), DC Circuit Pt1 No. 21, 3x black wires (banana ends), 2x red wires (banana ends), Keysight E36103B DC Supply (power source).

We then setup the experiment according to the above schematics using the listed equipments. Where the two multimeters become a voltmeter measuring in V, and an ammeter measuring in mA. However, instead of a resistor, we connected the wires to the Diode on the circuit box.
For the negative voltage, we switched the wires on the power supply. We also disconnected the voltmeter from the circuit, since the flipped current could go through the voltmeter instead of the diode due to the smaller resistance of the voltmeter compared to the large resistance on the diode with a reverse-bias. So, we have a circular setup between only the power supply, the Diode and the ammeter.

IMPORTANT NOTE: During the negative voltage reading, since the voltmeter was disconnected. Our measurement data points and uncertainty calculation for the voltage are both entirely based on the power supply. The calculation of the power supply uncertainty is included in Appendix per guidance by the TA.

2.3 Data gathering and processing methods

We began by measuring the positive voltage across the Diode following the setup in $Figure$ 1. Started with a source voltage at $0V$, then increased to $0.1V$, then increased to $0.4V$ where all subsequent are increment of $0.1V$ until we reached a maximum of $1.4V$ before the source supply shut down. During this shut down, the supply displayed message of "OCP tripped at $1.5V$".

We then disconnected and shut off the power supply, then setup the circuit like that of $Figure$ 2 following the setup menthod mentioned above (such as switching the wires connected to the power supply). We mirrored the negative measurement increments to be the same as the ones taken on the positive voltages to keep consistency.

3 Results and Plots

This section contains the plots (prediction versus measured values, and the residuals) and fitted parameters' results.

Note: For the saturation current, since from the class PDF and the *Introduction* section, the reverse current is equal to the negative saturation current. Given that our measurement yielded a constant value during the negative voltage reading, where our current values were $-0.00001mA$. We can estimate that the saturation current to be $I_s = -I \Rightarrow I_s = -1 \times -0.00001mA \Rightarrow I_s = 0.00001mA$.

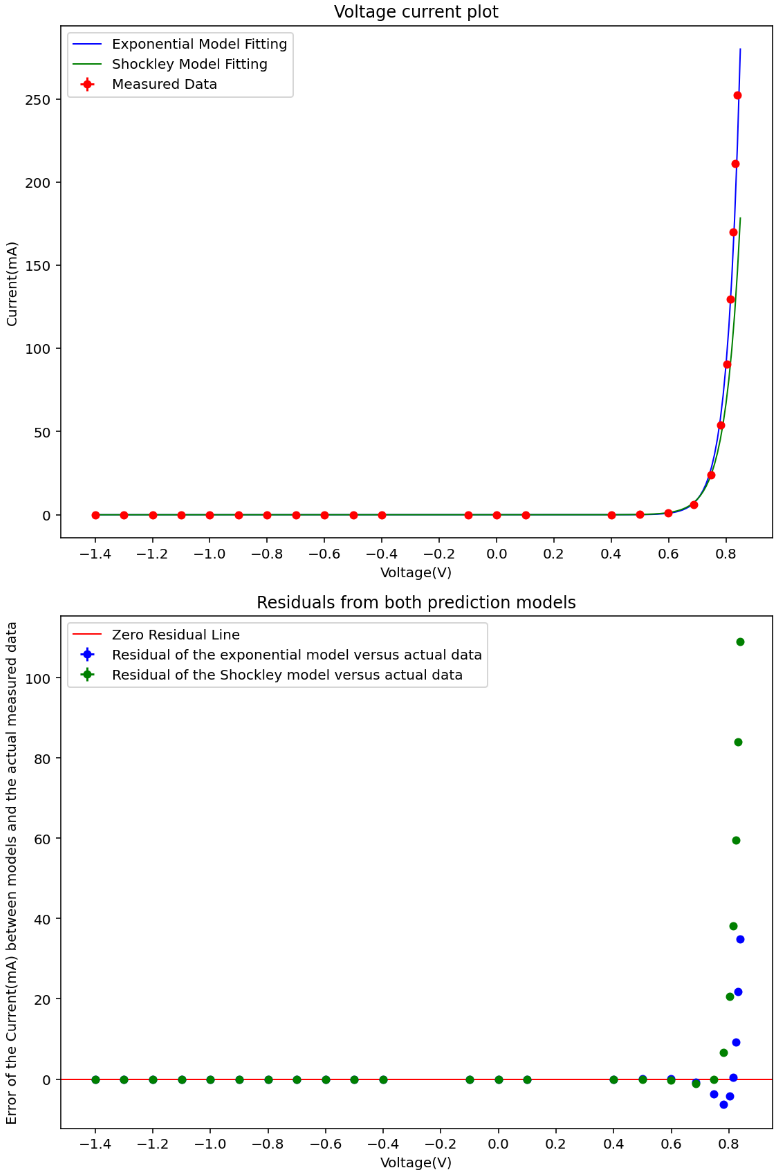

Figure 3: Plot of the model predictions and their respective residuals. Exponential Model: $I = 1166e^{22.702V}$*. Shockley Model:* $I = 0.00001(e^{19.643V} - 1)$*. Both models output in unit of milli-amp* (mA)*.*

3.1 Exponential Model Results

After performing the curve fitting function to our exponential model using the dataset, we have obtained the parameters with their respective uncertainties to be:

$$a \approx 1166 \times 10^{-9} \pm 6 \times 10^{-9}$$

$$b \approx 22.702 \pm 0.007$$

The reduced chi-squared value χ^2_{re} is ≈ 828.5.
Then our fitted exponential model is:

$$I = 1166e^{22.702V} \text{ in unit of milli-amps } (mA)$$

3.2 Shockley Model Results

Similar to the process for the fitting of the exponential model, we repeated it for the Shockley model. Where we obtained the fitted parameter and uncertainty to be:

$$a \approx 19.643 \pm 0.001$$

The reduced chi-squared value χ^2_{rs} is ≈ 10989.3.
Then our fitted Shockley model is:

$$I = 0.00001(e^{19.643V} - 1) \text{ in unit of milli-amps } (mA)$$

3.3 Reduced Chi-Squared Value Explanation

The large reduced chi-squared value for both models implies underfitting, but we can observe a close visual match between the dataset and our models. This large reduced chi-squared value could be due to the nature of the functions, where because we are fitting exponential functions, small deviations in voltages would cause large deviations in current at a sufficiently large voltage range, thus increasing the value of the residual. Getting a sufficiently small reduced chi-squared value (values close to 1) would require extremely precise tuning of the initial parameters of the python curve fitting function.

Furthermore, we could also get a smaller reduced chi-squared value by increasing the number of data points we measured (increasing the degrees of freedom). However, because the power supply has a current cap, we have a limited upper bound within our data set. However, since we can adequately determine which of the two models produces a better fit to answer the questions outlined in the handout, we will leave these values as they are.

4 Questions from the PDF

4.1 Does the Diode follow the Shockley equation?

While both of the reduced chi-square (χ^2_r) values are quite large, we can decisively observe that the Shockley equation's value is multiple magnitudes larger than that of the exponential equation. Furthermore, the exponential model on the residual plot shows a smaller deviation between the model and data point than the Shockley model at higher voltages.

Therefore, this Diode does not suffuciently satisfy the Shockley equation. It does however, more satisfy the exponential model, albeit only to a certain extent.

4.2 Estimated leakage current

Given that the leakage current is due to the current leaked when a reverse current is applied (in our negative voltage scenario). We can observe that in the dataset, while the negative voltage is applied, the measured current is constant at a value of $-0.00001mA$ or equivalently $-0.1\mu A$. Since we are only interested in the magnitude, therefore the leakage current is $0.00001mA = 0.01\mu A$.

4.3 Estimated thermal voltage V_T

Since in our Shockley model, and in the above *Equations\Models used* section, we expressed the parameter $a = \frac{1}{\eta V_T} \implies V_T = \frac{1}{\eta a}$. Through our parameter curve fitting, we obtained its value to be $a \approx 19.64$ in unit of V^{-1}. Furthermore, since no specification of the Diode was explicitly provided, we will assume that the ideality factor η is strictly between 1 and 2. This will provide us with a range, with a lower and upper estimations of the thermal voltage. Where for $1 \leq \eta \leq 2$, we have $25.454mV \leq V_T \leq 50.908mV$.

5 Conclusion

The goal of our lab is to examine whether the diode (on the DC Circuit Pt1 No. 21) provided in class follows the Shockley equation or not.

We have gathered measurement data for both the forward (positive) and reverse (negative) voltage readings based on the schematics in the *Methods* section. We then defined 2 models (exponential and Shockley) to be curve-fitted using the Python function *curve_fit* from the library *scipy.optimize*. Following this, we analyzed the fitted parameters and other details of the model highlighted in the *Results and Plots* section.

We have concluded that the given Diode does not follow the Shockley equation for an ideal diode[2]. This conclusion was reached by examining the residuals and reduced chi-squared value (χ^2_r) of our curve-fitted Shockley model. Then we used these metrics (such as $\chi^2_{rs} \approx 10989.3$) to compare with those from an exponential model ($\chi^2_{re} \approx 828.5$). We can clearly see that $\chi^2_{rs} > \chi^2_{re}$, indicating that between the two models, Shockley model's prediction is further away from the measured data than those from the exponential model. However it is important to note that, while the exponential model is a better fit, it still has a reduced chi-squared value much bigger than 1. This implies that, in order to obtain a truly accurate model for our measured dataset, further refinements to the exponential model and data gathering process are required.

Having performed the curve fitting of the models, we proceeded to estimate the diode's leakage current and the thermal voltage. The leakage current can be estimated by the observation of the ammeter during the negative voltage reading. This yielded a consistent reading of $-0.00001mA$ despite our multiple adjustments of the voltage setting on the power supply. This indicates that, within the range of the breakdown voltage, our diode leaks approximately $0.00001mA = 0.01\mu A$.

Regarding the estimation of the diode's thermal voltage, we used the fitted Shockley model. This is because our fitted Shockley parameter is in direct relationship with the thermal voltage (see *Introduction* section for the exact formula). One important thing to note: since we were not able to obtain the exact specification on the diode, we followed the class PDF for a potential range of the diode ideality factor η, where it is between 1 and 2. Therefore, our estimation of the thermal voltage has an upper and a lower value, and was found to be $25.454mV \leq V_T \leq 50.908mV$.

[2] We are not entirely sure why, but possible reasons explained in class include but are not limited to: the diode material, manufacturing process, extensive use, etc.

6 Appendix

Electrical Specifications

DC specifications for U1271A, U1272A, U1273A and U1273AX

Function	Range	Resolution	Accuracy ± (% of reading + counts of least significant digit)			Test current / Burden voltage
			U1271A	U1272A	U1273A / U1273AX	
Voltage[1]	30 mV	0.001 mV	—	0.05 + 20	0.05 + 20	—
	300 mV	0.01 mV	0.05 + 5	0.05 + 5	0.05 + 5	—
	3 V	0.0001 V	0.05 + 5	0.05 + 5	0.05 + 5	—
	30 V	0.001 V	0.05 + 2	0.05 + 2	0.05 + 2	—
	300 V	0.01 V	0.05 + 2	0.05 + 2	0.05 + 2	—
	1000 V	0.1 V	0.05 + 2	0.05 + 2	0.05 + 2	—
	Z_{LOW} (low impedance) enabled, applicable for 1000 V range and resolution only	0.1 V	—	1 + 20	1 + 20	—
Resistance[2]	30 Ω	0.001 Ω	—	0.2 + 10	0.2 + 10	0.65 mA
	300 Ω	0.01 Ω	0.2 + 5	0.2 + 5	0.2 + 5	0.65 mA
	3 kΩ	0.0001 kΩ	0.2 + 5	0.2 + 5	0.2 + 5	65 μA
	30 kΩ	0.001 kΩ	0.2 + 5	0.2 + 5	0.2 + 5	6.5 μA
	300 kΩ	0.01 kΩ	0.2 + 5	0.2 + 5	0.2 + 5	0.65 μA
	3 MΩ	0.0001 MΩ	0.6 + 5	0.6 + 5	0.6 + 5	93 nA/10 MΩ
	30 MΩ	0.001 MΩ	1.2 + 5	1.2 + 5	1.2 + 5	93 nA/10 MΩ
	100 MΩ	0.01 MΩ	2.0 +10	—	—	93 nA/10 MΩ
	300 MΩ	0.01 MΩ	—	2.0 + 10 @ < 100 MΩ 8.0 + 10 @ > 100 MΩ	.0 + 10 @ < 100 MΩ .0 + 10 @ > 100 MΩ	93 nA/10 MΩ
	300 nS	0.01 nS	1 + 10	1 + 10	1 + 10	93 nA/10 MΩ
Current[3]	300 μA	0.01 μA	0.2 + 5	0.2 + 5	0.2 + 5	< 0.04 V/100Ω
	3000 μA	0.1 μA	0.2 + 5	0.2 + 5	0.2 + 5	< 0.4 V/100 Ω
	30 mA	0.001 mA	0.2 + 5	0.2 + 5	0.2 + 5	< 0.08 V/1 Ω
	300 mA	0.01 mA	0.2 + 5	0.2 + 5	0.2 + 5	< 1.00 V/1 Ω
	3 A	0.0001 A	0.3 + 10	0.3 + 10	0.3 + 10	< 0.1 V/0.01 Ω
	10 A	0.001 A	0.3 + 10	0.3 + 10	0.3 + 10	< 0.3 V/0.01 Ω
	3 V	0.0001 V	0.5 + 5	[illegible]	0.5 + 5	Approximately

Figure 4: Uncertainty calculation of the devices' reading, more specifically used to calculate the uncertainties of the measured voltage and current.

Data Sheet

The Keysight E36103B is an accurate, 20 V, 2 A, 40 W compact power supply for your bench or system.

Highlights

Output Rating : 40 W

- Voltage: 20 V
- Current: 2 A
- Power: 40 W

Programming Accuracy : ± (% of output + offset)

- Voltage: 0.05% + 7 mV
- Current: 0.05% + 1 mA

Ripple and Noise (20 Hz to 20 MHz)

- Voltage rms: 0.8 mV
- Voltage peak-to-peak: 15 mV
- Current rms: 1 mA

Figure 5: Uncertainty calculation during the negative voltage reading, based off of the power supply specifications.