PHY224 Fitting Exercise 2 (Periodic Fitting): The annual average of CO_2 levels from the top of Mauna Loa in Hawaii

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Also note: Any constructive feedback or recommendation about formatting or otherwise for future improvement(s), is greatly appreciated.

In this fitting exercise, we will be implementing and fitting the datasets to a periodic model. Where we obtained our data from NOAA with the file named: *co2_mm_mlo.csv*. Note: as outlined in the pdf, the data points with exactly 0 or negative uncertainties have been removed for the curve fitting function to be called without running into errors.

1 The Periodic Model

1.1 Model Format:

For this exercise, we have adopted a model that uses the combination of exercise 1's models and added a periodic component to it. Counting an offset of 1960 to avoid runtime error, and letting x be the variable for time (in years) the resulting model we used is:

$$f_q(x) = A(x-1960)^4 + B(x-1960) + C$$

$$f_p(x) = f_q(x) + Dsin(2E\pi(x-1960) - \phi)$$

1.2 Curve Fitting, Parameters and Uncertainties

We first applied the curve fit function to the non-periodic model, and then used the result as the initial guesses for the periodic model's fitting. In the end we obtained:

$$A = (1,415 \times 10^{-9}) \pm (4 \times 10^{-9})$$

$$B = (1.407 \pm 0.001)$$

$$C = (309.65 \pm 0.03)$$

$$D = (3.027 \pm 0.009)$$

$$E = (100,050 \times 10^{-5} \pm 3 \times 10^{-5})$$

$$\phi = (0.439 \pm 0.009)$$

Our model written in a single function without the uncertainties is then:

$$\begin{aligned} f_p(x) &= (1,415 \times 10^{-9})(x-1960)^4 + 1.407(x-1960) + 309.65 \\ &+3.027sin\left(2\pi(100,050 \times 10^{-5})(x-1960) - 0.439\right) \text{ in unit of ppm} \end{aligned} \tag{1}$$

However, if we do include the uncertainties, then we would have:

$$f_p(x) = ((1,415 \times 10^{-9}) \pm (4 \times 10^{-9}))(x - 1960)^4$$
$$+(1.407 \pm 0.001)(x - 1960) + (309.65 \pm 0.03)$$
$$+(3.027 \pm 0.009)sin(2\pi((100,050 \times 10^{-5} \pm 3 \times 10^{-5}))(x - 1960) - (0.439 \pm 0.009))$$
$$\text{in unit of ppm} \tag{2}$$

From this model, we were also able to obtain the reduced chi-squared value of:

$$\chi_r^2 = 28.453$$

While this reduced chi-square value is still quite large, it is less than those from our models in Ex1. This indicates that by implementing the periodic model, we are a step closer at being able to formulate an appropriately accurate expression for the prediction of CO_2 levels at Mauna Loa.

1.3 Periodic Model Plots

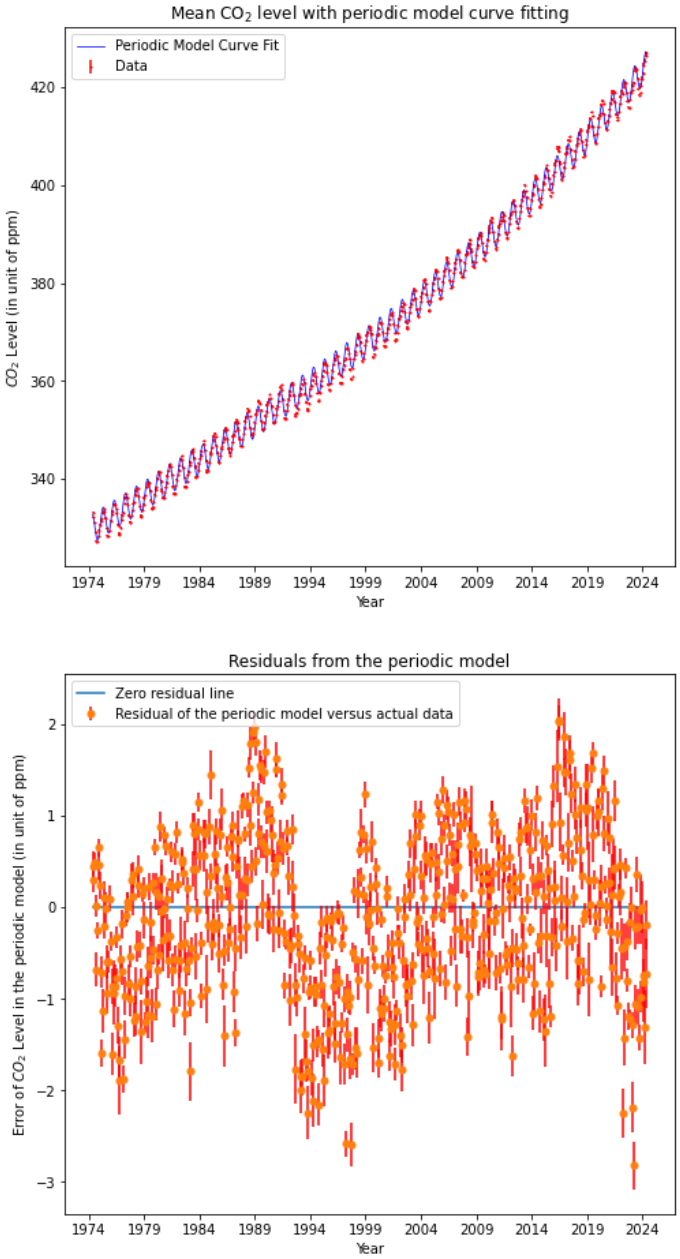

Figure 1: Periodic Model Fitting and Residuals (with uncertainties): $f_p(x) = ((1,415 \times 10^{-9}) \pm (4 \times 10^{-9}))(x - 1960)^4 + (1.407 \pm 0.001)(x - 1960) + (309.65 \pm 0.03) + (3.027 \pm 0.009)sin\left(2\pi((100,050 \times 10^{-5} \pm 3 \times 10^{-5}))(x - 1960) - (0.439 \pm 0.009)\right)$ in unit of ppm

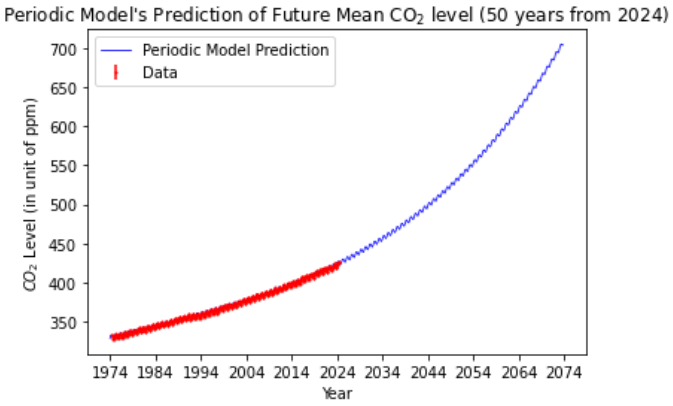

Figure 2: Using the same model as figure 1, we can make a prediction of the CO_2 levels (in ppm) over the next 50 years until 2074

2 Report Questions

2.1 Month of highest CO_2 level:

Based on the measurement dataset, and comparing the values of CO_2 of different months in the same year, the month with the highest CO_2 level is month 5 (presumably May) .
The overall CO_2 level has been increasing over time, however the periodic pattern seems to hold. Where at approximately the same month of the year, the level of CO_2 peaks. While we have no formal knowledge of growing season versus CO_2 level, what we can claim is that the overall CO_2 level increases before reaching May and decreases afterwards. Possibly indicating that May is the initial start of the next growth cycle.

2.2 When will the CO_2 level pass 2 times "Pre-Industrial" abundance of 285 (aka 570) ppm?

By applying our fitted periodic model, we predict that the CO_2 level will achieve 570ppm in the year of 2056. Our model indicates this level to be reached towards the latter half of the year, as the plants stop to grow and start to die (therefore not within the growing reason, since the CO_2 level is increasing rathern than decreasing).

2.3 How long until min CO_2 pass 2000's max CO_2?

Based on our calculation and observation of the measured dataset, the maximum CO_2 level in the year 2000 was 371.98ppm occurring at the decimal date of 2000.2917. The minimum CO_2 level of the closest year that is greater than the maximum CO_2 level of the year 2000, is 2003. More specifically, the minimum CO_2 level of 373.31ppm is reached at the decimal date of 2003.7917. Therefore, it would take 3.5 years from the date the maximum CO_2 level occurred in 2000 to be passed by the minimum CO_2 level occurred in 2003.