PHY224 Electron Charge-to-Mass Ratio.

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Abstract

This experiment measures the charge-to-mass ratio (e/m) of the electron by observing the curvature of an electron beam in a nearly uniform magnetic field produced by Helmholtz coils. To reduce systematic errors, we minimized parallax by aligning our line of sight perpendicular to the beam's circular path, keeping one eye closed to ensure consistent radius measurements. We also applied an off-axis correction for the magnetic field when the beam veered away from the coil center. By varying the coil current at constant voltage and, conversely, the accelerating voltage at constant current, we collected two data sets to model this off-axis correction factor to B_c and then fitted our models to predict the charge-to-mass ratio. We also investigated the anomalous effect of low-voltage/high-current conditions on the electron trajectory. The constant current prediction yielded $\frac{e}{m} \approx (1.57 \pm 0.06) \times 10^{11}\ C\ kg^{-1}$ with a $\chi_r^2 \approx 1.3$, and the constant voltage prediction yielded $\frac{e}{m} \approx (1.59 \pm 0.04) \times 10^{11}\ C\ kg^{-1}$ with a $\chi_r^2 \approx 0.37$. Compared to the literature's expected value of $\approx 1.76 \times 10^{11}\ C\ kg^{-1}$, our predicted charge-to-mass ratios fall slightly outside, indicating a small but notable discrepancy. Given our discussions on the present of known influences (parallax, external ferromagnetic sources), and the possibility of further setup improvements (including but not limited to magnetic shielding, repositioning of electronic devices), we consider our result to be in reasonable agreement within the context of known systematic uncertainties.

1 Introduction & Theory

In this experiment, we accelerate electrons through a magnetic field generated by Helmholtz coils, resulting in the electrons traveling in a circular trajectory. By measuring the radius of the beam r, the potential difference ΔV, and the current I, through the system, we are able to find the charge-to-mass ratio, $\frac{e}{m}$ of an electron.

1.1 Theoretical Background

An electron of charge e and mass m moving with velocity v in a magnetic field B experiences a force

$$\vec{F} = e\vec{v} \times \vec{B},$$

which, if $v \perp B$, provides the centripetal force for circular motion. Then, when the electron is accelerated from rest through ΔV, its kinetic energy is

$$e\Delta V = \frac{1}{2}mv^2.$$

We can describe the curvature of the electron's path in the beam by combining these equations:

$$\frac{1}{r} = \sqrt{\frac{e}{2m}}\frac{B}{\sqrt{\Delta V}}$$

In our setup, the total magnetic field is $B = B_c + B_e$, where B_c is due to the coils and B_e is an external field (e.g., Earth's field). We can plot this field in python as a B_c vs. r graph to isolate and find B_e according to:

$$B = B_c + B_e = \sqrt{\frac{2\,m\Delta V}{e}}\frac{1}{r},$$

and hence

$$B_c = \alpha\frac{1}{r} - B_e, \quad \text{where} \quad \alpha = \sqrt{\frac{2\,m}{e}\Delta V}. \tag{1}$$

Then, from the experiment manual given, we also have:

$$B_c = \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_0 n I}{R} \tag{2}$$

where n is the number of turns in each Helmholtz coil, R is the coil radius, and μ_0 is the permeability of free space (in SI units). We can then use this to find the relationship between r and the current/voltage measurements.

Before moving further, we must note that when we take our measurements of the electron beam (r), there might be a difference between the beam's center and the coil's central axis, ρ. To account for this, we apply a correction factor for every data point of B_c obtained:

$$\frac{B(\rho)}{B(0)} = 1 - \frac{\rho^4}{R^4(0.6583 + 0.29\frac{\rho^2}{R^2})^2} \tag{3}$$

where $B(0)$ is the axial field, giving us $B_c' = (\frac{B(\rho)}{B(0)})B_c$. Finally, we substitute this corrected B_c' value into the curvature equation, combine it with eq. (1) and rearrange to get:

$$\frac{1}{r} = \sqrt{\frac{e}{m}} k \left(I + \frac{1}{\sqrt{2}} I_0\right) \frac{1}{\sqrt{\Delta V}}. \tag{4}$$

Where

$$k = \frac{1}{\sqrt{2}} \left(\frac{4}{5}\right)^{\frac{3}{2}} \frac{\mu_0\, n}{R}, \quad I_0 = \frac{B_e}{k}. \tag{5}$$

represents the characteristic of the coil dimensions and proportionality constant for the external field respectively. We will plot three sets of data using python. One will be using the model equation (1). Two will be using eq.(4): one for constant current and one for constant voltage. We can use algebraic manipulations on the fitted coefficients to find our desired value of B_e, which will allow us to find $\frac{e}{m}$.

2 Methods

2.1 Setups

Equipments: 2x Multimeters (Keysight 34461A), Rheostat (No. 0530), HelmHoltz Coils with Powerbox (No. 1, with Ammeter Weston Model 81, Voltmeter Philips FM2513), Power Source 1 (Kikusui PMX350 DC Power Supply), Power Source 2 (Tripp-Lite Model PR-3/UL), Filament Power Supply 6.3V, Electron Gun (LD Didactic GmbH 0-50354. 555 571), Reflective Illuminated Ruler, 1-m Wooden Ruler, banana-ends wires. The circuit schematic is shown below:

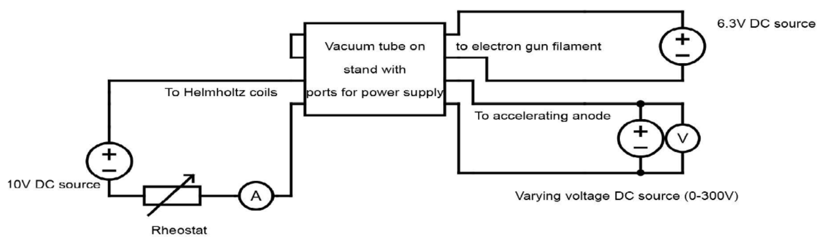

Figure 1: Diagram of the lab setup. Source: Lab manual.

We set up the apparatus according to the lab manual: turning on the filament supply first for about 30 s, then powering all other devices. After forming a circular beam, we measured diameters using the screen ruler, recorded ΔV and I, and collected two data sets (one with ΔV fixed, the other with I fixed). Finally, we shut down the set up by switching off the filament supply last. All diameters were initially in centimeters, then converted to meters in Python for curve fitting.

When taking diameter measurements, we must be careful of reading inaccuracies caused by the parallax of the ruler's reflection on the screen and the beam. We can mitigate this by closing one eye and observing perpendicular to the beam position, and taking the corresponding ruler measurement.

For low accelerating voltage and high current, the beam became non-circular, meaning different sections of its path were unequally affected. This led to larger errors, so we avoid this anomalous range by having voltage and current within a range such that the beam appears circular.

Although theory assumes the beam remains near the coil's center (where the field is uniform), practical setups allow for an off-axis distance ρ. For $0.2R < \rho < 0.5R$, the manual prescribes a correction factor (eq.(3)), which compensates for diminished field strength away from the central axis and improves the final accuracy of $\frac{e}{m}$.

We must also note the influence of nearby ferromagnetic materials. We did not notice a significant difference in the beam shape when we had electronic devices near the bulb, but we are able to determine the exact external field strength from the collected data in our analysis.

2.2 Python Models and Uncertainty Propagation

For the magnetic fit prediction model, we calculated the values of B_c using eq (2), then applied the correction factor from eq (3), then directly implemented eq (1) into the *curve_fit* function.

For the constant current prediction model, we used eq (4) to derive a model function with better curve fitting. Namely, inverting the formula to directly fit for r and simplifying the coefficient. Our model is then:

$$r = a\sqrt{V} \text{ in unit of meters. Where } a = \frac{1}{\sqrt{\frac{e}{m}}k\left(I + \frac{1}{\sqrt{2}}I_0\right)} \tag{6}$$

Similarly, for the constant voltage prediction model. We used eq (4) to derive:

$$r = \frac{a}{I + \frac{I_0}{\sqrt{2}}} \text{in unit of meters. Where } a = \frac{1}{\sqrt{\frac{e\Delta V}{m}}k} \tag{7}$$

In our experiment, the sources of uncertainties include: radius uncertainties (due to visual limitations, ruler resolution, parallax, electron beam thickness, etc), and multimeter uncertainties (calculated based on the manufacturer specification). We will use uncertainties from data collection and model uncertainties for our uncertainty propagation.

To propagate uncertainties for the data and model, we used the following equation from the Uncertainty Manual provided in class.

For measurand y and input values $x_1, x_2, ..., x_n$. The uncertainty of y is:

$$u(y) = \sqrt{\left(\frac{\partial y}{\partial x_1}\right)^2 (u(x_1))^2 + ... + \left(\frac{\partial y}{\partial x_n}\right)^2 (u(x_n))^2}$$

Using the constant voltage model as an example, we can obtain the uncertainty by

$$u(r) = u\left(\frac{a}{b}\right) = \frac{a}{b}\sqrt{\left(\frac{u(a)}{a}\right)^2 + \left(\frac{u(b)}{b}\right)^2}, \text{ for } b = I + \frac{I_0}{\sqrt{2}} \text{ and } u(a),\ u(b) \text{ are propagated accordingly.}$$

Note: for the multimeter uncertainties, we consulted the manufacturer's spec sheets, and the TA's recommendation of the column of 2 years. We applied the following formulas:

Voltage uncertainties: $\pm(0.006\%$ of reading $+0.001\%$ of range $)$.
Current uncertainties (between $100mA$ and $1A$): $\pm(0.12\%$ of reading $+0.01\%$ of range $)$.
Current uncertainties (between $1A$ and $3A$): $\pm(0.23\%$ of reading $+0.02\%$ of range $)$.

3 Results & Analysis

Following the process presented in the Methods section, we applied Python curve fitting to our datasets. First performing the magnetic fit model, then using the predicted parameter to perform the constant current and constant voltage curve fitting.

3.1 Magnetic Fit

We obtained the fitted external field value B_e, α and the prediction model to be:

$$B_e \approx (7 \pm\ 1) \times 10^{-5}\ T$$
$$\alpha \approx (427 \pm\ 6) \times 10^{-7}\ T * m$$
$$B_c(r) = 0.0000427\frac{1}{r} + 0.00007 \text{ in unit of Teslas (T)}$$

Our reduced chi square value is: $\chi_r^2 \approx 0.7$

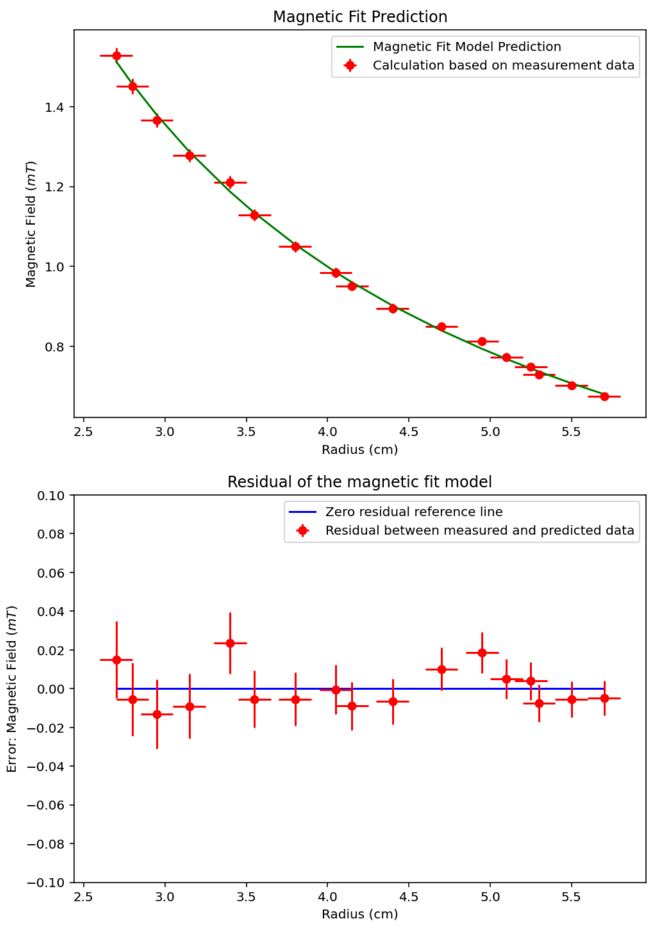

Figure 2: Plot of the magnetic field of the Helmholtz coil, B_c vs. beam radius r, and the corresponding residual plot. The fitted curve is calculated to be $B_c(r) = 0.0000427\frac{1}{r} + 0.00007$. Note: we adjusted the display units on each dimension for better visual (y-axis from T to mT, and x-axis from m to cm).

We see that while the magnetic curve fit is accurate, the χ_r^2 suggests slight overfitting, implying an overestimation of our uncertainty[1]. Given multiple error sources, a potential reason could be

[1] Uncertainty was obtained via propagation of those for B_c, R, ΔV, those associated with ρ, and k.

artificially large overestimation of measured values such as coil or beam radius.

The expected field in Toronto[2] is $\approx 5 \times 10^{-5}$ T, while our measured external field is $\approx (7 \pm 1) \times 10^{-5}$ T, implying nearby ferromagnets add between $(1 \text{ to } 3) \times 10^{-5}$ T. According to Seidman et al.[3], a single iPhone 12 Pro Max at 3 cm can generate around 3 Gs ($\approx 30 \times 10^{-5}$ T). With multiple adjacent electronic devices, labs and nearby radiation room, these sources likely account for the extra $(1 \text{ to } 3) \times 10^{-5}$ T on top of Earth's magnetic field influence.

3.2 Constant Current Fit

While fitting the magnetic model, we found outliers at low voltages within out constant current dataset. After consulting the manual and TA, we removed the first (lowest-voltage) data point in Python. We obtained the predicted parameter a, our prediction model (based on eq. (6)), and the calculated charge-to-mass ratio to be:

$$a \approx (402 \pm\ 2) \times 10^{-5}\ V^{-\frac{1}{2}}\ m$$
$$r = 0.00402(\Delta V)^{-\frac{1}{2}} \text{ in unit of meters (m)}$$
$$\frac{e}{m} \approx (1.57 \pm 0.06) \times 10^{11}\ C\ kg^{-1}$$

with reduced chi square value of: $\chi_r^2 \approx 1.3$

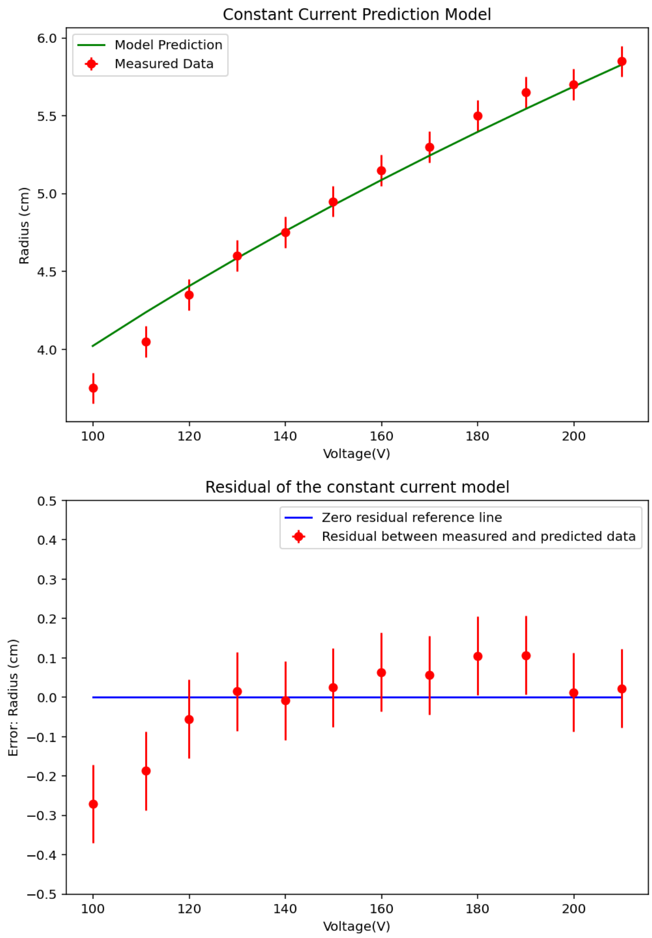

Figure 3: Plot showing varying potential difference ΔV against beam radius r, where current is kept constant. The corresponding residuals is also shown. The fitted prediction model is $r = 0.00402(\Delta V)^{-\frac{1}{2}}$ in units of meters. Again, note: we adjusted the display unit on the y-axis for better visual (y-axis from m to cm).

[2] Retrieved on April 05, 2025: https://www.magnetic-declination.com/CANADA/TORONTO/338447.html
[3] Retrieved on April 05, 2025: Seidman, S. et al. (2021). *Heart Rhythm, 18.* 10.1016/j.hrthm.2021.06.1203

Literature states that the charge-to-mass ratio value[4] is $\approx 1.76 \times 10^{11}\ C\ kg^{-1}$. Comparing this to our predicted charge-to-mass ratio of $\approx (1.57 \pm 0.06) \times 10^{11}\ C\ kg^{-1}$, we see that our value is slightly off.

Despite slight underfitting according to χ_r^2, our model still provides a reasonable e/m prediction. We observe larger deviations at lower voltages, even after removing the lowest point. Likely, data quality declined near the beam's anomalous range, affecting radius measurements. In future work, maintaining a safe voltage margin above this region would yield more reliable results.

3.3 Constant Voltage Fit

As in the previous fit, we now perform the fitting to the constant voltage dataset. Where we obtained the predicted parameter, the prediction model based on eq. (7), and the calculated charge-to-mass ratio to be:

$$a \approx (577 \pm 3) \times 10^{-4}\ A\ m$$
$$r = 0.0577\Big(\frac{1}{I + 0.0906}\Big) \text{ in unit of meters (m)}$$
$$\frac{e}{m} \approx (1.59 \pm 0.04) \times 10^{11}\ C\ kg^{-1}$$

This model yielded a reduced chi square value of: $\chi_r^2 \approx 0.37$

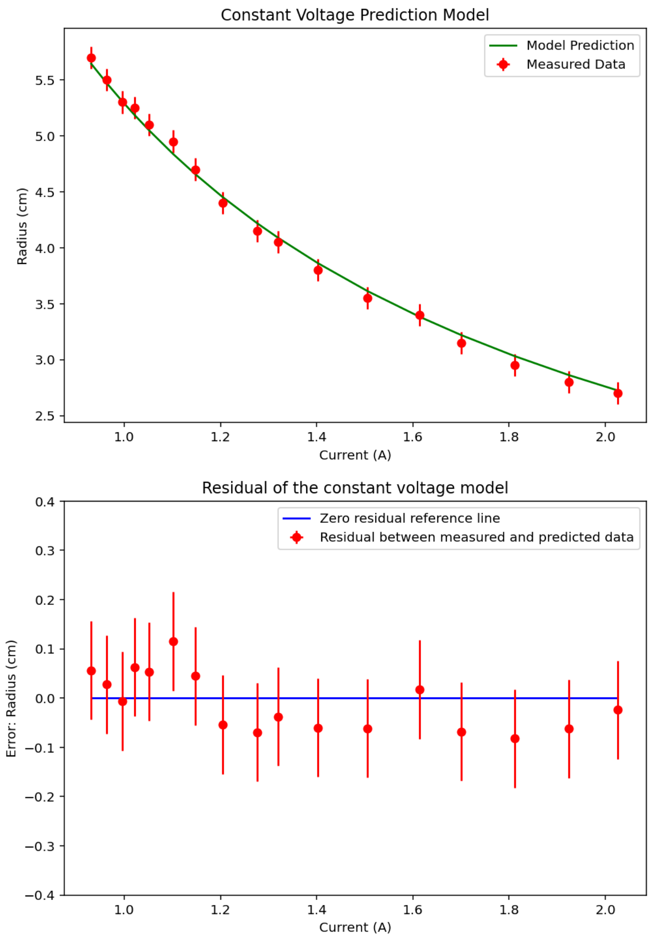

Figure 4: Plot showing varying current ΔV against beam radius r, where potential difference is kept constant. The corresponding residuals is also shown. The fitted prediction model is: $r = 0.0577(\frac{1}{I+0.0906})$ in unit of meters. Again, note: we adjusted the display unit on the y-axis for better visual (y-axis from m to cm).

Based on the χ_r^2 value, our model appears to overfit the data. The residual plot shows most point's uncertainty exceeding its deviation from the prediction, hinting at an overestimation of

[4]Retrieved April 05, 2025: https://physics.nist.gov/cgi-bin/cuu/Value?esme

uncertainties. This could potentially stem from the nature of our setup, involving many sources of uncertainties as discussed in section 2.3. To avoid this in a future exploration, we would attempt to hold this experiment in a more magnetically isolated environment.

Similar to the constant-current fit, we see a discrepancy from the accepted $1.76 \times 10^{11}\,\mathrm{C\,kg^{-1}}$; our estimate is $(1.59 \pm 0.04) \times 10^{11}\,\mathrm{C\,kg^{-1}}$, excluding the literature value despite propagated uncertainties. This could potentially be caused by coil misalignment or coil winding variations, or data quality issues. A more controlled, precisely aligned setup could improve agreement in subsequent experiments.

4 Conclusion

In this experiment, we investigated the electron's charge-to-mass ratio by examining how a beam of electrons, accelerated by a known potential difference, curves in a magnetic field generated by Helmholtz coils. We accounted for the Earth's field (and other stray fields) by isolating an external component B_e, applying an off-axis field correction for larger radii, and implementing careful parallax control in measuring beam diameters. Our reduced chi-square (χ^2_ν) values showed reasonably good agreement with the theoretical models, though one fit suggested some overestimation of uncertainty. Despite these nuances, both the constant current analysis yielded $\frac{e}{m} \approx (1.57 \pm 0.06) \times 10^{11}\ C\ kg^{-1}$, and constant voltage analysis yielded $\frac{e}{m} \approx (1.59 \pm 0.04) \times 10^{11}\ C\ kg^{-1}$. This result lies near the accepted literature value of $\approx 1.76 \times 10^{11}\ C\ kg^{-1}$, when given our discussion of potentially suboptimal data quality, it is to our best understanding, that the result is in reasonable agreement with the expected value. Especially when many external influences were present, all of which required corrections (notably known influences included: parallax, off-axis fields, and instrumental precision).

Potential improvements include more rigorous magnetic shielding to further minimize external fields (including but not limited to metal enclosures, repositioning of our electronic devices, etc), finer alignment of the electron gun with the coil axis, and measuring the coil radius R more precisely. Such refinements would reduce systematic error, especially at low electron energy and high magnetic field, where the beam can deviate from a clean circular path. These considerations underscore the importance of environmental control in high precision electromagnetism experiments. Nonetheless, the overall consistency of our measured ratio with the accepted value demonstrates the effectiveness of this experiment in illustrating classic principles of electromagnetism and experimental techniques for precision measurements.