PHY224 Circuit Exercise 2
Light Bulb

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Due February 16th, 2025

Also note: Any constructive feedback or recommendation about formatting or otherwise for future improvement(s), is greatly appreciated.

1 Introduction

In this exercise, we take a look at the resistance of a Tungsten filament bulb. Since the filament heats up when current passes through it, the bulb's resistance changes. We will collect data and then apply a few prediction models to fit the measured data. Of these models, we will be including a theoretical model provided in class to compare the accuracy of predictions based on the number of parameters and the format of our model.

1.1 Important Notations

1. Current I measured in the standard unit of amps (A). We will also use other scale of this unit such as milli-amps (mA) to obtain datapoints with adequate accuracy.
2. Voltage V measured in the standard unit of volts (V).
3. Reduced chi-squared values χ_r^2, unitless. Used to determine the accuracy of our prediction models below.

1.2 Equations/Models used

1. Prediction Power Model: $I_p = aV^b$ in unit of amps $(A), \forall (a, b) \in \mathbb{R}^2$
2. Prediction Logarithmic (Log) Model: $log(I) = b\, log(V) + log(a)$ in unit of log of amps, $\forall (a, b) \in \mathbb{R}^2$
3. Prediction Ideal Model: $I_i = aV^{\frac{3}{5}}$ in unit of amps $(A), \forall a \in \mathbb{R}$
4. Reduced chi-squared value: $\chi_r^2 = \frac{1}{N-m} \sum_{i=1}^{N} \left(\frac{y_i - f(x_i)}{u(x_i)} \right)^2$ where N is the number of data points, m is the number of model parameters, x_i is the specific time (independent variable), y_i is the corresponding measured data point, $f(x_i)$ is the corresponding predicted value, and $u(x_i$ is the uncertainty associated with that data point.
5. Data Uncertainty calculation[1]
6. Model Uncertainty for the power model: $\sigma_{I_p} = \sqrt{(V^b \sigma_a)^2 + (aV^b ln(V) \sigma_b)^2 + (abV^{b-1} \sigma_V)^2}$.
7. Model Uncertainty for the log model:$\sigma = \sqrt{(\frac{\sigma_a}{a})^2 + (ln(V)\sigma_b)^2 + \left(\frac{b\sigma_V}{V}\right)^2}$.
8. Model Uncertainty for the ideal model: $\sigma_{I_i} = \sqrt{(V^{3/5}\sigma_a)^2 + \left(\frac{3}{5} aV^{-2/5} \sigma_V\right)^2}$.

2 Methods

The methods involved with this experiment can be broken down into the following sections:

[1]See Appendix for a screenshot for the Multimeter manufacturer electrical specs used to calculate the uncertainty of the measured data.

2.1 Circuit Schematics

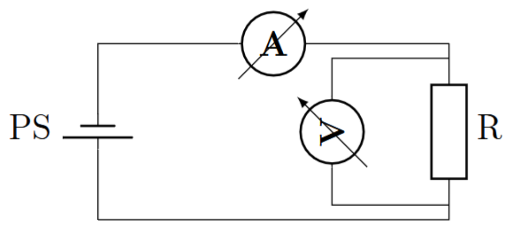

Figure 1: Circuit diagram of the setup, where the resistor has a 4 point connection.

2.2 Set Ups

Equipments: 2x Keysight U1272A (multimeters), DC Circuit Pt1 No. 21, 3x black wires (banana ends), 2x red wires (banana ends), Keysight E36103B DC Supply (power source).

We then setup the experiment according to the above schematics using the listed equipments. Where the two multimeters become a voltmeter measuring in V, and an ampmeter measuring in mA. However, instead of a resistor, we connected the wires to the Tungsten filament lightbulb on the circuit box.

2.3 Data gathering and processing methods

We began by measuring the lightbulbs resistance using positive current. Our initial non-zero current is applied at $0.1V$ (at the power source), then increased to $0.5V$ at the source. Then proceeded with an increment of $0.5V$ for each of the subsequent measurement data until we reached $13V$.

We then disconnected and turned off the power supply. After consulting with a TA, we reversed the positive and negative terminals to measure the negative current behaviour in the system. The time taken caused the lightbulb to cool down, changing the resistance in the bulb. Similar to the data gathering method for the positive current, we started off at $-0.1V$, then changed to $-0.5V$. Then proceeded with an increment of $-0.5V$ until we reached $-5V$ of current supplying to the system.

Having gathered all the measurement data, we proceeded to analyse and process them using the $curve\_fit$ function within the Python library *scipy.optimize* to create a curve fitting to the aforementioned prediction models.

Note: Due to the change in the bulb resistance from the positive measurements to the negative measurements, tweaks were added to the methodology outlined in the handout for the Python analysis. We separated the measurement data based on positive versus negative voltage reading. This is done to accommodate the resistance change and allows for successful curve fitting of the different models (especially for that of the logarithmic prediction), and avoids runtime errors.

3 Results and Plots

In this section we will be demonstrating and discussing the results obtained from our curve fitted prediction models, the parameters and their uncertainties, the residuals and the reduced chi-squared values for our circuit setup.

We have also separated the data based on positive and negative voltages, to avoid curve fitting runtime errors.

The following is the plot of the measured data with the x-axis being the voltage in unit of volts V and the y-axis being the current in unit of milli-amps mA.

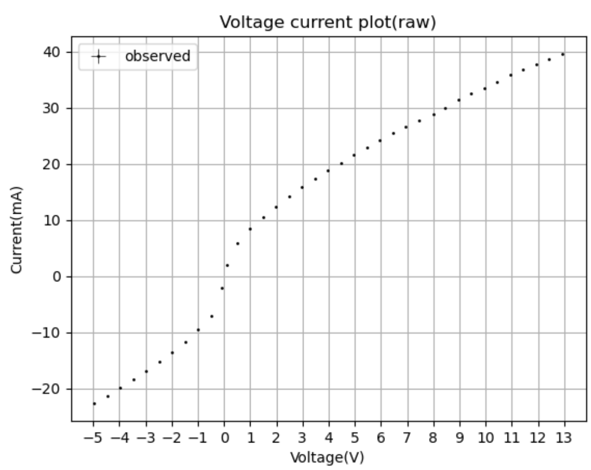

Figure 2: The raw data being plotted before we proceed with curve fitting.

3.1 Positive Voltages

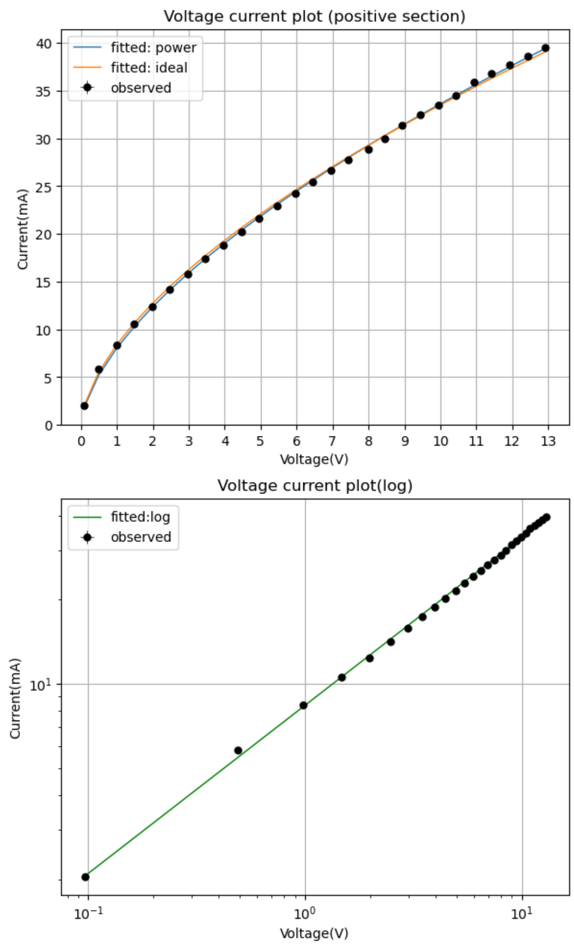

Figure 3: Models fitted to the positive voltage and current data. Details of each model are discussed in the following sections.

3.1.1 Power Model

We can see both curves fit to the shape of the data well, so our data is close to the ideal model. By curve fitting the positive voltage data, we obtained the fitted parameters for the power model to be:

$$a \approx 8.05 \pm 0.29$$

$$b \approx 0.62 \pm 0.02$$

Our fitted power prediction model is then:

$$I_{p^+} = 8.05V^{0.62} \text{ in unit of milli-amps } (mA)$$

With a reduced-chi squared value of: $\chi_r^2 \approx 0.7$.

3.1.2 Log Model

Similarly, for the logarithmic model, we see a close fit to the data points for the smaller voltages. We obtained the fitted parameters to be:

$$a \approx 8.3672 \pm 0.0004$$

$$b \approx 0.6010 \pm 0.0007$$

Our fitted log prediction model is then:

$$log(I) = 0.6010log(V) + log(8.3672) \text{ in unit of log of milli-amps } (mA)$$

With a reduced-chi squared value of: $\chi_r^2 \approx 8 \times 10^{-7}$.

3.1.3 Ideal Model

Following the same process, for the fitted parameters for the ideal model are:

$$a \approx 8.05 \pm 0.06$$

Our fitted ideal prediction model is then:

$$I_{i^+} = 8.05V^{\frac{3}{5}} \text{ in unit of milli-amps } (mA)$$

With a reduced-chi squared value of: $\chi_r^2 \approx 6.2$.

3.2 Negative Voltages

Exactly like the process used for the prediction and fitting of the positive voltages, we now perform curve fitting to the negative voltages and currents. Recall that from *Data gathering and processing methods* section, when we performed the prediction, we "made the current positive" in order to create a good fit and avoid runtime error.

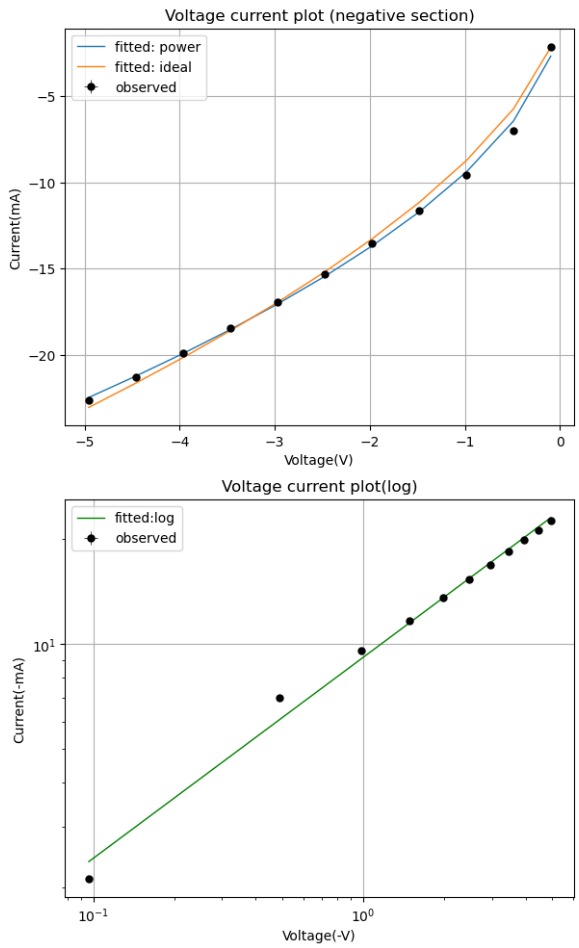

Figure 4: Models fitted to the processed negative voltage and current data. Details of each model are discussed in the following section.

3.2.1 Power Model

By curve fitting the positive voltage data, we obtained the fitted parameters for the power model to be:

$$a \approx 9.49 \pm 0.46$$

$$b \approx 0.54 \pm 0.04$$

Our fitted power prediction model is then:

$$I_{p^+} = 9.49V^{0.54} \text{ in unit of milli-amps } (mA)$$

With a reduced-chi squared value of: $\chi_r^2 \approx 0.8$.

3.2.2 Log Model

Similarly, for the logarithmic model we obtained the fitted parameters to be:

$$a \approx 9.1686 \pm 0.0004$$

$$b \approx 0.5767 \pm 0.0004$$

Our fitted log prediction model is then:

$$log(I) = 0.5767log(V) + log(9.1686) \text{ in unit of log of milli-amps } (mA)$$

With a reduced-chi squared value of: $\chi_r^2 \approx 9 \times 10^{-6}$.

3.2.3 Ideal Model

Following the same process, for the fitted parameters for the ideal model are:

$$a \approx 8.8 \pm 0.2$$

Our fitted ideal prediction model is then:

$$I_{i^+} = 8.8V^{\frac{3}{5}} \text{ in unit of milli-amps } (mA)$$

With a reduced-chi squared value of: $\chi_r^2 \approx 16.6$.

3.3 Residuals

Here, we plotted the residuals of the model predictions for both the set of positive and the set of negative voltages and currents. The large uncertainty in the power model data points arises from the propagation of uncertainty being applied to a power function, causing uncertainty to grow as voltage increases. We can see that the log model indeed seems to fit the data the closest, justifying the small reduced chi-squared value.

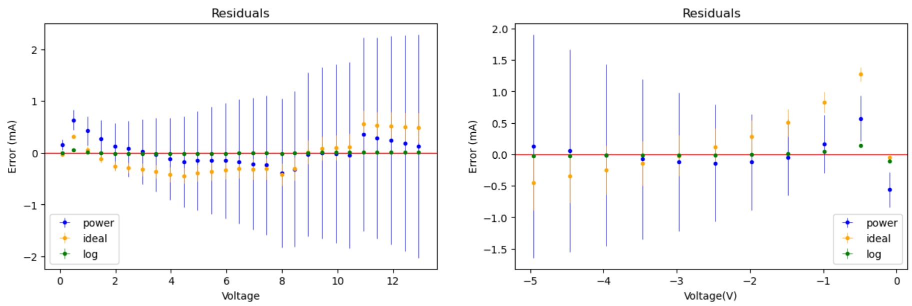

Figure 5: Residuals based on the different sets of data points. Left: based on the positive data set. Right: based on the negative data set.

4 Conclusion

In this experiment, we analyzed the voltage-current characteristics of a tungsten filament light bulb using three different models: the power model, the logarithmic model, and the ideal model. Through curve fitting, we determined the best-fit parameters and their associated uncertainties, and we evaluated the goodness of fit using reduced chi-squared values.

Our results show that the power model provided the most reasonable fit across both positive and negative voltage ranges, with reduced chi-squared values close to 1, indicating a relatively accurate representation of the data. The logarithmic model, while mathematically valid, produced very small chi-squared values, suggesting overfitting. On the other hand, the ideal model deviated more significantly, particularly for negative voltages. Given that the reduced-chi squared value is bigger than 1, this suggest that the theoretical value is underfitting the data. Keeping in mind that due to the nature of the Tungsten filament lightbulb and the fact that it heats up as current passes through. Our curve fitting parameters came surprisingly close to that of the theoretical values.

Overall, the experiment demonstrates the non-ohmic behavior of the light bulb due to temperature-dependent resistance changes, confirming that the power law model effectively captures this phenomenon.

5 Appendix

Electrical Specifications

DC specifications for U1271A, U1272A, U1273A and U1273AX

Function	Range	Resolution	Accuracy ± (% of reading + counts of least significant digit)			Test current / Burden voltage
			U1271A	U1272A	U1273A / U1273AX	
Voltage[1]	30 mV	0.001 mV	—	0.05 + 20	0.05 + 20	—
	300 mV	0.01 mV	0.05 + 5	0.05 + 5	0.05 + 5	—
	3 V	0.0001 V	0.05 + 5	0.05 + 5	0.05 + 5	—
	30 V	0.001 V	0.05 + 2	0.05 + 2	0.05 + 2	—
	300 V	0.01 V	0.05 + 2	0.05 + 2	0.05 + 2	—
	1000 V	0.1 V	0.05 + 2	0.05 + 2	0.05 + 2	—
	Z_{LOW} (low impedance) enabled, applicable for 1000 V range and resolution only	0.1 V	—	1 + 20	1 + 20	—
Resistance[2]	30 Ω	0.001 Ω	—	0.2 + 10	0.2 + 10	0.65 mA
	300 Ω	0.01 Ω	0.2 + 5	0.2 + 5	0.2 + 5	0.65 mA
	3 kΩ	0.0001 kΩ	0.2 + 5	0.2 + 5	0.2 + 5	65 µA
	30 kΩ	0.001 kΩ	0.2 + 5	0.2 + 5	0.2 + 5	6.5 µA
	300 kΩ	0.01 kΩ	0.2 + 5	0.2 + 5	0.2 + 5	0.65 µA
	3 MΩ	0.0001 MΩ	0.6 + 5	0.6 + 5	0.6 + 5	93 nA/10 MΩ
	30 MΩ	0.001 MΩ	1.2 + 5	1.2 + 5	1.2 + 5	93 nA/10 MΩ
	100 MΩ	0.01 MΩ	2.0 +10	—	—	93 nA/10 MΩ
	300 MΩ	0.01 MΩ	—	2.0 + 10 @ < 100 MΩ 8.0 + 10 @ > 100 MΩ	2.0 + 10 @ < 100 MΩ 8.0 + 10 @ > 100 MΩ	93 nA/10 MΩ
	300 nS	0.01 nS	1 + 10	1 + 10	1 + 10	93 nA/10 MΩ
Current[3]	300 µA	0.01 µA	0.2 + 5	0.2 + 5	0.2 + 5	< 0.04 V/100Ω
	3000 µA	0.1 µA	0.2 + 5	0.2 + 5	0.2 + 5	< 0.4 V/100 Ω
	30 mA	0.001 mA	0.2 + 5	0.2 + 5	0.2 + 5	< 0.08 V/1 Ω
	300 mA	0.01 mA	0.2 + 5	0.2 + 5	0.2 + 5	< 1.00 V/1 Ω
	3 A	0.0001 A	0.3 + 10	0.3 + 10	0.3 + 10	< 0.1 V/0.01 Ω
	10 A	0.001 A	0.3 + 10	0.3 + 10	0.3 + 10	< 0.3 V/0.01 Ω
	3 V	0.0001 V	0.5 + 5	[illegible]	0.5 + 5	Approximately

Figure 6: Uncertainty calculation of the devices' reading, more specifically used to calculate the uncertainties of the measured voltage and current.