IS5152 Assignment 2 Wang Ruicong A0244297W

1、(a) primal QP :

minimize $\frac{1}{2}(w_1^2+w_2^2) = \frac{1}{2}\|w\|^2$

subject to

$$5w_1+2w_2+b \le -1$$
$$6w_1+2w_2+b \le -1$$
$$6w_1+3w_2+b \le -1$$
$$3w_1+3w_2+b \le -1$$
$$8w_1+2w_2+b \ge 1$$
$$8w_1+4w_2+b \ge 1$$
$$5w_1+6w_2+b \ge 1$$
$$6w_1+4w_2+b \ge 1$$

(b) $w = \sum_{i=1}^{8} \alpha_i d_i x_i = -\alpha_1\begin{pmatrix}5\\2\end{pmatrix} - \alpha_2\begin{pmatrix}6\\2\end{pmatrix} - \alpha_3\begin{pmatrix}6\\3\end{pmatrix} - \alpha_4\begin{pmatrix}3\\3\end{pmatrix} + \alpha_5\begin{pmatrix}8\\2\end{pmatrix} + \alpha_6\begin{pmatrix}8\\4\end{pmatrix} + \alpha_7\begin{pmatrix}5\\6\end{pmatrix} + \alpha_8\begin{pmatrix}6\\4\end{pmatrix}$

$\sum_{i=1}^{8} \alpha_i d_i = 0 \Rightarrow -\alpha_1-\alpha_2-\alpha_3-\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8=0$

(c) x_3, x_5, x_8 are support vectors

$\therefore \begin{cases} 6w_1+3w_2+b=-1 \\ 8w_1+2w_2+b=1 \\ 6w_1+4w_2+b=1 \end{cases} \Rightarrow \begin{cases} w_1=2 \\ w_2=2 \\ b=-19 \end{cases} \Rightarrow$ The optimal hyperplane is $2x_1+2x_2-19=0$

(d) The support vectors are x_3, x_5, x_8, thus $\alpha_1=\alpha_2=\alpha_4=\alpha_6=\alpha_7=0$

~~and~~ $\begin{cases} -\alpha_3+\alpha_5+\alpha_8=0 \\ -6\alpha_3+8\alpha_5+6\alpha_8=2 \\ -3\alpha_3+2\alpha_5+4\alpha_8=2 \end{cases} \Rightarrow \begin{cases} \alpha_3=4 \\ \alpha_5=1 \\ \alpha_8=3 \end{cases}$

Therefore, $\alpha_1=0$ $\alpha_2=0$ $\alpha_3=4$ $\alpha_4=0$ $\alpha_5=1$ $\alpha_6=0$ $\alpha_7=0$ $\alpha_8=3$ satisfy all optimal conditions

Thus, the hyperplane $2x_1+2x_2-19=0$ is indeed optimal

2、 $K(x, x_i) = (x^T x_i + 1)^2 = (x_1 x_{i1} + x_2 x_{i2} + 1)^2 = 1 + x_1^2 x_{i1}^2 + 2x_1x_2x_{i1}x_{i2} + x_2^2 x_{i2}^2 + 2x_1x_{i1} + 2x_2x_{i2}$

$\therefore \Phi(x) = [1, x_1^2, \sqrt{2}x_1x_2, x_2^2, \sqrt{2}x_1, \sqrt{2}x_2]^T$

$\Phi(x_i) = [1, x_{i1}^2, \sqrt{2}x_{i1}x_{i2}, x_{i2}^2, \sqrt{2}x_{i1}, \sqrt{2}x_{i2}]^T$

$k(i,j) = (x_i^T x_j + 1)^2$

$\cancel{K(i,j) = k(x_i, x_j)}$

$\therefore \Phi(x_1) = [1, 0, 0, 1, 0, \sqrt{2}]^T$

$\Phi(x_2) = [1, 1, 0, 0, \sqrt{2}, 0]^T$

$\Phi(x_3) = [1, 0, 0, 1, 0, -\sqrt{2}]^T$

$\Phi(x_4) = [1, 1, 0, 0, -\sqrt{2}, 0]^T$

$$\Rightarrow K = \begin{bmatrix} 4 & 1 & 0 & 1 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 1 & 0 & 1 & 4 \end{bmatrix}$$

$Q(\alpha) = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 - \frac{1}{2}(4\alpha_1^2 + 4\alpha_2^2 + 4\alpha_3^2 + 4\alpha_4^2 - 2\alpha_1\alpha_2 - 2\alpha_1\alpha_4 - 2\alpha_2\alpha_3 - 2\alpha_3\alpha_4)$

Take the derivative of $Q(\alpha)$ with respect to $\alpha_1, \alpha_2, \alpha_3, \alpha_4$

$$\begin{cases} 1 - 4\alpha_1 + \alpha_2 + \alpha_4 = 0 \\ 1 - 4\alpha_2 + \alpha_1 + \alpha_3 = 0 \\ 1 - 4\alpha_3 + \alpha_2 + \alpha_4 = 0 \\ 1 - 4\alpha_4 + \alpha_1 + \alpha_3 = 0 \end{cases} \Rightarrow \begin{cases} \alpha_1 = \frac{1}{2} \\ \alpha_2 = \frac{1}{2} \\ \alpha_3 = \frac{1}{2} \\ \alpha_4 = \frac{1}{2} \end{cases}$$

$w = \sum_{i=1}^{4} \alpha_i d_i \Phi(x_i) = \alpha_1\Phi(x_1) - \alpha_2\Phi(x_2) + \alpha_3\Phi(x_3) - \alpha_4\Phi(x_4)$

$= \frac{1}{2}[0, -2, 0, 2, 0, 0]^T = [0, -1, 0, 1, 0, 0]^T$

The boundary is $-x_1^2 + x_2^2 = 0$

$w^T\Phi(x_1) = 1 > 0$ (√)

$w^T\Phi(x_2) = -1 < 0$ (√)

$w^T\Phi(x_3) = 1 > 0$ (√)

$w^T\Phi(x_4) = -1 < 0$ (√)

$w^T\Phi(x) > 0$ for class 1 samples, $w^T\Phi(x) < 0$ for class 2 samples

Therefore, the boundary $-x_1^2 + x_2^2 = 0$ is correct

3、(a) primal: max $z = 35t_1 + 20t_2$

s.t. $40t_1 + 4t_2 - 10w_1 - 20w_2 \le 0$

$35t_1 + 20t_2 - 20w_1 - 24.5w_2 \le 0$

$20t_1 + 36t_2 - 10w_1 - 12w_2 \le 0$

$20w_1 + 24.5w_2 = 1$

$t_1, t_2, w_1, w_2 \ge 0$

dual: min ε

s.t. $10\lambda_1 + 20\lambda_2 + 10\lambda_3 - 20\varepsilon \le 0$

$20\lambda_1 + 24.5\lambda_2 + 12\lambda_3 - 24.5\varepsilon \le 0$

$40\lambda_1 + 35\lambda_2 + 20\lambda_3 - 35 \ge 0$

$4\lambda_1 + 20\lambda_2 + 36\lambda_3 - 20 \ge 0$

$\lambda_1, \lambda_2, \lambda_3 \ge 0$

(b) we get $t_1 = 0.02$ $t_2 = 0.0025$ $w_1 = 0.001$ $w_2 = 0.04$

The efficiency is $\dfrac{35t_1 + 20t_2}{20w_1 + 24.5w_2} = \dfrac{0.7 + 0.05}{0.02 + 0.98} = 0.75 < 1$

Thus, we can conclude that center 2 is not efficient

(c) ① Reduce the number of tutors or reduce the rental cost

② Increase the number of students either in primary school or secondary school

4. (a) LP:

maximize $0.012x_1 + 0.014x_2 + 0.015x_3 + 0.017x_4 + 0.018x_5 + 0.02x_6 + 0.023x_7$

subject to

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 20$$
$$x_7 \le 2$$
$$x_1 + x_2 + x_4 \ge 10$$
$$x_1 + x_5 + x_7 \le 6$$
$$x_3 + x_6 \le x_1 + x_4$$
$$0 \le x_1, x_2, x_3, x_4, x_5, x_6, x_7 \le 8$$

(b) (i) $P_1 = 2P_4$ $P_2 = 2P_1$ $P_3 = P_4$ let $P_4 = 1$. $P_1 = 2$ $P_2 = 4$ $P_3 = 1$ $P_4 = 1$

LP: minimize $2S_1^- + 4S_2^- + S_3^- + S_4^-$

subject to

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 = 20$$
$$x_7 \le 2$$
$$x_1 + x_2 + x_4 \ge 10$$
$$x_1 + x_5 + x_7 \le 6$$
$$x_3 + x_6 \le x_1 + x_4$$
$$0 \le x_1, x_2, x_3, x_4, x_5, x_6, x_7 \le 8$$
$$0.012x_1 + 0.014x_2 + 0.015x_3 + 0.017x_4 + 0.018x_5 + 0.02x_6 + 0.023x_7 + S_1^- - S_1^+ = 0.36$$
$$x_2 + S_2^- - S_2^+ = 2$$
$$x_1 + S_3^- - S_3^+ = 1$$
$$x_1 + x_3 + S_4^- - S_4^+ = 4$$
$$S_1^-, S_1^+, S_2^-, S_2^+, S_3^-, S_3^+, S_4^-, S_4^+ \ge 0$$

(ii)

$x_1 = 1$, $x_2 = 2$, $x_3 = 3$, $x_4 = 7$, $x_5 = 0$, $x_6 = 5$, $x_7 = 2$

Goal 1: $0.012x_1 + 0.014x_2 + 0.015x_3 + 0.97x_4 + 0.018x_5 + 0.02x_6 + 0.023x_7 = 0.35 < 0.36$ (not achieve)

Goal 2: $x_2 = 2$ (≥ 2) (achieve)

Goal 3: $x_1 = 1$ (≥ 1) (achieve) Therefore, only Goal 1 is not achieved.

Goal 4: $x_1 + x_3 = 4$ (≥ 4) (achieve)

(c) (i) minimize $P_1 S_1^- + P_2 S_2^- + P_3 S_3^- + P_4 S_4^-$ where $P_1 >>> P_2 >>> P_3 >>> P_4$

(ii) Yes, the solution will be different.

The reason is that for (b-ii). Goal 1 is not achieved and the other three are achieved. For (c), Goal 1 has the highest priority, thus it should be achieved first compared with the other goals, thus the solution must be different.