IS5152 Data-driven Decision Making
SEMESTER 2, 2022-2023
Assignment 1
Due: Friday, 3 February 2023, 11.59pm

Instructions:

- Upload your answer as a single pdf file to Canvas.
- Name your file according to your student ID number, e.g. A1234567X.pdf.

1. (10 points) A company manufactures two products (1 and 2). Each unit of product 1 can be sold for \$15, and each unit of product 2 for \$25. Each product requires raw material and two types of labor (skilled and unskilled) as shown in the table below:

	Product 1	Product 2
Skilled labor	3 hours	4 hours
Unskilled labor	2 hours	3 hours
Raw material	1 unit	2 units

   At present, the company has available 100 hours of skilled labor, 70 hours of unskilled labor, and 30 units of raw material. Because of marketing considerations at least 3 units of product 2 must be produced.

   (a) (2 points) Let x_1 and x_2 be the number of units of product 1 and product 2 to be produced, respectively. Formulate a linear program to maximize total revenue.

   (b) (2 points) Show that the best production level is $x_1 = 24$ and $x_2 = 3$ by checking that all the necessary and sufficient conditions are satisfied.

   (c) (2 points) How much would the company be willing to pay for an additional unit of each type of labor?

   (d) (4 points) State the dual of the linear program from part (a). What is the solution of this dual linear program?

2. (10 points) Find the solution of the quadratic programming problem:

$$\min \ 2x_1^2 - x_2$$

   subject to

$$\begin{aligned} 2x_1 - x_2 &\leq 1 \\ x_1 + x_2 &\leq 1 \\ x_1, x_2 &\geq 0 \end{aligned}$$

   Show that all the necessary and sufficient optimality conditions are satisfied.

3. (10 points) Consider the following linearly constrained optimization problem:

$$\text{maximize } f(x) = -\ln(x_1 + 1) - x_2^2$$

subject to

$$\begin{aligned} x_1 + 2x_2 &\leq 3 \\ x_1, x_2 &\geq 0 \end{aligned}$$

where *ln* denotes natural logarithm.

(a) (5 points) Use the Kuhn-Tucker (KT) conditions to derive an optimal solution.

(b) (5 points) Is the solution you obtain in part (a) a global solution or a local solution? Explain your answer.