INDUSTRIAL ROBOT ARM

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Mathematical Model
Smart Methods

Problem Definition

We are attempting the mathematical models for the evaluation robot arm in order to be certain of the safe and danger regions, as well as the locations of the sensors, in place to ensure the optimal level of safety.

Variables

In 3D, we assumed the arm would move in three dimensions: x, y, and z and In 2D, therefore, x and y will be used.

$$F = \lambda(n - j - 1) + \sum_{i=1}^{j} f_i$$

Where F = number of DOF

n = number of links

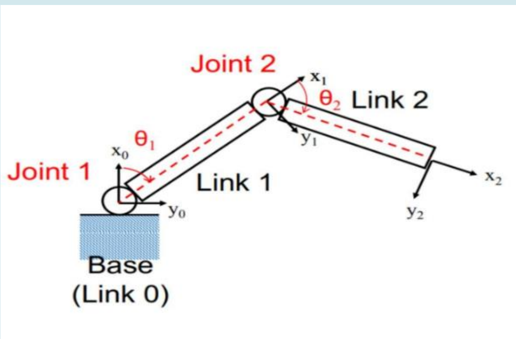
j = number of joints

λ = number of DOF in the space

f_i= number of DOF permitted by joint j_i

Operations

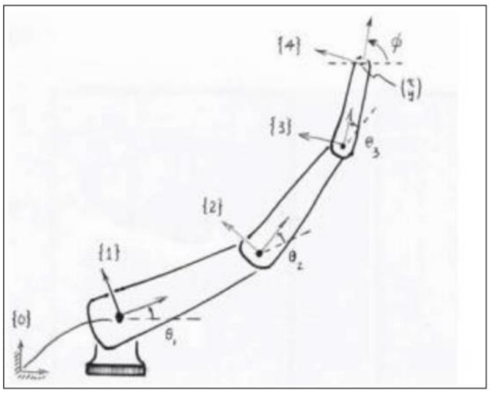
- The forward kinematics of a robot refers to the calculation of the position and orientation of its end-effector frame from its joint values.



Implementation

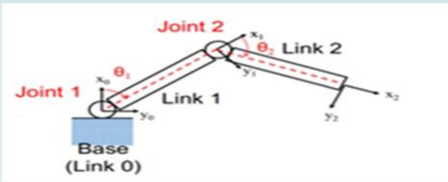
Basic trigonometry:

$$
\begin{aligned}
x &= L_1 \cos\theta_1 + L_2 \cos(\theta_1+\theta_2) + L_3 \cos(\theta_1+\theta_2+\theta_3) \\
y &= L_1 \sin\theta_1 + L_2 \sin(\theta_1+\theta_2) + L_3 \sin(\theta_1+\theta_2+\theta_3) \\
\phi &= \theta_1+\theta_2+\theta_3.
\end{aligned}
$$



Considering only the position of the end effector and ignoring its orientation, the forward kinematics can be expressed as:

$$
\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} L_1 \cos\theta_1 + L_2 \cos(\theta_1+\theta_2) \\ L_1 \sin\theta_1 + L_2 \sin(\theta_1+\theta_2) \end{bmatrix}.
$$



Inverse Kinematic:

Law of cosines:

$L1^2 + L2^2$ **− 2.L1.L2.cos β =** $x^2 + y^2$

from which it follows that:

$\beta = \arccos(\frac{L1^2 + L2^2 - x^2 - y^2}{2L1.L2})$ **So θ2 = π − β**

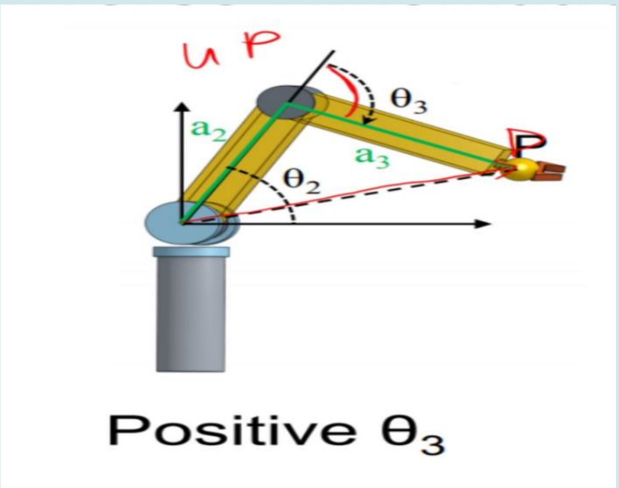
Also from the law of cosines,

$\alpha = \arccos(\frac{L1^2 - L2^2 + x^2 + y^2}{2L1.\sqrt{x^2+y^2}})$

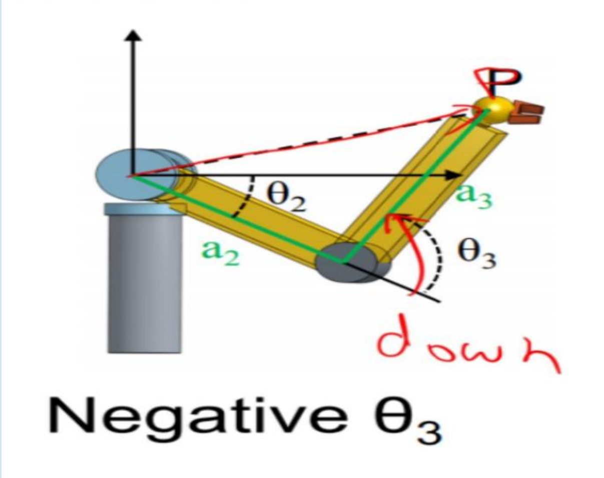
and since **tan(θ1 + α) = y/x** **θ1 =** $tan^{-1}(\frac{y}{x})$ **− α.**

Two solutions

The first solution



The second solution



SOLUTIONS METHODS

1-ANALYTICAL solution

- preferred, if it can be found
- use ad-hoc geometric inspection

2- NUMERICAL solution

- slower, but easier to be set up
- not a big deal to modern computer

WORKSPACE

- Workspace = { (x,y, z)| IK is available}