Q1

- $(ab + a) \cdot (1 + b) \leftarrow ab$

(Posix) $\boxplus$ Sequ (Left (Sequ (Char(a), Char(b))), Left (Emty))

$\boxplus$ Sequ (Right (Char(a)), Right (Char (b)))

- $(aa + a)^* \leftarrow aaa$

(Posix) $\boxplus$ Stars ([Left (Seq (Char(a), Char(a))), Right (Char(a))])

$\boxplus$ Stars ([Right (Char(a))])

$\boxplus$ Stars ([Right (Char(a)), Left (Sequ (Char(a), Char(a)))])

Q3

b.

Digits := RANGE ("0123456789")

START_Digits := RANGE ("123456789")

NUMBERS := Digits + (START_Digits . Digits*)

bcz → only 0 start or double digit non-0 start

LEFT := CHAR ('(')

RIGHT := CHAR (')')

ALPHABETS := RANGE ("abcdefgh .... xyz")

Id := ALPHABETS . ALPHABETS*

OPERATIONS := CHAR('+') + CHAR('-') + CHAR('*')

LANG = ("num" : NUMBERS) + ("id" : Id) +
("op" : OPERATIONS) + ("lp" : LEFT) + ("rp" : RIGHT)

(a+3)*b Yes → lp:(, id:a, op:+, num:3, rp:), op:*, id:b

)()++-33 yes → rp:), lp:(, rp:), op:+, op:+, op:-, num:33

(a/3)*3 No → there is no "/" in the language.

Q4

r is nullable

$$1 + r + r \cdot r \equiv r \cdot r$$

$$\hookrightarrow r + r \cdot r \equiv r \cdot r$$

because r is nullable

$$(r + r) \cdot r = r \cdot r$$

$$\begin{aligned} &((1 + r) + r) \cdot r \\ &= ((r) + r) \cdot r \\ &= (r + r) \cdot r \\ &= r \cdot r \end{aligned}$$

Q6

$$/ \cdot * \cdot \left(\sim (* \cdot /) \cdot \text{ALL} * \right) \cdot * \cdot /$$

$$/ \cdot * \cdot \left(\sim \left(\text{ALL}^* \cdot * \cdot / \cdot \text{ALL}^* \right) \right) \cdot * \cdot /$$

Q7

$$(0 \cdot (b \cdot c)) + ((0 \cdot c) + 1)$$

$$\Rightarrow 0 + (0 + 1)$$

$$\Rightarrow 0 + 1$$

$$\Rightarrow 1$$

Q8

- $(0 \cdot (b \cdot c)) + ((a \cdot c) + 1)$

  Right (Right (Empty))

- $(a + 1) \cdot (1 + 1)$

  Seq (Right (Empty), Left (Empty))

- a^*

  Stars (List()) / Stars (Nil)

Q9

When tokenizing an expression (e.g. splitting into its component words), the record expression is used for classying these tokens.

e.g. when lexing a block of code, we can produce a resulting expression of records which label each (notable) sub-expression with the token they matched.

Q10

$$
\begin{array}{lcl}
\mathit{nullable}(\mathbf{0}) & \stackrel{\text{def}}{=} & \mathit{false} \\
\mathit{nullable}(\mathbf{1}) & \stackrel{\text{def}}{=} & \mathit{true} \\
\mathit{nullable}(c) & \stackrel{\text{def}}{=} & \mathit{false} \\
\mathit{nullable}(r_1 + r_2) & \stackrel{\text{def}}{=} & \mathit{nullable}(r_1) \vee \mathit{nullable}(r_2) \\
\mathit{nullable}(r_1 \cdot r_2) & \stackrel{\text{def}}{=} & \mathit{nullable}(r_1) \wedge \mathit{nullable}(r_2) \\
\mathit{nullable}(r^*) & \stackrel{\text{def}}{=} & \mathit{true}
\end{array}
\qquad
\begin{array}{lcl}
\mathit{zeroable}(\mathbf{0}) & \stackrel{\text{def}}{=} & \mathit{true} \\
\mathit{zeroable}(\mathbf{1}) & \stackrel{\text{def}}{=} & \mathit{false} \\
\mathit{zeroable}(c) & \stackrel{\text{def}}{=} & \mathit{false} \\
\mathit{zeroable}(r_1 + r_2) & \stackrel{\text{def}}{=} & \mathit{zeroable}(r_1) \wedge \mathit{zeroable}(r_2) \\
\mathit{zeroable}(r_1 \cdot r_2) & \stackrel{\text{def}}{=} & \mathit{zeroable}(r_1) \vee \mathit{zeroable}(r_2) \\
\mathit{zeroable}(r^*) & \stackrel{\text{def}}{=} & \mathit{true}
\end{array}
$$

$$
\begin{array}{lcl}
\mathit{atmostempty}(\mathbf{0}) & \stackrel{\text{def}}{=} & \mathit{true} \\
\mathit{atmostempty}(\mathbf{1}) & \stackrel{\text{def}}{=} & \mathit{true} \\
\mathit{atmostempty}(c) & \stackrel{\text{def}}{=} & \mathit{false} \\
\mathit{atmostempty}(r_1 + r_2) & \stackrel{\text{def}}{=} & \mathit{atmostempty}(r_1) \vee \mathit{atmostempty}(r_2) \\
\mathit{atmostempty}(r_1 \cdot r_2) & \stackrel{\text{def}}{=} & \mathit{zeroable}(r_1) \vee \mathit{zeroable}(r_2) \vee (\mathit{atmostempty}(r_1) \wedge \mathit{atmostempty}(r_2)) \\
\mathit{atmostempty}(r^*) & \stackrel{\text{def}}{=} & \mathit{atmostempty}(r)
\end{array}
$$

$$
\begin{array}{lcl}
\mathit{somechars}(\mathbf{0}) & \stackrel{\text{def}}{=} & \mathit{false} \\
\mathit{somechars}(\mathbf{1}) & \stackrel{\text{def}}{=} & \mathit{false} \\
\mathit{somechars}(c) & \stackrel{\text{def}}{=} & \mathit{true} \\
\mathit{somechars}(r_1 + r_2) & \stackrel{\text{def}}{=} & \mathit{somechars}(r_1) \vee \mathit{somechars}(r_2) \\
\mathit{somechars}(r_1 \cdot r_2) & \stackrel{\text{def}}{=} & (\neg \mathit{zeroable}(r_1) \wedge \mathit{somechars}(r_2)) \vee \\
 & & (\mathit{somechars}(r_1) \wedge \neg \mathit{zeroable}(r_2)) \vee \\
 & & (\mathit{nullable}(r_1) \wedge \mathit{somechars}(r_2)) \vee \\
 & & (\mathit{somechars}(r_1) \wedge \mathit{nullable}(r_2)) \\
\mathit{somechars}(r^*) & \stackrel{\text{def}}{=} & \mathit{somechars}(r)
\end{array}
$$

$$
\begin{array}{lcl}
\mathit{infinitestrings}(\mathbf{0}) & \stackrel{\text{def}}{=} & \mathit{false} \\
\mathit{infinitestrings}(\mathbf{1}) & \stackrel{\text{def}}{=} & \mathit{false} \\
\mathit{infinitestrings}(c) & \stackrel{\text{def}}{=} & \mathit{false} \\
\mathit{infinitestrings}(r_1 + r_2) & \stackrel{\text{def}}{=} & \mathit{infinitestrings}(r_1) \vee \mathit{infinitestrings}(r_2) \\
\mathit{infinitestrings}(r_1 \cdot r_2) & \stackrel{\text{def}}{=} & (\neg \mathit{zeroable}(r_1) \wedge \mathit{infinitestrings}(r_2)) \vee (\neg \mathit{zeroable}(r_2) \wedge \mathit{infinitestrings}(r_1)) \\
\mathit{infinitestrings}(r^*) & \stackrel{\text{def}}{=} & \mathit{somechars}(r)
\end{array}
$$