Q1

Firstly, we need to consider if P(r) holds when r = 0, r = 1 and r = c.
Secondly, we need to show P (r) holds when r = r1 + r2, assuming P (r1) and P (r2) both hold(IH). Then, we need to prove P (r) holds when r = r1 · r2, assuming P (r1) and P (r2) both hold(IH). Finally, we need to show P(r∗) holds, assuming P(r) holds(IH).

Base Cases: $0 \mid 1 \mid c$

- $P(0)$

$$nullable(0) \rightarrow [\,] \in L(0)$$
$$false \rightarrow false$$

- $P(1)$

$$nullable(1) \rightarrow [\,] \in L(1)$$
$$[\,] \in \{[\,]\}$$
$$true \rightarrow true$$

- $P(c)$

$$nullable(c) \rightarrow [\,] \in L(c)$$
$$[\,] \in \{[c]\}$$
$$false \rightarrow false$$

- $P(r^*)$

$$nullable(r^*) \text{ iff } [\,] \in L(r^*)$$

$$P(r) : nullable(r) \text{ iff } [\,] \in L(r) \;\Big|\; IH$$

$$P(r^*) : nullable(r^*) \longrightarrow [\,] \in L(r^*)$$
$$[\,] \in (L(r))^*$$
$$true \longrightarrow true$$

Note: The IH wasn't even used here.

- $P(r_1 + r_2)$

$$nullable(r_1 + r_2) \text{ iff } [\,] \in L(r_1 + r_2)$$

$$\left.\begin{array}{l} P(r_1): nullable(r_1) \text{ iff } [\,] \in L(r_1) \\ P(r_2): nullable(r_2) \text{ iff } [\,] \in L(r_2) \end{array}\right\} IH$$

$$P(r_1+r_2): nullable(r_1 + r_2) \longrightarrow [\,] \in L(r_1 + r_2)$$
$$nullable(r_1) \vee nullable(r_2) \qquad [\,] \in L(r_1) \cup L(r_2)$$
$$\text{from(IH)} \quad [\,] \in L(r_1) \vee [\,] \in L(r_2) \qquad [\,] \in L(r_1) \vee [\,] \in L(r_2)$$

- $P(r_1 \cdot r_2)$

$$nullable(r_1 \cdot r_2) \text{ iff } [\,] \in L(r_1 \cdot r_2)$$

$$\left.\begin{array}{l} P(r_1): nullable(r_1) \rightarrow [\,] \in L(r_1) \\ P(r_2): nullable(r_2) \rightarrow [\,] \in L(r_2) \end{array}\right\} IH$$

$$P(r_1 \cdot r_2): nullable(r_1 \cdot r_2) \longrightarrow [\,] \in L(r_1 \cdot r_2)$$
$$nullable(r_1) \wedge nullable(r_2) \qquad [\,] \in L(r_1) \,@\, L(r_2)$$
$$\text{from(IH)} \quad [\,] \in L(r_1) \wedge [\,] \in L(r_2) \longrightarrow [\,] \in L(r_1) \wedge [\,] \in L(r_2)$$

Q2

~0

Q3

r^+ = r·r*
r ^ ? = 1+r r{n} =

ifn=0, r{0}=1 ifn=1, r{1}=r
...
if n = n+1, r{n+1}= r·r^n

r{m,n}=
for m = 0, n = 0, r{0,0} = 1

for m = n, r{m,n} = r{n}
for m < n, r{m}+r{m +1}......+r{n}

Q4

Yes, yes, no

Q5

$$S := A \cdot S \cdot B \mid B \cdot S \cdot A \mid \epsilon$$

$$A := a \mid \epsilon$$

$$B := b$$

a	No	ba	Yes
b	Yes	bb	Yes
ab	Yes	baa	No

Q6
All even-length palindromes over the alphabet {a,b}