ENPH 270

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Fundamentals

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

Center of mass

$$\bar{x} = \frac{\int_m x\,dm}{\int_m dm} \qquad \bar{y} = \frac{\int_m y\,dm}{\int_m dm}$$

Rolling Without Slipping when A is the ICR, the velocity and acceleration of the center of mass, G

$$\mathbf{v}_G = \boldsymbol{\omega} \times \mathbf{r}_{G/A}$$
$$\mathbf{a}_G = \boldsymbol{\alpha} \times \mathbf{r}_{G/A}$$

If $\mathbf{a}_{\text{ground}} = 0$, at the contact point A:

$$v_A = 0$$
$$a_A = -\omega^2 r_{A/G}$$

The acceleration of the contact point is zero along the direction of motion, but has a non-zero component towards to center G. If $\mathbf{a}_{\text{ground}} \neq 0$, then we cannot use $a_G = \alpha r_{G/A}$. Instead

$$a_{\text{ground}} = a_A$$

and

$$\mathbf{a}_G = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{G/A} - \underbrace{\omega^2 \mathbf{r}_{G/A}}_{\text{may be 0}}$$

Force and Acceleration

Mass moment of Inertia

$$I = \int_m r^2\,dm = \iint \rho r^2\,dA \qquad [\text{kg}\cdot\text{m}^2]$$

$I = mr^2$ Thin Ring $\qquad I = \frac{1}{2}mr^2$ Disk

$I = \frac{1}{12}ml^2$ Slender Rod

Parallel Axis Theorem I about an axis passing through a point other than the center of mass G is I about G plus the mass of the body times the distance between the parallel axes

$$I = I_G + md^2$$

Radius of Gyration

$$k = \sqrt{I/m} \qquad I = mk^2$$

Translational Equation of Motion For a body symmetric about a plane, the planar equation of motion is

$$\sum \mathbf{F} = m\mathbf{a}_G$$

Rotational Equation of Motion About an arbitrary point P, the sum of moments is

$$\begin{aligned}\sum \mathbf{M}_P &= \mathbf{r}_{G/P} \times m\mathbf{a}_P + \boldsymbol{\alpha} I_P \\ &= \mathbf{r}_{G/P} \times m\mathbf{a}_G + \boldsymbol{\alpha} I_G\end{aligned}$$

Scalar form:

$$\begin{aligned}\sum M_P &= -m(a_P)_x \int_m y\,dm + m(a_P)_y \int_m x\,dm + \alpha I_P \\ &= -m(a_P)_x \bar{y} + m(a_P)_y \bar{x} + \alpha I_P \\ &= -m(a_G)_x \bar{y} + m(a_G)_y \bar{x} + \alpha I_G \\ &= \sum (\mathcal{M}_k)_P\end{aligned}$$

If $P = G$, then

$$\sum M_G = \alpha I_G$$

Rotation about a Fixed Axis

$$\sum F_n = m(a_G)_n = m\omega^2 r_{G/O}$$
$$\sum F_t = m(a_G)_t = m\alpha r_{G/O}$$
$$\sum M_G = \alpha I_G$$

About the axis of rotation O:

$$\sum M_O = \alpha I_O$$

Work and Energy

Kinetic energy of a rigid body is the sum of its translational and rotational kinetic energy

$$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$$

If the body is rotating about axis A, then

$$T = \frac{1}{2}I_A\omega^2$$

Work done by a couple moment

$$W_{\text{couple}} = \int_{\theta_1}^{\theta_2} M\,d\theta$$

Principle of Work and energy Final total energy (kinetic energy T plus the potential energy V) of the system is the sum of the energy and the work done by non conservative forces.

$$T_1 + V_1 + \left(\sum U_{1-2}\right)_{\text{noncon}} = T_2 + V_2$$