ELEC 204

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Fundamentals

i is the rate of change in charge:

$$i = \frac{dq}{dt}$$

v is the energy required to move a positive unit charge through a circuit element. Charge moves from high to low potential

$$v = \frac{dW}{dq}$$

p is rate of change in energy per unit time

$$p = \frac{dW}{dt} = \frac{dW}{dq}\frac{dq}{dt} = vi$$

R is the ability to resist current

$$R \propto \rho\frac{l}{A} \qquad l\text{: Length of resistor; } A\text{: Cross section area}$$

G is the ability to conduct current

$$G = \frac{1}{R} \qquad \text{Units: Siemens [S]}$$

Node: Point where two or more elements join
Loop: Path whose last node is starting node
Mesh: A loop that does not enclose any other loops
Zeros: Root of the numerator
Pole: Root of the denominator

Cramer's Method

$$\mathbf{M} = \left(\begin{array}{cc|c} a+jb & \alpha & V_1 \\ \beta & c+jd & V_2 \end{array}\right)$$

$$V_1 = \frac{1}{\det \mathbf{M}}\begin{bmatrix} V_1 & \alpha \\ V_2 & c+jd \end{bmatrix} \qquad V_2 = \frac{1}{\det \mathbf{M}}\begin{bmatrix} a+jb & V_1 \\ \beta & V_2 \end{bmatrix}$$

Decibel Scale

$$\log 1 = 0 \qquad \log AB = \log A + \log B$$

$$\log\frac{A}{B} = \log A - \log B \qquad \log A^n = n\log A$$

The dB is used for measuring the ratio of variables of the same unit. Use the **magnitude** of quantities.

$$\text{1 Decibel: } 10\log_{10}\frac{A}{B}$$

Passive Sign Convention

Given the assumption that current flows from $+$ to $-$. Current that flow from $-$ to $+$ are multiplied -1

$$\text{Passive Component: Absorbing Power} \quad P \geq 0$$

$$\text{Active Component: Delivering Power} \quad P \leq 0$$

Tellegan's Theorem

Power consumed and produced by all elements in a circuit sum to zero at all time

$$\sum P = 0$$

KCL

Sum of currents entering any closed boundary is zero

KVL

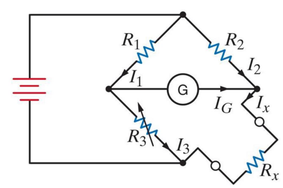
Sum of voltage drops around any closed path is zero

Resistors

Parallel Resistors

$$\frac{1}{R_{eq}} = \sum_{i=1}^{n}\frac{1}{R_i} \qquad G_{eq} = \sum_{i}^{n} G_i$$

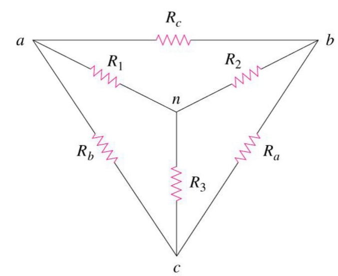
Wheatstone Bridge



Balanced when:

$$\frac{R_1}{R_3} = \frac{R_2}{R_x}$$

Y Δ Transformation



Let $R_a \leftrightarrow R_1$, $R_2 \leftrightarrow R_2$, $R_3 \leftrightarrow R_3$
Notice the pattern in the numerator and the denominator

$$R_1 = \frac{R_bR_c}{R_a + R_b + R_c} \qquad R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

If R values in the Y or Δ network are equal, then

$$R_Y = \frac{R_\Delta}{3} \qquad R_\Delta = 3R_Y$$

Nodal Analysis

1. Simplify the circuit if possible
Choose a node for ground which minimizes the number of floating sources
2. Construct KCL for all leftover nodes. Pure assign a current variable to each floating source
3. Use the convention that $\Delta V = V_{\text{start}} - V_{\text{end}}$
4. Solve equations

Superposition

1. Turn off all independent source, except one.

$$V_{\text{source}} \rightarrow \text{Ideal wire} \qquad I_{\text{source}} \rightarrow \text{Open circuit}$$

2. Calculate current or voltage contribution due to that source
3. Repeat for all independent sources
4. Sum up calculated values

Source Transformation

We can transform any voltage source in series with a resistor to a current source in parallel with that same resistor

$$V \rightarrow I \qquad I = VR$$

$$I \rightarrow V \qquad V = \frac{I}{R}$$

Thevenin's Theorem

A linear two terminal network can be transformed into one involving a resistor and a voltage source

R_{th} represents: Dependent sources, resistors
V_{th} represents: Independent sources

Methods for R_{th}

1. Turn off all independent source. Find the equivalent resistance. If there are dependent sources, then apply V_{test} or I_{test} at the terminal, and

$$\frac{V_{\text{terminal}}}{I_{\text{test}}} \text{ or } \frac{V_{\text{test}}}{I_{\text{terminal}}} = R_{th}$$

2. Given V_{th} and find I_{sc} we can find R_{th} (Without turning any sources off)
3. $1A2A$ method (V_{th} we can find R_{th} by solving only one equation in the $1A2A$ method)

Methods for V_{th}

1. $V_{oc} = V_{th}$ without turning off any sources
2. Find I_{sc} and R_{th} and apply ohm's law
3. $1A2A$ method (R_{th} we can find V_{th} by solving only one equation in the $1A2A$ method)

$1A2A$ Method

Find the voltage drop across a $1\,A$ source (V_1) and a $2\,A$ source (V_2), and

$$V_{th} + R_{th} = V_1 \qquad V_{th} + 2R_{th} = V_2$$

Maximum Power Transfer in Thevenin Circuit

$$p = i^2R_L = \left(\frac{V_{th}}{R_{th} + R_L}\right)^2 R_L \qquad p_{max} \text{ when } R_L = R_{th}$$

First Order Circuits

The number of capacitors and inductors that cannot be combined tells the order of the circuit.

Natural Response: No sudden application of DC source
Stepped Response: Sudden switching

Capacitors

$$q = CV_c \qquad i_c(t) = C\frac{dV_c}{dt} \qquad E_c(t) = \frac{1}{2}CV_c^2(t)$$

$$\text{In parallel: } C_{eq} = \sum_i^n C_i \qquad \text{In series: } \frac{1}{C_{eq}} = \sum_i^n \frac{1}{C_i}$$

RC Natural Response:

Capacitor Property: $v(0^-) = v(0) = v(0^+)$

Assuming $v_c(0) = V_o$

$$v_c(t) = V_o e^{\frac{-t}{\tau}} \qquad \tau = RC \qquad v_c(t) \approx 0 \text{ after } 5\tau$$

Inductors

$$v_L(t) = L\frac{di}{dt} \qquad E_L = \frac{1}{2}Li_L^2(t)$$

$$\text{In parallel: } \frac{1}{L_{eq}} = \sum_i^n \frac{1}{L_i} \qquad \text{In series: } L_{eq} = \sum_i^n C_i$$

RL Natural Response:

Inductor Property: $i(0^-) = i(0^+)$

Assuming $i(0) = I_o$

$$i_l(t) = I_o e^{\frac{-t}{\tau}} \qquad \tau = \frac{L}{R}$$

DC Step Response

For a step applied at $t = t_o$, and condition at time $t = t_0^+$

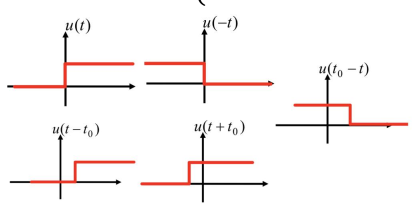
$$x(t) = [x(t_0^+) - x(\infty)]e^{-\frac{t-t_0}{\tau}} + x(\infty)$$

Second Order Circuits

Construct RCL series by using KVL, and RCL parallel by using KCL at top node.

Unit Step Function

$$u(t) = \begin{cases} 0 & t \leq 0 \\ 1 & t > 0 \end{cases}$$



Operation Amplifiers

Voltage output is limited by saturation

$$-V_{cc} \leq V_{out} \leq +V_{cc}$$

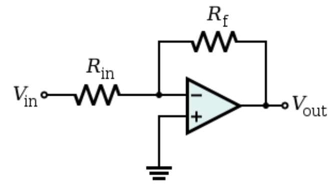
$-$ end is the inverting input, and $+$ end is the non-inverting input

An ideal opAmp has ∞ input impedance and 0 output impedance, and infinite gain

Given negative feedback, we can say

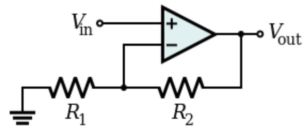
$$V_- = V_+ \qquad I_- = I_+ = 0$$

Inverting Amplifier



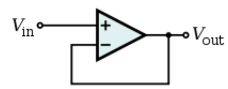
$$V_{out} = -\frac{R_f}{R_{in}}V_{in}$$

Non-Inverting Amplifier



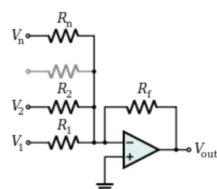
$$V_{out} = \left(1 + \frac{R_2}{R_1}\right)V_{in}$$

Buffer



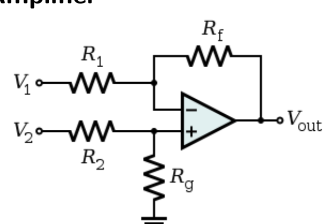
$$V_{out} = V_{in}$$

Summer



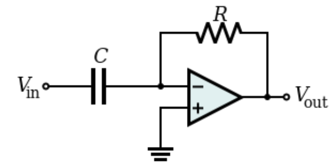
$$V_{out} = -R_f \sum_i^n \frac{V_i}{R_i}$$

Difference Amplifier



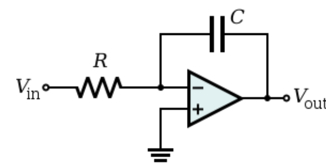
$$V_{out} = \frac{(R_f + R_1)R_g}{(R_g + R_2)R_1}V_2 - \frac{R_f}{R_1}V_1$$

Differentiator



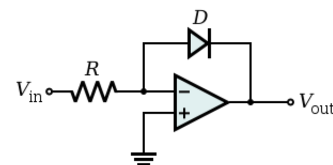
$$V_{out} = -RC\frac{dV_{in}}{dt}$$

Integrator



$$V_{out} = -\frac{1}{RC}\int v_i\, dt + V_{out}^{\text{initial}}$$

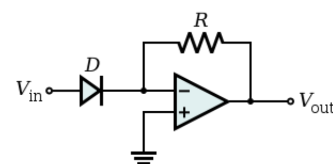
Logarithm



When $V_D > 0$, current through the diode is $I_D \approx I_S e^{\frac{V_D}{V_T}}$

$V_{out} = V_T \ln\left(\frac{V_{in}}{I_S R}\right)$ $\quad V_T$: Thermal Voltage; I_S: Saturation Current

Exponentiation

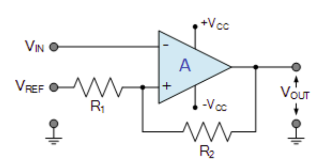


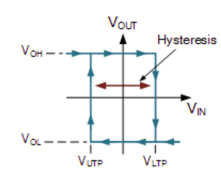
$V_{out} = -RI_S e^{\frac{V_{in}}{V_T}}$ $\quad V_T$: Thermal Voltage; I_S: Saturation Current

Comparator

$$V_{out} = \begin{cases} -V_{cc} & V_- > V_+ \\ V_{cc} & V_+ > V_- \end{cases}$$

A comparator with positive feedback





Identities

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$
$$\cos(-\alpha) = \cos(\alpha)$$
$$\sin(-\alpha) = -\sin(\alpha)$$
$$\cos(\omega t) = \sin(\omega t + \pi/2)$$
$$\sin(\omega t) = \cos(\omega t - \pi/2)$$
$$\cos(\omega t) = -\cos(\omega t \pm \pi)$$
$$\sin(\omega t) = -\sin(\omega t \pm \pi)$$
$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$
$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$
$$\cos(\alpha)\cos(\beta) = \cos(\alpha+\beta)/2 + \cos(\alpha-\beta)/2$$
$$\sin(\alpha)\cos(\beta) = \sin(\alpha+\beta)/2 + \sin(\alpha-\beta)/2$$

Combination of $\cos(\omega t)$ and $\sin(\omega t)$

$$A\sin(\omega t) + B\cos(\omega t) = \sqrt{A^2+B^2}\cos\left[\omega t - \tan(B/A)\right]$$

Phasors

$$X_M\cos(\omega t+\theta) = \text{Re}[X_M e^{j(\omega t+\theta)}] = X_M\angle\theta$$
$$|\mathbf{V}| = |V_M\angle\theta| = V_M$$

Multiplication by j shifts phasors by 90 degrees

$$jV\angle\theta = V\angle\theta + 90 = V\angle\theta + \frac{\pi}{2}$$

$$Z = X + jY = A\cos(\theta) + jB\sin(\theta) = A\angle\theta$$

Sinusoidal Steady State Analysis

$$\omega = 2\pi f \quad \text{[Rad/s]} \quad f = \frac{1}{T} \quad \text{[Hz]} \quad T = \frac{2\pi}{\omega} \quad x(t) = x(t+T)$$

Leading means approaching maximum first.

$$X_{rms} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} x(t)^2\,dT} = \frac{X_M}{\sqrt{2}}$$

The same average power is consumed for a $4\,V_{rms}$ and a $4\,V_{DC}$

Impedances

$$\mathbf{Z} = R + jX \qquad \text{Impedance} = \text{Resistance} + j(\text{Reactance})$$

Resistance: In Phase	R
Capacitor: I Lead V	$\frac{-j}{\omega C}$
Inductor: V Leads I	$j\omega L$

Admittance: $\frac{1}{\mathbf{Z}}$
A positive reactance is **inductive**, whereas a negative inductance is **capacitive**

AC Power

Power Factor: $pf = \cos(\theta_v - \theta_i)$
Reactive Factor: $rf = \sin(\theta_v - \theta_i)$
A lagging pf means that the load is **inductive**. A leading power factor means that the load is **capacitive**

$$P = \frac{V_M I_M}{2}pf = V_{rms}I_{rms}pf \qquad \text{Real/Active/Average [W]}$$

$$P = RI_{rms}^2 \qquad R\text{: Real Part of Impedance}$$

$$Q = \frac{V_M I_M}{2}rf = V_{rms}I_{rms}rf \qquad \text{Reactive [VAR]}$$

$$Q = XI_{rms}^2 \qquad X\text{: Complex Part of Impedance}$$

$$\tan(\theta_v - \theta_i) = \frac{Q}{P}$$

Instaneous Power $\quad p = P + P\cos(2\omega t) - Q\sin(2\omega t)$

Complex Power

$$\mathbf{S} = P + jQ = \mathbf{V}_{rms}\mathbf{I}_{rms}^* = \frac{1}{2}\mathbf{V}\mathbf{I}^* \qquad \text{[VA]}$$

AC Maximum Average Power Transfer (No Restriction)

Assuming $\mathbf{V}_{th}$ is not given in rms values, Select $\mathbf{Z}_L$ such that

$$\mathbf{Z}_L = R_{th} + jX_{th} = \mathbf{Z}_{th}^* \qquad P_{max} = \frac{|\mathbf{V}_{th}|^2}{8R_{th}} = \frac{|\mathbf{V}_{th}^{rms}|^2}{4R_{th}}$$

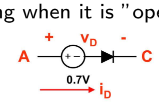
AC Maximum Average Power Transfer (Restricted)

1. Choose X_L as close as possible to $-X_{th}$
2. Adjust R_L as close as possible to

$$\sqrt{R_{th}^2 + (X_L + X_{th})}$$

Diodes

A diode is conducting if it is "closed" or "on".
A diode is not conducting when it is "open" or off



The pn junction of the diode results in a depletion region, which requires a barrier voltage to overcome ($\approx 0.7\,V$).

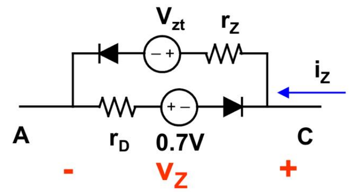
$$V_D - 0.7 \geq 0 \qquad \text{Operation Condition}$$

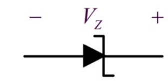
Diode current is a function of voltage $\quad i_D = I_0\left(e^{\frac{V_D}{nV_T}} - 1\right)$

$$V_T = \frac{kT}{q_e} = \frac{1.38\times10^{-23}T_{\text{Kelvin}}}{1.60\times10^{-19}}$$

$$T_{\text{K}} = T_{\text{Celcius}} + 273.15$$

For quality factor 1, and $v_D >> V_T$ we have $i_D = I_0 e^{\frac{V_D}{V_T}}$ **Zener Diode**





Zener diodes act like a perfect negative voltage source for a range of negative current values.

$\mathcal{L}$ aplace Transform

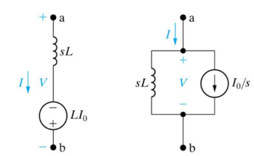
$$\mathcal{L}[f(t)] = \int_0^\infty f(t)e^{-st}\,dt = \mathbf{F}(s)$$

$$f(0) = \lim_{s\to\infty} sF(s) = \lim_{t\to0+} f(t) \qquad \text{Initial Value Theorem}$$

$$f(\infty) = \lim_{s\to0} sF(s) = \lim_{t\to\infty} f(t) \qquad \text{Final Value Theorem}$$

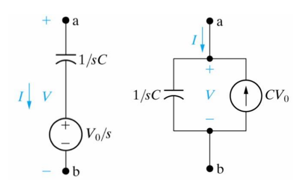
$F(s)$	$f(t)$
$\frac{A}{s+a}$	$Ae^{-at}u(t)$
$\frac{A}{(s+a)^2}$	$Ate^{-at}u(t)$
$\frac{\mathbf{V}}{s+a-jB} + \frac{\mathbf{V}^*}{s+a+jB}$	$2\lvert\mathbf{V}\rvert e^{-at}\cos(Bt+\theta)u(t)$
$\frac{\mathbf{V}}{(s+a-jB)^2} + \frac{\mathbf{V}^*}{(s+a+jB)^2}$	$2t\lvert\mathbf{V}\rvert e^{-at}\cos(Bt+\theta)u(t)$

S Domain Equivalents



$$V_L = L\frac{di}{dt} \qquad i = \frac{1}{L}\int_0^t V_L\,dt + I_0 \qquad \text{Time Domain}$$

$$V_L = sLI - LI_0 \qquad I = \frac{V}{sL} + \frac{I_0}{s} \qquad \text{S Domain}$$



$$V_c = \frac{1}{C}\int_0^t i\,dt + V_0 \qquad i = C\frac{dV_c}{dt} \qquad \text{Time Domain}$$

$$V_L = \frac{1}{sC} + \frac{V_0}{s} \qquad I = sCV - CV_0 \qquad \text{S Domain}$$

Impedance in the S Domain

Assuming zero initial conditions

Resistor	R
Capacitor	$\frac{1}{sC}$
Inductor	sL

Transfer Functions

Is the ratio of a ω dependent phasor output versus a phasor input.

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_{out}(\omega)}{\mathbf{V}_{in}(\omega)} \qquad \frac{\mathbf{I}_{out}(\omega)}{\mathbf{I}_{in}(\omega)} \qquad \text{Voltage/Current Gain}$$

$$\mathbf{H}(\omega) = \frac{\mathbf{V}_{out}(\omega)}{\mathbf{I}_{in}(\omega)} \qquad \frac{\mathbf{I}_{out}(\omega)}{\mathbf{V}_{in}(\omega)} \qquad \text{Transfer Impedance/Admittance}$$

We plot the magnitude of $\mathbf{H}$ versus frequency.
We may let $j\omega = s$ to ease analysis

Bode Plots

$$H_{db} = 20\log_{10}(\mathbf{H})$$

1. Place transfer function into standard form, involving constant parts as 1
2. Rewrite into phasor
3. Use reference table to plot the frequency (dB vs frequency)and phase (degrees vs frequency) plots. Add the sections graphically

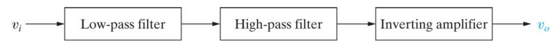
Magnitude	$20\log H$ [dB]
0.001	-60
0.01	-40
0.1	-20
0.5	-6
$\frac{1}{\sqrt{2}}$	-3
1	0
$\sqrt{2}$	3
2	6
10	20
20	26
100	40
1000	60

Frequency Selective Circuits

Passive: only involves R, L, or C
Active: uses opamps and transistors We can combine high pass and low pass filters via opAmps to obtain a band pass or band reject filter.
For a first order low/high pass filter

$$\omega_c \text{ Is found by setting } |\mathbf{H}(\omega)| = \frac{1}{\sqrt{2}}$$

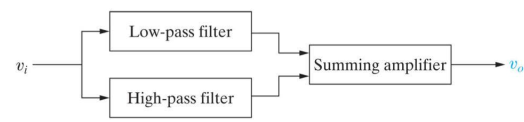
Band Pass



Given that ω_{CutOff} for the low pass filter is greater than the ω_{CutOff} for the high pass filter

$$\lim_{t\to 0} \mathbf{H} = 0 \qquad \lim_{t\to\infty} \mathbf{H} = 0$$

Band Reject



(a)

Frequencies lying between cut off frequency of the low pass and the cut off frequency for the high pass is rejected, given that $\omega_{low} < \omega_{high}$

$$\lim_{t\to 0} \mathbf{H} = C \qquad \lim_{t\to\infty} \mathbf{H} = C \qquad C > 0$$

BJT Transistors

BJT Modes of Operation

	Emitter-Base	Collector-Base
Cutoff	Reverse	Reverse
Active	Forward	Reverse
Saturation	Forward	Forward

BJT Active Region

Collector Current (i_C is independent of V_{CE})

$$i_C = I_s e^{\frac{V_{BE}}{V_T}} \qquad I_s \text{ Saturation Current}$$

$$i_C = \frac{\beta}{\beta+1} i_E = \alpha i_E \qquad \alpha \text{ Common Base Current Gain}$$

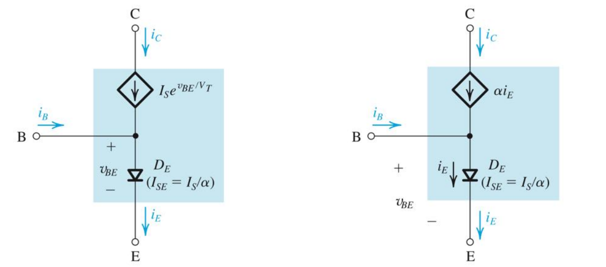
Base Current (Is a fraction of i_C)

$$i_B = \frac{i_C}{\beta} = \frac{i_s}{\beta} e^{\frac{V_{BE}}{V_T}} \qquad \beta \text{ Common Emitter Current Gain}$$

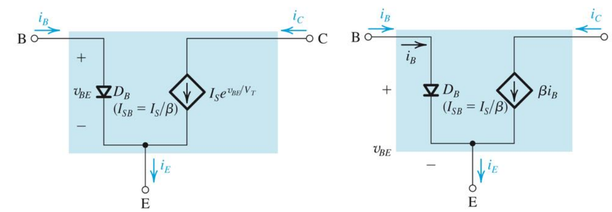
Emitter Current

$$i_E = i_C + i_B = \frac{\beta+1}{\beta} i_C$$

Large Signal Models



(a) (b)



For a pnp BJT, the diode in the model above is reversed, and so is the current source.
For large β, the base current can be assumed as zero.

Biasing

MOS

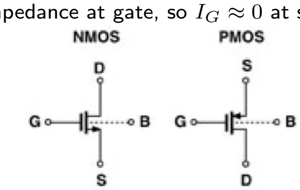
Common Source: In-Gate Out-Drain
Common Gate: In-Source Out-Drain
Common Drain: In-Gate Out-Source

BJT

Common Emitter: In-Base Out-Collector
Common Base: In-Emitter Out-Collector
Common Collector: In-Base Out-Emitter

MOS Transistors

A MOS transistor has Gate, Source and Drain.
Infinite input impedance at gate, so $I_G \approx 0$ at steady state



Current flow determies the source and drain terminals
NMOS: I flows from Drain $\to$ Source
PMOS: I flows from Source $\to$ Drain

NMOS Current Equations

$$\Delta V = V_{GS} - V_{TH}$$

$$I_D = I_{DS} =$$

Cut Off

$$0 \qquad V_{GS} < V_{TH}$$

Deep Triode

$$\mu_n C_{ox}\frac{W}{L}(\Delta V)V_{DS} \qquad V_{GS} > V_{TH}, V_{DS} << 2(\Delta V)$$

Triode

$$\mu_n C_{ox}\frac{W}{L}\left[(\Delta V)V_{DS} - \frac{V_{DS}^2}{2}\right] \qquad V_{GS} > V_{TH}, V_{DS} < \Delta V$$

Saturation

$$\frac{\mu_n C_{ox}}{2}\frac{W}{L}(\Delta V)^2 \qquad V_{GS} > V_{TH}, V_{DS} > \Delta V$$

PMOS Current Equations

$$\Delta V = V_{SG} - |V_{TH}|$$

$$I_D = I_{SD} =$$

Cut Off

$$0 \qquad V_{SG} < |V_{TH}|$$

Deep Triode

$$\mu_p C_{ox}\frac{W}{L}(\Delta V)V_{SD} \qquad V_{SG} > |V_{TH}|, V_{SD} << 2(\Delta V)$$

Triode

$$\mu_p C_{ox}\frac{W}{L}\left[(\Delta V)V_{SD} - \frac{V_{SD}^2}{2}\right] \qquad V_{SG} > |V_{TH}|, V_{SD} < \Delta V$$

Saturation

$$\frac{\mu_p C_{ox}}{2}\frac{W}{L}(\Delta V)^2 \qquad V_{SG} > |V_{TH}|, V_{SD} > \Delta V$$

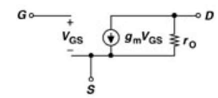
Transconductance

As I_D in saturation is function of overdrive or effective voltage $(V_{GS} - V_{TH})$, we define transconductance, g_m as how well the device converts voltage into current.

$$g_m = \frac{\partial I_D}{\partial V_{GS}}$$

g_m is max in saturation, and drops in the triode region

Small Signal Model



$$i_D = g_m V_{GS} + \frac{V_{DS}}{R}$$