MECH 260

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Cosine Law

$$c_L^2 = a_L^2 + b_L^2 - 2a_L b_L \cos(C_{\text{angle}})$$

Small Angle Approximation

$$\cos(\theta) \approx 1 - \frac{\theta^2}{2} \qquad \sin(\theta) \approx \theta \qquad \tan(\theta) \approx \theta$$

Polar Moment of Inertia

$$J = \iint r^2\, dA \qquad r = \text{Distance from Torsional Axis}$$

$$J = \frac{\pi}{2}(r_{out}^4 - r_{in}^4)\text{Hollow Shaft}$$

Second Moments

$$I_x = \iint y^2\, dA \qquad I_y = \iint x^2\, dA$$

For square cross sections

$$I_x = \frac{1}{12}bh^3 \qquad I_y = \frac{1}{12}hb^3$$

For circular cross sections

$$I = \frac{\pi}{4}r^4$$

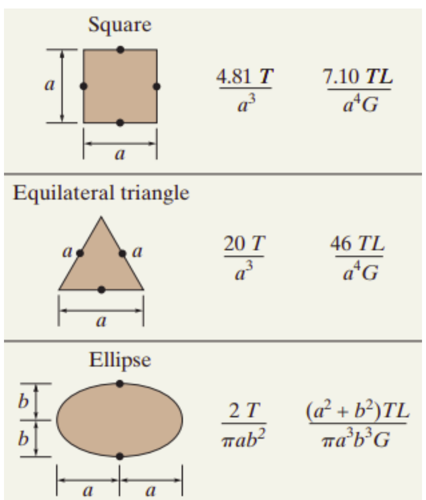
For composite areas:

$$I_y = \sum \bar{I}_y + Ad_x^2 \qquad I_x = \sum \bar{I}_x + Ad_y^2$$

$d_{x,y}$ is distance to a new axis, usually the computed $\bar{y}$

τ_{max} and ϕ for non-circular cross sections

τ_{max} ϕ



Factor of Safety

$$\text{F.S.} = \frac{P_U}{P_A} = \frac{\sigma_U}{\sigma_A} = \frac{\tau_U}{\tau_A} \qquad \text{F.S.} \geq 1$$

Normal Stress σ and Strain ϵ

$$\sigma = \frac{F}{A} \qquad \epsilon = \frac{\delta}{L_o}$$

In Linear Elastic Region:

$$\sigma = E\epsilon$$

Mod of Toughness: Area under entire σ vs ϵ curve curve

Up to yield point, in the linear region:

$$\text{Mod of Resilience: } = \frac{\sigma_y \epsilon_y}{2} = \frac{\sigma_y^2}{2E}$$

Volume under a normal stress distribution gives resultant load

$$\text{Constant Distribution: } P = \sigma A$$

$$\text{Varying Distribution: } P = \iint \sigma(x, y)\, dA$$

Ductile / Brittle

Ductile: Can be subjected to large strains before fracturing

$$\%\text{ Elongation} = \frac{L_{\text{fracture}} - L_o}{L_o}$$

$$\%\text{ Reduction of Area} = \frac{A_o - A_{\text{fracture}}}{A_o}$$

Brittle: Can support little strain before fracturing on planes normal to tensile load. Support compression much better

Poisson's Ratio

Ratio of latitudinal ϵ and longitudinal ϵ

$$v = \frac{-\epsilon_y}{\epsilon_x}$$

Shear Stress τ and Strain γ

$$\gamma = \frac{\pi}{2} - \theta_{\text{final}}$$

Up to proportional limit:

$$\tau = G\gamma$$

Axial Load

Linear elastic deformation of uniform rod:

$$\delta = \frac{PL}{AE}$$

Discontinuous cross sections:

$$\delta = \sum \frac{P_i L_i}{A_i E_i}$$

Continuous varying cross section:

$$\delta = \int_0^L \frac{P(x)}{A(x)E(x)}\, dx$$

Thermal Effects

Thermal deformation:

$$\delta = \alpha(\Delta T)L \qquad \delta = \int_0^L \alpha \Delta T\, dx$$

Stress Concentration

At geometrical discontinuities,

$$K = \frac{\sigma_{\text{max}}}{\sigma_{\text{ave}}} \qquad \sigma_{\text{ave}} = \frac{P}{A_{\text{smallest}}}$$

Torsion

Angle of twist: ϕ

$$\gamma = r\frac{d\phi}{dx} \qquad \gamma_{max} = R\frac{\phi}{L}$$

Torsion Formula

$$\tau = r\frac{T(x)}{J(x)}$$

Maximum shear stress occurs at the outer surface

$$\tau_{max} = \frac{RT}{J} \qquad \tau = \tau_{max}\frac{r}{R}$$

Total ϕ:

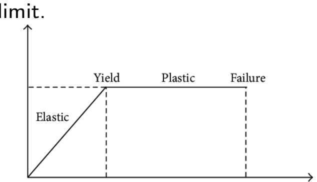
$$\phi = \int_0^L \frac{T(x)}{J(x)G(x)}\, dx \qquad \phi = \sum \frac{TL}{JG}$$

T and ϕ when thumb in RHR is directed outward from shaft

$$\text{Power: } 2\pi f T$$

Elastic Plastic

Elastic-plastic: The σ-ϵ curve is linear, then becomes constant beyond elastic limit.



Elastic-Plastic Torsion

$$T = \frac{\pi \tau_{\text{yield}}}{6}(4R^3 - r^3) \qquad r = \text{radius of elastic core}$$

When $r \to 0$, T is plastic torsion

Beam Bending

Sign convention:

- Shear Force: $\downarrow$ and $\uparrow$
- Moment is ccw on the left and cw on the right

Shear force and moment diagrams

$$\frac{dV}{dx} = w(x) \qquad w(x) \text{ distributed load intensity}$$

$$\frac{dM}{dx} = V(x) \qquad \text{Slope of the } T \text{ diagram gives } V$$

Neutral axis (NA) separates the compression and tension regions, and passes though the centroidal axis. For composite areas

$$\bar{y} = \frac{\sum \hat{y}A}{\sum A} \qquad \hat{y} = \text{distance to reference edge}$$

Pure bending causes linear variation in ϵ and σ

$$\sigma = \frac{My}{I} \qquad y \text{ is the distance from the NA}$$

Composite Beam

Stiffer material must decrease in width, softer must increase in width

$$n' = \frac{E_1}{E_2} \qquad b' = bn' \qquad \sigma = n'\sigma'$$

Unsymmetric Bending

M makes θ with the $+x$ axis. Break M into components

$$\sigma = -\frac{M_x y}{I_x} + \frac{M_y x}{I_y}$$

Orientation of NA, making α with $+x$ axis

$$\tan(\alpha) = \frac{I_x}{I_y}\tan(\theta)$$

Inelastic Bending

$$\text{Shape Factor} = \frac{M_p}{M_y}$$

Shear Stress in Beams

$$\tau = \frac{VQ(y)}{It} \; t\text{: Width of cross section}$$

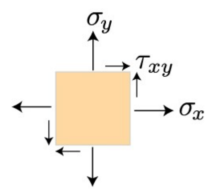
A: Area of cross section a distance $\bar{y}$ from NA
τ is max at the NA

$$Q = \iint y\,dA = \bar{y}A \qquad \bar{y}\text{: Distance } \bar{y} \text{ of A to NA}$$

When cross section geometry suddenly changes, shear formula is inaccurate

Stress Transformation

A plane stress element with positive normal and shear stress.



Sign convention

- Shear is positive if it is positive in the first quadrant
- Compressive normal is negative, tensile is positive

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) + \tau_{xy}\sin(2\theta)$$

$$\sigma_y' = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos(2\theta) - \tau_{xy}\sin(2\theta)$$

$$\tau_{xy}' = -\frac{\sigma_x - \sigma_y}{2}\sin(2\theta) + \tau_{xy}\cos(2\theta)$$

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$$

Principal Stresses

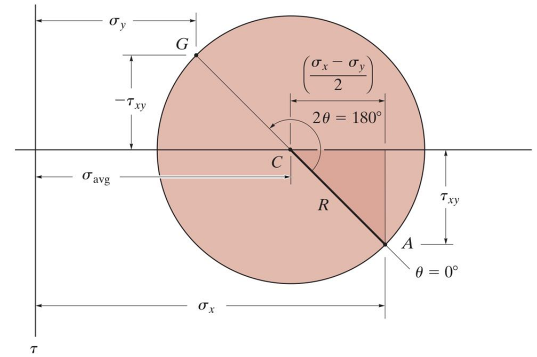
$$\sigma_1, \sigma_2 = \sigma_{avg} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

σ_1, σ_2 are 90 degrees apart on plane diagram, and 180 apart on mohr circle

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Principal Angle

$$\tan(2\theta_p) = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$



Mohr Circle

$$(\sigma_x' - \sigma_{avg})^2 + \tau_{xy}' = \tau_{max}^2$$

Positive shear is plotted by convention in the $-y$ direction
Rotation by 2θ on the circle rotates the plane element by θ in the same direction

Brittle Material Failure

Material fails when $\sigma \geq \sigma_U$

Ductile Material Failure

Material fails when $\tau_{\text{absmax}} \geq \tau_{max} = \frac{\sigma_Y}{2}$
If we are trying to find a suitable If principal σ have the same sign

$$|\sigma_1| = \sigma_{\text{Allowed}} \qquad |\sigma_2| = \sigma_{\text{Allowed}}$$

If different signs

$$|\sigma_1 - \sigma_2| = \sigma_{\text{Allowed}}$$

$$\tau_{\text{absmax}} = \frac{\sigma_{\text{Allowed}}}{2}$$

Maximum Distortion Energy Criterion We want σ_1 and σ_2 be such that

$$(\sigma_1^2 - \sigma 1\sigma_2 + \sigma_2^2) \leq \sigma_Y^2$$