MECH 431

Ruiheng Su 2021

Chapters 1/2

General economics concepts

Economics is study of the use of scarce resources that have alternative uses

- We don't have enough resources to give everyone what they want (time, budget, facilities, data, labor, people with expertise...)
- Managing scarce resource requires making choices and often value judgements
- Making a choice involves "not" doing another choice - trade-offs

Opportunity cost the cost of forgoing the next best thing

- Economics quantifies resource management
- Econometrics is statistics used to analyze economic data

Decision framework and current cost models

Problems come in levels of difficulty

- Simple: not much effort required; small amount of variables; obvious solutions exist
- **Intermediate**: economics problem; needs structured thought;
- Complex: involves a mixture of economic, political, social, and ethical elements; (annual budget of a corporation; building a pipeline; choosing a partner...)

Intermediate questions focus on costs, revenues and benefits that occur at different times

- Which projects are worthwhile?
- How should projects be designed?

Not all problems require engineering economics analysis. Problems that do should

- be important enough to justify serious thought
- have economics issues as a significant component of the analysis leading to a decision
- require organization
- the problem requires decision variables and their consequences be well understood
- there are non-financial factors involved (first nations rights, safety, ethics...)

Decision-making process

1. Recognize the problem
2. Define the goal
   - Wide scope: "make the business more profitable"
   - Narrow scope: "Determine the most economical machine to buy"
3. Assemble relevant data
4. Identify feasible alternatives
5. Select the criterion to determine the best alternative
6. Construct a model
7. Predict the outcomes or consequences for each alternative
8. Choose the best alternative
9. Audit the results

Current cost models simple arithmetic models that compare anticipated costs and benefits over a short period of time. (Such as costs and revenue per unit of product produced and sold.)

In current cost models we only consider the costs, but we neglect

- Currency inflation
- Cost of time, cheaper option may take a longer time
- Depreciation of instruments
- Quality of product
- Functionality of product
- Ethics
- Environmental impacts
- Differentiate between business and pleasure activities (going to Hawaii for training - is this appropriate?)
- Safety - what is safe enough, what is too safe?
- Issue that arise from globalization - different countries have different ethical expectations

Costs and cost estimating

Costs the economic value of the resources used in the production of goods and services. (Such as land, labour, capital, utilities, . . .)

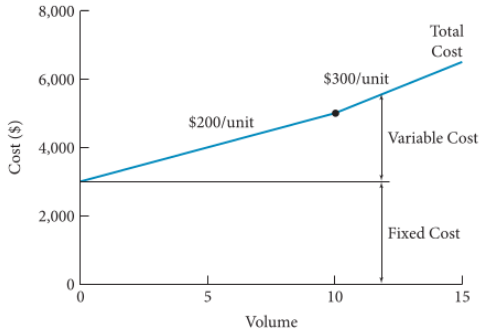
There can also be **social costs**. These are also called externalities. An **externality** is a cost (or benefit) to a third party caused by a producer that is not financially incurred or received by that producer.

There are various types of costs:

- **Fixed costs:** costs that remain constant
  The rent for a storage unit stays the same regardless how much of that unit you utilize.
- **Variable costs:** opposite of fixed costs. They depend on the level of output, or activity
  The amount of raw material a manufacturer need to purchase varies with the number of items produced.
- **Total cost** is sum of the fixed and variable cost.
- **Marginal cost:** variable cost per unit
- **Average cost:** total cost per unit

On a units-produced versus total cost plot,

- the fixed cost is a constant offset to the curve
- the variable cost manifests itself in the **changing** slope of the curve as more/less units are produced
- the variable cost at unit n is cost curve evaluated at n minus the fixed cost
- the marginal cost for the n^{th} unit is reflected in the slope of the curve at n



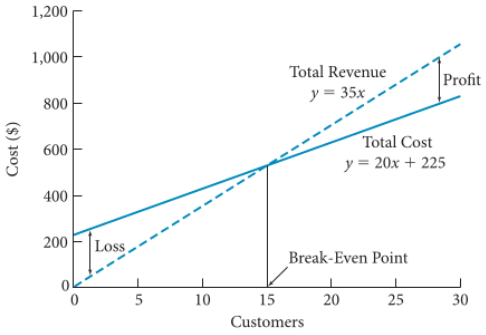
The same terms and definitions apply when we consider **revenues**.

We

- **Break even:** when the total revenue is equal to the total costs
- **Profit:** when the total revenue exceeds the total costs
- **Loss:** when the total costs exceed the total revenue

On a units-produced versus costs plot, the

- **Break even point** is at the intersection between the revenue and costs curve
- the profit/loss (at n^{th} unit sold) is the difference between the revenue and cost curve at n



Since the cost curve at $n = 0$ has a non-zero offset, we say that our production has fixed costs. In the plot shown, we do not have fixed revenue.

Sunk cost: money spent in the past that cannot be recovered.

- disregarded in engineering economic analysis

Example: money paid to buy a car two years ago. The money you spent shouldn't affect you decision when you decide to buy a new car.

Opportunity cost: cost associated with a resource being used for an alternative task; cost of forgoing the next-best thing.

Overhead cost: "indirect costs of running a company that cannot be tied to any particular task that the company executes."

Looking at costs on a timescale,

- **Recurring costs:** costs that occurs at regular intervals

  An Paying rent for your storage unit.

- **Non-recurring cost:** unique costs that occur at irregular intervals

  Costs due accidents, unexpected illness, capital expenditure (buying new equipment)

  To prevent large non-recurring costs, we may choose to buy insurance. This leads to a recurring cost of the insurance premium.

When comparing different options, the **incremental cost** is the cost difference between two alternatives.

When considering how costs are actually "paid out"

- **Cash costs:** require money to move from one party to another
- **Book costs:** are recorded ("accounting costs"), but are not transactions, and do not involve cash flow from one party to another

  Book costs are often accounting exercises. They do not directly involve cash flow, so they are not accounted for in an analysis. But they can generate cash costs/revenue. How you report your spending can lead to different tax percentages.

The **life cycle** is all the time from conception to the retirement of a product/process.

- **Life cycle cost:** total of all the costs incurred over the life cycle of the process/product
- **Life cycle costing:** designing products, goods, and services recognizing their associated costs over their life cycles.
- The later a design change is made, the higher the cost.
- Early design decisions "lock in" costs that will be incurred later - 70% to 90% of all costs are set during the design phases

Cost models

We can classify cost estimating into three levels:

1. **Rough estimates:** back of the envelope calculations, accuracy can vary widely
2. **Budget/Semi-detailed estimates:** based on historical data
3. **Detailed estimates:** estimates made from detailed designs using quantitative models and vendor quotes; Highly accurate

Estimation accuracy are affected by

- **Resource constraints:** whether there is enough time and labor to retrieve all the information we need for a precise estimate
- **Experience:** Whether the estimator is experienced
- **One-of-a-kind estimates:** whether similar projects has been done in the past that can be used as a reference for the current estimate; (the cost of sending astronauts to the moon for the first time)

Cost estimating models

1. **Per-unit model:** uses a cost per unit factor
2. **Segmenting model:** divided a problem into item, estimate each, and add together
3. **Cost indexes:** cost indexes are dimensionless values that record the historical change in costs. The ratio of cost indexes are of primary interest. They are not absolute measures

$$\frac{\text{Cost at } T = a}{\text{Cost at } T = b} = \frac{\text{Index value at } T = A}{\text{Index value at } T = b}$$

4. **Power sizing model:** used to estimate the costs of industrial plants or equipment. We account for the economies of scale using a **power sizing exponent**, x, which determines how much to scale up or down costs of an product at two different capacities

$$\frac{\text{Cost at capacity A}}{\text{Cost at capacity B}} = \left(\frac{A}{B}\right)^x$$

5. **Learning curve:** accounts for cost improvements. The time required to produce the N^{th} unit is related to the time required to produce the first unit, and a **learning curve exponent**

$$T(N) = N^b T(1)$$

Learning curves are often referred to by its **percentage learning slope**. A curve with $b = -0.074$ is a "95%" learning curve since $2^b = 0.95$.

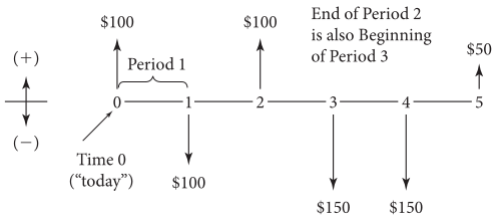
We can calculate b by

$$b = \frac{\log(\text{learning curve expressed as a decimal})}{\log 2}$$

Cashflow diagrams

A cash flow diagram is a chart of class flows over a period of time. It consists of

- discrete time points $(0, 1, 2, 3, \dots)$
- arrows to indicate cash flow
- the arrow points up (positive) if it represents revenue
- the arrow points down (negative) if it represents expenses
- the length of the arrow is proportional the amount of cash involved
- Instead of arrows, we can use bars instead



The categories of cash flow:

- **First cost:** cost that occurs at time 0
- **Operations and maintence:** ongoing/recurring expenses
- **Salvage value:** receipt at project termination
- **Revenues:** annual receipts (annual sales/ reduced costs)
- **Overhaul:** major capital expenditure occuring during life of asset

Simple cashflow analysis techniques

Do not provide a comprehensive view of a project. They help set minimum financial viability requirements before doing more detail analysis.

Payback period is the time required to break-even, neglecting any interest rates and any economic consequences that occur after the payback period. (Time to required to break even.)

Two options may have the same payback period, but one option might provide further benefits, but this is not reflected in the payback period.

Two options may have the same payback period, but one option might provide a consistent return, while the returns of the other option may be back weighted. (They have different benefits vs time curves.) The payback period does not reflect this.

The pay back period does no account for the **time value of money**.

Cost benefit ratio (CBR) the ratio of total costs divided by total benefits. If CBR ≥ 1, then the project may be worthwhile. Otherwise, further consideration is required.

Chapter 3

Time value of money

Engineering projects often take up over multiple years.

Time value of money the idea that the value of cash today will be worth more than in the future because of the present day's earning potentials.

- The stronger the preference for current consumption, the more important time is in investing
- When time becomes important, we also need to consider **interest**, inflation, depreciation, other costs

Interest two interpretations of interest:

1. rate at which we consider how the value of money changes over time (related to the time value of money, and uncertainty and risk)
2. an amount paid by a borrower to get access to money; an amount received by a lender to lose access to a sum of money

Another interpretation is based on **uncertainty and risk**. The later we receive the benefits, the less certain we can be about what the money will be worth, and what it can be used for.

Money is a has value, and so they can be leased or rented.

- **Interest payments:** is the "rent" that a party pays to whenever they borrow a sum of money; this also reflects the risk of the lender losing the money entirely
- **Interest rate:** is the rate of return received by a lender for lending money; they quantity how a lender/borrower values money over time

To a lender,

- A high interest rate loan means that the lender values the same amount of money higher today, than they do in the future
- A low interest rate loan means that we value money today similarly than we do in the future
- A zero interest rate means that the lender values money at exactly the same between today and the future
- A negative interest rate means that the lender values the money more in the future than they do today (extremely rare in an engineering scenario)

Inflation refers to the trend that the amount of goods and services we can purchase with the same amount of money decreases over time.

Depreciation most assets tend to lose value over time.

We rather have money today since

- Having money today means that we can collect rent on it (Money is a commodity)
- Less risk
- money tend to lose value over time assets depreciate over time
- Having money today means that we can invest it today

Simple and compound interest

Consider the following definitions

- **P:** present value. This is the value at the initial time of an analysis
- **F:** future value. This is the value of as asset in the future relative to time of PV
- **i:** interest rate per period
- $\mathbf{I_n}$**:** interest paid in the single period n
- **n:** index for the period

For discrete cast flows, we can compute

Simple interest: interest that is applied once to the "principal amount" and paid at the end of the term (on the **maturity date**). Given the principal value P, interest rate per period i, and n periods,

$$I = Pin$$

The total amount that a lender who lends out P for n periods receives is the future value F.

$$F = I + P$$

Simple interest is easy to compute, but rarely seen in real life.

Compound interest: interest that is computed on the outstanding amount after every compounding period. The outstanding amount includes the unpaid principal and unpaid interest that has been accumulated.

$$I_n = P(1+i)^n - P$$

At the end of the first period:

$$I_1 = Pi$$

and the future value at the end of the first period is

$$F_1 = I_1 + P = P(1+i)$$

At the end of the second period,

$$I_2 = i(P(1+i))$$

and the future value at the end of the second period is

$$\begin{aligned} F_2 &= F_1 + I_2 \\ &= P(1+i) + Pi(1+i) \\ &= P(1+i)^2 \end{aligned}$$

The total amount we need to pay after n compounding periods is

$$F_n = P(1+i)^n$$

Finally,

$$I_n = F_n - P$$

We can define the **single payment compound amount factor**:

$$\text{SPCAF} = (1+i)^n$$

which is constant for any combinations of i, n.

The inverse of the SPCAF is the **single payment present worth factor**:

$$\text{SPPWF} = \text{SPCAF}^{-1}$$

- Values of SPCAF are often tabulated in compound interest tables
- We compute F from P by multiplying P by the SPCAF
- An interest rate that is used to convert F to P is a **discounting rate**
- An interest rate used to convert a P to F is a **compounding rate**

Nominal and effective interest rates

Other than per year, interest rates can be specifed "semi-annually", or "quarterly", et cetera.

- **Nominal interest rate (r):** is an interest rate computed by directly scaling an interest rate such that it becomes a "per year" interest rate, with no consideration of compounding.

Example: a bank bond pays 1% every quarter. The nominal rate is $1\% \times 4$ which gives 4% per year.

"12% per year, compounded semi-annually" is also a nominal interest rate.

- **Effective interest rate ($\mathbf{i_a}$):** takes compounding into consideration - a bond that pays 1% quaterly will end up paying you more than 4% annually. For m compounding periods (per year),

$$i_a = \left(1 + \frac{r}{m}\right)^m - 1$$

  r/m is the **interest rate per compounding period**.

Example: investing 1000 dollars at 6% per year, compounded semi-annually. (The 6% is nominal interest rate.)

The future value of our investment after one year is

$$F = P(1+i)^n$$

where i is the interest rate per compounding period, and n is the number of periods.

In this case, we have 6% per year, but its compounded semi-annually. So i is $6\%/2 = 3\%$. A year consists of two compounding periods, so $n = 2$. Then the future value is

$$F = 1000(1+0.03)^2 = 1060.90$$

Recall that the interest amount is

$$I = F - P = 60.90$$

So the effective annual interest rate is $I/P = 6.09\%$.

Cashflow equivalence

A sum of money at one time period may have the same "value" as a more/less sum of money at another period in time, with respect to an interest rate.

We can use `Excel` functions, `FV`, `PV` to help us.

- If we want the future value on an investment, then we should put the present value as negative
- If we want the present value of a future amount (that is a revenue), then the value returned by `PV` is negative, since we have to invest this present amount to receive a future amount

Chapters 4/5

Uniform Series

Previously, we considered single payments.

Uniform series: A series of cash payments received in uniform period, amount A, and interest rate i.

For example, \$200 dollars received every year at an interest rate of 2%.

If payments are given in a **uniform series**, then we can compute the present value of n payment using the **uniform series present worth factor**:

$$\textbf{USPWF} = \left(\frac{(1+i)^n - 1}{i(1+i)^n}\right)$$

where

- i is the annual interest
- n is the number of annual payments received

Such that the present value of n uniform annuities (denoted A) can be found by

$$P = A\,(\textbf{USPWF}(i, n))$$

Consider the present value a payment of A received n years from now, at an interest rate of i. We know that its present value can be found by

$$P_n = A(1+i)^{-n}$$

For n annual payments equal to A, the present value is

$$\begin{aligned} P &= A(1+i)^{-1} + A(1+i)^{-2} + \cdots + A(1+i)^{-n} \\ &= A\left(\sum_{j=1}^{n} \frac{1}{(1+i)^j}\right) \end{aligned}$$

The partial sum can be simplified using the geometric series partial sum formula.

- This answers: "what do I need to invest today to get an annuity payment of A for n years?"

The inverse of **USPWF** is the **capital recovery factor**. This factor allows us to solve for the annuity amount A, given an investment P.

If we invest \$1000 today at \$10 percent interest, and we want to withdraw it over 5 years, then the capital recovery factor is

$$\frac{1}{\textbf{USPWF}}(0.1, 5) = 0.263797481$$

So the annuity amount we would haveeceived is \$263.8.

Annuity: a fixed sum of money paid to someone each year

The future value of an annuity is the product of the annuity amount and the **uniform series compound amount factor**.

$$\textbf{USCAF} = \left(\frac{(1+i)^n - 1}{i}\right)$$

where

$$F = A\,(\textbf{USCAF}(i, n))$$

For example, if we make an annual deposit of \$1500 for ten years at an interest rate of 8%, the balance of our investment is

$$F = 1500\,(\textbf{USCAF}(0.08, 10)) = \$21729.84$$

The inverse of **USCAF** is the **uniform series sinking fund factor**, **USSFF**. The product of **USSFF** and F gives the annuity amount.

Arithmetic Gradient Cashflow Series

Instead of a uniform series, what if the annual payments we receive linearly increases? For example, receiving \$100 every year, plus \$50 extra for every year after the first year. This can be though of as a uniform series of \$100, and also a growing portion, increasing by \$50 every year.

Arithmetic Gradient Series: consists of a uniform series component A, and a **gradient** component G.

In our previous example, $A = 100$, and $G = 50$. The amount of payment received on year n is

$$\begin{cases} A + (n-1)G & n \geq 1 \\ 0 & n = 0 \end{cases}$$

The present value of an arithmetic gradient series is the sum of the present worth of uniform component and the present worth of gradient component.

$$P = P_{\text{uniform}} + P_{\text{gradient}}$$

The present value of the gradient portion is the product of the **arithmetic gradient present worth factor** and G

$$\begin{aligned} P_{\text{gradient}} &= G\left(\frac{(1+i)^n - in - 1}{i^2(1+i)^n}\right) \\ &= G\,(\textbf{AGPWF}(i, n)) \end{aligned}$$

Example: Consider \$1500 maintenance cost for every 6 months, and the maintenance costs grows by \$75 every 6 months. What is the present value of the maintenance costs over a ten-year period, if the interest rate is 11.25% compounded semi-annually?

The 11.25% is the nominal interest rate. The interest rate per compounding period is $0.1125/2 = 0.05625$. The number of compounding periods in 10 years is 20. The uniform portion is $A = 1500$, the gradient portion is $G = 75$. So

$$\begin{aligned} P &= 1500\,(\textbf{USPWF}(0.05625, 20)) \\ &\quad + 75\,(\textbf{AGPWF}(0.05625, 20)) \\ &= 24585.49 \end{aligned}$$

So our total maintenance spending in the next ten years to equal to \$24585.49 to us today.Once we find the present value of an arithmetic gradient series, we can use the capital recovery factor to convert the present value into an equivalent annuity payment.

The product of the **arithmetic gradient uniform series factor** and G gives the equivalent annuity amount of a uniform series equivalent to the gradient portion.

$$A_{\text{eq}} = G\left(\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right)$$

With our previous example, $G = 75$ for $n = 20$, $i = 0.05625$ gives

$$A_{\text{eq}} \approx 578.69$$

The present value of our total maintenance cost over ten years can also be found by

$$P = (1500 + 578.69)\,\textbf{USPWF}(0.05625, 20)$$

Geometric Gradient Cashflow Series

The payment on period $n = 0, 1, 2, \ldots$ is given by

$$\begin{cases} 0 & n = 0 \\ A_n(1+g)^{n-1} & n = 1 \end{cases}$$

where $g \in \mathbb{R}$, is the percent that the annuity amount grows every year.

The present value of a geometric gradient series is the product of A_1 and the **geometric series present worth factor**:

$$P = A_1\left(\frac{1 - (1+g)^n(1+i)^{-n}}{i - g}\right)$$

If $i = g$, then

$$P = \frac{A_1 n}{(1+i)}$$

The future value is the product of A_1 and the **geometric series compound amount factor**:

$$F = A_1\left(\frac{(1+i)^n - (1+g)^n}{i - g}\right)$$

Models are simple, but

- It's easier to start with simple models
- Quick, gives approximate constraints and bounds
- Sometimes no enough detail is known about the future

Annuities Due

We have been looking at cash flows that occur at the end of the period. For example, we take out a loan at the end of period 0. Period 1 begins. At the end of period 1, we accrue some interest on the loan. Period 2 begins . . .

Alternatively, cashflows are occur at the beginning of a period. For example, we take out $5000 loan at 12% annual interest compounded monthly.

1. Receive cash at the end of period 0
2. Period 1 begins and 1% interest accrues on the loan payment. Our loan balance is now $5050.
3. Any payments we make this month are then subtracted from the loan balance
4. Period 2 begins and the process repeats

Ordinary annuity/Annuity in arrears: annuity payments occur at the end of the period. (The first is one period from now.)

Annuity in advance/Annuity Due: annuity payments occur at the beginning of the period. (The first occurs now.)

To get a formula for an annuity due, multiply the formula by a factor of $(1+i)$.

For ordinary annuity, consider the future value of investing $25 per year for three years at 9% interest.

1. At the end of period 1, we make the $25 payment. At the end of period 3, and our investment becomes $1.09^2 \times 25$
2. At the end of period 2, we make another $25 payment. At the end of period 3, this $25 becomes 1.09×25.
3. At the end of period 3, we make the last $25 payment. No interest is accrued on this payment

So the future value is $81.95.

For annuity due,

1. We pay 25 now, at period 0, and by the beginning of period 3, this payment becomes 25×1.09^3
2. We pay 25 in beginning of period 1, by the beginning of period 3, this payment becomes 25×1.09^2
3. We pay 25 in beginning of period 2, by the beginning of period 3, this payment becomes 25×1.09

The future value at the beginning of period 3 is 89.33. We end up collecting one extra period of interest.

Perpetuities

A series with perpetual payments are call perpetuities.

The future value of a perpetual series is ∞, at a non-zero, positive interest rate.

We will be only consider present values. If we take a current payment at "compound" it out forever, the payment has ∞ future value.

If we can "discount" payments that occurs in the future, starting from a time infinitely away to one period away, what would be the value of those payments now?

For constant, ordinary, annuity payments:

$$P = A/i$$

For constant, annuity due payments:

$$P = A(1+i)/i$$

For a geometric series annuity where $i > g$:

$$P = A/(i-g)$$

Perpetuities are considered when we look at endowment funds by universities or churches. If a project life is very long, it might be possible for us to consider it using perpetuities.

Differing Periods

If the period of annuity payments differ from the compounding interest periods, we have **general annuities**.

To account for general annuities, we either

1. compute an effect interest for the payment periods
2. compute the equivalent payment amounts for each compounding period and apply the interest rate

The equivalent interest rate per payment period and the effective rate (nominal annual interest rate divided by the number of compounding periods per year) must satisfy

$$(1+i_{\text{eq}})^p = (1+i)^c$$

where p is the number of payment periods per year, and c is the number of compounding periods per year.

Rearraning,

$$i_{\text{eq}} = (1+i)^{c/p} - 1$$

In Canada, mortgage interest rates are specified semi-annually.

Consider a loan of $295000 that requires monthly payments for 25 years at an interest rate of 5.35% compounded semi-annually. Find the monthly payment.

The equivalent interest rate is

$$i_{\text{eq}} + 1 = (1 + 5.35/2)^{2/12} = 1.0044\cdots + 1$$

We know the present value of the loan, so we can solve for the constant annuity payment.

$$P = A\left(\frac{(i_{\text{eq}}+1)^n - 1}{i_{\text{eq}}(i_{\text{eq}}+1)^n}\right)$$

Sources of Capital

A firm's interest rate depend on its sources of capital, the investment opportunities, and risk.

Sources of capital available to a firm generally fall into: money generated from operation, borrowed money, and money from selling stock.

Internal: money generated from the firm's operations; retained from profits.

External: money raised from sources outside the firm.

- Short term debt: bank loans, credit cards, line of credit
- Long term debt: bonds, pension funds, mortgages
- One-time: issue now stock - selling equity
- Funds from customers - a customer might be happy to fund you for other benefits, in the form a contract

Bond: is a loan taken out by the firm. The firm receives money from investors, and pays the investors interest on regular, agreed-upon intervals, and returns the capital on the maturity date. Investors need to be convinced that the firm will still be in business when the bond is due.

One example where buyers receive funds from suppliers is between grocery stores and suppliers. The suppliers pay to get their product onto the eye-level shelve space

Selling equity: refers to the salve of the common shares of a company, instead of only the assets

The mix of externally provided funds is the **capital structure.** This depends on the needs of the firm and the decision of its managers.

Cost of debt/borrowed money: is the interest rate at which money can be borrowed

- the lender charges a higher rate for more risky loans
- A large, profitable firm might be able to borrow money at the **prime rate** - the interest rate banks charge their best and most sought-after customers

Cost of equity: the rate of return that investors require on the shares of a firm

- a higher cost of equity reflects the market's assessment of the risk level associated with the firm being able to provide returns in the form of dividend and/or increase in the price of its shares
- Someone could offer to buy your company, only if you can double the sum in two years and pay them back. This corresponds to a 100% rate of return
- Another way to sell equity is in the form of "preferred shares". In exchange for your investment, the firm pays you a certain amount of dividends every year

A high risk firm has a highly uncertain cast flow. Lenders will want a higher rate of return.

Weighed average cost of capital (WACC): a firm's overall cost of capital is the average of the rates of return required by the providers, weighted by the fraction of the total capital provided

- Also known simply as the **cost of capital**
- The rate of return on a firm's common stock and retained earnings is called **return on equity**

If a firm's stockholder's expect a 15% rate of return, we should set the return on equity to be 15%.

Investment opportunities

Firms need to ensure that all selected projects are better than the best rejected project.

With a fixed capital, the firm invests in opportunities that have a greater rate of return than the rest.

- The opportunity cost is the rate fo return on the best rejected project

Minimum attractive rate of return (MARR): can be taken as the firm's interest rate. We require that the MARR is

- greater than the cost of capital (WACC)
- greater than the opportunity cost
- equal to the greatest of: cost of borrowed money, WACC, opportunity cost

The MARR may be adjusted based on risk. A higher MARR may be assigned to a project with higher risk.

Present worth analysis and equal lives

Better things to do: when analyzing whether a single project is worth pursuing or not, the underlying assumption is that there are other alternatives that we could be using our funds for.

In present worth analysis, we assume:

- All cash flows occur that the end of the period. Period 0 represents now
- No sunk costs - these should not affect our decisions going foward
- Two viewpoints - investor and borrower
- Money is obtained at interest rate i despite there might be multiple sources of capital

In general, we also need to consider the scope of the problem. On manufacturing scope, a process improvement might be welcomed, but on a company scale, the improvement cause have unexpected interactions. This would be infavourable on this scale.

The criterion that we will use to judge between projects depends on whether the inputs and outputs are fixed.

1. Neither input nor output fixed: maximize the difference between output and input ("No limit on input, but at least 500 units required." Maximize our benefit by minimizing the input to cut costs and maximizing the output)
2. Fixed input: maximize the output/revenue ("$100 dollars available." Maximize the benefits we receive for $100)
3. Fixed output: minimize the input ("1000 units required." Maximize our benefits by minimizing the production cost)

Three **project lives** (**planning horizon**, or **analysis period**) are possible when analyzing a project:

1. **Equal lives:** the useful lifetimes of the alternatives are equal to the analysis period
2. **Not equal lives:** the alternatives have useful lifetimes different from the analysis period
3. **Infinite lives:** the analysis period is ∞

Multiple alternatives when different lifespans might call for different analysis periods.

When alternatives have the same analysis periods ("equal lives"), the chosen alternative has the maximum **net present value (NPV)** - the difference between the present value of their benefits, and the present value of their costs.

In `Excel`, the function we use is `NPV(rate, cashflows)`, where cashflows include the net cash flows starting at year 1. Any year 0 net cashflows should added to the results of the `NPV` function.

Unequal lives and infinite lives

When alternative have different lives (they provide benefits and costs on different time scales), we cannot consider them using **NPV**.

Instead, we can consider

Least common multiple: we choose the analysis period to be the least common multiple of all the project lives of the alternatives considered, assuming that the alternatives are **repeatable**.

$$\text{LCM}(a, b, \dots)$$

is the smallest number evenly divisible by all of $(a, b, \dots)$.

If we need to choose between a 5 year and a 10 year project, the LCM is 10 (since $10/10 = 1$, and $10/5 = 2$) years. So we will consider the NPV of the two projects assuming the 5 year project repeats twice.

The LCM of 7 and 13 is 91 years.

Consider a firm choosing between two instruments to buy for a 10 year project. Instrument A has a lifespan of 7 years, and instrument B has a lifespan of 13 years. We can choose our analysis period to be 10 years, but account for purchasing another copy of instrument A at year 7. The second purchased instrument A would still have some salvage value by the end of year 10, and instrument B will also have some salvage value remaining.

- We assumed that we will still be able to buy instrument A at the same price we buy it now, in 7 years

The least common multiple method may not make sense (alternatives may be not repeatable).

Another method is to "brute force". We forcibly choose an analysis period.

Consider a machine with $600 installation cost, $200 annual maintenance cost, lifetime of 5 years, and an interest rate of 10%. We decided to analyse the present worth of the machine over 10 years.

$$PV = 600 + \frac{600}{(1+0.1)^5} + 200\left(\frac{(1+0.1)^{10}-1}{0.1(1+0.1)^{10}}\right)$$

There is an additional term involving 600 since we will need to purchase the machine again at year 5, assuming that it can still be purchased at $600.

Infinite lives

Large infrusture projects can almost always be assumed to be "permanent". We can assume the analysis periods to be ∞.

For projects with infinite lives, we can compute it's associated **capitalized cost** - the present value of the initial cost, and what we need to set aside in order to provide the service indefinitely at some interest rate.

Given a present investment P, at an interest rate i, the investment grows to $P + Pi$ by the end of the period. As long as we withdraw Pi, the principal amount will always remain P.

This can be thought of as an end of period withdrawal/annuity,

$$A = Pi$$

If we want an annuity A to support our project, then we must invest at least

$$P = A/i$$

for this to be possible.

Multiple Alternatives

Given a **common analysis period**, we can compute the NPV for each alternative. The best project is project with the most positive NPV.

The following options all have an expected lifetime of 20 years.

Gas fire furnace: $8000 installation cost, $1200 per year, rising 5% per year.

Geothermal heat pump: $23000 installation cost, $400 per year, rising 4% per year.

Electric baseboard heater: $2200 installation cost, $1900 per year, rising 4% per year.

None of the options provide any revenues. So our objective is to simply minimize the costs, and choose the project with the most positive NPV.

The present value of each option is can be found by adding the installation cost, and the present value of the growing annual costs for the next 20 years.

Salvage value/costs: one-time cash amount recoverable/required at the end of a project. Specified as a future value.

- A piece of equipment has salvage value when we are done with it - we can resell it
- A mine has salvage costs - we need to clean up, and restore damage environment
- Dangerous chemicals have salvage costs - we need to pay to get rid of it

Chapter 6

Equivalent Annual Cashflow Analysis

Instead of using the net present value, we can compare projects based on **equivalent annual cash flows (EACF)**.

EACF Analysis: Assuming repeatability, we can compare projects with different lifetimes using their EACF instead of choosing a LCM analysis period. The EACF remains the same for the same project repeated an arbitrary amount of time.

For a given analysis period and interest rate, if we know any one-time values, we can convert them into a uniform series on of annuities in the analysis period.

- Given an interest rate, we can always recover our NPV from the EACF
- The EACF increases when the benefits increase, and decrease when costs increase

The best alternative is the one that maximizes the EACF.

Consider the following tires purchase options. Assuming a 12% interest.

- \$30.95 per tire, with a life of 12 months
- \$59.95 per tire, with a life of 48 months

Under the assumption of annuities due, for the first option, our equivalent annual cost is

$$30.95(1+0.12)^1 = \$34.66$$

For the second option, our equivalent annual cost is

$$59.95\left(\frac{0.12(1+0.12)^4}{(1+0.12)^4-1}\right) = \$19.74$$

So the second option is better, since it has the least annual cost.

When the analysis period is specified, and not an integer multiple of the useful lifetimes of the alternatives, it's easier to compute the NPV first, and convert into a EACF

Chapter 7

Internal Rate of Return

With net present value analysis, our entire computation depends on a single interest rate, that is often assumed to be equal to the MARR.

The best project is the one with the most positive NPV, even though spendings in some projects might be more **efficient** than others.

Consider two alternatives at $i = 10\%$. For option 1, we invest \$1000000 and receive \$1190000 at the end of year 1.

For option 2, we invest \$100000, and receive \$200000 at the end of year 1.

Both options have the same NPV! Yet option 2 is much better, since we get the same NPV despite a much smaller investment than option 1.

Internal Rate of Return (IRR): is the interest rate at which the present worth of the benefits equal the present worth of the costs.

Given the PV of the costs, and the FV that we will receive after n periods, the IRR is such that

$$\text{NPV} = \text{PV}_{\text{benefit}} - \text{PV}_{\text{cost}} = 0$$
$$\implies \text{EUCF} = 0$$

- To the borrower: the IRR is the interest rate that causes the unpaid balance on a loan to equal 0 when the final payment has been made
- To the investor: the IRR is the interest rate that the unrecovered investment is equal to zero at the end of the life of the investment

For investments: maximize the IRR. A higher IRR means a higher net cash flow.

For loans: minimize the IRR.

You are planning to get a \$15,000 loan from your bank. If you are limited to 60 payments of \$300 per month, which of the following bank rate of return is acceptable for you?

We can compute the suitable monthly interest rate. Any rate of return below the computed monthly IRR is acceptable. In these cases, we would actually end up with a positive net present value!

Fixed cash flows: For a fixed set of cash flows related to an investment/loan a higher interest rate means: the value of investments goes down, the value of loans go up

Internal rate of return does not account for

- Difference rates used for investing and borrowing
- Does do distinguish between investing and borrowing

Incremental Rate of Return

When we compare two acceptable potential projects, we want to know whether it is worth it to spend an extra initial amount to get more benefit compared to the project with the smaller intial cost.

Consider three options:

1. 3×45 yard shovels at \$80 million each - revenue \$122 million per year
2. 4×30 yard shovels at \$58 million each - revenue \$106 million per year

The MARR of the mine is 40%, and will operate for 20 years.

Each option must have an IRR of at least equal to the MARR. Otherwise, we can remove it from consideration.

Optional 2 is the cheapest, at \$232 million and an IRR of 46%.

Option 1 is the next cheapest, at the cost of \$240 million. Compared to option 2, option 1 is \$8 million more expensive, but every year, option 1 yields $122 - 106 = 16$ million more than option 2.

The ΔIRR for option 1 compared to option 2 is the interest rate such that the PV of the \$16 million annuity for 20 years equal \$8 million present investment. So it is worth taking option 1 over option 2.

Incremental IRR (ΔIRR): is computed between two alternatives. The ΔIRR is the interest rate such that the net present value between the two projects are equal - the intersection between two NPV versus interest rate curves

For an investment:

- If $\Delta\text{IRR} \geq \text{MARR}$, choose the higher initial cost project - the additional cost is justified
- If $\Delta\text{IRR} < \text{MARR}$, choose the lower initial cost project

A Warning: upgrading to an option with a greater initial cost is only while if the investor COULD earn a rate of return at the ΔIRR. Otherwise, whether switching is worthwhile depends on the investor's maximum possible rate of return, and her MARR.

If the MARR is 35% per year and the IRR of cost alternative A is 40%, the IRR of cost alternative B is 50% and ΔIRR (B-A) is 30%, what is the best alternative?

Alternative A, since the incremental IRR between optional B and A is less than the MARR, so option B is not worth pursuing over A.

1. Find IRR for every project, discard any for which IRR < MARR - this step compares each project to the "**do-nothing**" alternative
2. Arrange the remaining alternatives in ascending order of initial cost
3. Find the ΔIRR to upgrade from the lowest initial cost option to the option with the next-lowest initial cost
4. If IRR > MARR, upgrade
5. Otherwise, stick the with the lowest cost option
6. Repeat the previous steps until all alternatives have been considered

Modified Internal Rate of Return

The problem with IRR:

- Investing and borrow rates are assumed to be equal, yet most profiting firms often invest at a higher rate than they borrow at
- More than one solution for the IRR - NPV = 0 is a polynomial

Descartes' Rule: If a real polynomial has m sign changes, then it has $m - 2k$ positive roots, where $k \in \mathbb{Z}$, $k \in (0, m/2)$.

When cashflows have multiple sign changes, the IRR may non-unique.

Modified internal rate of return (MIRR): accounts differences in the borrowing (or financing) rate, e_{fin}, and the investing rate (MARR, e_{inv}).
The MIRR is the interest rate such that for n compounding periods, makes the present worth of all the expenses calculated using e_{fin} equal to the future worth of all revenue calculated using e_{inv}.

We can interpret the MIRR as

- Discounting all our expenses to a present value, at the borrowing rate
- Investing all revenues, compounding all revenues to a future value the end of the projects using the investment rate
- The MIRR is the interest rate such that

$$P = \frac{F}{(1+i)^n}$$

holds

A firm normally borrows money at 8%, and invests them at 15%. In n periods, the present worth of all the expenses was found to be -6.744. The future worth of all the "receipts" were found to be 16.5.

The MIRR is the interest rate that makes the present and future worths equivalent.

$$0 = (1 + \text{MIRR})^n \, (\text{PV}_{\text{cost}}) + \text{FV}_{\text{revenue}}$$

Incremental Analysis

There are three types of projects we can analyze.

1. **Independent:** selection of this project is independent of the decision to undertake any other projects
2. **Mutallly Exclusive:** at most 1 project, or do nothing, can be selected among competing alternatives. We cannot directly compare the projects with each other. Instead, we need to look at their increments
3. **Contingent:** selection of a project depends on the selection of at least one other project

When project A is contingent on project B, this means we must do project B in order to do project A.

For example, a firm asks to purchase 20 of our products, and wants the option to purchase 60 more in the next three years. The option to purchase 60 more is contingent on whether the first 20 sales were made.

When comparing different alternatives using the IRR method, **our goal is to maximize our net weath, not our rate of return**, so we need to consider the **incremental IRR** between different options.

Incremental Analysis: the examination of differences between alternatives to determine if the increased costs are justified by the increased benefits

- When the interest rate is high: we tend to favour options with lower initial costs
- When the interest rate is low: we can afford to pursue projects with high initial costs

Example: A firm wishes to purchase an electromagnet for their production facility. They have two quotes:

1. Three beginning-of-year annual payments of $1000, or
2. Single payment of $2783

The firm has an MARR of 9%. Which of the two options should be chosen?

We should first decide whether the do-nothing alternative is the better choice. To do this, we compare the do nothing option against option 1. This means that the first step is to reject any alternatives whose IRR is less than the MARR.

If option 1 is worth pursuing over doing nothing, then we need to consider whether it is worth to pursue option 2 (more expensive option) over option 1. To do this, we can compute the incremental IRR by subtracting the net cashflows of option 1 from the net cash flows of option 1.

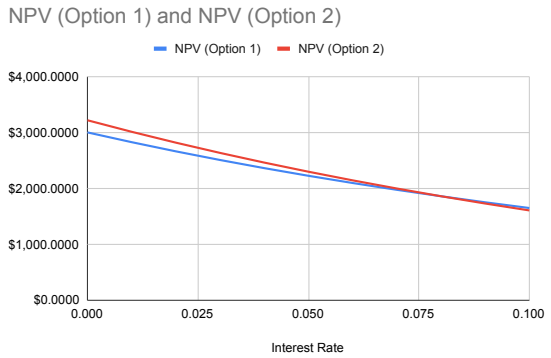
In this case, we find an incremental IRR of 8%, which is less than our MARR, so it is not worth it to pursue option 2 over option 1.

Graphical Method/Sensitivity Analysis

Given a list of alternatives, and when the MARR of a firm is yet to be determined, we can get an idea of which alternative to pursue.

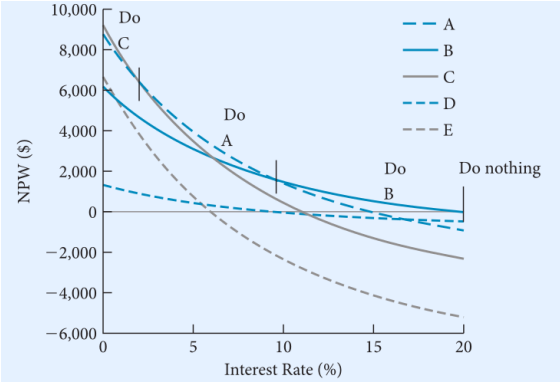
1. Choose a metric we will consider - NPV, EACF
2. For every project, plot the metric over 0% to 100% interest rate
3. The insection between the curves is the incremental IRR of the projects
4. The best project at a given interest rate maximizes the NPV, or EACF
5. Generate a choice table - list the best project to pursue between different intervals of interest values

For the previous example, we could have found the incremental IRR of the two options by considering the NPV versus interest rate graph of the two options.



Which shows an intersection at approximately 8%, which is less than the 10% MARR.

Example: an example of a sensitivity analysis:



For some given MARR, our goal is to always maximize our NPV.

- Between 0 and 3%, the option C has the maximum NPV
- Eventhough there is an intersection of the curves for option E and option B at around 1%, we do not care about this intersection since option C has the greatest NPV at MARR equal to 1%
- At MARR equal to 20%, the maximum NPV of all the options is 0. Beyond an MARR of 20%, none of the options become worth it to purse, so we stick to the do-nothing option

Chapter 8

Future Worth and Benefit Cost Redux

Similar to the present worth analysis, future worth analysis also requires deciding on a common analysis period.

Net Future Value (NPV): is the sum of the unrecovered capital and the total return on investment at the given interest rate, i, at a given time in the future. After paying for all the operating costs, we consider ourselves to invest the amount left over at the given i.

At a given MARR, an alternative is acceptable when

- NPV ≥ 0
- NFV ≥ 0
- EACF ≥ 0

Under these conditions, we can define the benefit-cost ratios

- $\text{PV}_{\text{benefit}}/\text{PV}_{\text{costs}} \geq 1$
- $\text{FV}_{\text{benefit}}/\text{FV}_{\text{costs}} \geq 1$
- $\text{EUAB}/\text{EUAC} \geq 1$

where **EUAB** and **EUAC** stands for **equivalent uniform annual benefit/cost**, and all factors magnitudes.

Chapter 10

Sensitivity Analysis

All of our analysis will never be perfect. The actual cash flow is often different from our projected cashflows due to

- Techology change: changes to production costs and quality of product
- Changes in number of size of competitors
- Introduction of new products: our customers may have more/less options
- Changes to **macroeconomic variables**: inflation, unemployment, economic growth, exchange rate
- International events

Sensitivity analysis: evaluates how much a particular decision metric depends on its input parameters. This gives us a feel for our accurate our estimates might be

Break-even Analysis

A form of sensitivity analysis represented as a **break-even chart**.

For a given range of a particular input paramter

- time
- initial cost
- benefits . . .

The **"breakeven point" between two alternatives** is the point in that given range for which both options are equally considerable - we become indifferent about which project will be chosen.

Breakeven analysis: for a given range of input parmeter, the project is **sensitive** to that paramter if the range contains a breakeven point

We have a choice between constructing a project at full capacity now, or construct a first stage now, and a second stage at n years later. We can plot the net present value of both options as a function of n. The intersection of the curves represent a breakeven point.

The project choice is sensitive if the range of options for n include the break even point.

. . . . . .

Using estimated initial costs, we determined one of three projects was the preferred alternative.

For option 1, if we want to see how sensitive is our result to the accuracy of its initial cost estimate, we can plot the NPV of each option as a function of intial cost of option 1 and find the breakeven points.

What-if Analysis

Our decision parameters (NPV, IRR) is dependent on uncertain cost estimates.

Similar to breakeven analysis, we "sweep" an input variable and see how our preferred option changes.

			Cashflows		
Property	-30%	-15%	Base Case	15%	30%
Initial Investment	$210,000	$255,000	$300,000	$345,000	$390,000
Annual Benefits	$59,500	$72,250	$85,000	$97,750	$110,500
Salvage Value	$42,000	$51,000	$60,000	$69,000	$78,000
Project Lifetime	4	5	6	7	8
Interest Rate	9.8%	11.9%	14.0%	16.1%	18.2%

			Net Present Values		
	-30%	-15%	Base Case	15%	30%
Initial Investment	$147,872	$102,872	$57,872	$12,872	($32,128)
Annual Benefits	($41,289)	$8,291	$57,872	$107,452	$157,033
Salvage Value	$49,671	$53,772	$57,872	$61,972	$66,072
Project Lifetime	($8,431)	$26,673	$57,872	$85,600	$110,244
Interest Rate	$106,617	$81,023	$57,872	$36,874	$17,779

1. Find the most sensitive parameters
2. Consider what happens to the NPV for the **best case** - the best set of reasonable parameter estimates tend to maximize our NPV
3. Consider what happens to the NPV for the **worst case** - the worst set of reasonable parameter estimates that tend to minimize our NPV
4. Compare between the worst case, base case, and best case

As a rule of thumb, given an optimistic, most likely, and pessimistic estimate, the mean estimate can be found by putting 4 times as much weight on the mostlikely estimate:

$$\text{mean value} = \frac{1}{6}(\text{best} + 4 \cdot \text{base} + \text{worst})$$

Expected Values

For sensitivity analysis, we look at how our decisions change depending on the range of input parameters we consider - whether our decision parameters are "sensitive" to a range of input parameters.

All input parameters are uncertain. Future estimates are more uncertain than near-term estimates,

Parameters can be

- **Deterministic** - values are known with certainty, specified through contracts
- **Random** - the parameter is instrinsically uncertain

Probability

In economic analysis, our probability distributions often involve 2 to 5 **discrete outcomes**.

An oil company might consider three outcomes when drilling for a new well: 70% dry well, 20% average production well, 10% highly productive well.

Statistically independent: we will often assume that random variables are statistically independent. For example, the annual benefits of project is not correlated with the project lifetime. This is not always valid.

Joint probability of independent random variables: given the probability of two statistically independent events, $P(A)$, $P(B)$, the probability of both events ocurring is

$$P(A \wedge B) = P(A)P(B)$$

The probability fo rolling a 6 and flipping heads is $1/12$.

Expected value: the sum of the possible outcomes $\{O_1, O_2, \ldots, O_n\}$ weighted by its probability $P(O_i)$. If the project was repeated for ∞ number of times, then on average, the outcomes will equal our expected value.

$$\text{EV}(O) = \sum_{i=1}^{n} O_i P(O_i)$$

If project A has a 50% chance of having an EUAB of \$1000, and 50% chance of having an EUAB of \$500, then the expected value of EUAB for project A is \$750.

Standard Deviation

The risk of a project can be quantified using the **standard deviation**. For a given expected value,

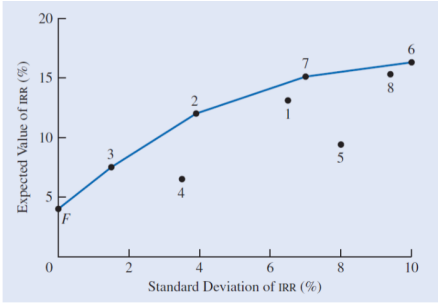
- the outcomes of a **risky/volatile** project can greatly differ from our expected value
- the outcomes of a **safe/stable** project is often not far off from our expected value

Standard deviation: given n outcomes represented by $\{O_1, O_2, \ldots, O_n\}$, each associated with probability P_i, from which we compute the expected value (EV), the standard deviation is defined as

$$\sigma = \sqrt{\mathrm{EV}(O^2) - \mathrm{EV}^2(O)}$$

Risk versus return

We can assess the risks of our projects by plotting them on the standard deviation versus IRR plane. The best project maximizes the IRR, while minimizing the standard deviation.



In the figure, projects 4 is dominated by project 3, which has a better IRR, and lower risk. Project 5 is dominated by projects 1 and 2.

The line connecting projects F, 3, 2, 7, 6 is the **efficient frontier**. Depending on the trade-off of risk and return, the best choice lies on the line.

Safe project: if the expected value is at lease 2 times the standard deviation of expected value, then the project is safe.

Risk Mitigation

There are two main ways to migitate risk.

1. Hedging - you give up some benefits and any best possible outcome cases, in exchange to "shave off" the worst potential outcomes
2. Insurance - paying a fee to an insurance provider for protection from the financial implications of accidents

Hedging

A famous writer has a choice to switch to a new platform. In the neww platform he will be able to keep 90% of his subscription earnings, and the platform will keep the remaining 10%.

- But he might not get enought subscribers in the new platform

The platform offers the following option:

- the writer is paid \$270, 000 in his first year
- the platform will keep 85% of his subscription revenue
- the writer will ony be able to keep 15%

If the writer does not take the platform's offer, he estimates his annual revenue will be

- \$40,000 in the worst case
- \$480,000 in the normal case
- \$700,000 in the best ccase

with equal probability to end up in any one of these cases. We can compute the standared deviatio of the quality. Given the probabilities, we can compute a standard deviation for this set of values.

On the otherhand, if the writer takes up the plan, then he estimates his annual revenue to be

- \$236,000 in the worst case
- \$330,000 in the expected casde
- \$360000 for the best case

all with equal probability. We can then compute a standard deviation for the writer if he switchees platoform.

Now that we have the standard deviations for both options, we will find that by taking up the platform's offer, the writer's expected incomes decreases, but his worst possible income increases.

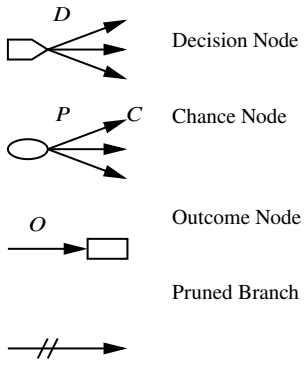
Insurance

A plan we buy to mitigate a potential bad outcome. Using insurance, we don't give up the best possible outcomes like we do in hedging. But in exchange, we must pay a premium.

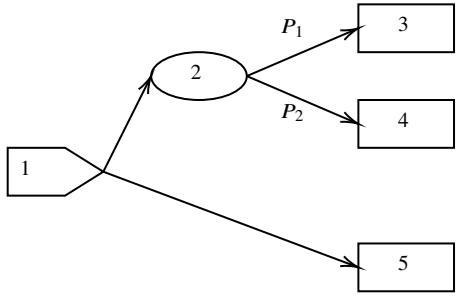
High insurance rates might render the projeect unfeasible.

Decision Trees

Decision Trees: a logical structure of a problem in terms of the sequence of decisions and outcomes of chance events



- Decision trees span from left ot right
- All decision trees start with a single decision node
- All decision trees end in more than 1 outcome nodes
- The horizontal axis may or may not represent time
- Each node is numbered so they are easily located
- Pruned branches indicate a discarded path



Starting from the right

1. Determine the expected value at every chance node
2. Determine the value at every decision node using the best immediate subnode, discount for interest growth if necessary
3. Move to the left and "roll back" until we reach the initial decision node
4. The value at the left most node is the expected value of the overall decision

Simulation

1. Formulate the models for determining the project outcome from the project componenents
   - Variable of interest: NPV, IRR, Benefit/Cost ratio, future value, other?
2. Determine the probability distributions of all project componenets that are random variables
3. Sample distributions to produce values for random project componenents and calculate the project outcome
4. Repeat step 3 until a large enough sample has been taken
5. Produce a frequency distribution and a histogram to estimate the probability distribution of the project outcome
6. Produce summary statistics of the project outcome, mean median, standard deviation, range, minimum, maximum

Common distributions

- Fixed
- Uniform distribution
- Normal distribution
- Cases

Chapter 11

Depreciation

Depreciation: the decrease in value of an asset due to deterioration, of obsolescence

Assets depreciate due to

- Physical losses - deterioration of an equipment, thermal cycling
- Time-related losses -
- Functional losses - the asset no longer or barely meets represent requirements

Instead of incurring the total costs of purchasing an equipment in a single year, the costs can be spread out over the time of life time fo the equipment.

Depreciation is a **book cost** and act as a **tax shield**.

Tax shield: depreciation is an expense over time that is deducted from our revenue. Depreciation reduces the taxable income of a firm. Subsequently, the firm's tax payments decrease, resulting in a "tax shield"

Consider a $5 million investment ot purchase a piece of equipment that result in a $1 million per year savings for 20 years, at 15% MARR.

If we consider the $5 million upfront, then the NPV is the total PV of $1 million dollar savings over 20 years, subtracted by $5 million.

Let's say the investment has a life of 20 years, and it depreciates all of its value over 20 years. In this case, the the NPV is the total PV of the difference between $1 million and $250000 over 20 years. The $5 million dollar equipment losses $250000 in value every year for 20 years.

Effect of Depreciation on Taxes

Legitimate business expenses are not taxed

- Spendings to buy materials for production
- Hire labour

These expenditures typically provide a benefit over time, and **the government require capital expenditures to be spread out over a period of time** to reflect this.

If the government allowed you to deduct your full capital expenditure at once, then they lose a great amount of tax incomes early on.

On the otherhand, if you are required to deduct your expenditures over time, then they have a steadier flow of tax incomes.

Book value of assets: businesses depreciate the book value of their assets. Greater capital expenditure reduces the asset base and improves the return on assets

- A lender will value your assets more than your return on assets. Greater capital expenditure is bad
- A company that wants to acquire your business will value your return on assets

Types of Property

Tangible property: real property such as land, buildings, equipment, vehicles.

Intangible property: such as patents, copyrights, trademarks, franchises. Other examples include brand and custometer loyalty

Depreciable properties primarily involve hard assets that is used to produce income.

- Computers, software, equipment
- Has a useful lifetime that can be determined

Some assets are not subject to depreciation

- Land
- Leased properties - you do not own the leased building
- Factory inventory - raw materials that go into production; These are considered expenses

General depreciation guidelines

- Depreciate an asset as rapidly as legally possible to derive the largest benefit from tax shields early on
- Faster depreciation reduces our base assets, and gives us a higher return on asset

Book value: is the difference between the **cost basis/initial capital cost** and the sum of all depreciation expenses incurred so far

- The cost basis is the total cost of acquiring an asset and putting it into service
- The **salvage value** is the book value at the end of a project's depreciable life

Linear depreciation

Given the cost basis, B, salvage value, S, and N depreciable periods, the amount for which the asset depreciates by after period n is equal to

$$\frac{B-S}{N}n$$

Sum of years digits depreciation

Let SOYD be the "sum of years' digits",

$$\text{SOYD} = \frac{N(N+1)}{2}$$

Given N depreciable periods, the cost basis, B, and the salvage value, S, the amount the asset will depreciate by after period n is

$$\frac{N-n+1}{\text{SOYD}}(B-S)$$

- Tends to front-load depreciation

Declining balance depreciation

The asset depreciates at a constant depreciation rate, D. The amount the asset depreciates by for year n is given by

$$d_n = DB(1-D)^{n-1}$$

So the book balue of the asset after n years is

$$B(1-D)^n$$

- Front-loads depreciation event more than SOYD
- Front-loading means the greatest present value of depreciation tax shields

In year 1, the asset loses

$$d_1 = DB$$

So the book value at the end of year 1 is

$$B - DB = B(1-D)$$

In the following year, the asset loses

$$d_2 = DB(1-D)$$

So the book value of the asset is

$$\begin{aligned} B(1-D) - DB(1-D) &= B(1-D)(1-D) \\ &= B(1-D)^2 \end{aligned}$$

Capital Cost Allowance

The CCA is the Canadian mandated method for depreciating assets for **income tax purposes**.

Capital Cost Allowance (CCA): assets are grouped into classes with prescribed depreciation rates. Every period, the total value of all the assets in each class is depreciated

- There are specific rules about how asset values gets added/deducted from the total value of a class of assets, when an asset is purchased/disposed

Undepreciated Capital Cost (UCC): given a class, for year n, is the **book value** of your assets that is eligible for depreciation

Determining the CCA

1. Find or determine the UCC **for the beginning of the period**
2. Add the **cost basis** of any assets acquired to the UCC using the **1/2 year rule**
    - Only 1/2 of the cost basis of an asset may be added to the UCC in the year of the purchase
    - The remainder is added next year
    - There are some exceptions to the rule, but for insignificant purchases
    - Recall the cost basis includes all costs associated with getting asset running
3. Reduce the UCC by how much you sold your assets for
    - **Loss on disposal:** If the book value at disposition is greater than what we sold for, then we have depreciated too little, and we can collect tax credits
    - **Recaptured CCA:** occurs when we sell our asset for more than its current book value. In this case, we have depreciated too much, and we claimed more tax credits than we were entitled for. We will need eventually need to pay taxes on this difference called the recaptured CCA
    - **Capital gain:** If we sold our assets for more than we initially paid for it, then we will eventually need to pay taxes on the difference between what we sold it for, what is book value of the asset is. This difference is equal to

      $$\text{Recaptured CAA} + \text{Capital Gain}$$
4. Find the CCA for the period based on the UCC balance and the CCA rate for that class; Reduce the UCC by this amount. Given a CCA rate, d, for the class, the CCA for year n is equal to

$$\text{CCA} = \text{UCC} \times d$$

The government wants to collect the most taxes, as early on as possible. By restricting 1/2 of the cost basis be added the next year, the government reduces the income tax shield you can immediately gain from making a capital expense.

Determination of taxes: the CCA is our allowable depreciation expense we claim for tax purposes. We reduce our income by the CCA amount when determining how much tax to pay

While the CCA is a book cost, the tax credits are cash benefits

Class: Equipment, Class 8			Rate:	20%	Tax Rate	26.50%		
Year	Opening UCC Balance	Purchases	UCC Additions	Dispositions	Available UCC	CCA	Ending UCC Balance	Tax Shield
1	$0	$22,000	$11,000	$0	$11,000	$2,200	$8,800	$583
2	$8,800	$5,000	$13,500.0	$0	$22,300.0	$4,460	$17,840.0	$1,182
3	$17,840	$4,000	$4,500.0	$0	$22,340.0	$4,468	$17,872.0	$1,184
Class: Vehicles, Class 10			Rate:	30%	Tax Rate	26.50%		
Year	Opening UCC Balance	Purchases	UCC Additions	Dispositions	Available UCC	CCA	Ending UCC Balance	Tax Shield
1	$0	$15,000	$7,500	$0	$7,500	$2,250	$5,250	$596
2	$5,250	$0	$7,500.0	$0	$12,750.0	$3,825	$8,925.0	$1,014
3	$8,925	$34,000	$17,000.0	$7,500	$18,425.0	$5,528	$12,897.5	$1,465
	=Previous Period Ending UCC Balance	Total of all purchases for this class	1/2 of this years purchases plus unadded 1/2 from previous year	Proceeds of all dispositions	=Opening UCC banalce + UCC Additions - Dispositions	=Available UCC * CCA Rate	=Available UCC - CCA	=CCA * Tax Rate

1. Determine the opening UCC balance - 0 or the previous ending UCC balance
2. Determine the purchases
3. Determine the UCC additions. Recall the 1/2 year rule
4. Determine any dispositions to subtract from the UCC balance
5. Determine the available UCC
6. Determine the CCA by multiply available UCC by the CCA rate
7. Subtract the availabe UCC by the CCA amount
8. Determine the tax shield, equal to CCA times the tax rate
9. The ending UCC becomes the opening UCC for the next period

---

Neglecting the 1/2 year rule, using declining balance decpreciation, at CCA rate d, and cost basis P, the CCA for the asset at year n is given by

$$\text{CCA}_n = \begin{cases} (P/2)d & n = 1 \\ P(1 - d/2)(1 - d)^{n-2} & n \geq 2 \end{cases}$$

The UCC at the end of year n is

$$\text{UCC}_n = P(1 - d/2)(1 - d)^{n-1}$$

Chapter 12

Introduction to Taxes

The federal and provincial goverment in Canada both levy taxes on individuals and corporations.

- Income tax
    - Major difference between personal and corporate income taxes in how they are calculated
- Sales tax
    - Paid whenever a sale is made
    - GST (Goods and services tax)
    - HST (Harmonized sales tax)
    - PST (Provincial sales tax)
- Property/Wealth tax
- Tariffs
- Other taxes

Taxes fund the government. On the federal level, taxes go towards

- National defence
- Indigenous affairs
- Fisheries and oceans

On the provincial level, taxes fund

- Education
- Health care
- Social assistance

On the municipal level, taxes fund

- Roads and transit
- Police and fire
- Waste management

Income tax

Income taxes have the most significance when it comes to financial analysis.

Income tax: provide the government with a portion of net income received by an individual or corporation. Income taxes are the main source revenue for the government.

Sales Tax

Sales tax can effect the economics of a project. HST/GST are **value added taxes (VAT)**.

Value added tax: the amount paid to the government is equal to a percentage times the value added to the product by the manufacturer.

A manufacturer buys $100 worth in raw material, produces parts, and sells the parts for a total of $150.

During the purchase, the manufacturer must pay 5% worth in sales tax. Therefore, the manufacturer pays $105 in total in the purchase of raw material.

When the manufacturer sells the part, the manufacturer collects a same 5% sales tax from it's buyers. So the buys pays $157.5 in total.

At this point, the manufacturer must pay the goverment the $2.5, the difference between the tax that you collected, and the tax that you paid.

In BC, the provincial sales tax is a not a VAT. The VAT becomes an additional added to production.

When the manufacturer buys raw material, they paid the full PST in addition to the nominal cost.

The sales tax that the manufacturer collects for the government must be fully paid by manufacturer.

Property tax

A tax levied on the value of land and the attached property.

- Used by the municipal government
- The responsibility is often passed on from land owners to renters in commercial leases

Tariffs

A tax imposed by a country on the goods and services imported from another country.

- Tariffs increase the price of goods and services purchased from another country, making them less attractive to domestic consumers
- Increases price of imports might shy away domestic consumers from foreign products
- Could be fixed amount per units of a type of goods, or a percentage based on the value of the item

The stated purpose is to protect industries from anti-competitive practices, from countries from pouring cheap good into the country.

Income tax

Income taxes decrease the benefits of our profits.

Personal income taxes:

- Individual investors are concerned with tax efficiency. How do we allocate our assets to minimize the amount of taxes we have to pay
- Taxed amount is calculated based on **income only**, without taking into account our expenses
- Rates increase with the before-tax amount you earn
- Tax credits (tuition, RRSP) exist to reduce the amount you actually have to pay
- Your after-tax earnings is your **gross income**

Corporate income taxes:

- Taxed amount is computed based on the difference between income and legitimate capital expenses
- Tax rates very depending on the size of the firm
- A business of the same size have the same income rate regardless of how much money they make

A size-dependent income tax model might discourage corporations to grow. Increasing their revenues beyond a certain point causes a jump in the tax rate. Business will want to always stay below a certain threshold so they so not have to pay a sudden increase in income tax

Taxable income of individuals

Different provinces all have their own income tax rates.

Average tax rate is equal to

$$\text{Total taxes payable}/\text{Taxable income}$$

Marginal tax rate is the tax rate that applies to the next taxable dollar. The marginal tax rate refers to different tax rates payed by individuals who earn different amounts.

- If earning one additional dollar sets you into the next **tax bracket**, your marginal tax rate increases

Consider the following hypothetical tax brackets.

- 5%: 0 to 50000
- 10%: 5000.01 to 100000
- ⋮

If Bob earns $6000 per year. He will have to pay 5% on the $5000, and 10% on the additional $1000 that sets him over first tax bracket.

Bob's marginal tax rate is still 10%.

- To low income individuals, social benefits might make up a significant portion of their income. Earning more income from their job might make that individual ineligible to the social benefits resulting in a lower income overall
- There are also income taxes on the federal level

Income Taxes

Corporate Income Taxes

The federal tax rate is

- Small business: 10%
- Corporate: Nominal 38%, but realistically 15%

For BC specifically,

- Small business (with revenue less than $500,000): 2%
- Corporate: 12%

- **Income** is our net revenue before tax
- **Profit** is our is what is left after-tax

Corporate income taxes are prepared by accountants who follow the **Generally Accepted Accounting Principles (GAAP)**.

In Canada, accountants also follow the **Income Tax Act** when declaring and maintaining the value of assets. It defines what are considered legitimate capital expenses.

- The income tax act serves us for tax purposes only
- Companies can keep different sets of "books"/records for other purposes such as to show to banks or investors
- One way of keeping record isn't more correct than others

Operating revenue are routine, positive cash flows into the business.

- This does not include one-time revenues such as
  - Selling equity
  - Getting a loan
  - Selling assets

Operating costs are routine negative cash flows into the business.

- Fixed costs and variable costs
- Rent, leases
- This does not include one-time costs such as
  - Paying out dividends
  - Buying back equity

The difference between the operating revenue and costs give the **before-tax cash flow (BTCF)**.

The **taxable income** is equal to the BTCF minus the capital cost allowance and debt interest.

Negative taxable income: as long as the business has other taxables, we can assume that the negative income tax amount becomes a "benefit"

Net Profit: Given the tax rate, t, the net after-tax profit for our business is equal to

$$(\text{BTCF} - \text{CCA} - \text{I})(1 - t)$$

- This computation involves a book-value - the CCA
- This means that in general, the net profit is also a book value, and do not represent our actual cash flows

Let the tax rate of a firm be 34%. The firm has

- Gross income of $300000
- Operating expense of $120000
- Assets worth $800000
- CCA rate of 12%

To find the net profit for the year, we first find our BTCF. This happens to be

$$300000 - 120000$$

The capital cost allowance is equal to the depreciation expense of our assets

$$800000 \times 0.12$$

We have not stated any debts here, so we won't consider any interests in our taxable income

$$300000 - 120000 - 800000 \times 0.12 - 0 = 84000$$

The firm's net profits for the year is equal to $84000(1 - 0.34)$

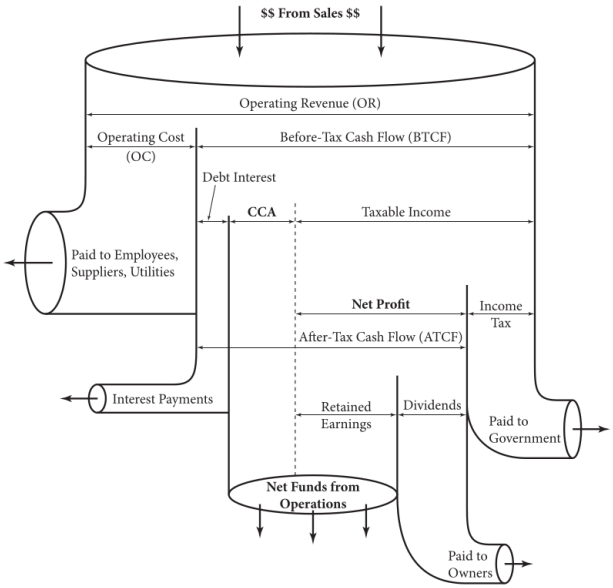
The **after-tax cashflow** is equal to

$$\text{BTCF} - \text{Income Tax}$$

The **net cashflow from operations** is equal to

$$\text{BTCF} - \text{Income Tax} - \text{Interest Payments} - \text{Dividends}$$

Interest Payments: are paid from the after-tax cash flow. They are not a part of the taxable income



The **net cash flow** is equal to our

- Net cash flow from operations, plus
- New equity sales, plus
  - We are giving up a portion of our company for money into our business
- New debt, plus
  - We gain some cash, but we now still owe the same amount
- Proceeds from asset disposal, minus
- New purchase of assets, minus
  - We gain an asset, but we also lose the same amount in cash. There is no change in value so there are no income taxes payable
- Repurchase of equity, minus
- Repayment of debt principle

Dividends

Portions of the profit that are paid to the shareholders.

- Expressed in $ per share every period
- Stable companies offer dividends - banks, oil, energy companies
  - The share prices of stable companies do not change much
  - By paying out dividends, these companies make their shares more worth owning
- High growth companies rarely offer dividends since all their profits are reinvested to help sustain higher than average growth

Working Capital: Money we need to cover the lag between selling something, and actually getting paid for it.

The working capital may be made of

- Internal cash from the bank
- Previously purchased raw materials
- Lines of credit

Capital tax factor: assuming that the full capital cost allowance will be taken every year, the present value of the net capital investment is given by

$$P = B\left(1 - \left(\frac{td}{i+d}\right)\left(\frac{1+i/2}{1+i}\right)\right)$$

where B is the cost basis, d is the CCA rate for the asset class, t is the firm's marginal tax rate, i is the interest rate

Similarly, the present value of the tax shield is equal to

$$P = B\left(\frac{td}{i+d}\right)\left(\frac{1+i/2}{1+i}\right)$$

Disposing of assets

Books-open assumption: the amount received upon deposition is simply subtracted from the UCC of that year. In effect, any recaptured CCA or loss on disposal "will continue to be allocated at the regular CCA rate"

Books-closed assumption: Any recapatured CCA or loss on disposal (i.e. the difference between the book value at the dispoal price) is taxed at the marginal tax rate of that year and applied to the final year's income statement

- If we sell the asset for more than our original cost basis, then we have a capital gain
- Capital gains are taxed at a different rate than regular income

The books-open assumption is used more often compared to books-closed assumption.

Books-open assumption

The net salvage value is

$$\text{NSV} = S\left(1 - \frac{td}{i+d}\right)\left(\frac{1}{1+iN}\right)$$

where

- S is the salvage value
- d is the CCA rate for the asset class
- t is the marginal tax rate
- i is the interest rate
- N is the lifetime of the asset, or years until disposal

Books-closed assumption

The net salvage value for the books-closed assumption is

$$\begin{aligned}\text{NSV} &= S + \text{DTE} \\ &= S(1-t) + B_d t\end{aligned}$$

where

- DTE is the **disposal tax effect**, equal to

$$\text{DTE} = t(B_d - S)$$

- S is the salvage value
- t is the marginal tax rate
- B_d is the book value at disposal

Under the books closed assumption, the net salvage value is equal to the taxed salvage value, and the book taxed book value atIf the book value at disposal, B_d was greater than what we sold the the asset for, S, then we get a tax shield equal to the tax rate times the difference.

After-tax Rate of Return

The after-tax MARR is often complex to compute. As a first estimate, it is approximately equal to the before-tax MARR times one less of our marginal tax rate.

$$\text{MARR}(1-t)$$

Example: Consider a small bakery with an after-tax MARR of 15%. The bakery is investing on a $80000 piece of equipment. But accounting for the electrical and structure improvements that need to be made to the facility, the cost basis for this purchase is $80000 + $14000.

The CCA rate for the asset is 30%.

From the purchase, we expect to increase sales by $23000 per year, though we expect the costs to increase by

- $500 per year in electricity
- $1500 in year one, increasing $250 per year

The asset will last for 10 years, with no salvage value. The marginal tax rate for the bakery is 12%.

Cost Basis		$94,000									
MARR	15%	Tax Rate		12%	CCA Rate	30%		Cost Basis		$94,000	
Period	Revenue	Op Costs	BTCF	Opening UCC	CCA	Ending UCC	Taxable Income	Income Tax	Net Profit	ATCF	NCFO
0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0	$0
1	$23,000	($2,000)	$21,000	$47,000	$14,100	$32,900	$6,900	$828	$6,072	$20,172	$20,172
2	$23,000	($2,250)	$20,750	$79,900	$23,970	$55,930	($3,220)	($386)	($2,834)	$21,136	$21,136
3	$23,000	($2,500)	$20,500	$55,930	$16,779	$39,151	$3,721	$447	$3,274	$20,053	$20,053
4	$23,000	($2,750)	$20,250	$39,151	$11,745	$27,406	$8,505	$1,021	$7,484	$19,229	$19,229
5	$23,000	($3,000)	$20,000	$27,406	$8,222	$19,184	$11,778	$1,413	$10,365	$18,587	$18,587
6	$23,000	($3,250)	$19,750	$19,184	$5,755	$13,429	$13,995	$1,679	$12,315	$18,071	$18,071
7	$23,000	($3,500)	$19,500	$13,429	$4,029	$9,400	$15,471	$1,857	$13,615	$17,643	$17,643
8	$23,000	($3,750)	$19,250	$9,400	$2,820	$6,580	$16,430	$1,972	$14,458	$17,278	$17,278
9	$23,000	($4,000)	$19,000	$6,580	$1,974	$4,606	$17,026	$2,043	$14,983	$16,957	$16,957
10	$23,000	($4,250)	$18,750	$4,606	$1,382	$3,224	$17,368	$2,084	$15,284	$16,666	$16,666
		NPV BTCF	$7,149						NPV ATCF	$1,977.17	

1. Begin by finding the BTCF
2. Recall the 1/2 year rule to determine the opening UCC

3. Compute the CCA, and deduct the CCA from the opening UCC to find the ending UCC for that period
4. The taxable income is equal to the BTCF minus the CCA and any interest payments (none in this case)
5. We can find the income tax, and subtract the income tax from the taxable income to find the net profit
6. The ATCF is equal to the net profit plus the CCA and any interest payments (none in this case)
7. The net cash flow from operations is the after-tax cash flow minus dividends (none in this case)
8. Repeat the computation for every period
9. The after-tax NPV is equal to difference between the cost basis and the sum of all the present values of the net cash flows from operations

Example: You purchase a $4000 shed with a CCA rate of 4%. In six years, you expect to retire, and will shell the shed for $2000 then.

You have a $1500 annual profit, and a tax rate of 22%.

You have a interest rate (MARR) of 9%. Use closed books for disposition. What is the after-tax NPV for the investment?

To find the present value of the shed, we multiply the cost basis of the shed by the captial tax factor. Assuming that the cost basis was $4000:

$$4000\left(1-\left(\frac{0.22\cdot 0.04}{0.09+0.04}\right)\left(\frac{1+0.09/2}{1+0.09}\right)\right)=3740$$

The present value of the shed is less than $4000**. The shed gives us a series of tax shields for the next 6 years that has a present value of** $260**.**

We can then find the after-tax present value of our uniform series annual profits. Every year, our profit is taxed at 22%, so the PV is

$$1500\,(1-0.22)\,(P/A,9\%,6)=5249$$

Using the books-closed assumption, the net salvage value is

$$2000+(0.22)\left(4000\,(1-0.04)^6-2000\right)=2249$$

We neglected the 1/2 year rule when computing the book balue at disposal.

We can discount $2249 back to a present value using the 9% interest rate.

Chapter 13

Inflation Concepts and CPI

Inflation: is a rise in the prices of goods and services in an economy over a period of time. Inflation is also an erosion in the purchasing power of fixed amount of money over time

- If inflation is non-neligible, then the actual dollars returned by a project does not reflect the actual purchasing power of the future cashflow.

Reason for Inflation

1. Money supply - there is too much money in the system relative to goods and services. In this case, the value of money tends to decrease (supply and demand)
2. Exchange rates - prices change to reflect the comparative value currency between countries
3. Cost push - producer of goods and serives "push" the cost of increasing operating costs to consumers
4. Demand pull - when demand > supply, prices tend to increase
    - Real estate
    - Early releases of popular products like iphones
    - Concert tickets get sold out quickly, and resaler sell the same tickets for a higher price

Some samples

- The average price of bread increase from $3.49 last year to $3.63 this year - this represent a 4% inflation per year

$$\frac{3.63-3.49}{3.49}\approx 0.04$$

To find the **average inflation rate** over n periods, we first find the inflated price at the end of n periods, then solve for the inflation rate such that the final price is equal to the initial price compounded at the inflation rate.

Example: Consider a base price of $100. Over two years, the inflation rates were 4% and 8%.

The inflated price at the end of year 2 is

$$100(1+0.04)(1+0.08)=P_f$$

The average inflation rate is f, such that

$$100(1+f)^2=P_f$$

where we find 5.98%.

Example: A fund was established 25 years ago, with $10000. This year, the fund was increased to $18000. The average inflation rate over the 25 years was 3%. Was this increase enough to offser inflation?

Using 3% as our "interest rate", we can find the future value $10000. This comes out to be $20938, which is greater than $18000. So the increase was not enough to offset inflation.

- **Disinflation** is a reduction in inflation rate over time
- **Deflation** occur when the inflation is negative
- **Hyperinflation** occurs when a country experiences ver high and usually accelerating rates of inflation, rapidly eroding real value of the local currency

Measure of Inflation

Price indexes taken at two instances of time reflect inflation/deflation that occurred between the two points in time

- Consumer price index (CPI)
    - Measures price changes in food, shelter, medical care, transportation, apparel, and other selected goods and services used by average individuals
    - Statistics Canada periodically measure the cost to purchase a standard bunle of goods and services based on prevailing prices
    - May be specified for down to cities
- Producer price index (PPI)
- Commodity specific indexes

Given index value at period n, and the index value at period $n-1$, the % increase in price between period n and $n-1$ is equal to

$$100\%\times\frac{\text{Index}_n-\text{Index}_{n-1}}{\text{Index}_{n-1}}$$

Consumer Price Index (CPI)

The CPI for any point in time, relative to the CPI of the base year, is simply scaled by a factor equal to the ratio of the current price of the standard bundle, and the base year price of the standard bundle.

$$\text{CPI}_{\text{current}}=\text{CPI}_{\text{base}}\frac{P_{\text{current}}}{P_{\text{base}}}$$

Example: An item with cost $1200 in 1997 is estimated to cost $2100 in 2009. If the cost index in 1997 was 435, what is the cost index for 2009?

If we take 2009 to be the "current year" in the formulat given above, we find that

$$\text{CPI}_{2009}=435\left(\frac{2100}{1200}\right)=761.25$$

Inflation Analysis

One method to account for inflation is to adjust the MARR.

Both outside and project-related inflation affect engineering projects.

- Project-related inflation: increase in price of project related costs, thereby reducing our NPV
- Outside inflation: decrease in purchasing power for the same cash amount

Inflation rate (f): annual rate of increase in the dollar amount needed to pay for the same goods/services

Real interest rate (i_R or i′): measures the real growth of money, neglecting effect of inflation. Also known as the **inflation-free rate**

Real, or constant dollars are inflation-free dollars expressed using a base year. Real dollars are fictitious, and have constant purchasing power. Real dollars are useful for comparing projects that occured at different times.

Market interest rate (i): adjusted interest rate that accounts for both inflation and real interest. Also known as the **nominal interest rate**.

Given the inflation rate and the real interest rate, the market interest rate is given by

$$1 + i = (1 + i_R)(1 + f)$$

Nominal, inflated, or actual dollars carry the effect of inflation. Reflects the actual cash flow of the project.

Example: If the UBC meal plan costs \$1500 now. Based on a general inflation rate of 2% each year, we expect the plan to cost \$1530 in the next year.

If we do not account for inflation, we would expect the meal plan to cost \$1500 next year. In this case, \$1500 is the real dollar cost.

The actual/nominal dollar cost is \$1530.

Example: A bank offers a nominal interest rate of 5.5% for investing in their bank. We consider the real interest rate for the case of an inflation rate of 2% versus 8%.

$$i_R = \frac{1 + i}{1 + f} - 1$$

Using our formula, for 2% inflation, we will find a real interest rate of 3.4%. This means that our real value of our investments/ our purchasing power will grow by 3.4% in the next year.

On the other hand, for an inflation rate of 8%, we find a real interest rate of 2.3%. Our purchasing power declines by 2.3% in the next year.

Convert Between Real and Actual Dollars

Given the inflation rate per period over N periods, we can convert actual/nominal dollars in period N, A_N to real dollars, $R_{b,N}$. (The script b represents the year number of the base year.)

Assuming a constant inflation rate per year over the N periods,

$$R_{b,N} = \frac{A_N}{(1 + f)^N}$$

Which resembles the present value formula with an interest rate equal to the inflation rate per year.

Consider a gift of 1.2 million, invested at 8.0% market interest rate 55 years ago. Over the 55 years, the inflation rate averaged to 6.0% from 1960 to 2015.

In this case, the market interest rate is 8.0%. The real interest rate is given by

$$i_R = i' = \frac{(1 + 0.08)}{(1 + 0.06)} - 1 = 0.01887$$

To find the worth of the gift in actual dollars, we compound 1.2 million using market interest rate, for 55 years.

$$1200000(1 + 0.08)^{55}$$

To find the worth of the gift in real dollars, we can

1. Compound 1.2 million using the real interest rate, for 55 years, or
2. Find worth of the gift in actual dollars, and discount the worth for 55 years using the inflation rate (to remove the contribution from inflation)

Erosion of Purchasing Power: given an actual dollar amount A at a base year, and a positive inflation rate, f, if A **was not used in any investments, then the markey interest rate is** 0, and the real interest rate is equal to

$$\frac{-f}{1 + f}$$

So real dollars erode in value over time.

Actual and Real Dollars Analysis

Analysis must onyl be performed using real dollars and real interest rates, or nominal dollars with nominal interest rates, but never both.

Same NPV: Regardless of the method of analysis, and whether there is tax involved, the NPV should be the same

Different IRR: While the before tax IRR will be the same regardless of whether inflation applies, the after-tax IRR differs. While the benefits increase at the same rate as inflation does, the depreciation schedule remains the same. This generates more taxable income, decreasing our after-tax IRR.

Ignore Inflation: we are safe to ignore inflation if we assume that the costs and benefits will increase at the same rate of inflation as the economy as a whole. This is often invalid when a raw componenet to a product is rapidly deflating due to cheaper production costs.

Comparison between projects are made by comparing NPVs. We can use the

1. Deflation method:
    - Given a series of actual/nominal cashflows
    - Convert the series into their real/constant dollar amount using inflation rate
    - Compute the NPV using the real/constant interest rate
2. Adjusted discount method:
    - Given a series of actual/nominal cashflows
    - Compute the market interest rate
    - Use the market interest rate to find the actual/nominal NPV

The NPV computed using both methods should be the same.

Effect of Inflation on Cashflow

1. Depreciation Expense
    - The depreciation expense is always expressed in actual dollars
    - The depreciation expense does not change with inflation, since they are completely determined by the initial cost basis
    - Inflation increases actual cashflows, while depreciation expense remains the same
    - This results in higher taxes
2. Salvage value
    - Salvage value also gets inflated
    - Book values do not get inflated
    - Results in higher taxable gains
3. Loan payments
    - Borrowers repay their loan with dollars of decreasing purchasing power
    - Lenders trys to account for this using a suitable financing rate
    - This is way lenders often prefer a stable inflation rate, so their financing rate predictions remain accurate over the typical period of a loan
4. Working capital drain
    - The cost of the working capital increases with inflation
5. Rate of return and NPV
    - Unless revenues are sufficiently increased to keep pace with inflation, tax effects and/or a working capital drain, firms will get a lower rate of return and NPV

Chapter 14

Replacement Analysis

The physical useful lifetime of a project is always greater than or equal to the economic lifetime.

A pressure vessels are regulated instruments that have a well-defined useful lifetime related to safety considerations.

But the economic lifetime of the vessel maybe less than its physical lifetime if the cost to continue servicing the vessel until its end-of-life is greater than simply replacing the vessel.

Economic life: measured from the point of purchase until the point at which we save money by replacing the asset

Replacement analysis helps us determine when an asset should be replaced.

- **Defender:** is the existing equipment, building, or decision that was previously implemented
- **Challenger:** the proposed replacement that is under consideration

Considerations of replacment analysis

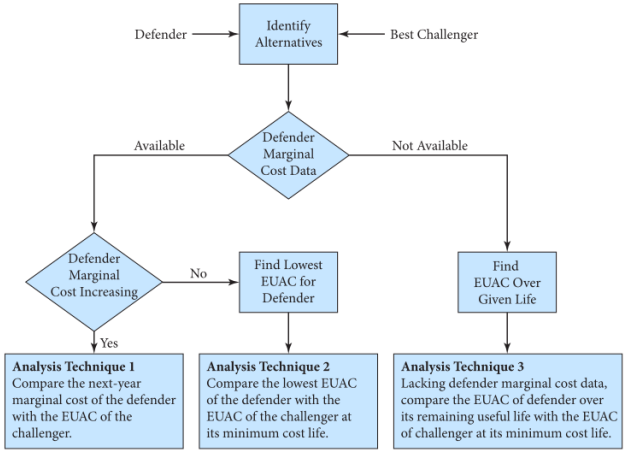
- Can a unit be repaired if it fails? Or do we need to remove it from service?
- Availability of standby units
- Do componenets or units fail independently?
- Budget constraint - limits how many replacements are allowed over the planning horizon
- Planning horizon?
- How will production be impacted during a repair?
- Sociotechnical issues
- Income tax consequences
- Is there preventative maintenance?
- Can the unit return to its full capacity after repairs?
- Could future replacements be more cost-efficient replacements than what is currently offered?

Minimum cost life: the number of years that an asset should operate for such that it minimizes the equivalent annual cost (EUAC) of the asset.

The EUAC of an asset has contributions from

1. Initial cost - decreases as the number of years the asset operates for increases
2. Maintenance costs - generally increases with the number of years an asset operates for
3. Salvage value - on which interest also acts on

We can visualize the minimum cost lifetime by plotting the years versus the (total, initial, and running) costs expressed in an EUAC.



Marginal cost: for a given year, is the cost the run the asset for that year.

The marginal cost typically includes:

- Capital recovery costs - the opportunity cost of keeping the asset for that year; This is often equal to the loss in **market value/salvage value and lost interest for the year**
- Yearly operating and maintenance costs
- Yearly taxes and insurance

An example of finding before-tax marginal cost:

Example: Consider an asset with the following costs

- Investment: $25000
- Annual operating costs: $2000 in year 1, increasing $500 per year
- Annual cost for risk of breakdown: $5000 per year for 3 years, then increasing by $15000 per year

The asset has a useful life of 7 years, and the company has an MARR of 15%.

Year	Market value (MV)	Loss in MV	Loss in interest	O&M Cost	Cost of breakdown risk	Total Marginal Cost
0	25000	-	-	-	-	-
1	18000	7000	3750	2000	5000	17750
2	13000	5000	2700	2500	5000	15200
3	9000	4000	1950	3000	5000	13950
4	6000	3000	1350	3500	6500	14350
5	4000	2000	900	4000	8000	14900
6	3000	1000	600	4500	9500	15600
7	2500	500	450	5000	11000	16950
		***Market value the year before times the current market value	***Market value the year before times the MARR			

- Between years n and $n-1$, if we keep the asset for year n, and sell it at the end of year n, then we lost the difference in market value if we sold the asset at the end of year $n-1$
- If we sold the asset at the year $n-1$, then we could have collected interest at the end of year n equal to the interest rate times the market value at year $n-1$. This is the interest forgone

Let's look at the previous example accounting for taxes this time. Let's assume that

- The asset depreciates by $5000 per year
- Income tax rate of 40%
- After-tax MARR of 10%

Year	Market value (MV)	Book Value	Recaptured capital/Loss on disposal	Taxes Or Tax Savings	After-tax market value	Loss in after-tax market value	Loss in interest	O&M Cost	Cost of breakdown risk	Total O&M + Cost of breakdown risk	After-tax annual Expense	Tax savings due to depreciation	Total after-tax marginal cost
0	25000	25000	0	0	25000			-	-	-			
1	18000	20000	-2000	800	18800	6200	2500	2000	5000	7000	4200	2000	10900
2	13000	15000	-2000	800	13800	5000	1880	2500	5000	7500	4500	2000	9380
3	9000	10000	-1000	400	9400	4400	1380	3000	5000	8000	4800	2000	8580
4	6000	5000	1000	-400	5600	3800	940	3500	6500	10000	6000	2000	8740
5	4000	0	4000	-1600	2400	3200	560	4000	8000	12000	7200	2000	8960
6	3000	0	3000	-1200	1800	600	240	4500	9500	14000	8400	0	9240
7	2500	0	2500	-1000	1500	300	180	5000	11000	16000	9600	0	10080
			** Difference between the market value and the book value	** Tax rate times the recaptured capital or loss on disposal	** Market value plus any tax savings or taxes we should gain/have to pay	** Difference between market value last year, and the market value this year	** Market value last year times the after-tax MARR				** Total annual cost times (1-taxrate)	** Taxshield we gain for this year	** Annual expense + Loss in interest and market value - tax savings

Technique #1

Assumes

- The best challenger will continue to be available in all later years at the same economic cost
- Defender marginal cost is increasing

We keep the defender as long as it's marginal cost for the year is less than the minimum EUAC of the challenger. Otherwise, the defender is replaced.

Technique #2

Assumes

- The best challenger will continue to be available in all later years at the same economic cost
- Defender marginal is not increasing initially. This is often the case when the defender is early on in its useful life

Keep the defender in operation if its minimum EUAC is less than the minimum EUAC for the challenger. Otherwise, the defender is replaced.

Technique #3

Assumes

- Marginal cost of the defender is unknown. The defender is based on agign technology with a shrinking market

Keep the defender if the EUAC of the defender over its **remaining stated useful lifetime** is less than the minimum EUAC of the challenger.

Opportunity cost perspective: replacement analysis should be using the opportunity cost persepective. All costs/benefits associated with an asset are costs/benefits we would forgo if we replaced the asset

Example: Given an interest rate of 11%, consider the defender

- Purchased at $100000
- Currently valued at $30000
- In 10 years, we estimate its value to be $2000
- Costs $6000 per year to operate this asset

and the challenger

- Has a purchase price of $50000
- Its estimate value in 10 years is $12000
- A $2000 dollars per year operating cost

To determine the EUAC of both options, let's use the defender as an example

- Over the next 10 years, the value of asset will reduce to 0. To account for the gradual loss, we convert the present value of $30000 to an annual cost

- Actually, with more information, we know that the value of the asset in 10 years is equal to \$2000. So we convert the future value of \$2000 to an annual benefit to account for this in our EUAC.
  - Depending on our sign convention, we might have the EUAC from the initial value to be positive. In which case our annual benefits will be negative
- The operational cost is already given in terms of real dollars, in annual terms

Which ever EUAC is the more positive, we should keep that option in operation for the next 10 years

Example: Consider a more complex example. Given an interest rate of 10%, we first make a plot of the marginal cost of the project over time.

	Data		Calculating Marginal Costs		
Year	Salvage Value	O&M Cost	$S_{t-1}(1+i)$	$-S_t$	Marginal Cost
0	\$5,000				
1	4,000	\$ 0	\$5,500	−\$4,000	\$1,500
2	3,500	100	4,400	−3,500	1,000
3	3,000	200	3,850	−3,000	1,050
4	2,500	300	3,300	−2,500	1,100
5	2,000	400	2,750	−2,000	1,150
6	2,000	500	2,200	−2,000	700
7	2,000	600	2,200	−2,000	800
8	2,000	700	2,200	−2,000	900
9	2,000	800	2,200	−2,000	1,000
10	2,000	900	2,200	−2,000	1,100
11	2,000	1,000	2,200	−2,000	1,200

We can see that the marginal cost of the project behaves in a complex manner. This is why we will use analysis method #3.

Let's follow the convention that the EUAC is reported in terms of its magnitude.

The EUAC of each year consists of

- Capital recovery - what we gain from the salvage value. Given the initial cost, P (salvage value at year 0), and the salvage value that year S, the contribution from capital recovery if the project was retired at the end of year n is

$$(P-S)(A/P, 10\%, n) + Si$$

- Maintenance - given by the total cost of maintence for keeping the asset for n years, converted to an annual cost

			If Retired at End of Year n		
Years Kept, n	Salvage Value (S) at End of Year n	Maintenance Cost for Year	EUAC of Capital Recovery $(P-S)\times(A/P, 10\%, n)+Si$	EUAC of Maintenance $100(A/G, 10\%, n)$	Total EUAC
0	P = \$5,000				
1	4,000	\$ 0	\$1,100 + 400	\$ 0	\$1,500
2	3,500	100	864 + 350	48	1,262
3	3,000	200	804 + 300	94	1,198
4	2,500	300	789 + 250	138	1,177
5	2,000	400	791 + 200	181	1,172
6	2,000	500	689 + 200	222	1,111
7	2,000	600	616 + 200	262	1,078
8	2,000	700	562 + 200	300	1,062
9	2,000	800	521 + 200	337	1,058
10	2,000	900	488 + 200	372	1,060
11	2,000	1,000	462 + 200	406	1,068

Chapter 8

Accounting

Three functions within business

1. Engineering economy: analyzes economic impact of alternatives and project over life cycles
   - Tries to look into the future
2. **Accounting**: Determines dollar impact of past decisions, reports economic viability of the firm, and evaluates funding sources
   - Records all transactions
   - Analyzes past transactions, summarize, and reports on them
   - Provides data for general accounting and cost accounting
   - Forecasts future cashflows
3. Management: allocates investment funds to projects, evaluates performance, allocates resources, and directs personnel
   - Takes information from engineering economic analysis and accounting to make decisions

Accounting statements

Consists of the **balance sheet** and the **income statement**.

Balance sheet: a snapshot of the firm's financial condition at a point in time.

Income statement: the firm's performance specified over a period of time. Also known as the **profit loss statement**

The fundamental relation in accounting is that **assets equal liabilities plus equity**.

- **Assets** are properties of the firm that have monetary value. They can be physical or virtual
- **Liabilities** are dollar claims against the firm. This is money that the firm owes to someone else. Liabilities can also be products that the firm promised to deliver
- **Equity** represents the available funding from the firm. But this isn't necessary cash on hand

In a balance sheet **current assets** include

- Cash - things that go into this category is straight-up cash, or things taht can be turned into cash relatively quickly
- Accounts receivable - money we can collect, but haven't yet (coming from products we have sold)
- Securities - stocks we have in another company, or other holdings
- Inventories - includes the raw materials, and along with completed products that the company current has

Fixed assets include things that cannot be turned into cash quickly

- Land
- Plant and equipment
- Minus any accumulated depreciation

Other assets include

- Prepays or deferred charges
- Intangibles

Current liabilities include money that can be taken back on very short order

- Accounts payable - products, materials, services we received, but haven't paid for yet
- Notes payable - money we received, but haven't paid for
- Accrued expense - for example, paid vacations for employees. If the employees decide to defer their vacation to a later date, this becomes a liability to the firm

Long term liabilities might include long term loans.

Equity include

- Preferred shares - do not give shareholders voting rights, but preferred shareholders are paid dividends before holders of common share
- Common shares - common shares give the shareholder 1 vole per share they own
- Capital shares - the total number of common and preferred shares
- Retained shares

Working capital: different from our previous definition, the working capital in accounting is equal to the difference between current assets and current liabilities

The starting and ending equity in a **balance sheet** is linked to the overall profit/loss specified by an **income statement**

To find the **retained earnings** in a balance sheet at the end of a period, we need to look at

- Net income/loss given by the income statement
- Any capital aquisitions
- Any depreciation of capital assets
- New stocks and dividends

Let RE_{end} be the ending retained earnings,

$$RE_{end} = RE_{beg} + \text{Net income or loss} + \text{New Stock} - \text{Dividends}$$

- Issuing new stock will get us more equity on hand
- Includes book costs, so retained earnings will differ from actual cashflows

Cost accounting

Cost accouting: used to develop product costs, determine the mix of labour, materials, and other costs in a production setting, and evaluate outsourcing and subconstracting possibilities

The two main ways to do cost accounting is

- Cash accouting
- Accural accounting

In **cash accounting**, revenues and expenses are only recorded when there is cashflow.

- works well for simple transactions and possibly small businesses, but can get complicated quickly

In **accural accounting**, revenues and expenses are recorded as they are incurred

- Revenues are recorded when a sale is invoiced, but not when the payment is received
- Expenses are entered when an order is made
- Reduces the room for "shenanigans"

In cost accounting, we have

- **Direct costs**: due to activities directly associated with the final product
- **Indirect costs**: due to activities not easily linked directly to a project
  - Machine depreciation: a single machine might be used to produce multiple products. How much of the machine's depreciation was due to producing "product A" as opposed to "product B"?
  - Management
  - Sales
  - Overhead costs like adminstration

One way to accounting for indirect costs is through **Absorption costing**, where we allocate overhead costs between different product based on the relative ratio of factor such as

- Number of direct-labour costs each project has
- Direct-labour hours
- Direct-material cost
- Total direct cost

If producing product A costs twice as many hours than to produce product B, then we will allocate twice has much overhead costs to product A as do to product B.

Example: lets say that a company produces 50,000 units of a product. It has

- Direct material costs of $20 per unit
- $17 per unit in direct labour costs
- $3 per unit in variable manufacturing costs

And there is a fixerd overhead cost for a total of $100,000 per year.

The direct cost per unit is

$$20 + 17 + 3 = 40$$

Since the firm only produces this single product, the overhead cost per unit would be

$$100,000/50,000 = 2$$

But this can potential distort product costs. Firms are shifting towards the use of **activity based costing**, which provides better insights into "true" costs and the profitablility of products and services at the expense of being more difficult to implement.

Example: a firm produces two types of generators:

- Type 1: 100 units; Has direct costs fo $500 per unit
- Type 2: 500 units; Has direct costs of $300 per unit

Both type 1 and 2 generators use the same assembly line, for a total overhead cost of $60,000. It takes twice as long to assemble type 1 generators compared to type 2, per unit.

If assembling each type 2 generator takes up "1 unit of time", then assembling all 600 generators take up

$$500 + 100 \times 2 = 700$$

units of time. So we can say that the type 1 generator uses $2/7$ of the line time, and the type 2 generator uses $5/7$ of the line time.

Using activity based costing, we would say that the type 1 generators have

$$(2 \cdot 60,000/7)\ /100 = \$171$$

in overhead costs per unit produced (as opposed to $100), and the type 2 generators uses

$$(5 \cdot 60,000/7)\ /500 = \$86$$

in overhead costs per unit (as opposed to $100).

Using activity based costing, our overhead cost allocation became more reasonable. Using absorption costing, type 2 generator was more expensive than it needed to be, and the type 1 generator was too cheap.

Product pricing

Challenges:

- Availability and ease of access of data that should be used in analysis - data might be tucked away, not easily exported . . .
- Inventory or land maybe valued too-low since it was based on acquisition cost instead of what it was actually worth in the "market"
  - This might work to our benefit when we try to evaluate our financing to sell our campany. In this case, our return on capital may appear higher - A company that is considering acquiring our firm will value our return on captial more than the total worth of our assets
  - It might be bad when we try to sell this asset
  - It might make the company appear like it has less value in its assets - a lender will value your asset worth more than your return on capital
- Valuation of capital equipments may differ depending on the depreciation method and company police used.

Public Sector Analysis

Engineering project analysis will differ when we consider projects from a public perspective.

Purpose of investments in public projects can be ambiguous.

In the private sector, we might consider making a profit as the main purpose of a project. Any potential external consequences (**externalities**) are often neglected, unless there is "corporate social responsibility" or government regulations in this area.

Externality: external consequence often neglected in engineering economic analysis. The same action can lead to positive (good) externalities for one group, and negative (bad) externalities for another group

In the public sector, we might consider

- general welfare of society
  - how do we evaluate the benefits of having a road, an army or a communtiy center?
- provide benefits to some community
- "to re-distribute wealth"

as the main purpose of the project.

Governments are attempting to **internalize externalitiess**

- Imposing fines for going over a certain emission limit
- Imposing road taxes to reduce congestion
- Establishing patents to encourage inventors to invent
- Copyright to allow artists to make benefits from their creations

Viewpoint for analysis

The government tries to take the viewpoint of its constituents - the municiple government, provincial government, and the federal government

- This can lead to conflicts when its constituents have different priorities

Recall the Canada line. The provincial government was focused on getting the Canada line built. But Translink was more focused on providing regional benefits by getting the millenium line built to Coquitlam.

Building a dam in BC benefits the people of BC and will be funded by the people of BC. So the viewpoint we should take is from the people living in BC.

As a rule of thumb, take the viewpoint at least as broad as those who pay the costs and those who receive the benefits.

Interest rate selection

The goal of public investment involves *the use of public resources to promote the general welfare and to secure the benefits of a given project ot whomsoever they may accrue,* ***as long as those benefits outweigh the costs***.

- **Time value of money:** argues that the interest rate should be 0% since there is very little time/no time at all between collecting money from taxpayers and spending tax dollars
  - Problem is that while cashflows occur quickly, the benefits of investment does not accrue immediately
- **Cost of capital:** argues that the interest rate should be the cost for government to borrow money. On top of collecting taxes, governments also borrow money at a very low interest rate compared to the private sector
  - Canada Savings Bonds: 1 to 2.5% interest rate. You are basically borrowing money to the government
  - For BC, the average interest for the money the govenment borrowed was 4%
- **Opportunity cost:** the opportunity cost for the government is the best opportunity foregone by the government. The **taxpayer opporunity cost** is the opportunity foregone by the taxpayers from paying taxes and not investing
  - One example of taxpayer opportunity cost is the Canada pension plan, which yields a lower rate of return than most profiles of stocks and bonds. Instead of putting your money into the CPP, you could have been invested it and receiving a greater rate of return instead

Cost benefit ratio

The benefit-cost ratio is often used in public sector investment analysis. It is the ratio of the (present/future/annual) value of our benefits and our costs.

- If the ratio is > 1, then we should invest
- If the ratio is < 1, then we should not invest
- If the ratio is ≈ 1, then the decision requires further consideration
  - A ratio equal to 1 suggests a net present value of 0 and that the project's IRR is equal to our interest rate
  - A ratio of 1 indicates that the project does not have a significant benefit. It is especially difficult to quantify and estimate cashflows for public sector projects

Incremental benefit-cost analysis

When there are multiple (mutually exclusive) alternatives, we need to also consider the incremental benefit-cost ratio - is it worth it to take on an option that is worth more initially?

1. Identify all possible alternatives
2. Find the B/C ratio for each alternative
3. Eliminate all projects with a B/C that is less than 1
4. Rank the project in ascending order of cost (present/future or annual will work)
5. Find the incremental B/C ratio for the two lowest cost alternatives
6. If the ratio is greater than 1, then switching to the higher cost alternative is justified
7. Iterate through all the possible alternatives until there is only one left

Project financing

- Smaller projects are fully funded through taxation
- Larger projects may require borrowing through bonds
- **Public-private partnerships** may also be formed - for example, the Canada line was a "P3"

Most public sector projects have very long project lifetimes.

- At a fixed project lifetime, increasing the project interest rate decreases the B/C ratio of the project
- At a fixed interest rate, increasing the project lifetime increases the B/C ratio of the project

Quantifying and valuing benefits and costs

- Many projects have difficulty quantifying benefits. How do you weigh the benefits of building a school?
- As a result, different parties often choose different ways to quantify these benefits depending on their viewpoint

Project politics

Politics can significantly influence project decisions.