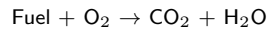


ENPH 257

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Fundamentals

- Density of water: 997 kg/m^3
- $R = 8.314 \text{ J/mol/K}$
- $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$
- $k_b = N_a R$: Boltzmann's constant is related to R through Avagadro's constant.
- Dew Point: Temperature at which local vapour pressure is equal to the saturated vapour pressure
- Parts per million by volume: $V_1 \times 10^5 / V_2$
- Combustion Equation:



- **Vapour Pressure**: pressure at which a gas can coexist with its solid or liquid phase
- Solid angle: $\Omega = A/r^2$

The "3" and "5" in C_V is related to the degree of freedom of monatomic and diatomic molecules

$$\begin{array}{lll} C_V = \frac{3}{2}R & C_P = \frac{5}{2}R & \text{monatomic} \\ C_V = \frac{5}{2}R & C_P = \frac{7}{2}R & \text{diatomic} \end{array}$$

Ideal Gas Law

$$\rho = \frac{m}{V} = \frac{pM}{RT}$$

p : pressure; ρ : density; m : mass of gas in g; M : mean molar mass in g/mol; R : ideal gas constant (8.314 J/mol/K); T : temperature

Work

$$dL = pS dh = p dV$$

Work between states A and B is

$$L = \int_{V_A}^{V_B} p dV$$

Isobarics

$$Q_H = nc_p(T_2 - T_1)$$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

Isothermals

$$pV = \text{constant.}$$

Work done by Isothermals

$$L = \frac{m}{M} RT \ln \left(\frac{V_2}{V_1} \right) = \frac{m}{M} RT \ln \left(\frac{p_1}{p_2} \right)$$

Adiabatics

$$TV^{\gamma-1} = \text{constant}$$

$$PV^{\gamma} = \text{constant}$$

$$P^{1-\gamma} T^{\gamma} = \text{constant}$$

γ : Heat capacity ratio (C_v/C_p)

First Law

Change in internal energy plus the work done by the system is equal to heat absorbed.

$$\Delta U + L = Q$$

ΔU is a function of temperature alone for gases.

ΔU is zero for cycles.

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

Q_2, Q_1 : heat absorbed, heat given off by system; T_2, T_1 : temperature of high and low source

Second Law

1. Lord Kelvin: "A transformation whose **only** final result is to transfer into work heat extracted from a source which is at the same temperature throughout is impossible."
2. Clausius (Restated): "If heat flows by conduction from a body A to another body B then a transformation whose **only** final result is to transfer heat B to A is impossible."

Cycle Efficiency

$$\eta = \frac{W}{Q_H}$$

For Carnot cycle

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \frac{Q_1}{Q_2}$$

Entropy

For reversible cycles, the equality holds

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{Q_i}{T_i} = \oint \frac{dQ}{T} \leq 0$$

dQ : heat absorbed; T : temperature of the cycle

Transformation between equilibrium states

$$S(B) - S(A) \geq \int_A^B \frac{dQ}{T}$$

- Integral is path independent.
- Equality for reversible transformation
- Greatest change in S arise from reversible transformations

$$\begin{aligned} \Delta S &= nc_v \ln \left(\frac{T_2}{T_1} \right) + nR \ln \left(\frac{V_2}{V_1} \right) \\ &= nc_p \ln \left(\frac{T_2}{T_1} \right) + nR \ln \left(\frac{P_2}{P_1} \right) \end{aligned}$$

Barometric Equation

$$\frac{dp}{dz} = -\rho g = \frac{-Mg}{RT} p$$

p : pressure; ρ : density; M : mean molar mass in g/mol; g : gravitational acceleration in m/s^2 ; R : ideal gas constant (8.314 J/mol/K); T : temperature

$$p(z) = p(0) \exp \left(\frac{-Mgz}{RT} \right) \quad \text{constant temperature}$$

Scale Height

Altitude when pressure drops to $1/e$ of pressure at sea level ($z = 0$).

$$\frac{RT}{Mg} \quad [\text{km}]$$

T : mean surface temperature

Dry Adiabatic Lapse Rate

$$\Gamma_d = \frac{dT}{dP} \frac{dP}{dz} = \frac{dT}{dz} = \frac{Mg}{R} \left(1 - \frac{1}{\gamma} \right)$$

For K/km: M : mean molar mass in g/mol (use kg/mol if you want K/m); g : gravitational acceleration in m/s^2 ; R : ideal gas constant (8.314 J/mol/K)

Clapeyron's Equation

$$\frac{\partial p}{\partial T} = \frac{\lambda}{T(v_2 - v_1)}$$

λ : latent heat of evaporation

v_1, v_2 : inverse of density (volume per unit mass)

Take $v_2 \gg v_1$, and let v_2 be

$$pv_2 = \frac{RT}{M} \quad \frac{dp}{dT} \approx \frac{\lambda M}{RT^2} p$$

And the solution is

$$p(T) = K \exp \left(\frac{-\lambda M}{RT} \right)$$

M : molar mass

Relative Humidity

$$RH = \frac{P_p}{SVP}$$

P_p : Partial vapour pressure; SVP : Saturated vapour pressure

Moisture Content

$$\frac{\rho_w}{\rho} = \frac{(SVP)(RH)r_w}{P_t} = \frac{P_p}{P_t} r_w$$

P_t : Total pressure

r_w : molar mass ratio between water and molecule of atmosphere

Enthalpy

Enthalpy is the internal energy of the system plus the work needed to make room for it.

$$H = U + PV$$

This is energy needed to create the system out of nothing and put it in such an environment.

$H > 0$: Endothermic

$H < 0$: Exothermic

Gibb's Free Energy

Is the work you need to do in an environment of constant pressure and temperature in order to create that system. It is enthalpy minus the energy TS can flow in as heat from the environment.

$$\begin{aligned} G &= U - TS + PV \\ &= H - TS \end{aligned}$$

At constant temperature and pressure

$$\begin{aligned} \Delta G &= \Delta U - T\Delta S + P\Delta V \\ &= \Delta H - T\Delta S \end{aligned}$$

Voltage of a Fuel Cell

$$V = \frac{\Delta G}{nF}$$

where ΔG is the Gibb's free energy per mole, $F = 9.65 \times 10^4$ C, and n is the number moles of electrons in the redox reaction

Heat and Electrical Work in a Fuel Cell

Electrical Work is the negative Gibb's free energy, $T\Delta S$ is the hear from environment.

Mean Kinetic Energy

$$\frac{3}{2}k_B T = \frac{1}{2}m\langle v^2 \rangle$$

For a point like particle, monatomic particle. 3 is related to the 3 degrees of freedom of such a particle.

Bulk Modulus

Is the radio pressure change to the fractional change in volume

$$B = -V \frac{\partial P}{\partial V}$$

Sound

Tramission of sound in air is adiabatic.

$$c = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma k_B T}{m}} = \sqrt{\frac{\gamma}{3}} v_{rms}$$

where

$$v_{rms} = \sqrt{\frac{3k_B T}{m}}$$

Maxwell-Boltzmann Distribution

$$f(v) = \sqrt{\left(\frac{m}{2\pi k_B T}\right)^3} 4\pi v^2 \exp\left(\frac{-mv^2}{2k_B T}\right)$$

$$v_{rms}^2 = \int_0^\infty v^2 f(v) dv = \frac{3k_B T}{m}$$

m is the mass of the molecule.

Spectral Radiance

Is power per wavelength per solid angle going out. (Irradiance is radiation coming in.)

$$B_\lambda = \frac{2hc^2}{\lambda^2} \frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right) - T} \quad [\text{W}/\text{sr}/\text{m}^2/\text{m}]$$

Radiance is spectral radiance integrated over the entire spectrum.

Stefan-Boltzmann Law

Total power radiated by a black body is

$$P = \epsilon A \sigma T^4$$

Wien's Displacement Law

$$\lambda_{min} \propto \frac{1}{T} = \frac{b}{T}$$

where $b \approx 0.003$ mK.

Photovoltaics

All of photon energy at maximum wavelength corresponding to the band gap voltage of the cell is converted into electrical work.

$$\lambda_{max} = \frac{hc}{qV_{band}}$$

Any larger wavelength equals zero conversion. Any smaller wavelength sees rapidly decreasing efficiency.

Direct Heating

For a sphere of radius a , emissivity ϵ , and reflectance r (or albedo).

$$P_{in} = (1 - r)I\pi a^2 + \sigma 4\pi a^2 T_{amb}^4$$

$$P_{out} = \epsilon \sigma 4\pi a^2 t^4 + k_c 4\pi a^2 (t - T_{amb})$$

Power in

$$(1 - r)A_{eff}ST^4$$