MECH 431

Ruiheng Su 2021

Chapters 1/2

General economics concepts

Economics is study of the use of scarce resources that have alternative uses

- We don't have enough resources to give everyone what they want (time, budget, facilities, data, labor, people with expertise...)
- Managing scarce resource requires making choices and often value judgements
- Making a choice involves "not" doing another choice - trade-offs

Opportunity cost the cost of forgoing the next best thing

- Economics quantifies resource management
- Econometrics is statistics used to analyze economic data

Decision framework and current cost models

Problems come in levels of difficulty

- Simple: not much effort required; small amount of variables; obvious solutions exist
- **Intermediate**: economics problem; needs structured thought;
- Complex: involves a mixture of economic, political, social, and ethical elements; (annual budget of a corporation; building a pipeline; choosing a partner...)

Intermediate questions focus on costs, revenues and benefits that occur at different times

- Which projects are worthwhile?
- How should projects be designed?

Not all problems require engineering economics analysis. Problems that do should

- be important enough to justify serious thought
- have economics issues as a significant component of the analysis leading to a decision
- require organization
- the problem requires decision variables and their consequences be well understood
- there are non-financial factors involved (first nations rights, safety, ethics...)

Decision-making process

1. Recognize the problem
2. Define the goal
   - Wide scope: "make the business more profitable"
   - Narrow scope: "Determine the most economical machine to buy"
3. Assemble relevant data
4. Identify feasible alternatives
5. Select the criterion to determine the best alternative
6. Construct a model
7. Predict the outcomes or consequences for each alternative
8. Choose the best alternative
9. Audit the results

Current cost models simple arithmetic models that compare anticipated costs and benefits over a short period of time. (Such as costs and revenue per unit of product produced and sold.)

In current cost models we only consider the costs, but we neglect

- Currency inflation
- Cost of time, cheaper option may take a longer time
- Depreciation of instruments
- Quality of product
- Functionality of product
- Ethics
- Environmental impacts
- Differentiate between business and pleasure activities (going to Hawaii for training - is this appropriate?)
- Safety - what is safe enough, what is too safe?
- Issue that arise from globalization - different countries have different ethical expectations

Costs and cost estimating

Costs the economic value of the resources used in the production of goods and services. (Such as land, labour, capital, utilities, . . .)

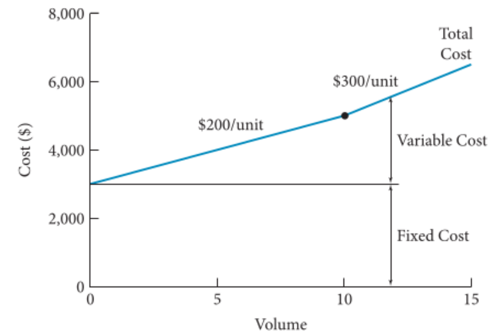
There can also be **social costs**. These are also called externalities. An **externality** is a cost (or benefit) to a third party caused by a producer that is not financially incurred or received by that producer.

There are various types of costs:

- **Fixed costs:** costs that remain constant
  The rent for a storage unit stays the same regardless how much of that unit you utilize.
- **Variable costs:** opposite of fixed costs. They depend on the level of output, or activity
  The amount of raw material a manufacturer need to purchase varies with the number of items produced.
- **Total cost** is sum of the fixed and variable cost.
- **Marginal cost:** variable cost per unit
- **Average cost:** total cost per unit

On a units-produced versus total cost plot,

- the fixed cost is a constant offset to the curve
- the variable cost manifests itself in the **changing** slope of the curve as more/less units are produced
- the variable cost at unit n is cost curve evaluated at n minus the fixed cost
- the marginal cost for the n^{th} unit is reflected in the slope of the curve at n



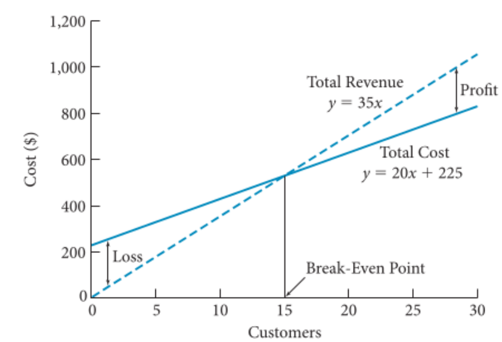
The same terms and definitions apply when we consider **revenues**.

We

- **Break even:** when the total revenue is equal to the total costs
- **Profit:** when the total revenue exceeds the total costs
- **Loss:** when the total costs exceed the total revenue

On a units-produced versus costs plot, the

- **Break even point** is at the intersection between the revenue and costs curve
- the profit/loss (at n^{th} unit sold) is the difference between the revenue and cost curve at n



Since the cost curve at $n = 0$ has a non-zero offset, we say that our production has fixed costs. In the plot shown, we do not have fixed revenue.

Sunk cost: money spent in the past that cannot be recovered.

- disregarded in engineering economic analysis

Example: money paid to buy a car two years ago. The money you spent shouldn't affect you decision when you decide to buy a new car.

Opportunity cost: cost associated with a resource being used for an alternative task; cost of forgoing the next-best thing.

Overhead cost: "indirect costs of running a company that cannot be tied to any particular task that the company executes."

Looking at costs on a timescale,

- **Recurring costs:** costs that occurs at regular intervals

  An Paying rent for your storage unit.

- **Non-recurring cost:** unique costs that occur at irregular intervals

  Costs due accidents, unexpected illness, capital expenditure (buying new equipment)

  To prevent large non-recurring costs, we may choose to buy insurance. This leads to a recurring cost of the insurance premium.

When comparing different options, the **incremental cost** is the cost difference between two alternatives.

When considering how costs are actually "paid out"

- **Cash costs:** require money to move from one party to another
- **Book costs:** are recorded ("accounting costs"), but are not transactions, and do not involve cash flow from one party to another

  Book costs are often accounting exercises. They do not directly involve cash flow, so they are not accounted for in an analysis. But they can generate cash costs/revenue. How you report your spending can lead to different tax percentages.

The **life cycle** is all the time from conception to the retirement of a product/process.

- **Life cycle cost:** total of all the costs incurred over the life cycle of the process/product
- **Life cycle costing:** designing products, goods, and services recognizing their associated costs over their life cycles.
- The later a design change is made, the higher the cost.
- Early design decisions "lock in" costs that will be incurred later - 70% to 90% of all costs are set during the design phases

Cost models

We can classify cost estimating into three levels:

1. **Rough estimates:** back of the envelope calculations, accuracy can vary widely
2. **Budget/Semi-detailed estimates:** based on historical data
3. **Detailed estimates:** estimates made from detailed designs using quantitative models and vendor quotes; Highly accurate

Estimation accuracy are affected by

- **Resource constraints:** whether there is enough time and labor to retrieve all the information we need for a precise estimate
- **Experience:** Whether the estimator is experienced
- **One-of-a-kind estimates:** whether similar projects has been done in the past that can be used as a reference for the current estimate; (the cost of sending astronauts to the moon for the first time)

Cost estimating models

1. **Per-unit model:** uses a cost per unit factor
2. **Segmenting model:** divided a problem into item, estimate each, and add together
3. **Cost indexes:** cost indexes are dimensionless values that record the historical change in costs. The ratio of cost indexes are of primary interest. They are not absolute measures

$$\frac{\text{Cost at } T = a}{\text{Cost at } T = b} = \frac{\text{Index value at } T = A}{\text{Index value at } T = b}$$

4. **Power sizing model:** used to estimate the costs of industrial plants or equipment. We account for the economies of scale using a **power sizing exponent**, x, which determines how much to scale up or down costs of an product at two different capacities

$$\frac{\text{Cost at capacity A}}{\text{Cost at capacity B}} = \left(\frac{A}{B}\right)^x$$

5. **Learning curve:** accounts for cost improvements. The time required to produce the N^{th} unit is related to the time required to produce the first unit, and a **learning curve exponent**

$$T(N) = N^b T(1)$$

Learning curves are often referred to by its **percentage learning slope**. A curve with $b = -0.074$ is a "95%" learning curve since $2^b = 0.95$.

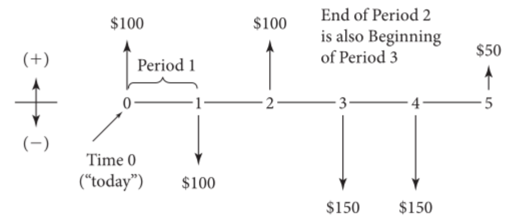
We can calculate b by

$$b = \frac{\log(\text{learning curve expressed as a decimal})}{\log 2}$$

Cashflow diagrams

A cash flow diagram is a chart of class flows over a period of time. It consists of

- discrete time points $(0, 1, 2, 3, \ldots)$
- arrows to indicate cash flow
- the arrow points up (positive) if it represents revenue
- the arrow points down (negative) if it represents expenses
- the length of the arrow is proportional the amount of cash involved
- Instead of arrows, we can use bars instead



The categories of cash flow:

- **First cost:** cost that occurs at time 0
- **Operations and maintence:** ongoing/recurring expenses
- **Salvage value:** receipt at project termination
- **Revenues:** annual receipts (annual sales/ reduced costs)
- **Overhaul:** major capital expenditure occuring during life of asset

Simple cashflow analysis techniques

Do not provide a comprehensive view of a project. They help set minimum financial viability requirements before doing more detail analysis.

Payback period is the time required to break-even, neglecting any interest rates and any economic consequences that occur after the payback period. (Time to required to break even.)

Two options may have the same payback period, but one option might provide further benefits, but this is not reflected in the payback period.

Two options may have the same payback period, but one option might provide a consistent return, while the returns of the other option may be back weighted. (They have different benefits vs time curves.) The payback period does not reflect this.

The pay back period does no account for the **time value of money**.

Cost benefit ratio (CBR) the ratio of total costs divided by total benefits. If CBR ≥ 1, then the project may be worthwhile. Otherwise, further consideration is required.

Chapter 3

Time value of money

Engineering projects often take up over multiple years.

Time value of money the idea that the value of cash today will be worth more than in the future because of the present day's earning potentials.

- The stronger the preference for current consumption, the more important time is in investing
- When time becomes important, we also need to consider **interest**, inflation, depreciation, other costs

Interest two interpretations of interest:

1. rate at which we consider how the value of money changes over time (related to the time value of money, and uncertainty and risk)
2. an amount paid by a borrower to get access to money; an amount received by a lender to lose access to a sum of money

Another interpretation is based on **uncertainty and risk**. The later we receive the benefits, the less certain we can be about what the money will be worth, and what it can be used for.

Money is a has value, and so they can be leased or rented.

- **Interest payments:** is the "rent" that a party pays to whenever they borrow a sum of money; this also reflects the risk of the lender losing the money entirely
- **Interest rate:** is the rate of return received by a lender for lending money; they quantity how a lender/borrower values money over time

To a lender,

- A high interest rate loan means that the lender values the same amount of money higher today, than they do in the future
- A low interest rate loan means that we value money today similarly than we do in the future
- A zero interest rate means that the lender values money at exactly the same between today and the future
- A negative interest rate means that the lender values the money more in the future than they do today (extremely rare in an engineering scenario)

Inflation refers to the trend that the amount of goods and services we can purchase with the same amount of money decreases over time.

Depreciation most assets tend to lose value over time.

We rather have money today since

- Having money today means that we can collect rent on it (Money is a commodity)
- Less risk
- money tend to lose value over time assets depreciate over time
- Having money today means that we can invest it today

Simple and compound interest

Consider the following definitions

- **P:** present value. This is the value at the initial time of an analysis
- **F:** future value. This is the value of as asset in the future relative to time of PV
- **i:** interest rate per period
- **I_n:** interest paid in the single period n
- **n:** index for the period

For discrete cast flows, we can compute

Simple interest: interest that is applied once to the "principal amount" and paid at the end of the term (on the **maturity date**). Given the principal value P, interest rate per period i, and n periods,

$$I = Pin$$

The total amount that a lender who lends out P for n periods receives is the future value F.

$$F = I + P$$

Simple interest is easy to compute, but rarely seen in real life.

Compound interest: interest that is computed on the outstanding amount after every compounding period. The outstanding amount includes the unpaid principal and unpaid interest that has been accumulated.

$$I_n = P(1+i)^n - P$$

At the end of the first period:

$$I_1 = Pi$$

and the future value at the end of the first period is

$$F_1 = I_1 + P = P(1+i)$$

At the end of the second period,

$$I_2 = i(P(1+i))$$

and the future value at the end of the second period is

$$\begin{aligned} F_2 &= F_1 + I_2 \\ &= P(1+i) + Pi(1+i) \\ &= P(1+i)^2 \end{aligned}$$

The total amount we need to pay after n compounding periods is

$$F_n = P(1+i)^n$$

Finally,

$$I_n = F_n - P$$

We can define the **single payment compound amount factor**:

$$\text{SPCAF} = (1+i)^n$$

which is constant for any combinations of i, n.

The inverse of the SPCAF is the **single payment present worth factor**:

$$\text{SPPWF} = \text{SPCAF}^{-1}$$

- Values of SPCAF are often tabulated in compound interest tables
- We compute F from P by multiplying P by the SPCAF
- An interest rate that is used to convert F to P is a **discounting rate**
- An interest rate used to convert a P to F is a **compounding rate**

Nominal and effective interest rates

Other than per year, interest rates can be specifed "semi-annually", or "quarterly", et cetera.

- **Nominal interest rate (r):** is an interest rate computed by directly scaling an interest rate such that it becomes a "per year" interest rate, with no consideration of compounding.

Example: a bank bond pays 1% every quarter. The nominal rate is $1\% \times 4$ which gives 4% per year.

"12% per year, compounded semi-annually" is also a nominal interest rate.

- **Effective interest rate (i_a):** takes compounding into consideration - a bond that pays 1% quaterly will end up paying you more than 4% annually. For m compounding periods (per year),

$$i_a = \left(1 + \frac{r}{m}\right)^m - 1$$

r/m is the **interest rate per compounding period**.

Example: investing 1000 dollars at 6% per year, compounded semi-annually. (The 6% is nominal interest rate.)

The future value of our investment after one year is

$$F = P(1+i)^n$$

where i is the interest rate per compounding period, and n is the number of periods.

In this case, we have 6% per year, but its compounded semi-annually. So i is $6\%/2 = 3\%$. A year consists of two compounding periods, so $n = 2$. Then the future value is

$$F = 1000(1+0.03)^2 = 1060.90$$

Recall that the interest amount is

$$I = F - P = 60.90$$

So the effective annual interest rate is $I/P = 6.09\%$.

Cashflow equivalence

A sum of money at one time period may have the same "value" as a more/less sum of money at another period in time, with respect to an interest rate.

We can use `Excel` functions, `FV`, `PV` to help us.

- If we want the future value on an investment, then we should put the present value as negative
- If we want the present value of a future amount (that is a revenue), then the value returned by `PV` is negative, since we have to invest this present amount to receive a future amount

Chapters 4/5

Uniform Series

Previously, we considered single payments.

Uniform series: A series of cash payments received in uniform period, amount A, and interest rate i.

For example, $200 dollars received every year at an interest rate of 2%.

If payments are given in a **uniform series**, then we can compute the present value of n payment using the **uniform series present worth factor**:

$$\mathbf{USPWF} = \left(\frac{(1+i)^n - 1}{i(1+i)^n}\right)$$

where

- i is the annual interest
- n is the number of annual payments received

Such that the present value of n uniform annuities (denoted A) can be found by

$$P = A\left(\mathbf{USPWF}(i, n)\right)$$

Consider the present value a payment of A received n years from now, at an interest rate of i. We know that its present value can be found by

$$P_n = A(1+i)^{-n}$$

For n annual payments equal to A, the present value is

$$\begin{aligned} P &= A(1+i)^{-1} + A(1+i)^{-2} + \cdots + A(1+i)^{-n} \\ &= A\left(\sum_{j=1}^{n} \frac{1}{(1+i)^j}\right) \end{aligned}$$

The partial sum can be simplified using the geometric series partial sum formula.

- This answers: "what do I need to invest today to get an annuity payment of A for n years?"

The inverse of **USPWF** is the **capital recovery factor**. This factor allows us solve for the annuity amount A, given an investment P.

If we invest $1000 today at $10 percent interest, and we want to withdraw it over 5 years, then the capital recovery factor is

$$\frac{1}{\mathbf{USPWF}}(0.1, 5) = 0.263797481$$

So the annuity amount we would received is $263.8.

Annuity: a fixed sum of money paid to someone each year

The future value of an annuity is the product of the annuity amount and the **uniform series compound amount factor**.

$$\mathbf{USCAF} = \left(\frac{(1+i)^n - 1}{i}\right)$$

where

$$F = A\left(\mathbf{USCAF}(i, n)\right)$$

For example, if we make an annual deposit of $1500 for ten years at an interest rate of 8%, the balance of our investment is

$$F = 1500\left(\mathbf{USCAF}(0.08, 10)\right) = \$21729.84$$

The inverse of **USCAF** is the **uniform series sinking fund factor**, **USSFF**. The product of **USSFF** and F gives the annuity amount.

Arithmetic Gradient Cashflow Series

Instead of a uniform series, what if the annual payments we receive linearly increases? For example, receiving $100 every year, plus $50 extra for every year after the first year. This can be though of as a uniform series of $100, and also a growing portion, increasing by $50 every year.

Arithmetic Gradient Series: consists of a uniform series component A, and a **gradient** component G.

In our previous example, $A = 100$, and $G = 50$. The amount of payment received on year n is

$$\begin{cases} A + (n-1)G & n \geq 1 \\ 0 & n = 0 \end{cases}$$

The present value of an arithmetic gradient series is the sum of the present worth of uniform component and the present worth of gradient component.

$$P = P_{\text{uniform}} + P_{\text{gradient}}$$

The present value of the gradient portion is the product of the **arithmetic gradient present worth factor** and G

$$\begin{aligned} P_{\text{gradient}} &= G\left(\frac{(1+i)^n - in - 1}{i^2(1+i)^n}\right) \\ &= G\left(\mathbf{AGPWF}(i, n)\right) \end{aligned}$$

Example: Consider $1500 maintenance cost for every 6 months, and the maintenance costs grows by $75 every 6 months. What is the present value of the maintenance costs over a ten-year period, if the interest rate is 11.25% compounded semi-annually?

The 11.25% is the nominal interest rate. The interest rate per compounding period is $0.1125/2 = 0.05625$. The number of compounding periods in 10 years is 20. The uniform portion is $A = 1500$, the gradient portion is $G = 75$. So

$$\begin{aligned} P &= 1500\left(\mathbf{USPWF}(0.05625, 20)\right) \\ &\quad + 75\left(\mathbf{AGPWF}(0.05625, 20)\right) \\ &= 24585.49 \end{aligned}$$

So our total maintenance spending in the next ten years to equal to $24585.49 to us today.Once we find the present value of an arithmetic gradient series, we can use the capital recovery factor to convert the present value into an equivalent annuity payment.

The product of the **arithmetic gradient uniform series factor** and G gives the equivalent annuity amount of a uniform series equivalent to the gradient portion.

$$A_{\text{eq}} = G\left(\frac{1}{i} - \frac{n}{(1+i)^n - 1}\right)$$

With our previous example, $G = 75$ for $n = 20$, $i = 0.05625$ gives

$$A_{\text{eq}} \approx 578.69$$

The present value of our total maintenance cost over ten years can also be found by

$$P = (1500 + 578.69)\,\mathbf{USPWF}(0.05625, 20)$$

Geometric Gradient Cashflow Series

The payment on period $n = 0, 1, 2, \ldots$ is given by

$$\begin{cases} 0 & n = 0 \\ A_n(1+g)^{n-1} & n = 1 \end{cases}$$

where $g \in \mathbb{R}$, is the percent that the annuity amount grows every year.

The present value of a geometric gradient series is the product of A_1 and the **geometric series present worth factor**:

$$P = A_1\left(\frac{1 - (1+g)^n(1+i)^{-n}}{i - g}\right)$$

If $i = g$, then

$$P = \frac{A_1 n}{(1+i)}$$

The future value is the product of A_1 and the **geometric series compound amount factor**:

$$F = A_1\left(\frac{(1+i)^n - (1+g)^n}{i - g}\right)$$

Models are simple, but

- It's easier to start with simple models
- Quick, gives approximate constraints and bounds
- Sometimes no enough detail is known about the future

Annuities Due

We have been looking at cash flows that occur at the end of the period. For example, we take out a loan at the end of period 0. Period 1 begins. At the end of period 1, we accrue some interest on the loan. Period 2 begins . . .

Alternatively, cashflows are occur at the beginning of a period. For example, we take out \$5000 loan at 12% annual interest compounded monthly.

1. Receive cash at the end of period 0
2. Period 1 begins and 1% interest accrues on the loan payment. Our loan balance is now \$5050.
3. Any payments we make this month are then subtracted from the loan balance
4. Period 2 begins and the process repeats

Ordinary annuity/Annuity in arrears: annuity payments occur at the end of the period. (The first is one period from now.)

Annuity in advance/Annuity Due: annuity payments occur at the beginning of the period. (The first occurs now.)

To get a formula for an annuity due, multiply the formula by a factor of $(1 + i)$.

For ordinary annuity, consider the future value of investing \$25 per year for three years at 9% interest.

1. At the end of period 1, we make the \$25 payment. At the end of period 3, and our investment becomes $1.09^2 \times 25$
2. At the end of period 2, we make another \$25 payment. At the end of period 3, this \$25 becomes 1.09×25.
3. At the end of period 3, we make the last \$25 payment. No interest is accrued on this payment

So the future value is \$81.95.

For annuity due,

1. We pay 25 now, at period 0, and by the beginning of period 3, this payment becomes 25×1.09^3
2. We pay 25 in beginning of period 1, by the beginning of period 3, this payment becomes 25×1.09^2
3. We pay 25 in beginning of period 2, by the beginning of period 3, this payment becomes 25×1.09

The future value at the beginning of period 3 is 89.33. We end up collecting one extra period of interest.

Perpetuities

A series with perpetual payments are call perpetuities.

The future value of a perpetual series is ∞, at a non-zero, positive interest rate.

We will be only consider present values. If we take a current payment at "compound" it out forever, the payment has ∞ future value.

If we can "discount" payments that occurs in the future, starting from a time infinitely away to one period away, what would be the value of those payments now?

For constant, ordinary, annuity payments:

$$P = A/i$$

For constant, annuity due payments:

$$P = A(1 + i)/i$$

For a geometric series annuity where $i > g$:

$$P = A/(i - g)$$

Perpetuities are considered when we look at endowment funds by universities or churches. If a project life is very long, it might be possible for us to consider it using perpetuities.

Differing Periods

If the period of annuity payments differ from the compounding interest periods, we have **general annuities**.

To account for general annuities, we either

1. compute an effect interest for the payment periods
2. compute the equivalent payment amounts for each compounding period and apply the interest rate

The equivalent interest rate per payment period and the effective rate (nominal annual interest rate divided by the number of compounding periods per year) must satisfy

$$(1 + i_{\text{eq}})^p = (1 + i)^c$$

where p is the number of payment periods per year, and c is the number of compounding periods per year.

Rearraning,

$$i_{\text{eq}} = (1 + i)^{c/p} - 1$$

In Canada, mortgage interest rates are specified semi-annually.

Consider a loan of \$295000 that requires monthly payments for 25 years at an interest rate of 5.35% compounded semi-annually. Find the monthly payment.

The equivalent interest rate is

$$i_{\text{eq}} + 1 = (1 + 5.35/2)^{2/12} = 1.0044 \cdots + 1$$

We know the present value of the loan, so we can solve for the constant annuity payment.

$$P = A\left(\frac{(i_{\text{eq}} + 1)^n - 1}{i_{\text{eq}}(i_{\text{eq}} + 1)^n}\right)$$

Sources of Capital

A firm's interest rate depend on its sources of capital, the investment opportunities, and risk.

Sources of capital available to a firm generally fall into: money generated from operation, borrowed money, and money from selling stock.

Internal: money generated from the firm's operations; retained from profits.

External: money raised from sources outside the firm.

- Short term debt: bank loans, credit cards, line of credit
- Long term debt: bonds, pension funds, mortgages
- One-time: issue now stock - selling equity
- Funds from customers - a customer might be happy to fund you for other benefits, in the form a contract

Bond: is a loan taken out by the firm. The firm receives money from investors, and pays the investors interest on regular, agreed-upon intervals, and returns the capital on the maturity date. Investors need to be convinced that the firm will still be in business when the bond is due.

One example where buyers receive funds from suppliers is between grocery stores and suppliers. The suppliers pay to get their product onto the eye-level shelve space

Selling equity: refers to the salve of the common shares of a company, instead of only the assets

The mix of externally provided funds is the **capital structure.** This depends on the needs of the firm and the decision of its managers.

Cost of debt/borrowed money: is the interest rate at which money can be borrowed

- the lender charges a higher rate for more risky loans
- A large, profitable firm might be able to borrow money at the **prime rate** - the interest rate banks charge their best and most sought-after customers

Cost of equity: the rate of return that investors require on the shares of a firm

- a higher cost of equity reflects the market's assessment of the risk level associated with the firm being able to provide returns in the form of dividend and/or increase in the price of its shares
- Someone could offer to buy your company, only if you can double the sum in two years and pay them back. This corresponds to a 100% rate of return
- Another way to sell equity is in the form of "preferred shares". In exchange for your investment, the firm pays you a certain amount of dividends every year

A high risk firm has a highly uncertain cast flow. Lenders will want a higher rate of return.

Weighed average cost of capital (WACC): a firm's overall cost of capital is the average of the rates of return required by the providers, weighted by the fraction of the total capital provided

- Also known simply as the **cost of capital**
- The rate of return on a firm's common stock and retained earnings is called **return on equity**

If a firm's stockholder's expect a 15% rate of return, we should set the return on equity to be 15%.

Investment opportunities

Firms need to ensure that all selected projects are better than the best rejected project.

With a fixed capital, the firm invests in opportunities that have a greater rate of return than the rest.

- The opportunity cost is the rate fo return on the best rejected project

Minimum attractive rate of return (MARR): can be taken as the firm's interest rate. We require that the MARR is

- greater than the cost of capital (WACC)
- greater than the opportunity cost
- equal to the greatest of: cost of borrowed money, WACC, opportunity cost

The MARR may be adjusted based on risk. A higher MARR may be assigned to a project with higher risk.

Present worth analysis and equal lives

Better things to do: when analyzing whether a single project is worth pursuing or not, the underlying assumption is that there are other alternatives that we could be using our funds for.

In present worth analysis, we assume:

- All cash flows occur that the end of the period. Period 0 represents now
- No sunk costs - these should not affect our decisions going foward
- Two viewpoints - investor and borrower
- Money is obtained at interest rate i despite there might be multiple sources of capital

In general, we also need to consider the scope of the problem. On manufacturing scope, a process improvement might be welcomed, but on a company scale, the improvement cause have unexpected interactions. This would be infavourable on this scale.

The criterion that we will use to judge between projects depends on whether the inputs and outputs are fixed.

1. Neither input nor output fixed: maximize the difference between output and input ("No limit on input, but at least 500 units required." Maximize our benefit by minimizing the input to cut costs and maximizing the output)
2. Fixed input: maximize the output/revenue ("\$100 dollars available." Maximize the benefits we receive for \$100)
3. Fixed output: minimize the input ("1000 units required." Maximize our benefits by minimizing the production cost)

Three **project lives** (**planning horizon**, or **analysis period**) are possible when analyzing a project:

1. **Equal lives:** the useful lifetimes of the alternatives are equal to the analysis period
2. **Not equal lives:** the alternatives have useful lifetimes different from the analysis period
3. **Infinite lives:** the analysis period is ∞

Multiple alternatives when different lifespans might call for different analysis periods.

When alternatives have the same analysis periods ("equal lives"), the chosen alternative has the maximum **net present value (NPV)** - the difference between the present value of their benefits, and the present value of their costs.

Unequal lives and infinite lives

When alternative have different lives (they provide benefits and costs on different time scales), we cannot consider them using **NPV**.

Instead, we can consider

Least common multiple: we choose the analysis period to be the least common multiple of all the project lives of the alternatives considered, assuming that the alternatives are **repeatable**.

$$\mathrm{LCM}(a, b, \dots)$$

is the smallest number evenly divisible by all of $(a, b, \dots)$.

If we need to choose between a 5 year and a 10 year project, the LCM is 10 (since $10/10 = 1$, and $10/5 = 2$) years. So we will consider the NPV of the two projects assuming the 5 year project repeats twice.

The LCM of 7 and 13 is 91 years.

Consider a firm choosing between two instruments to buy for a 10 year project. Instrument A has a lifespan of 7 years, and instrument B has a lifespan of 13 years. We can choose our analysis period to be 10 years, but account for purchasing another copy of instrument A at year 7. The second purchased instrument A would still have some salvage value by the end of year 10, and instrument B will also have some salvage value remaining.

- We assumed that we will still be able to buy instrument A at the same price we buy it now, in 7 years

The least common multiple method may not make sense (alternatives may be not repeatable).

Another method is to "brute force". We forcibly choose an analysis period.

Consider a machine with \$600 installation cost, \$200 annual maintenance cost, lifetime of 5 years, and an interest rate of 10%. We decided to analyse the present worth of the machine over 10 years.

$$PV = 600 + \frac{600}{(1+0.1)^5} + 200\left(\frac{(1+0.1)^{10} - 1}{0.1(1+0.1)^{10}}\right)$$

There is an additional term involving 600 since we will need to purchase the machine again at year 5, assuming that it can still be purchased at \$600.

Infinite lives

Large infrusture projects can almost always be assumed to be "permanent". We can assume the analysis periods to be ∞.

For projects with infinite lives, we can compute it's associated **capitalized cost** - the present value of the initial cost, and what we need to set aside in order to provide the service indefinitely at some interest rate.

Given a present investment P, at an interest rate i, the investment grows to $P + Pi$ by the end of the period. As long as we withdraw Pi, the principal amount will always remain P.

This can be thought of as an end of period withdrawal/annuity,

$$A = Pi$$

If we want an annuity A to support our project, then we must invest at least

$$P = A/i$$

for this to be possible.

Multiple Alternatives

Given a **common analysis period**, we can compute the NPV for each alternative. The best project is project with the most positive NPV.

The following options all have an expected lifetime of 20 years.

Gas fire furnace: \$8000 installation cost, \$1200 per year, rising 5% per year.

Geothermal heat pump: \$23000 installation cost, \$400 per year, rising 4% per year.

Electric baseboard heater: \$2200 installation cost, \$1900 per year, rising 4% per year.

None of the options provide any revenues. So our objective is to simply minimize the costs, and choose the project with the most positive NPV.

The present value of each option is can be found by adding the installation cost, and the present value of the growing annual costs for the next 20 years.

Salvage value/costs: one-time cash amount recoverable/required at the end of a project

- A piece of equipment has salvage value when we are done with it - we can resell it
- A mine has salvage costs - we need to clean up, and restore damage environment
- Dangerous chemicals have salvage costs - we need to pay to get rid of it

Chapter 6

Equivalent Annual Cashflow Analysis

Instead of using the net present value, we can compare projects based on **equivalent annual cash flows (EACF)**.

EACF Analysis: Assuming repeatability, we can compare projects with different lifetimes using their EACF instead of choosing a LCM analysis period. The EACF remains the same for the same project repeated an arbitrary amount of time.

For a given analysis period and interest rate, if we know any one-time values, we can convert them into a uniform series on of annuities in the analysis period.

- Given an interest rate, we can always recover our NPV from the EACF
- The EACF increases when the benefits increase, and decrease when costs increase

The best alternative is the one that maximizes the EACF.

Consider the following tires purchase options. Assuming a 12% interest.

- \$30.95 per tire, with a life of 12 months
- \$59.95 per tire, with a life of 48 months

Under the assumption of annuities due, for the first option, our equivalent annual cost is

$$30.95(1+0.12)^1 = \$34.66$$

For the second option, our equivalent annual cost is

$$59.95\left(\frac{0.12(1+0.12)^4}{(1+0.12)^4-1}\right) = \$19.74$$

So the second option is better, since it has the least annual cost.

When the analysis period is specified, and not an integer multiple of the useful lifetimes of the alternatives, it's easier to compute the NPV first, and convert into a EACF

Chapter 7

Internal Rate of Return

With net present value analysis, our entire computation depends on a single interest rate, that is often assumed to be equal to the MARR.

The best project is the one with the most positive NPV, even though spendings in some projects might be more **efficient** than others.

Consider two alternatives at $i = 10\%$. For option 1, we invest \$1000000 and receive \$1190000 at the end of year 1.

For option 2, we invest \$100000, and receive \$200000 at the end of year 1.

Both options have the same NPV! Yet option 2 is much better, since we get the same NPV despite a much smaller investment than option 1.

Internal Rate of Return (IRR): is the interest rate at which the present worth of the benefits equal the present worth of the costs.

Given the PV of the costs, and the FV that we will receive after n periods, the IRR is such that

$$\begin{aligned} \text{NPV} = \text{PV}_{\text{benefit}} - \text{PV}_{\text{cost}} = 0 \\ \implies \text{EUCF} = 0 \end{aligned}$$

- To the borrower: the IRR is the interest rate that causes the unpaid balance on a loan to equal 0 when the final payment has been made
- To the investor: the IRR is the interest rate that the unrecovered investment is equal to zero at the end of the life of the investment

For investments: maximize the IRR. A higher IRR means a higher net cash flow.

For loans: minimize the IRR.

You are planning to get a \$15,000 loan from your bank. If you are limited to 60 payments of \$300 per month, which of the following bank rate of return is acceptable for you?

We can compute the suitable monthly interest rate. Any rate of return below the computed monthly IRR is acceptable. In these cases, we would actually end up with a positive net present value!

Fixed cash flows: For a fixed set of cash flows related to an investment/loan a higher interest rate means: the value of investments goes down, the value of loans go up

Internal rate of return does not account for

- Difference rates used for investing and borrowing
- Does do distinguish between investing and borrowing

Incremental Rate of Return

When we compare two acceptable potential projects, we want to know whether it is worth it to spend an extra initial amount to get more benefit compared to the project with the smaller intial cost.

Consider three options:

1. 3×45 yard shovels at \$80 million each - revenue \$122 million per year
2. 4×30 yard shovels at \$58 million each - revenue \$106 million per year

The MARR of the mine is 40%, and will operate for 20 years.

Each option must have an IRR of at least equal to the MARR. Otherwise, we can remove it from consideration.

Optional 2 is the cheapest, at \$232 million and an IRR of 46%.

Option 1 is the next cheapest, at the cost of \$240 million. Compared to option 2, option 1 is \$8 million more expensive, but every year, option 1 yields $122 - 106 = 16$ million more than option 2.

The ΔIRR for option 1 compared to option 2 is the interest rate such that the PV of the \$16 million annuity for 20 years equal \$8 million present investment. So it is worth taking option 1 over option 2.

Incremental IRR (ΔIRR): is computed between two alternatives. The ΔIRR is the interest rate such that the net present value between the two projects are equal - the intersection between two NPV versus interest rate curves

For an investment:

- If $\Delta\text{IRR} \geq \text{MARR}$, choose the higher initial cost project - the additional cost is justified
- If $\Delta\text{IRR} < \text{MARR}$, choose the lower initial cost project

A Warning: upgrading to an option with a greater initial cost is only while if the investor COULD earn a rate of return at the ΔIRR. Otherwise, whether switching is worthwhile depends on the investor's maximum possible rate of return, and her MARR.

If the MARR is 35% per year and the IRR of cost alternative A is 40%, the IRR of cost alternative B is 50% and ΔIRR (B-A) is 30%, what is the best alternative?

Alternative A, since the incremental IRR between optional B and A is less than the MARR, so option B is not worth pursuing over A.

1. Find IRR for every project, discard any for which IRR < MARR
2. Arrange the remaining alternatives in ascending order of initial cost
3. Find the ΔIRR to upgrade from the lowest initial cost option to the option with the next-lowest initial cost
4. If IRR > MARR, upgrade
5. Otherwise, stick the with lowest cost option
6. Repeat the previous steps until all alternatives have been considered

Modified Internal Rate of Return

The problem with IRR:

- Investing and borrow rates are assumed to be equal, yet most profiting firms often invest at a higher rate than they borrow at
- More than one solution for the IRR - NPV = 0 is a polynomial

Descartes' Rule: If a real polynomial has m sign changes, then it has $m - 2k$ positive roots, where $k \in \mathbb{Z}$, $k \in (0, m/2)$.

When cashflows have multiple sign changes, the IRR may non-unique.

Modified internal rate of return (MIRR): accounts differences in the borrowing (or financing) rate, e_{fin}, and the investing rate (MARR, e_{inv}).
The MIRR is the interest rate such that for n compounding periods, makes the present worth of all the expenses calculated using e_{fin} equal to the future worth of all revenue calculated using e_{inv}.

We can interpret the MIRR as

- Discounting all our expenses to a present value, at the borrowing rate
- Investing all revenues, compounding all revenues to a future value the end of the projects using the investment rate
- The MIRR is the interest rate such that

  $$P = \frac{F}{(1+i)^n}$$

  holds

A firm normally borrows money at 8%, and invests them at 15%. In n periods, the present worth of all the expenses was found to be -6.744. The future worth of all the "receipts" were found to be 16.5.

The MIRR is the interest rate that makes the present and future worths equivalent.

$$0 = (1 + \text{MIRR})^n \left(\text{PV}_{\text{cost}}\right) + \text{FV}_{\text{revenue}}$$

Incremental Analysis

There are three types of projects we can analyze.

1. **Independent:** selection of this project is independent of the decision to undertake any other projects
2. **Mutallly Exclusive:** at most 1 project, or do nothing, can be selected among competing alternatives. We cannot directly compare the projects with each other. Instead, we need to look at their increments
3. **Contingent:** selection of a project depends on the selection of at least one other project

When project A is contingent on project B, this means we must do project B in order to do project A.

For example, a firm asks to purchase 20 of our products, and wants the option to purchase 60 more in the next three years. The option to purchase 60 more is contingent on whether the first 20 sales were made.

Incremental Analysis: the examination of differences between alternatives to determine if the increased costs are justified by the increased benefits

- When the interest rate is high: we tend to favour options with lower initial costs
- When the interest rate is low: we can afford to pursue projects with high initial costs

Graphical Method

1. Choose a metric we will consider - NPV, EACF
2. For every project, plot the metric over 0% to 100% interest rate
3. The insection between the curves is the increment IRR
4. The best project at a given interest rate maximizes the NPV, or EACF
5. Generate a choice table - list the best project to pursue between different intervals of interest values

Chapter 8

Future Worth and Benefit Cost Redux

Similar to the present worth analysis, future worth analysis also requires deciding on a common analysis period.

Net Future Value (NPV): is the sum of the unrecovered capital and the total return on investment at the given interest rate, i, at a given time in the future. After paying for all the operating costs, we consider ourselves to invest the amount left over at the given i.

At a given MARR, an alternative is acceptable when

- NPV ≥ 0
- NFV ≥ 0
- EACF ≥ 0

Under these conditions, we can define the benefit-cost ratios

- $\text{PV}_{\text{benefit}}/\text{PV}_{\text{costs}} \geq 1$
- $\text{FV}_{\text{benefit}}/\text{FV}_{\text{costs}} \geq 1$
- $\text{EUAB}/\text{EUAC} \geq 1$

where **EUAB** and **EUAC** stands for **equivalent uniform annual benefit/cost**, and all factors magnitudes.

Sensitivity Analysis

All of our analysis will never be perfect. The actual cash flow is often different from our projected cashflows due to

- Techology change: changes to production costs and quality of product
- Changes in number of size of competitors
- Introduction of new products: our customers may have more/less options
- Changes to **macroeconomic variables**: inflation, unemployment, economic growth, exchange rate
- International events

Sensitivity analysis: evaluates how much a particular decision metric depends on its input parameters. This gives us a feel for our accurate our estimates might be

Break-even Analysis

A form of sensitivity analysis represented as a **break-even chart**.

For a given range of a particular input paramter

- time
- initial cost
- benefits . . .

The **"breakeven point" between two alternatives** is the point in that given range for which both options are equally considerable - we become indifferent about which project will be chosen.

Breakeven analysis: for a given range of input parmeter, the project is **sensitive** to that paramter if the range contains a breakeven point

We have a choice between constructing a project at full capacity now, or construct a first stage now, and a second stage at n years later. We can plot the net present value of both options as a function of n. The intersection of the curves represent a breakeven point.

The project choice is sensitive if the range of options for n include the break even point.

. . . .

Using estimated initial costs, we determined one of three projects was the preferred alternative.

For option 1, if we want to see how sensitive is our result to the accuracy of its initial cost estimate, we can plot the NPV of each option as a function of intial cost of option 1 and find the breakeven points.

What-if Analysis

Our decision parameters (NPV, IRR) is dependent on uncertain cost estimates.

Similar to breakeven analysis, we "sweep" an input variable and see how our preferred option changes.

Property	-30%	-15%	Base Case	15%	30%
			Cashflows		
Initial Investment	$210,000	$255,000	$300,000	$345,000	$390,000
Annual Benefits	$59,500	$72,250	$85,000	$97,750	$110,500
Salvage Value	$42,000	$51,000	$60,000	$69,000	$78,000
Project Lifetime	4	5	6	7	8
Interest Rate	9.8%	11.9%	14.0%	16.1%	18.2%

	-30%	-15%	Base Case	15%	30%
			Net Present Values		
Initial Investment	$147,872	$102,872	$57,872	$12,872	($32,128)
Annual Benefits	($41,289)	$8,291	$57,872	$107,452	$157,033
Salvage Value	$49,671	$53,772	$57,872	$61,972	$66,072
Project Lifetime	($8,431)	$26,673	$57,872	$85,600	$110,244
Interest Rate	$106,617	$81,023	$57,872	$36,874	$17,779

1. Find the most sensitive parameters
2. Consider what happens to the NPV for the **best case** - the best set of reasonable parameter estimates tend to maximize our NPV
3. Consider what happens to the NPV for the **worst case** - the worst set of reasonable parameter estimates that tend to minimize our NPV
4. Compare between the worst case, base case, and best case

Expected Values

For sensitivity analysis, we look at how our decisions change depending on the range of input parameters we consider - whether our decision parameters are "sensitive" to a range of input parameters.

All input parameters are uncertain. Future estimates are more uncertain than near-term estimates,

Parameters can be

- **Deterministic** - values are known with certainty, specified through contracts
- **Random** - the parameter is instrinsically uncertain

Probability

In economic analysis, our probability distributions often involve 2 to 5 **discrete outcomes**.

An oil company might consider three outcomes when drilling for a new well: 70% dry well, 20% average production well, 10% highly productive well.

Statistically independent: we will often assume that random variables are statistically independent. For example, the annual benefits of project is not correlated with the project lifetime. This is not always valid.

Joint probability of independent random variables: given the probability of two statistically independent events, $P(A)$, $P(B)$, the probability of both events ocurring is

$$P(A \wedge B) = P(A)P(B)$$

The probability fo rolling a 6 and flipping heads is $1/12$.

Expected value: the sum of the possible outcomes weighted by its probability. If the project was repeated for ∞ number of times, then on average, the outcomes will equal our expected value.

If project A has a 50% chance of having an EUAB of $1000, and 50$ chance of having an EUAB of %500, then the expected value of EUAB for project A is $750.

Standard Deviation

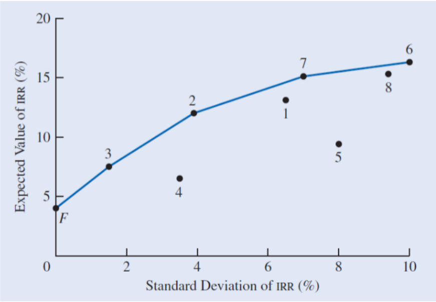
The risk of a project can be quantified using the **standard deviation**. For a given expected value,

- the outcomes of a **risky/volatile** project can greatly differ from our expected value
- the outcomes of a **safe/stable** project is often not far off from our expected value

Standard deviation: given n outcomes represented by $\{O_1, O_2, \ldots, O_n\}$, each associated with probability P_i, from which we compute the expected value (EV), the standard deviation is defined as

$$\sigma = \sqrt{\sum_{i=1}^{n} O_i^2 P_i - \text{EV}}$$

We can assess the risks of our projects by plotting them on the standard deviation versus IRR plane. The best project maximizes the IRR, while minimizing the standard deviation.



Safe project: if the expected value is at lease 2 times the standard deviation of expected value, then the project is safe.