Simulation1_result

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0 Simulation setting

$$W_1 \sim Unif(-1, 1)$$
$$W_2 \sim Unif(-1, 1)$$
$$W_3 \sim Bernoulli(0.5)$$
$$A \sim Bernoulli(\pi_0) \ where \ \pi_0 = expit(0.5 + \frac{1}{3}W_1)$$
$$Y \sim N(\mu_0, 1) \ where \ \mu_0 = 0.1 + \beta_1 A + \beta_2 A(W_1^2 + W_3) + W_1 + W_2^2$$

$$\tau(W) = \beta_1 + \beta_2(W_1^2 + W_3)$$

$$\beta = (\beta_1, \beta_2) \in \{(0, 0.25),\ (0.25, 0.25),\ (0.25, 0),\ (0, 0),\ (0.25, 0.75)\}$$
$$n \in \{100,\ 250,\ 500,\ 750,\ 1000\}$$

$$\begin{aligned}\psi_0 &= E[\tau(W)^2] \\ &= (E[\tau(W)])^2 + Var[\tau(W)] \\ &= (\beta_1 + \beta_2 E(W_1^2) + \beta_3 E(W_3))^2 + \beta_2^2 Var(W_1^2) + \beta_2^2 Var(W_3) \\ &= (\beta_1 + \frac{5}{6}\beta_2)^2 + (\frac{1}{5} - \frac{1}{9} + \frac{1}{4})\beta_2^2\end{aligned}$$

$$\begin{aligned}\theta_0 &= Var[\tau(W)] \\ &= (\frac{1}{5} - \frac{1}{9} + \frac{1}{4})\beta_2^2\end{aligned}$$

Table 1: Simulated data distribution under different beta settings

beta	pi0	psi0	theta0
(0, 0.25)	0.5	0.065	0.021
(0.25, 0.25)	0.5	0.231	0.021
(0.25, 0)	0.5	0.062	0.000
(0, 0)	0.5	0.000	0.000
(0.25, 0.75)	0.5	0.956	0.191

SL Library

```
# SL3
# learners = create.Learner("SL.earth", params = list(penalty=-1))
# mu.SL.library = c("SL.gam.interaction", "SL.glm.interaction", learners$names)
```

1 Estimators

1.6 $\beta = (0.25, 0.75)$

2 Confidence Intervals

2.6 $\beta = (0.25, 0.75)$

Table 2: Wald-type CI coverage for psi0 with correction (SL3) linear

n	type	na	coverage	coverage.c	cnt
100	psi.est	0	0.890	0.890	1000
250	psi.est	0	0.902	0.902	1000
500	psi.est	0	0.915	0.915	1000
750	psi.est	0	0.922	0.922	1000
1000	psi.est	0	0.928	0.928	1000

Table 3: Wald-type CI coverage for psi0 with correction (SL3)

n	type	na	coverage	coverage.c	cnt
100	psi.est	0	0.906	0.906	1000
250	psi.est	0	0.910	0.910	1000
500	psi.est	0	0.936	0.936	1000
750	psi.est	0	0.946	0.946	1000
1000	psi.est	0	0.947	0.947	1000

Table 4: Bootstrap CI coverage for psi0 with correction (SL3) linear

n	type	na	coverage	coverage.c	cnt
100	psi.est	0	0.877	0.877	1000
250	psi.est	0	0.907	0.907	1000
500	psi.est	0	0.912	0.912	1000
750	psi.est	0	0.916	0.916	1000
1000	psi.est	0	0.924	0.924	1000

Table 5: Bootstrap CI coverage for psi0 with correction (SL3)

n	type	na	coverage	coverage.c	cnt
100	psi.est	0	0.899	0.899	1000
250	psi.est	0	0.909	0.909	1000
500	psi.est	0	0.935	0.935	1000
750	psi.est	0	0.943	0.943	1000
1000	psi.est	0	0.946	0.946	1000

Table 6: Wald-type CI coverage for theta0 with correction (SL3) linear

n	type	na	coverage	coverage.c	cnt
100	theta.est	0	0.925	0.932	1000
250	theta.est	0	0.908	0.911	1000
500	theta.est	0	0.912	0.912	1000
750	theta.est	0	0.916	0.916	1000
1000	theta.est	0	0.923	0.923	1000

Table 7: Wald-type CI coverage for theta0 with correction (SL3)

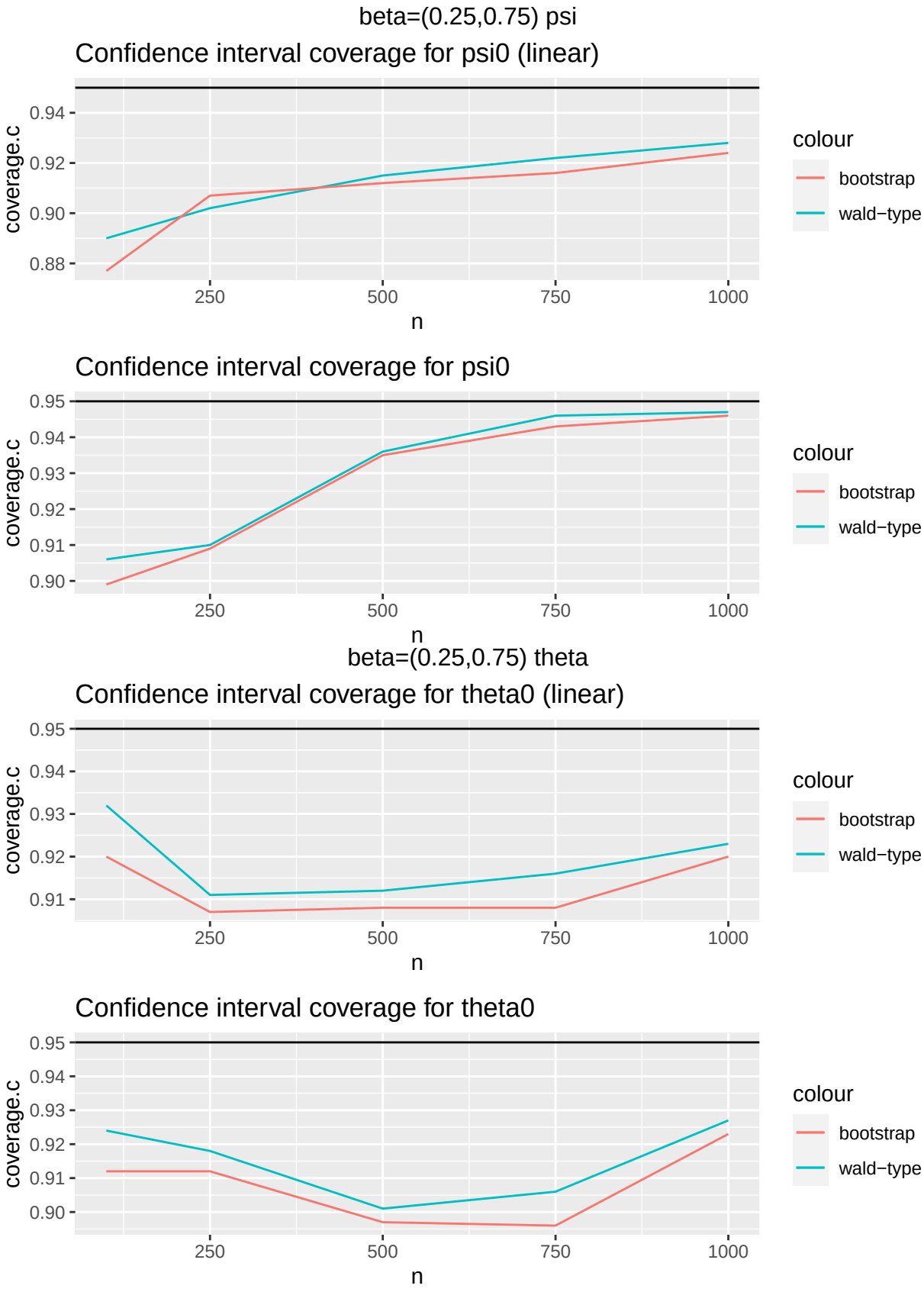
n	type	na	coverage	coverage.c	cnt
100	theta.est	0	0.917	0.924	1000
250	theta.est	0	0.918	0.918	1000
500	theta.est	0	0.901	0.901	1000
750	theta.est	0	0.906	0.906	1000
1000	theta.est	0	0.927	0.927	1000

Table 8: Bootstrap CI coverage for theta0 with correction (SL3) linear

n	type	na	coverage	coverage.c	cnt
100	theta.est	0	0.910	0.920	1000
250	theta.est	0	0.902	0.907	1000
500	theta.est	0	0.907	0.908	1000
750	theta.est	0	0.908	0.908	1000
1000	theta.est	0	0.920	0.920	1000

Table 9: Bootstrap CI coverage for theta0 with correction (SL3)

n	type	na	coverage	coverage.c	cnt
100	theta.est	0	0.905	0.912	1000
250	theta.est	0	0.912	0.912	1000
500	theta.est	0	0.897	0.897	1000
750	theta.est	0	0.896	0.896	1000
1000	theta.est	0	0.923	0.923	1000


beta=(0.25,0.75) psi
Confidence interval coverage for psi0 (linear)
coverage.c
0.94
0.92
0.90
0.88
250
500
750
1000
n
colour
bootstrap
wald−type
Confidence interval coverage for psi0
coverage.c
0.95
0.94
0.93
0.92
0.91
0.90
250
500
750
1000
n
colour
bootstrap
wald−type
beta=(0.25,0.75) theta
Confidence interval coverage for theta0 (linear)
coverage.c
0.95
0.94
0.93
0.92
0.91
250
500
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1000
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colour
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wald−type
Confidence interval coverage for theta0
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0.93
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0.91
0.90
250
500
750
1000
n
colour
bootstrap
wald−type

3 Testing

3.6 $\beta = (0.25, 0.75)$

Table 10: Testing for psi under beta=(0.25, 0.75) linear

n	type	na	cnt	reject.rate
100	Gamma.stat	0	1000	0.937
250	Gamma.stat	0	1000	1.000
500	Gamma.stat	0	1000	1.000
750	Gamma.stat	0	1000	1.000
1000	Gamma.stat	0	1000	1.000

Table 11: Testing for psi under beta=(0.25, 0.75)

n	type	na	cnt	reject.rate
100	Gamma.stat	0	1000	0.944
250	Gamma.stat	0	1000	1.000
500	Gamma.stat	0	1000	1.000
750	Gamma.stat	0	1000	1.000
1000	Gamma.stat	0	1000	1.000

Table 12: Testing for theta under beta=(0.25, 0.75) linear

n	type	na	cnt	reject.rate
100	Omega.stat	0	1000	0.276
250	Omega.stat	0	1000	0.518
500	Omega.stat	0	1000	0.872
750	Omega.stat	0	1000	0.977
1000	Omega.stat	0	1000	0.997

Table 13: Testing for theta under beta=(0.25, 0.75)

n	type	na	cnt	reject.rate
100	Omega.stat	0	1000	0.253
250	Omega.stat	0	1000	0.599
500	Omega.stat	0	1000	0.896
750	Omega.stat	0	1000	0.992
1000	Omega.stat	0	1000	0.996