Simulation.rewrite.rst

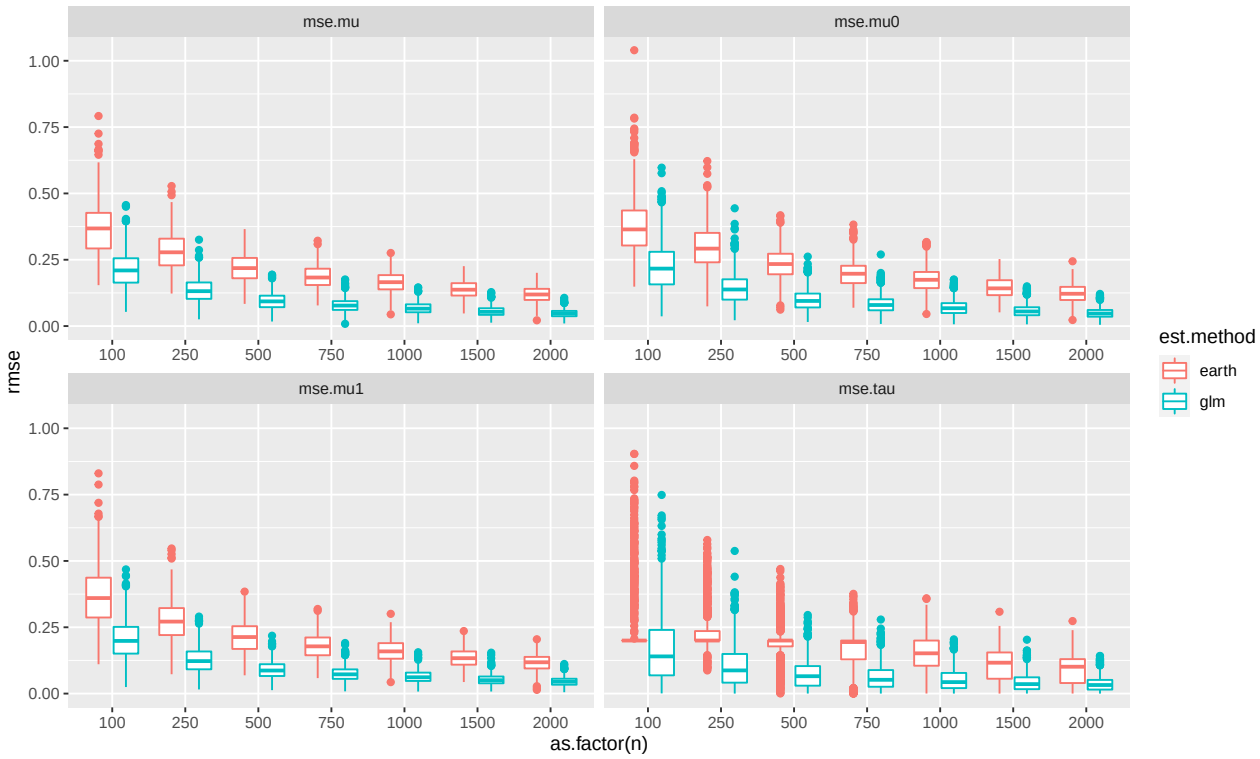
1 Scenario 1

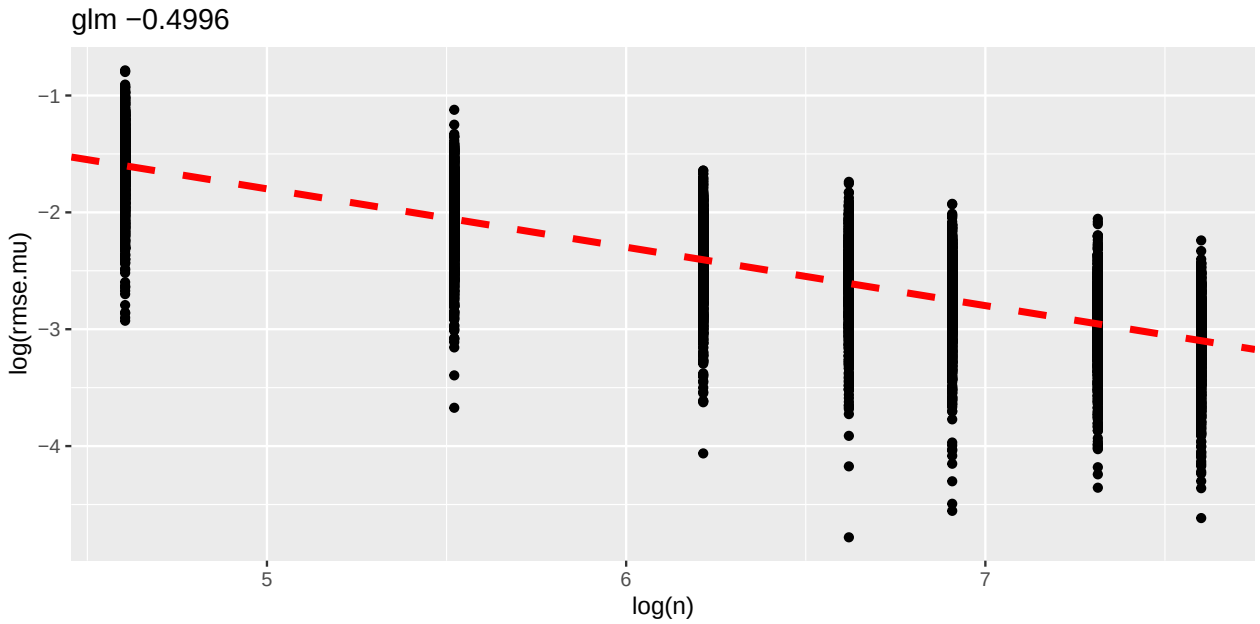
- W_1, W_2 and W_3 are all continuous random variables
- no interaction term in $Y \sim A + W$, i.e., $\tau(W)$ is a constant

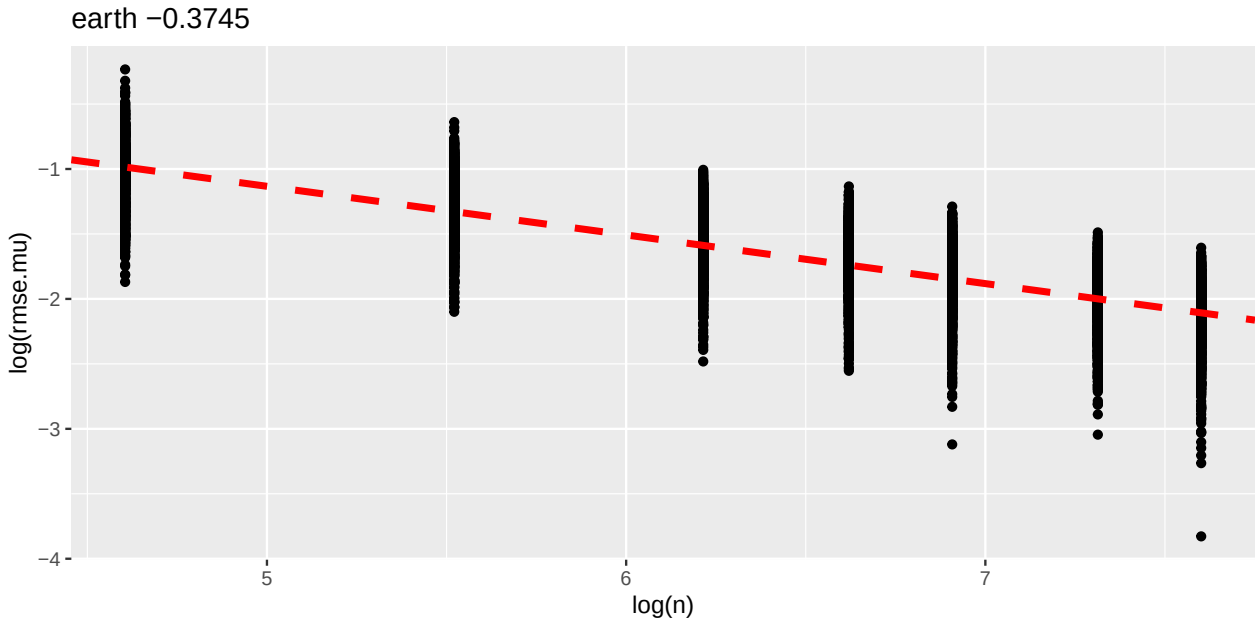
$$W_1 \sim Unif(-1, 1)$$
$$W_2 \sim Unif(-1, 1)$$
$$W_3 \sim Unif(-1, 1)$$
$$A \sim Bernoulli(\pi_0) \ where \ \pi_0 = expit(0.5 + \frac{1}{3}W_1)$$
$$Y \sim N(\mu_0, 1)$$
$$\mu_0(A, W) = 0.1 + 0.2 * A + 0.5 * W_1 - 0.3 * W_2 + 0.1 * W_3$$
$$\mu_0(1, W) = 0.3 + 0.5 * W_1 - 0.3 * W_2 + 0.1 * W_3$$
$$\mu_0(0, W) = 0.1 + 0.5 * W_1 - 0.3 * W_2 + 0.1 * W_3$$
$$\tau(W) = 0.2$$
$$\psi_0 = 0.04$$
$$\theta_0 = 0$$

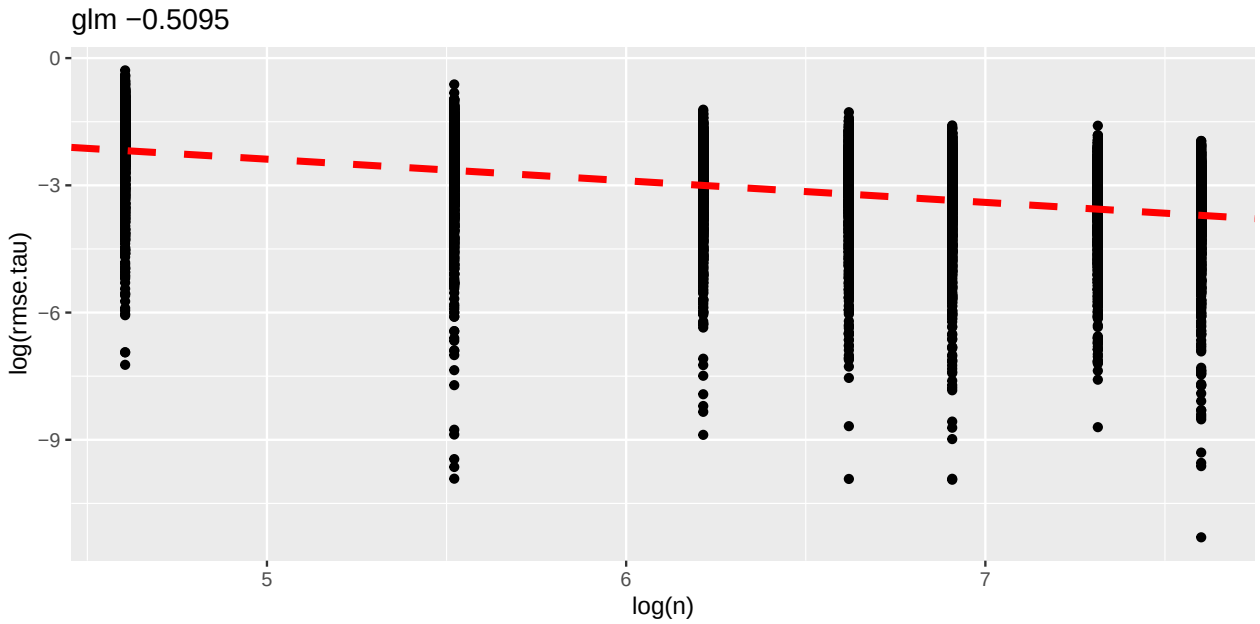
```
# glm: glm(Y ~ ., data=AW, family='gaussian')
# earth: SL.library = c("SL.earth")
```

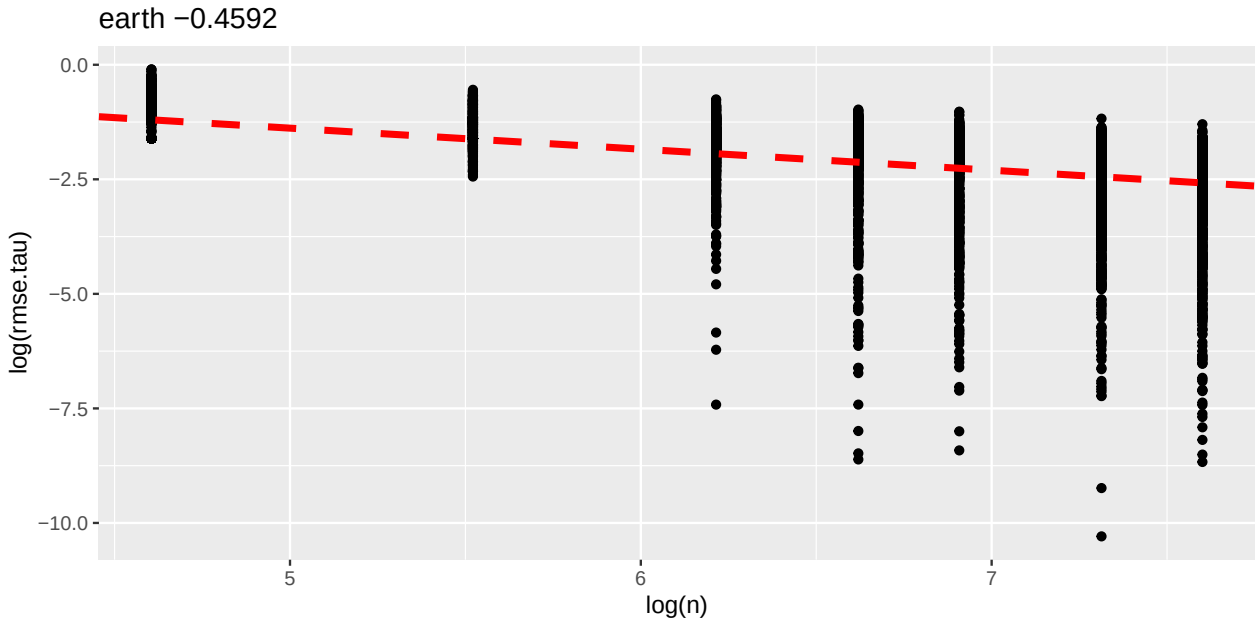
$$rmse = \frac{\sqrt{\sum(\hat{\mu} - \mu_0)^2}}{\sqrt{n}}$$

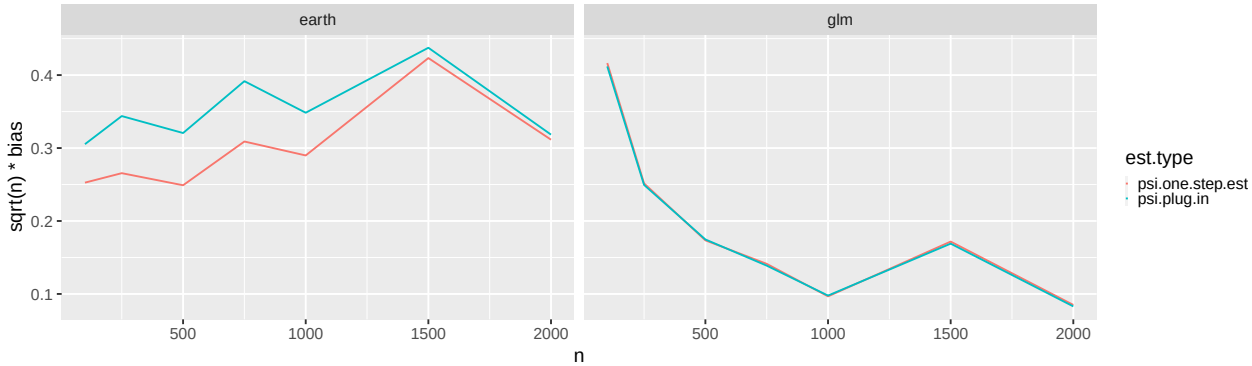

mse.mu
mse.mu0
mse.mu1
mse.tau
rmse
1.00
0.75
0.50
0.25
0.00
100
250
500
750
1000
1500
2000
as.factor(n)
est.method
earth
glm


glm −0.4996
log(rmse.mu)
−1
−2
−3
−4
5
6
7
log(n)


earth −0.3745
log(rmse.mu)
−1
−2
−3
−4
5
6
7
log(n)


glm −0.5095
log(rmse.tau)
0
−3
−6
−9
5
6
7
log(n)

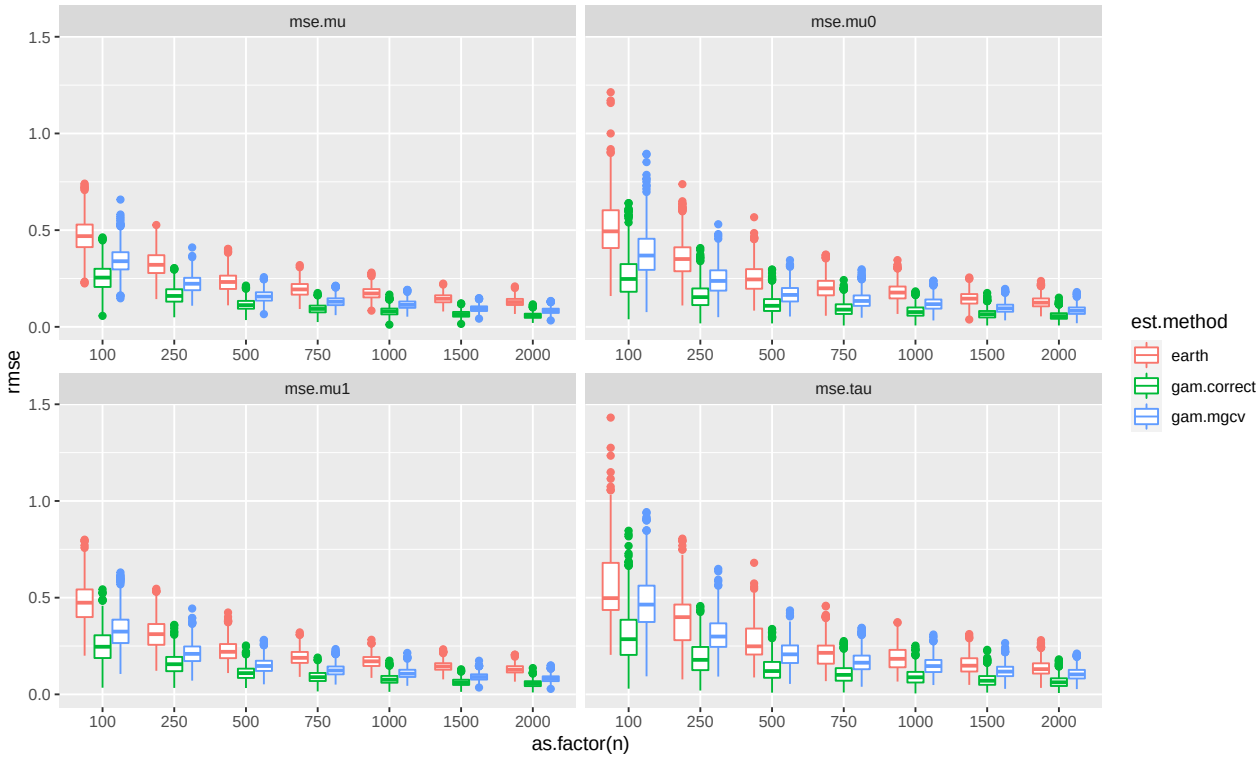

earth −0.4592
log(rmse.tau)
0.0
−2.5
−5.0
−7.5
−10.0
5
6
7
log(n)

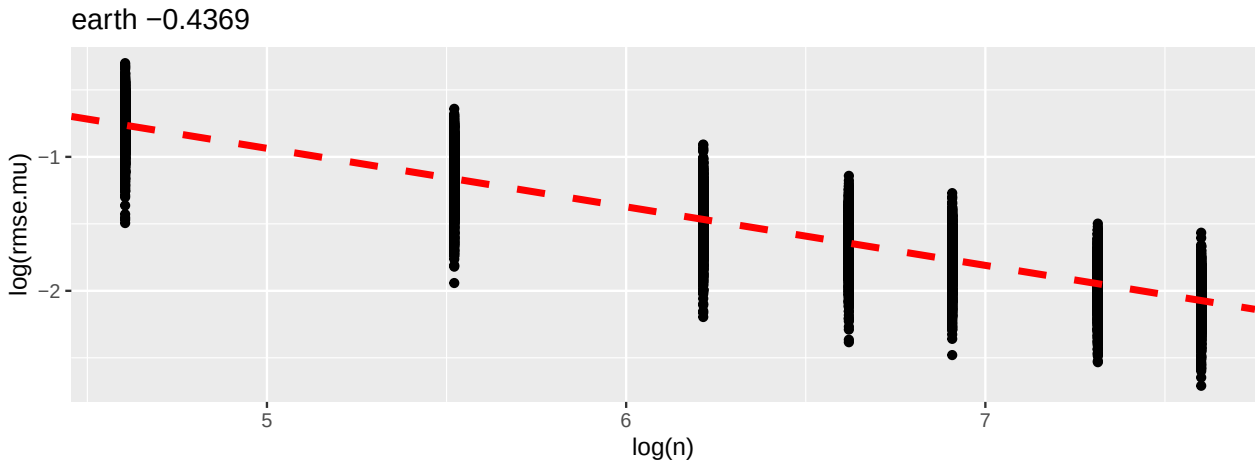

earth
glm
sqrt(n) * bias
0.4
0.3
0.2
0.1
500
1000
1500
2000
n
est.type
psi.one.step.est
psi.plug.in

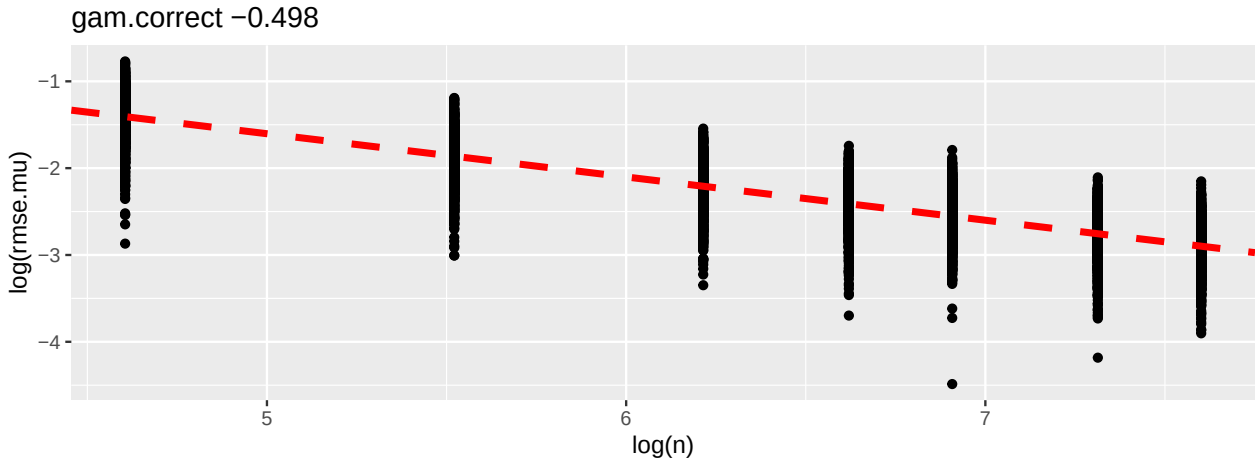
3 Scenario 3

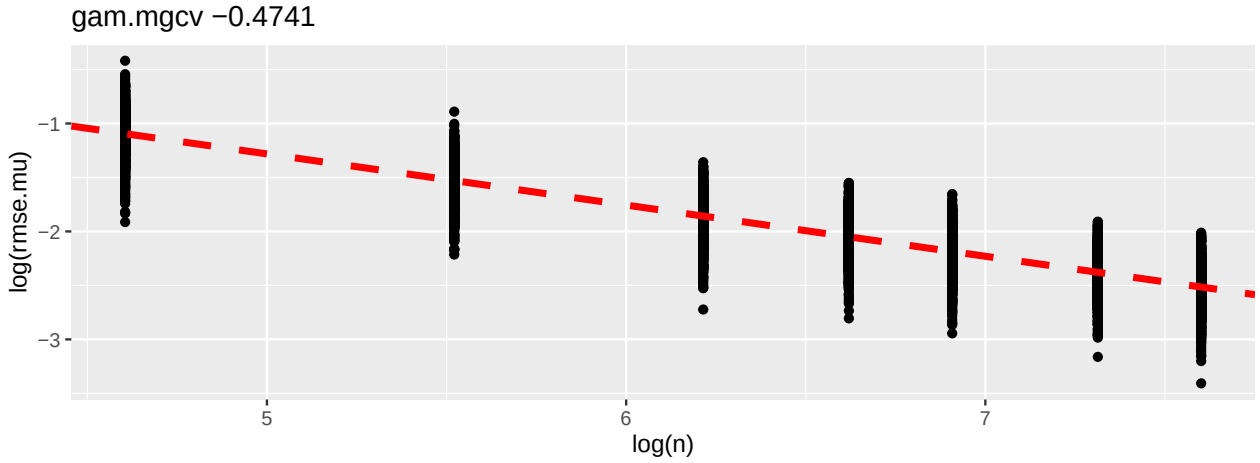
$$W_1 \sim Unif(-1, 1)$$
$$W_2 \sim Unif(-1, 1)$$
$$W_3 \sim Bernoulli(0.5)$$
$$A \sim Bernoulli(\pi_0) \ where \ \pi_0 = expit(0.5 + \frac{1}{3}W_1)$$
$$Y \sim N(\mu_0, 1) \ where \ \mu_0 = 0.1 + 0.25 * A + 0.75A(W_1^2 + W_3) + W_1 + W_2^2$$
$$\tau(W) = 0.25 + 0.75 * (W_1^2 + W_3)$$
$$\psi_0 = 0.956$$
$$\theta_0 = 0.191$$

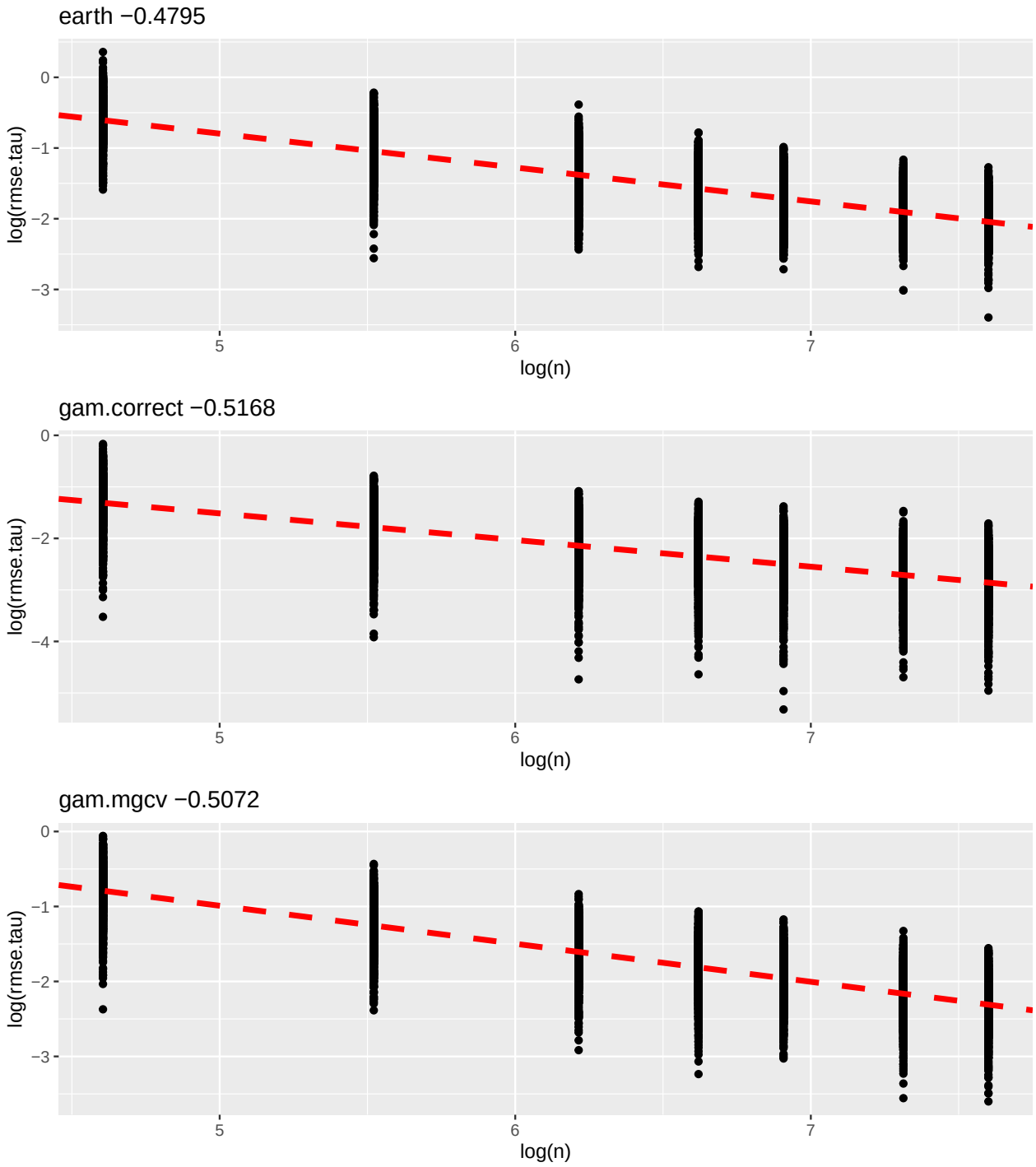
```
# gam.correct: gam.model <- as.formula("Y ~ W1 + I(W2^2) + I(W1^2):A + A*W3")
# earth: SL.library = c("SL.earth")
# gam.mgcv: gam.model <- as.formula("Y ~ s(W1) + s(W2) + s(W1, by=A) + s(W2, by=A) + A*W3")
# mu.reg <- mgcv::gam(gam.model, data = AW, method = "REML")
```

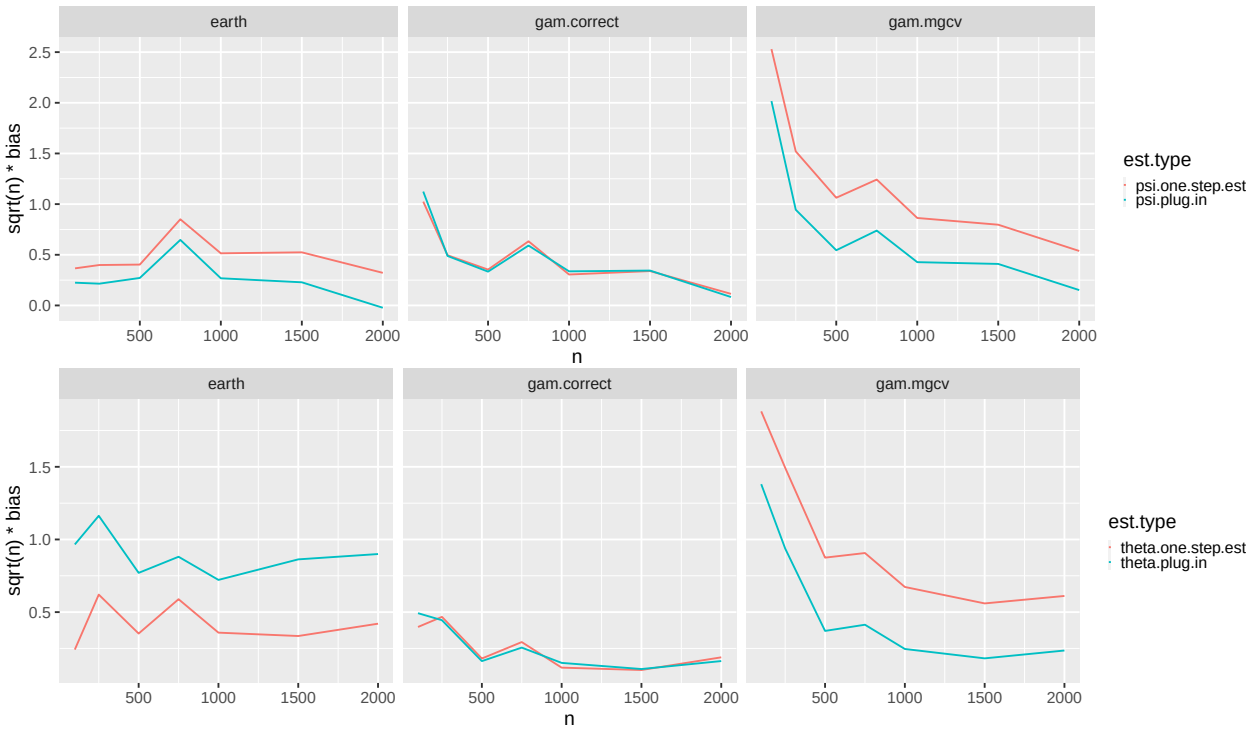



earth −0.4369
log(rmse.mu)
−1
−2
5
6
7
log(n)


gam.correct −0.498
log(rmse.mu)
−1
−2
−3
−4
5
6
7
log(n)


gam.mgcv −0.4741
log(rmse.mu)
−1
−2
−3
5
6
7
log(n)


earth −0.4795
log(rmse.tau)
0
−1
−2
−3
5
6
7
log(n)
gam.correct −0.5168
log(rmse.tau)
0
−2
−4
5
6
7
log(n)
gam.mgcv −0.5072
log(rmse.tau)
0
−1
−2
−3
5
6
7
log(n)


earth
gam.correct
gam.mgcv
sqrt(n) * bias
2.5
2.0
1.5
1.0
0.5
0.0
500
1000
1500
2000
n
est.type
psi.one.step.est
psi.plug.in
earth
gam.correct
gam.mgcv
sqrt(n) * bias
1.5
1.0
0.5
500
1000
1500
2000
n
est.type
theta.one.step.est
theta.plug.in