

# Simulation1\_result

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## 0 Simulation setting

$$W_1 \sim Unif(-1, 1)$$

$$W_2 \sim Unif(-1, 1)$$

$$W_3 \sim Bernoulli(0.5)$$

$$A \sim Bernoulli(\pi_0) \text{ where } \pi_0 = \text{expit}(0.5 + \frac{1}{3}W_1)$$

$$Y \sim N(\mu_0, 1) \text{ where } \mu_0 = 0.1 + \beta_1 A + \beta_2 A(W_1^2 + W_3) + W_1 + W_2^2$$

$$\tau(W) = \beta_1 + \beta_2(W_1^2 + W_3)$$

$$\beta = (\beta_1, \beta_2) \in \{(0, 0.25), (0.25, 0.25), (0.25, 0), (0, 0), (0.25, 0.75)\}$$

$$n \in \{100, 250, 500, 750, 1000\}$$

$$\begin{aligned}\psi_0 &= E[\tau(W)^2] \\ &= (E[\tau(W)])^2 + Var[\tau(W)] \\ &= (\beta_1 + \beta_2 E(W_1^2) + \beta_3 E(W_3))^2 + \beta_2^2 Var(W_1^2) + \beta_2^2 Var(W_3) \\ &= (\beta_1 + \frac{5}{6}\beta_2)^2 + (\frac{1}{5} - \frac{1}{9} + \frac{1}{4})\beta_2^2\end{aligned}$$

$$\begin{aligned}\theta_0 &= Var[\tau(W)] \\ &= (\frac{1}{5} - \frac{1}{9} + \frac{1}{4})\beta_2^2\end{aligned}$$

Table 1: Simulated data distribution under different beta settings

beta	pi0	psi0	theta0
(0, 0.25)	0.5	0.065	0.021
(0.25, 0.25)	0.5	0.231	0.021
(0.25, 0)	0.5	0.062	0.000
(0, 0)	0.5	0.000	0.000
(0.25, 0.75)	0.5	0.956	0.191

SL Library

```
# SL3
# learners = create.Learner("SL.earth", params = list(penalty=-1))
# mu.SL.library = c("SL.gam.interaction", "SL.glm.interaction", learners$names)
```

## 1 Estimators

1.6  $\beta = (0.25, 0.75)$

## 2 Confidence Intervals

2.6  $\beta = (0.25, 0.75)$

Table 2: Wald-type CI coverage for psi0 with correction (SL3)

n	type	na	coverage	coverage.c	cnt
100	psi.est	0	0.906	0.906	1000
250	psi.est	0	0.910	0.910	1000
500	psi.est	0	0.936	0.936	1000
750	psi.est	0	0.946	0.946	1000
1000	psi.est	0	0.947	0.947	1000
1500	psi.est	0	0.940	0.940	1000
2000	psi.est	0	0.942	0.942	1000

Table 3: Bootstrap CI coverage for psi0 with correction (SL3)

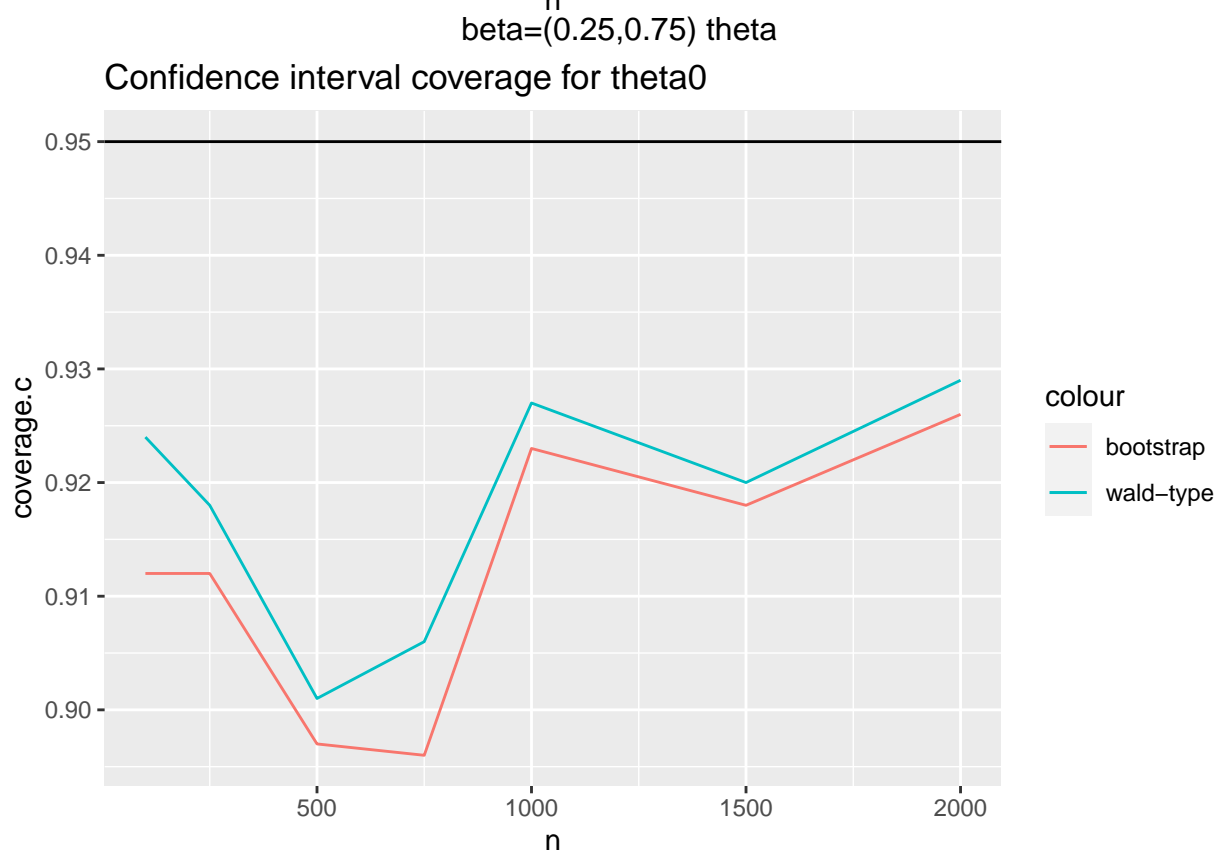
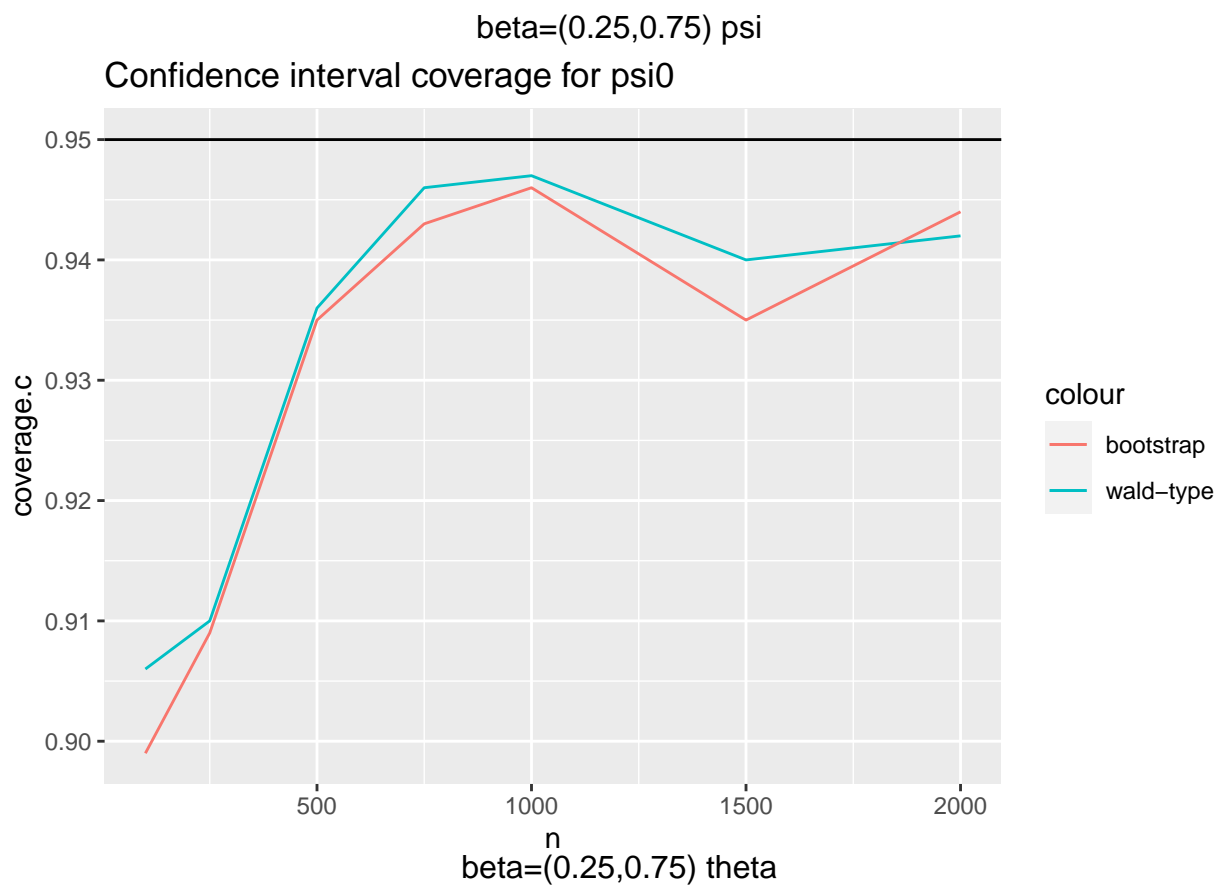
n	type	na	coverage	coverage.c	cnt
100	psi.est	0	0.899	0.899	1000
250	psi.est	0	0.909	0.909	1000
500	psi.est	0	0.935	0.935	1000
750	psi.est	0	0.943	0.943	1000
1000	psi.est	0	0.946	0.946	1000
1500	psi.est	0	0.935	0.935	1000
2000	psi.est	0	0.944	0.944	1000

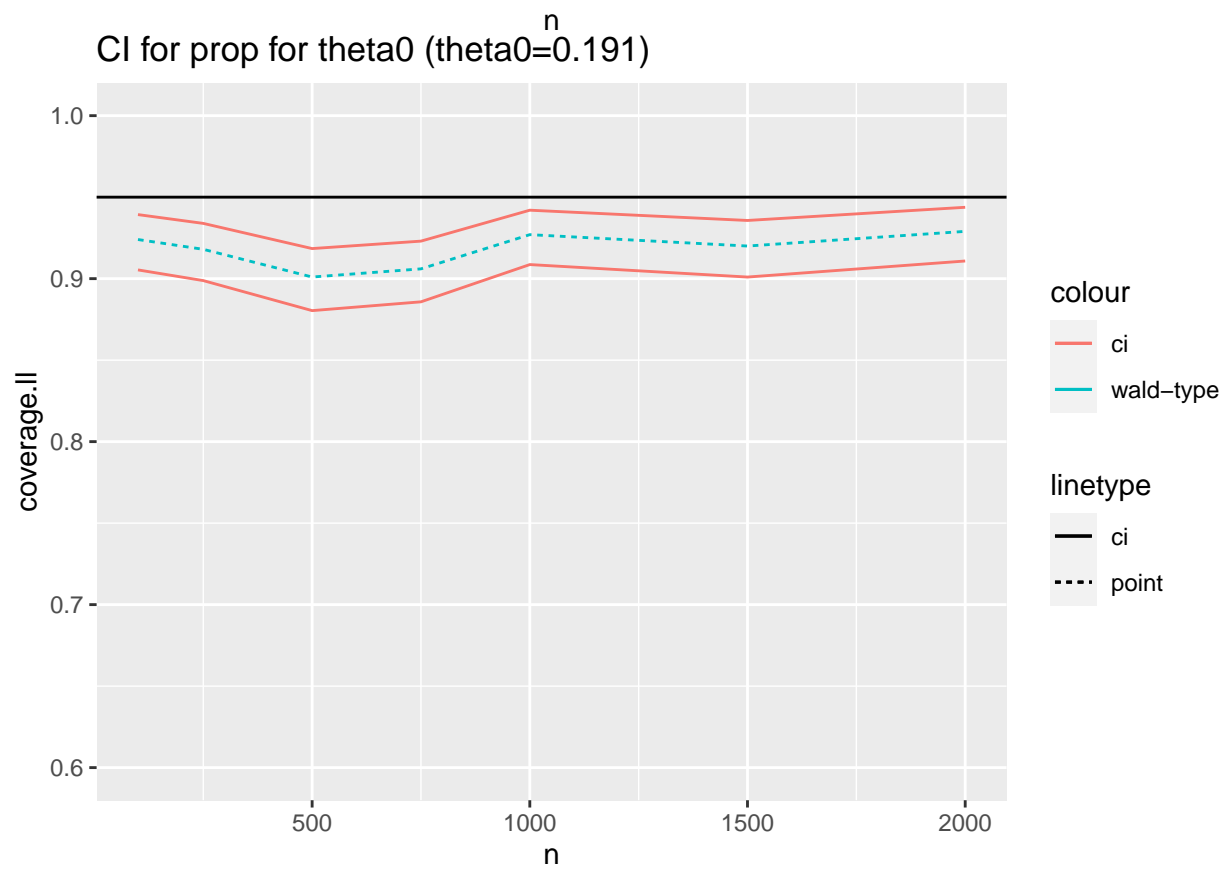
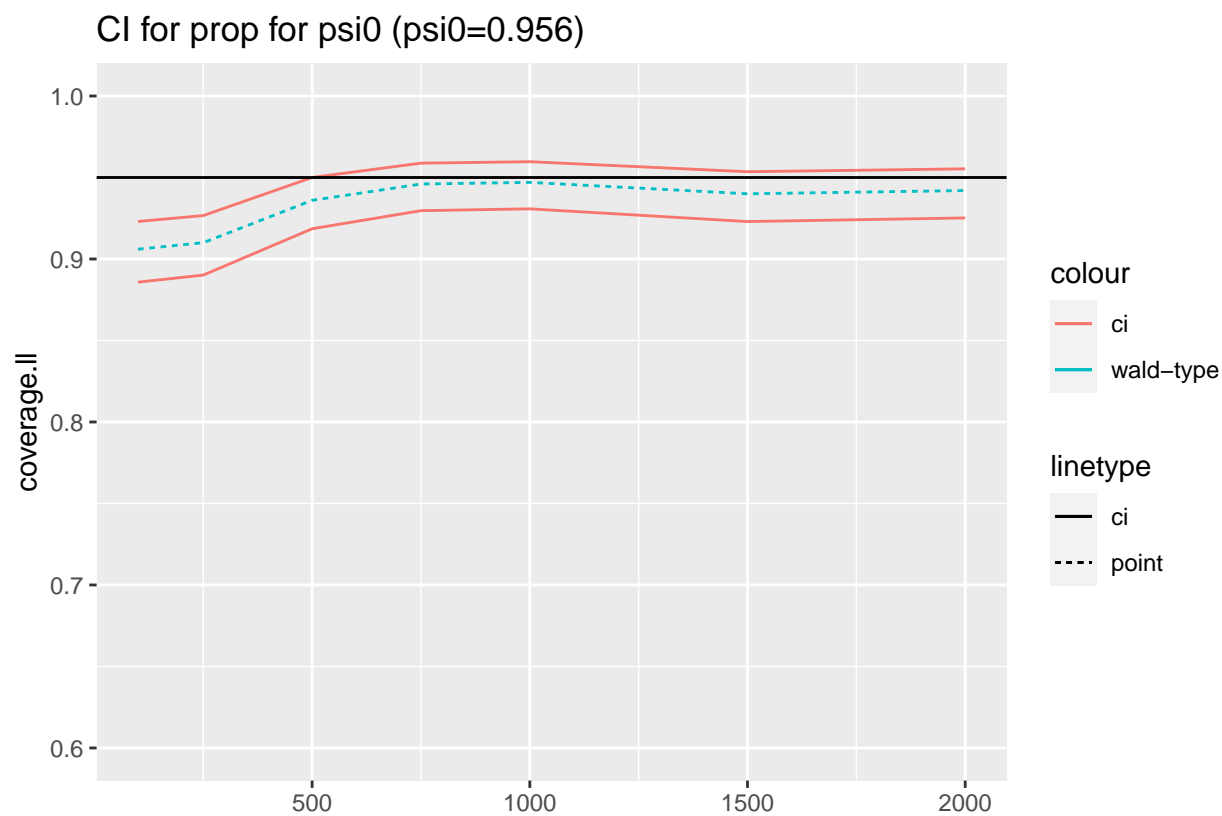
Table 4: Wald-type CI coverage for theta0 with correction (SL3)

n	type	na	coverage	coverage.c	cnt
100	theta.est	0	0.917	0.924	1000
250	theta.est	0	0.918	0.918	1000
500	theta.est	0	0.901	0.901	1000
750	theta.est	0	0.906	0.906	1000
1000	theta.est	0	0.927	0.927	1000
1500	theta.est	0	0.920	0.920	1000
2000	theta.est	0	0.929	0.929	1000

Table 5: Bootstrap CI coverage for theta0 with correction (SL3)

n	type	na	coverage	coverage.c	cnt
100	theta.est	0	0.905	0.912	1000
250	theta.est	0	0.912	0.912	1000
500	theta.est	0	0.897	0.897	1000
750	theta.est	0	0.896	0.896	1000
1000	theta.est	0	0.923	0.923	1000
1500	theta.est	0	0.918	0.918	1000
2000	theta.est	0	0.926	0.926	1000





### 3 Testing

**3.6**  $\beta = (0.25, 0.75)$

Table 6: Testing for psi under beta=(0.25, 0.75)

n	type	na	cnt	reject.rate
100	Gamma.stat	0	1000	0.944
250	Gamma.stat	0	1000	1.000
500	Gamma.stat	0	1000	1.000
750	Gamma.stat	0	1000	1.000
1000	Gamma.stat	0	1000	1.000
1500	Gamma.stat	0	1000	1.000
2000	Gamma.stat	0	1000	1.000

Table 7: Testing for theta under beta=(0.25, 0.75)

n	type	na	cnt	reject.rate
100	Omega.stat	0	1000	0.253
250	Omega.stat	0	1000	0.599
500	Omega.stat	0	1000	0.896
750	Omega.stat	0	1000	0.992
1000	Omega.stat	0	1000	0.996
1500	Omega.stat	0	1000	0.999
2000	Omega.stat	0	1000	1.000