Hindley-Milner
type inference

$$\text{exp} ::= \text{fun } x \to \text{exp} \mid \text{exp exp} \mid \text{var}$$

subset of untyped λ-calculus

type rules: same as ones on the slides from Tuesday.

$\Gamma \vdash x : t$

$$\boxed{\text{Var}: \dfrac{}{\Gamma \vdash x : t} \quad \Gamma(x) = t}$$ axiom

$$\text{App}: \dfrac{\Gamma \vdash (f : s) \to t \qquad \Gamma \vdash a : s}{\Gamma \vdash f\ a : t}$$

⇐ issue: f takes in a of type s but return type is t.

well-formed type s

$$\text{Fun}: \dfrac{\text{Wf } s \qquad \Gamma\{x \to s\} \vdash e : t}{\Gamma \vdash \text{fun } x \to e : s \to t}$$

TVar: ____________

TArrow: ____________

Infering a type or fraction:

After principal type of a term

a type of a term s.t.
substitute we can substitute another
well-formed type w/ the principal type

- Start by introducing type variables and constraint system.

empty $\vdash e$: 'a

Solve for this 'a.

& Use inference rules bottom-up.

e.g. (if fun x → e) : 'a → Introduce 'b, 'c.

Prove that Γ (

Solving type constraints

a = b → Removing b from the system
by removing w/ a.

(a → b) = (c → d) → replace w/
a = c ∧ b = d.

Well-known fact :