

Prove by Contrapositive $\forall n \in \mathbb{N}$ $n^2 + 2n + 2$ is even $\rightarrow n$ is even

Contrapositive: n is odd $\rightarrow n^2 + 2n + 2$ is odd.

let $n \in \mathbb{N}$ be odd st $n = 2k + 1$ for some $k \in \mathbb{N}$.

$$\begin{aligned} \text{then } n^2 + 2n + 2 &= (2k+1)^2 + 2(2k+1) + 2 = 4k^2 + 4k + 1 + 4k + 2 + 2 \\ &= 4k^2 + 8k + 4 + 1 = 2(2k^2 + 4k + 2) + 1 \text{ which is odd.} \end{aligned}$$

Hence the statement is true by contrapositive.