

OPTIMIZATION OF A FIXED VIRTUAL SENSING FEEDBACK ANC CONTROLLER FOR IN-EAR HEADPHONES WITH MULTIPLE LOUDSPEAKERS

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ABSTRACT

In this paper we consider an in-ear headphone equipped with an inner microphone and multiple loudspeakers and we propose an optimization procedure with a convex objective function to derive a fixed multi-loudspeaker ANC controller aiming at minimizing the sound pressure at the ear drum. Based on the virtual microphone arrangement (VMA) technique and measured acoustic paths between the loudspeakers and the ear drum, the FIR filters of the ANC controller are jointly optimized to minimize the power spectral density at the ear drum, subject to design and stability constraints. For an in-ear headphone with two loudspeakers, the proposed multi-loudspeaker VMA controller is compared to two single-loudspeaker VMA controllers. Simulation results with diffuse noise show that the multi-loudspeaker VMA controller effectively improves the attenuation by up to about 10 dB for frequencies below 300 Hz when compared to both single-loudspeaker VMA controllers.

Index Terms— Active noise cancellation, Headphones, Feedback control, Virtual sensing

1. INTRODUCTION

Active Noise Cancellation (ANC) applied to headphones aims at minimizing the environmental noise at the listener's ears [1]. The working principle of ANC is based on the destructive superposition of sound waves, in this context the sound wave of the environmental noise reaching the listener's ear and the sound wave generated by the loudspeaker of the headphone. Whereas the passive attenuation achieved by the construction materials of the headphone is mainly effective in the mid and high frequencies, ANC is mainly effective in the low frequencies.

Several ANC approaches have been proposed in the literature [2, 3], either based on fixed or adaptive feedforward or feedback controllers. In this paper we consider in-ear

ANC headphones with an inner microphone and multiple loudspeakers, and we aim at minimizing the sound pressure at the ear drum by means of a fixed multi-loudspeaker feedback ANC controller. Since the sound pressure at the inner microphone and at the ear drum are not equal, several virtual sensing techniques have been proposed (see the literature review in [4]) to move the zone of quiet from the position of the inner microphone to the position of the ear drum. In [5] a virtual sensing technique called feedback virtual microphone arrangement (VMA) technique was proposed that exploits the proximity of the inner microphone to the ear drum. Using measured acoustic transfer functions between the loudspeaker and the inner microphone and the ear drum, the VMA technique calculates its control signal using a real-time estimate of the sound pressure at the ear drum with ANC on.

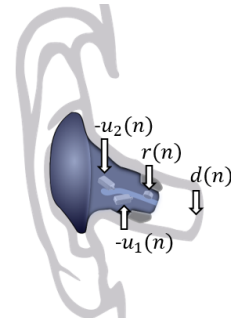


Fig. 1. In-ear headphone with an inner microphone and two loudspeakers, $r(n)$ and $d(n)$ denote the signals generated by the incident noise arriving at the inner microphone and the ear drum, respectively, and $-u_1(n)$ and $-u_2(n)$ denote the loudspeaker signals generated by the ANC controller.

In [6] and [7] optimization procedures of an FIR filter as ANC controller have been proposed, which were formulated using objective functions that are convex w.r.t. the FIR filter coefficients. In [6] the optimization procedure used the internal model control structure [8] to transform the non-convex objective function of the feedback ANC controller into a convex objective function of an equivalent feedforward ANC

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controller. In [7] the optimization procedure approximated the non-convex minimization of the objective function of the feedback ANC controller by the convex maximization of the objective function's denominator. In [5, 9–13] optimization procedures of IIR filters have been proposed, which are formulated using objective functions that are non-convex w.r.t. the IIR filter coefficients. In [5] the objective function was similarly formulated as in [7], but without yielding a convex objective function. The author assumed a twin-biquad filter and applied an exhaustive search to find the optimal parameter values. In [9] a different optimization procedure was proposed that uses the sequential quadratic programming (SQP) method to minimize the non-convex objective function of the feedback ANC controller. In [10, 11] an open-loop zero-pole placement method was used to design a reasonable initial solution for the optimization. A two-step optimization procedure was proposed in [12] that uses a genetic algorithm to find an initial solution and the Nelder-Mead simplex method to find the local optimum in its vicinity. In [13] a differential evolution algorithm was used to find a local minimum after several optimization repetitions. All the above mentioned methods make use of only one loudspeaker and, except for [5] and [13], aim at minimizing the sound pressure at the inner microphone and not at the ear drum.

In this work, we propose an optimization procedure to derive a fixed multi-loudspeaker feedback ANC controller using the VMA technique. To formulate a convex objective function, we assume the ANC controller to be an FIR filter per channel and use the approximation proposed in [7] to overcome the non-convexity of the objective function.

This paper is structured as follows: Firstly, a description of the virtual sensing technique used in this work will be presented. Secondly, a theoretical analysis of the problem based on the PSD of the signal at the ear drum is made. Thirdly, the proposed optimization procedure for the design of the ANC controller is described. Fourthly, simulations results based on acoustic transfer functions measured with an in-ear ANC headphone prototype are presented to validate the proposed optimization procedure.

2. FEEDBACK VIRTUAL SENSING

Fig. 1 depicts an in-ear headphone equipped with an inner microphone and two loudspeakers. The incident noise arrives first at the inner microphone as $r(n)$ and then at the ear drum as $d(n)$. Based on the inner microphone signal, the multi-loudspeaker feedback ANC controller calculates the signals $-u_1(n)$ and $-u_2(n)$ to drive the loudspeakers, aiming

at generating a sound wave that destructively overlaps with the sound wave of the incident noise at the ear drum.

The block diagram of the proposed multi-loudspeaker feedback VMA technique is presented in Fig. 2 for the general case of L loudspeakers. Its working principle is the same as the single-loudspeaker VMA technique proposed in [5], with the exception that the controller

$$\mathbf{W}(z) = [W_1(z), \dots, W_L(z)]^T \quad (2)$$

is a single-input multiple-output system, which generates individual signals for each loudspeaker, and the measured acoustic functions $\hat{\mathbf{B}}_r(z) = [\hat{B}_{r,1}(z), \dots, \hat{B}_{r,L}(z)]^T$ and $\hat{\mathbf{S}}(z) = [\hat{S}_1(z), \dots, \hat{S}_L(z)]^T$ are multiple-input single-output systems. The technique starts with a calibration stage, in which a probe tube microphone is placed at the ear drum (depicted in pink) to measure the acoustic transfer functions from the loudspeakers to the ear drum $\mathbf{S}(z)$ and to the inner microphone $\mathbf{B}_r(z)$. For this, the L loudspeaker channels are measured in a sequential way. Afterwards, the probe tube microphone is removed, the fixed controller $\mathbf{W}(z)$ is derived (see Section 4) and the control stage using the DSP (in blue) starts. In the control stage, the sound pressure at the ear drum $e(n)$ is estimated in real-time using the measured acoustic transfer functions $\hat{\mathbf{S}}(z)$ and $\hat{\mathbf{B}}_r(z)$ in a three-step approach. First, $\hat{\mathbf{B}}_r(z)$ is used to estimate $r(n)$ by subtracting the estimated sound pressure generated by the loudspeakers of the headphone at the position of the inner microphone from the microphone signal. Second, the incident noise is assumed to be equal at the inner microphone and at the ear drum, i.e. $\hat{d}(n) = \hat{r}(n)$. Third, the estimated sound pressure at the ear drum $\hat{e}(n)$ with ANC on is estimated by using the estimated sound pressure generated by the incident noise at the ear drum $\hat{d}(n)$ and the measured acoustic transfer functions $\hat{\mathbf{S}}(z)$. The resulting signal $\hat{e}(n)$ is then used by the controller $\mathbf{W}(z)$ to calculate the control signals $u_1(n)$ to $u_L(n)$.

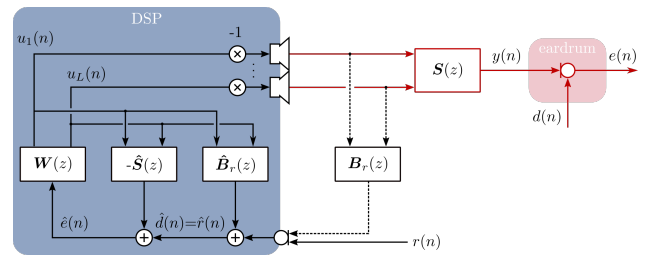


Fig. 2. Block diagram of the proposed ANC controller implementing a feedback virtual microphone arrangement technique with multiple loudspeakers.

$$\Phi_{ee}(f) = \left(1 - \frac{|\Phi_{dr}(f)|^2}{\Phi_{dd}(f)\Phi_{rr}(f)}\right)\Phi_{dd}(f) + \left|\frac{\Phi_{dr}(f)}{\Phi_{rr}(f)} - \frac{\mathbf{W}^T(f)\mathbf{S}(f)}{1 + \mathbf{W}^T(f)(\hat{\mathbf{S}}(f) + \mathbf{B}_r(f) - \hat{\mathbf{B}}_r(f))}\right|^2 \Phi_{rr}(f), \quad (1)$$

3. SYSTEM ANALYSIS

The power spectral density (PSD) of the sound pressure at the ear drum $e(n)$ at frequency f can be written as in (1), where $\Phi_{dr}(f)$ denotes the cross-power spectral density (CPSD) between $d(n)$ and $r(n)$, and $\Phi_{dd}(f)$ and $\Phi_{rr}(f)$ denote the PSDs of the respective signals. The left-hand addend determines the minimum achievable PSD $\Phi_{ee}(f)$, which is only equal to zero when the magnitude squared coherence between $d(n)$ and $r(n)$ is equal to 1. The right-hand addend defines the scope of optimization of the controller $\mathbf{W}(z)$. Here, the controller $\mathbf{W}(z)$ and the acoustic transfer functions $\mathbf{B}_r(z)$, $\hat{\mathbf{B}}_r(z)$ and $\hat{\mathbf{S}}(z)$ build a transfer function, which in series connection with the acoustic paths $\mathbf{S}(z)$ should approximate $\Phi_{dr}(z)/\Phi_{rr}(z)$ as well as possible.

In the VMA technique [5, 14], it is assumed that the sound pressure generated by the incident noise is equal at the inner microphone and at the ear drum, i.e. $d(n) = r(n)$. In addition, it is assumed that the measured acoustic transfer functions from the loudspeakers to the ear drum are time-invariant and that after removing the probe tube microphone they remain unchanged, i.e. $\hat{\mathbf{S}}(z) = \mathbf{S}(z)$. We also assume that changes in the measured acoustic transfer functions to the inner microphone can be compensated with state-of-the-art online estimation methods [15], i.e. $\hat{\mathbf{B}}_r(z) = \mathbf{B}_r(z)$. If these assumptions are applied to (1), the PSD $\Phi_{ee}(f)$ reduces to

$$\Phi_{ee}^{\text{vma}}(f) = \left| \frac{1}{1 + \mathbf{W}^T(f)\hat{\mathbf{S}}(f)} \right|^2 \Phi_{rr}(f). \quad (3)$$

Hence, in this case, the scope of optimization of the feedback ANC controller depends only on the PSD of the incident noise at the inner microphone $\Phi_{rr}(f)$ and the measured acoustic transfer functions $\hat{\mathbf{S}}(f)$.

4. CONTROLLER DESIGN

Designing the controller $\mathbf{W}(z)$ by direct minimization of (3) is a non-convex optimization problem w.r.t. the parameters of $\mathbf{W}(z)$. Moreover, a stability constraint is required to restrict the solution space to controllers $\mathbf{W}(z)$ that yield a stable system. Similarly as in [7] for the minimization of the sound pressure at the inner microphone using one loudspeaker, we propose to optimize the multi-loudspeaker controller $\mathbf{W}(z)$ by solving the alternative problem of maximizing the denominator in (3) in the discrete Fourier transform (DFT) domain. The proposed convex objective function is defined as

$$\max_{\mathbf{w}} \sum_{k=0}^{L_{\text{DFT}}/2-1} \left| 1 + \mathbf{W}^T(\Omega_k)\hat{\mathbf{S}}(\Omega_k) \right|^2 G_1^2(\Omega_k), \quad (4)$$

where L_{DFT} denotes the DFT length, $\mathbf{w} = [\mathbf{w}_1^T, \dots, \mathbf{w}_L^T]^T$ denotes the concatenated filter coefficient vector of length $L \cdot N$, with N the FIR filter length per channel, $\Omega_k =$

$2\pi k/L_{\text{DFT}}$ denotes the normalized frequency, $\mathbf{W}(\Omega_k) = [W_1(\Omega_k), \dots, W_L(\Omega_k)]^T$ contains the frequency response of the ANC controller for each channel, $\hat{\mathbf{S}}(\Omega_k)$ contains the frequency response of the measured acoustic transfer functions $\hat{\mathbf{S}}(z)$, and $G_1(\Omega_k)$ denotes a frequency weighting function. As in [7], we use $G_1(\Omega_k)$ to weight the low frequencies more than the mid and high frequencies.

Aiming at deriving a controller $\mathbf{W}(z)$ that yields a stable system a stability constraint is imposed. Similarly as for the single-loudspeaker case in [7], the solution space is restricted to avoid the encirclement of the Nyquist point by means of the following hyperbolic-boundary constraint

$$|\varrho - \mathbf{W}^T(\Omega_k)\hat{\mathbf{S}}(\Omega_k)| \leq |\varrho + \mathbf{W}^T(\Omega_k)\hat{\mathbf{S}}(\Omega_k)| + 2\rho, \forall \Omega_k, \quad (5)$$

where ϱ determines the focus $(-\varrho, 0)$ and ρ determines the x-axis intersect $(-\rho, 0)$ of a single-sided hyperbola, with $0 < \rho < 1$ and $\rho < \varrho$. A similar constraint with a fixed focus $\varrho = 1$ was considered in [9, 16]. The advantage of parameterizing the focus ϱ is that after designing the gain margin (GM) using

$$\text{GM} \geq \frac{1}{\rho}, \quad (6)$$

the phase margin (PM) is not fixed and can be designed with ϱ using [7]

$$\text{PM} \geq \arccos \left(\sqrt{(\rho^2 \cdot \varrho^2 + \rho^2 - \rho^4)/\varrho^2} \right). \quad (7)$$

Feedback ANC approaches are subject to the water-bed effect and therefore prone to produce amplifications outside the attenuation bandwidth, because the associated acoustic transfer functions are typically non-minimum phase systems [17]. Aiming at restricting the amplification, the design constraint

$$1 \leq G_2(\Omega_k)|1 + \mathbf{W}^T(\Omega_k)\hat{\mathbf{S}}(\Omega_k)|, \forall \Omega_k \quad (8)$$

is introduced, where $G_2(\Omega_k)$ denotes the maximum allowed amplification over frequency. In addition, for implementation considerations the gain of the controller is limited by applying the constraint

$$W_l(\Omega_k) \leq G_3(\Omega_k), \forall \Omega_k, \quad (9)$$

where the subindex $l \in \{1, \dots, L\}$ denotes the loudspeaker channel, and $G_3(\Omega_k)$ denotes the maximum allowed gain over frequency.

The proposed optimization problem maximizing the convex objective function in (4), subject to the constraints in (5), (8) and (9) can then be solved using SQP algorithms, e.g., implemented in the MATLAB function `fmincon()`.

5. SIMULATION RESULTS

The performance of the proposed feedback ANC controller was evaluated using the in-ear headphone prototype described in [18, 19], which is equipped with $L = 2$ loudspeakers. The acoustic transfer functions between the loudspeakers and the inner microphone $\hat{B}_r(z)$ and the ear drum $\hat{S}(z)$ were measured for a subject wearing the in-ear headphone prototype and using a probe tube microphone. For the simulations, a cylindrically diffuse noise field was considered. All simulations were performed using 10 s signals at a sampling frequency $f_s = 44.1$ kHz. For the proposed optimization procedure we used $L_{\text{DFT}} = 8192$ in (4), and $\rho = 0.8$ and $\varrho = 2$ in (5). Similarly to [7], we used frequency-dependent functions $G_1(f)$, $G_2(f)$ and $G_3(f)$ depicted in Fig. 3. We compared the performance of the multi-loudspeaker VMA controller, using $N = 64$ filter coefficients for each loudspeaker channel, with two single-loudspeaker VMA controllers, using $N = 128$ filter coefficients.

Fig. 4 depicts the PSDs of the sound pressure at the ear drum $\Phi_{ee}(f)$ with ANC off and with ANC on, either using single-loudspeaker VMA controllers (ANC Ch1, ANC Ch2) or using the proposed multi-loudspeaker controller (Proposed). It can be observed from ANC off that the passive attenuation of the in-ear headphone is very high for frequencies above 4 kHz. The single-loudspeaker VMA controllers ANC Ch1 and ANC Ch2 achieve an attenuation starting at about 60 Hz and ending at about 800 Hz or 1 kHz. The proposed multi-loudspeaker VMA controller clearly outperforms both single-loudspeaker VMA controllers by extending the attenuation bandwidth to frequencies below 20 Hz and incrementing the attenuation magnitude by about 10 dB for frequencies below 300 Hz. For all VMA controllers (single- and multi-loudspeaker) it can be observed that above about 1 kHz an amplification is produced. Whereas this amplification remains below the maximum amplification of 4 dB specified by $G_2(f)$ up to about 4 kHz, it exceeds this value for higher frequencies. This may be due to the fact that the main assumption of the VMA technique, i.e. $d(n) = r(n)$, does not hold in this frequency region, because the wave lengths are comparable to the distance between the inner microphone and ear drum. However, it should be realized that this amplification happens in a frequency region where the passive attenuation of the in-ear headphone is very high.

6. CONCLUSIONS

In this paper we considered an in-ear headphone equipped with an inner microphone and multiple loudspeakers and we proposed an optimization procedure to derive a multi-loudspeaker feedback ANC controller aiming at minimizing the sound pressure at the ear drum. We proposed to jointly optimize the FIR filters of the controller by minimizing a convex objective function, subject to design and stability

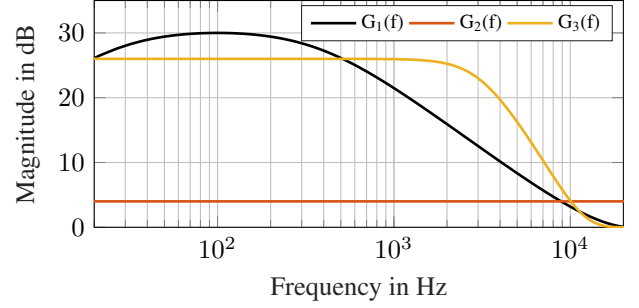


Fig. 3. Frequency-dependent functions, where $G_1(f)$ is the frequency weighting in the objective function, $G_2(f)$ is the maximum allowed amplification, and $G_3(f)$ is the maximum controller gain.

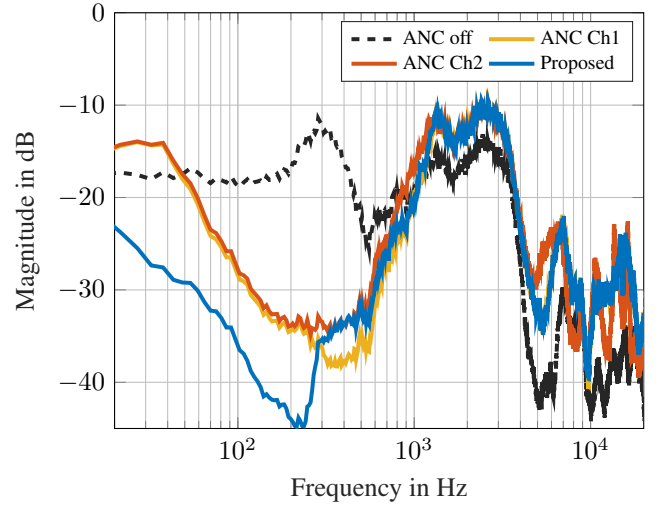


Fig. 4. PSD at the left ear of the subject when ANC is off, when the single-loudspeaker VMA technique with either channel 1 or 2 is on, and when the proposed multi-loudspeaker VMA technique (channel 1 and channel 2) is on.

constraints. Using acoustic transfer functions measured with an in-ear headphone prototype with two loudspeakers, the proposed multi-loudspeaker VMA controller was compared to two single-loudspeaker VMA controllers using the same total amount of filter coefficients. Simulation results with diffuse noise field show that the multi-loudspeaker VMA controller effectively improves the attenuation by up to about 10 dB for frequencies below 300 Hz when compared to both single-loudspeaker VMA controllers. In future work the optimization procedure will be extended by integrating uncertainty models of the acoustic transfer functions, e.g., produced by inter-subject variability.

7. REFERENCES

- [1] E. D. Simshauser and M. E. Hawley, "The noise cancelling headset—an active ear defender," *The Journal of the Acoustical Society of America*, vol. 27, no. 1, pp. 207–207, 1955.
- [2] S.-M. Kuo and D. R. Morgan, *Active noise control systems: algorithms and DSP implementations*. New York, NY: Wiley, 1996.
- [3] S. J. Elliott, *Signal Processing for Active Control*. Signal Processing and its Applications, London: Academic Press, 2001.
- [4] D. Moreau, B. Cazzolato, A. Zander, and C. Petersen, "A review of virtual sensing algorithms for active noise control," *Algorithms*, vol. 1, p. 69–99, Nov 2008.
- [5] M. Pawelczyk, "Analog active control of acoustic noise at a virtual location," *IEEE Transactions on Control Systems Technology*, vol. 17, no. 2, pp. 465–472, 2009.
- [6] B. Rafaely and S. J. Elliott, " H_2/H_∞ active control of sound in a headrest: design and implementation," *IEEE Transactions on Control Systems Technology*, vol. 7, pp. 79–84, 1999.
- [7] P. Rivera Benois, *Hybrid and Pseudo-Cascaded Active Noise Control Applied to Headphones*. PhD thesis, Helmut-Schmidt-Universität / Universität der Bundeswehr Hamburg, Hamburg, Dec. 2020.
- [8] M. Morari and E. Zafiriou, *Robust process control*. Prentice-Hall, NJ, 1989.
- [9] S.-H. Yu and J.-S. Hu, "Controller design for active noise cancellation headphones using experimental raw data," *IEEE/ASME Transactions on Mechatronics*, vol. 6, no. 4, pp. 483–490, 2001.
- [10] K.-W. Liang and J.-S. Hu, "An open-loop pole–zero placement method for active noise control headphones," *IEEE Transactions on Control Systems Technology*, vol. 25, no. 4, pp. 1278–1283, 2017.
- [11] K.-W. Liang and J.-S. Hu, "Optimal controller design for virtual sensing with independent noise source measurement," *IEEE Transactions on Control Systems Technology*, vol. 27, no. 1, pp. 363–369, 2019.
- [12] J. Wang, J. Zhang, J. Xu, C. Zheng, and X. Li, "An optimization framework for designing robust cascade bi-quad feedback controllers on active noise cancellation headphones," *Applied Acoustics*, vol. 179, p. 108081, 2021.
- [13] F. An, Q. Wu, and B. Liu, "Feedback controller optimization for active noise control headphones considering frequency response mismatch between microphone and human ear," *Applied Sciences*, vol. 12, no. 3, 2022.
- [14] S. J. Elliott and David, "A virtual microphone arrangement for local active sound control," in *International Conference on Motion and Vibration Control*, vol. 2, (Tokyo), pp. 1027–1031, Japan Society of Mechanical Engineers, 1992.
- [15] S. Ahmed, M. T. Akhtar, and X. Zhang, "Robust auxiliary-noise-power scheduling in active noise control systems with online secondary path modeling," *IEEE Transactions on Audio, Speech, and Language Processing*, vol. 21, no. 4, pp. 749–761, 2013.
- [16] E. Polak, "A modified nyquist stability test for use in computer-aided design," *IEEE Transactions on Automatic Control*, vol. 29, no. 1, pp. 91–93, 1984.
- [17] M. Pawelczyk, "Analogue active noise control," *Applied Acoustics*, vol. 63, no. 11, pp. 1193 – 1213, 2002.
- [18] F. Denk, M. Lettau, H. Schepker, S. Doclo, R. Roden, M. Blau, J.-H. Bach, J. Wellmann, and B. Kollmeier, "A one-size-fits-all earpiece with multiple microphones and drivers for hearing device research," in *Proc. AES Conference on Headphone Technology*, Aug 2019.
- [19] F. Denk and B. Kollmeier, "The hearpiece database of individual transfer functions of an in-the-ear earpiece for hearing device research," *Acta Acust.*, vol. 5, p. 2, 2021.