INSTANTANEOUS LINEAR DIMENSIONALITY REDUCTION OF MULTICHANNEL TIME-SERIES SIGNAL FOR ARRAY SIGNAL PROCESSING

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ABSTRACT

Linear dimensionality reduction of signals observed by a sensor array is often useful in balancing the accuracy and speed of post-stage processing, especially in real-time systems with limited computational resources. However, for multichannel time-series signals having time-invariant intertemporal and interchannel correlations, the direct application of frequency-wise linear dimensionality reduction method requires a large number of digital filters with large filter lengths, which is still unpreferable in the viewpoint of computational cost. We propose a frequency-independent, i.e., instantaneous, linear dimensionality reduction method that achieves low computational cost and latency and high restoration accuracy. We also show several results of numerical experiments to compare the proposed method with other instantaneous linear dimensionality reduction methods, i.e., the principal component analysis and element selection method, and demonstrate the effectiveness of the proposed method.

Index Terms— Dimensionality reduction, multichannel time-series signal, array signal processing, coordinate descent

1. INTRODUCTION

Array signal processing has realized a wide variety of applicational techniques exploiting spatial and temporal information of target physical quantity. In acoustic signal processing, which we mainly consider as an expected application of this work, various methods for beamforming [1–3], source separation and enhancement [4–7], sound field estimation [8, 9], and source localization [1] using microphone array have been proposed in the literature, and these techniques are used in further applications such as hearing aid [1], speech recognition [10, 11], and sound field reproduction by loudspeakers or headphones [8, 12].

The recent development of miniaturization and integration technologies of sensors realized a sensor array with a large number of channels (e.g., dozens to over a hundred channels for a microphone array [13–15]). While the increase of the number of channels generally improves the accuracy of these methods, it is not necessarily preferable in the viewpoint of computational cost since many of the above techniques involve computational complexity highly dependent on an input dimension. In particular, for example, most of the state-of-the-art blind source separation algorithms require as many times of scalar multiplications as cubic or larger order of the input dimension [16], which makes it difficult to apply these algorithms in real-time systems with limited computational resources, such as embedded systems. For such situations, it is useful to arbitrarily balance the dimension of the observed signal by a simple calculation in accordance with the requirement of accuracy and latency in the post-stage processing.

Linear dimensionality reduction, i.e., dimensionality reduction by linear map, is one of the effective approaches for the above tradeoff problem. The principal component analysis (PCA) [17] and the element selection method [18] are representative special cases of the linear dimensionality reduction, where the compression matrix (i.e., the linear map representing dimensionality reduction) is determined on the basis of various constraints and criteria. In whatever methods, a linear map of the observed signal essentially corresponds in a physical sense to the observation by virtual sensors having different directivities and frequency responses. Therefore, a compressed signal by linear dimensionality reduction can be used directly as an input signal to many of the above-mentioned post-stage applicational processings unless they require specific directivity and frequency response of the sensors. This is, in such a context, an advantage over nonlinear dimensionality reduction methods.

In this work, we deal with linear dimensionality reduction of multichannel time-series signals having time-invariant intertemporal and interchannel correlation, which is a natural setting in acoustic measurement by a microphone array. Owing to the time-invariant property, such signals can be modeled in the frequency domain using only frequency-wise interchannel correlation (without interfrequency correlation). However, frequency-wise direct application of the PCA or element selection method makes the dimensionality reduction process be a multi-input multi-output linear time-invariant system. Therefore, it requires a large number of digital filters with large filter lengths (dozens to thousands in many practical cases) in its implementation, which is still undesirable in low-latency systems with limited computational resources.

To overcome the above problems, we propose a frequency-independent linear dimensionality reduction method, which can be implemented as an instantaneous mixing of the observed signals in the digital or even possibly analog domain. Although detailed studies on linear dimensionality reduction with analog circuits are left for future work, the dimensional reduction before analog-to-digital conversion may also contribute to low-cost manufacturing of a sensor array while keeping its spatial sensing performance. First, we formulate the observation and dimensionality reduction model in the frequency domain, where the compression matrix (i.e., the linear map representing dimensionality reduction) is defined to be frequency-independent. Then, the optimization problem is formulated so that the original observed signal can be restored well from the compressed signal. In contrast to the dimensionality reduction process, the restoration process in the optimization problem is allowed to be frequency-dependent; here, note that we need not consider the computational cost or latency of the restoration process because it is only required virtually in the optimization problem but unnecessary in the practical application. This inconsistency setting

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between the frequency-independent dimensionality reduction process and the frequency-wise restoration process makes a difference between the proposed method and the PCA; we will also give their comparisons in detail. As far as we know, it is the first attempt to introducing this problem setting in instantaneous but nonbinary dimensionality reduction, which is one of our main contributions in this work. We refer to [19] for a frequency-independent element selection method considering wideband signals, although the constraint and criterion for an optimal dimensionality reduction are essentially different from that of the proposed method. The proposed optimization problem is nonconvex and difficult to solve in an explicit computable form due to the above-mentioned inconsistent formulation. To solve this complex problem, we propose the iterative algorithm based on the coordinate descent method, which is another contribution of this work. Finally, results of numerical experiments for different two array geometries are reported to evaluate and compare the proposed method, PCA, and element selection method.

2. FORMULATION

2.1. Notation

Throughout this paper, $\mathbb{N}$, $\mathbb{R}$, and $\mathbb{C}$ denote the set of all natural numbers, real numbers, and complex numbers, respectively. For vector and matrix notation, $(\cdot)^*$, $(\cdot)^{\mathsf{T}}$, and $(\cdot)^{\mathsf{H}}$ denote the element-wise complex conjugate, transpose, and Hermitian transpose, respectively, $\|\cdot\|_2$ denotes the Euclidean norm of a vector, $\mathrm{tr}(\cdot)$ denotes the trace of a square matrix, $\mathrm{vec}(\cdot)$ denotes the vectorization of a matrix by stacking its columns, and the operator $\otimes$ denotes the Kronecker product.

2.2. Problem statement

Suppose $M \in \mathbb{N}$ channel time-series signals are observed by a sensor array. These signals are represented in the frequency domain in this paper, and the set of frequency indices is denoted by Ω, which is assumed to be a finite set for theoretical convenience. Let $y_m(\omega) \in \mathbb{C}$ denote the observed signal of the $m \in \{1, \dots, M\}$th channel at frequency $\omega \in \Omega$. Moreover, it is assumed to be a superposition of the ideal (noise-free) component $x_m(\omega) \in \mathbb{C}$ and the noise component $\epsilon_m(\omega) \in \mathbb{C}$, as modeled as the following equation:

$$y_m(\omega) = x_m(\omega) + \epsilon_m(\omega) \quad (m \in \{1, \dots, M\},\ \omega \in \Omega). \tag{1}$$

Hereafter, we also use the vector notation as

$$\mathbf{y}(\omega) = \mathbf{x}(\omega) + \boldsymbol{\epsilon}(\omega) \quad (\omega \in \Omega) \tag{2}$$

with $\mathbf{y}(\omega) := [y_1(\omega), \dots, y_M(\omega)]^{\mathsf{T}}$, $\mathbf{x}(\omega) := [x_1(\omega), \dots, x_M(\omega)]^{\mathsf{T}}$, and $\boldsymbol{\epsilon}(\omega) := [\epsilon_1(\omega), \dots, \epsilon_M(\omega)]^{\mathsf{T}}$. Since we here consider real-valued signals in the time domain, we assume $-\omega \in \Omega$, $\mathbf{x}(-\omega) = \mathbf{x}(\omega)^*$, and $\boldsymbol{\epsilon}(-\omega) = \boldsymbol{\epsilon}(\omega)^*$ for each $\omega \in \Omega$.

Let K be a natural number smaller than M, and we consider linear dimensionality reduction represented as

$$\mathbf{z}(\omega) = \mathbf{P}\mathbf{y}(\omega) \quad (\omega \in \Omega), \tag{3}$$

where $\mathbf{z}(\omega) \in \mathbb{C}^K$ is the compressed signal and $\mathbf{P} \in \mathbb{R}^{K \times M}$ is a frequency-independent compression matrix, which will be determined on the basis of a criterion described in the following paragraph so that the original signal $\mathbf{x}(\omega)$ can be restored well. Note that this dimensionality reduction process can be achieved instantaneously in the time domain owing to the frequency independence of $\mathbf{P}$.

The optimal compression matrix is determined as follows. First, we here deal with the variables in (2) as random variables and assume that the observed signal has no interfrequency correlation (i.e., the observed signal has time-invariant correlation) and that the ideal and noise components of the observed signal have no correlation with each other. Moreover, for each $\omega \in \Omega$, the mean of $\mathbf{x}(\omega)$ and $\boldsymbol{\epsilon}(\omega)$ is assumed to be zero, and their covariances, denoted hereafter by $\mathbf{V}(\omega) := \mathrm{E}[\mathbf{x}(\omega)\mathbf{x}(\omega)^{\mathsf{H}}]$ and $\boldsymbol{\Sigma}(\omega) := \mathrm{E}[\boldsymbol{\epsilon}(\omega)\boldsymbol{\epsilon}(\omega)^{\mathsf{H}}]$, are assumed to be given, where $\mathrm{E}[\cdot]$ denotes the expectation of a random variable. Then, let $R : \mathbb{R}^{K \times M} \times \Omega \to [0, \infty)$ be defined as

$$R(\mathbf{P}, \omega) := \min_{\mathbf{Q}(\omega) \in \mathbb{C}^{M \times K}} \mathrm{E}[\|\mathbf{x}(\omega) - \mathbf{Q}(\omega)\mathbf{P}\mathbf{y}(\omega)\|_2^2] \quad (\mathbf{P} \in \mathbb{R}^{M \times K},\ \omega \in \Omega), \tag{4}$$

which means the minimum expected restoration error at frequency ω when the original signal is restored from the compressed signal $\mathbf{P}\mathbf{y}(\omega)$ by an optimal linear map (referred hereafter to as restoration matrix). We define the objective function as the sum of these expected restoration errors with respect to frequencies and determine the optimal compression matrix by solving the following minimization problem:

$$\underset{\mathbf{P} \in \mathbb{R}^{K \times M}}{\text{minimize}}\ F(\mathbf{P}) := \sum_{\omega \in \Omega} R(\mathbf{P}, \omega). \tag{5}$$

Although we can also consider the weighted sum in the right-hand side of (5) with a slight modification in later formulations and algorithms, we here do not deal with a frequency-wise weighting to clarify our basic idea and avoid unnecessary notational complexity.

In practical situations, it is often difficult to model accurate $(\mathbf{V}(\omega))_{\omega \in \Omega}$ and $(\boldsymbol{\Sigma}(\omega))_{\omega \in \Omega}$. When no useful information is available, $(\mathbf{V}(\omega))_{\omega \in \Omega}$ is modeled using "noninformative" prior only exploiting physical property of target quantity (e.g., diffuse field model for sound field [20]), and when additional information on the target quantity and environment is given, it may be exploited for more accurate modeling. On the other hand, the observation noise is originated in many cases from a thermal noise or quantization error and generally has no interchannel correlation. In such cases, $(\boldsymbol{\Sigma}(\omega))_{\omega \in \Omega}$ can be well modeled by diagonal matrices.

2.3. Comparison with principal component analysis

The proposed formulation is closely related to the PCA, which is one of the most commonly used dimensionality reduction methods for general multivariate data. The PCA aims to find the linear map maximizing the transformed data's variance called the principal component score [13, 17]. It is well known that the PCA is equivalent to a specific type of minimization problem of the restoration error in a similar form to (11), and it can be solved explicitly by the eigenvalue decomposition of the data covariance matrix [17]. If we define the compression and restoration matrices to be both dependent on each frequency and assume $\boldsymbol{\Sigma}(\omega)$ to be a scalar matrix for each $\omega \in \Omega$, the optimization problem

$$\underset{\mathbf{P}(\omega) \in \mathbb{C}^{K \times M}}{\text{minimize}} \left(\min_{\mathbf{Q}(\omega) \in \mathbb{C}^{M \times K}} \mathrm{E}[\|\mathbf{x}(\omega) - \mathbf{Q}(\omega)\mathbf{P}(\omega)\mathbf{y}(\omega)\|_2^2] \right) \tag{6}$$

is equivalent to a certain PCA (referred hereafter to as the frequency-wise PCA) and the optimal solution $\hat{\mathbf{P}}(\omega) \in \mathbb{C}^{K \times M}$ is derived explicitly as

$$\hat{\mathbf{P}}(\omega) = \mathrm{eigvec}\left(\mathbf{V}(\omega), K\right)^{\mathsf{H}}. \tag{7}$$

Here, for any Hermitian matrix $\mathbf{A} \in \mathbb{C}^{M\times M}$, $\mathrm{eigvec}(\mathbf{A}, K)$ denotes the $M \times K$ matrix whose column vectors consist of the eigenvectors of $\mathbf{A}$ corresponding to the first K largest eigenvalues. Conversely, if we define the compression and restoration matrices to be both frequency-independent and assume $\mathbf{\Sigma}(\omega)$ to be a scalar matrix for each $\omega \in \Omega$, the optimization problem

$$\underset{\mathbf{P}\in\mathbb{R}^{K\times M}}{\mathrm{minimize}} \left(\min_{\mathbf{Q}\in\mathbb{R}^{M\times K}} \sum_{\omega\in\Omega} \mathrm{E}[\|\mathbf{x}(\omega) - \mathbf{Q}\mathbf{P}\mathbf{y}(\omega)\|_2^2] \right) \tag{8}$$

is equivalent to another certain PCA [13] (referred hereafter to as the instantaneous PCA), and the optimal solution $\hat{\mathbf{P}} \in \mathbb{R}^{K\times M}$ is derived explicitly as

$$\hat{\mathbf{P}} = \mathrm{eigvec}\left(\sum_{\omega\in\Omega} \mathbf{V}(\omega), K\right)^{\mathsf{H}}. \tag{9}$$

From the practical point of view, implementation of the frequency-wise PCA requires a large number of digital filters with large filter length, although it may achieve well restorable dimensionality reduction. On the other hand, the dimensionality reduction process based on (9) can be achieved instantaneously as well as that based on the proposed formulation; however, an unnecessary constraint, i.e., the frequency independence, is imposed on the restoration matrix, which prevents us from finding the best restorable compression matrix. The proposed formulation in (5) allows inconsistency between the frequency-independent dimensionality reduction and the frequency-wise restoration process, which is expected to lead to better restorable dimensionality reduction than the instantaneous PCA.

3. PROPOSED ALGORITHM

The problem in (5) is nonconvex and difficult to solve in an explicit computable form, unlike the PCA. Here, we present an iterative algorithm based on the coordinate descent method that guarantees the monotonic nonincreasing property of the objective function.

First, we define an essentially equivalent optimization problem to (5) as

$$\underset{\mathbf{P}\in\mathbb{R}^{K\times M}, (\mathbf{Q}(\omega)\in\mathbb{C}^{M\times K})_{\omega\in\Omega}}{\mathrm{minimize}} \; G(\mathbf{P}, (\mathbf{Q}(\omega))_{\omega\in\Omega}) \tag{10}$$

with

$$\begin{aligned} &G(\mathbf{P}, (\mathbf{Q}(\omega))_{\omega\in\Omega}) \\ &:= \sum_{\omega\in\Omega} \mathrm{E}[\|\mathbf{x}(\omega) - \mathbf{Q}(\omega)\mathbf{P}\mathbf{y}(\omega)\|_2^2] \\ &= \sum_{\omega\in\Omega} \mathrm{tr}\Big((\mathbf{I} - \mathbf{Q}(\omega)\mathbf{P})\mathbf{V}(\omega)(\mathbf{I} - \mathbf{Q}(\omega)\mathbf{P})^{\mathsf{H}} \\ &\qquad\qquad + \mathbf{Q}(\omega)\mathbf{P}\mathbf{\Sigma}(\omega)\mathbf{P}^{\mathsf{H}}\mathbf{Q}(\omega)^{\mathsf{H}} \Big) \\ &\qquad (\mathbf{P} \in \mathbb{R}^{K\times M},\ \mathbf{Q}(\omega) \in \mathbb{C}^{M\times K},\ \omega \in \Omega), \end{aligned} \tag{11}$$

where $\mathbf{I}$ denotes the $M \times M$ identity matrix. This is still a nonconvex problem; however, the objective function G can be regarded as a quadratic function of the first and second variables if the other is fixed, as easily seen in (11). Therefore, we can apply the iterative algorithm based on the coordinate descent method to (10) with arbitrary initial solutions. We show the proposed optimization algorithm in Algorithm 1. Note that the computational cost of the optimization algorithm has no influence on that of the proposed linear dimensionality reduction process itself since the optimization can be completed before the use of the sensor array.

Algorithm 1 Algorithm for optimizing compressing matrix

Input: $\left(\mathbf{V}(\omega) \in \mathbb{C}^{M\times M},\ \mathbf{\Sigma}(\omega) \in \mathbb{C}^{M\times M}\right)_{\omega\in\Omega}$

1: Initialize:

$$\mathbf{P} = \mathrm{eigvec}\left(\sum_{\omega\in\Omega} \mathbf{V}(\omega), K\right)^{\mathsf{H}}. \tag{12}$$

2: **repeat**

3: Update:

$$\mathbf{Q}(\omega) \leftarrow (\mathbf{V}(\omega)\mathbf{P}^{\mathsf{H}}) \left[\mathbf{P}(\mathbf{V}(\omega) + \mathbf{\Sigma}(\omega))\mathbf{P}^{\mathsf{H}}\right]^{-1}, \tag{13}$$

$$\begin{aligned} \mathrm{vec}(\mathbf{P}) \leftarrow &\left[\sum_{\omega\in\Omega} (\mathbf{V}(\omega) + \mathbf{\Sigma}(\omega))^{\mathsf{T}} \otimes (\mathbf{Q}(\omega)^{\mathsf{H}}\mathbf{Q}(\omega))\right]^{-1} \\ &\cdot \mathrm{vec}\left(\sum_{\omega\in\Omega} \mathbf{Q}(\omega)^{\mathsf{H}}\mathbf{V}(\omega)\right). \end{aligned} \tag{14}$$

4: Normalize $\mathbf{P}$ using an arbitrary nonsingular matrix $\mathbf{A} \in \mathbb{R}^{K\times K}$ (e.g., row-wise normalization or orthonormalization):

$$\mathbf{P} \leftarrow \mathbf{A}\mathbf{P}. \tag{15}$$

5: **until** stopping condition is met

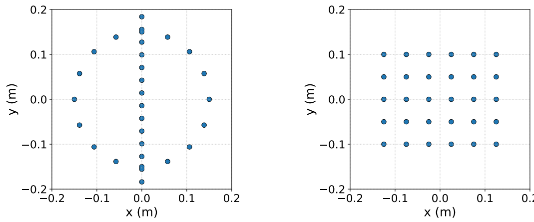
6: **return** $\mathbf{P}$

In (12), note that we can take $\mathbf{P}$ as a real-valued matrix since $\sum_{\omega\in\Omega} \mathbf{V}(\omega)$ is a real-valued symmetric matrix by assumption. There can be various other ways of initialization; in this paper, we use the solution of (8), i.e., instantaneous PCA, as an initial solution. Owing to the monotonic nonincreasing property of the objective function, the proposed method guarantees an equivalent or better solution than this initial solution. The update rules in (13) and (14) are derived by the minimization of the quadratic function in (11). Here, we use the formula [21] $\mathrm{vec}(\mathbf{A}\mathbf{X}\mathbf{B}) = (\mathbf{B}^{\mathsf{T}} \otimes \mathbf{A})\mathrm{vec}(\mathbf{X})$ for $\mathbf{A} \in \mathbb{C}^{K\times K}$, $\mathbf{B} \in \mathbb{C}^{M\times M}$, and $\mathbf{X} \in \mathbb{C}^{K\times M}$ to derive (14). The normalization step in (15) prevents the numerical instability of the algorithm, although it is unnecessary in a theoretical sense; it can be seen easily that $G(\mathbf{A}\mathbf{P}, (\mathbf{Q}(\omega)\mathbf{A}^{-1})_{\omega\in\Omega}) = G(\mathbf{P}, (\mathbf{Q}(\omega))_{\omega\in\Omega})$ holds for any invertible $\mathbf{A} \in \mathbb{C}^{K\times K}$.

4. NUMERICAL EXPERIMENTS

Numerical simulations were conducted to evaluate and compare the proposed method and other linear dimensionality reduction methods, i.e., the frequency-wise PCA, instantaneous PCA [13], and element selection method. The compression matrix for the proposed method was derived using Algorithm 1, where the row-wise normalization was used in each normalization step of (15). The compression matrices for the frequency-wise PCA and instantaneous PCA were derived by (7) and (9), respectively. We reemphasize that the linear dimensionality reduction process based on the frequency PCA requires a higher computational cost than the others, although it theoretically achieves the best restoration accuracy. The compression matrix for the element selection method was derived as a binary matrix representing element selection of the sensors' indices, where the objective function was defined as (5) and the greedy algorithm was used for the optimization.

The experimental settings were designed as follows. In the



(a) **Circle + Line** (b) **Plane**

Fig. 1: Array configuration.

three-dimensional space with the Cartesian coordinates $(\mathrm{x}, \mathrm{y}, \mathrm{z})\,\mathrm{m}$, an array of $M = 30$ omnidirectional microphones was located, where the following two different array geometries, **Circle + Line** and **Plane**, were investigated. In **Circle + Line**, on the xy-plane ($z = 0\,\mathrm{m}$), 16 microphones were located equiangularly on a circle with a radius of $0.15\,\mathrm{m}$ centered at the origin. In addition, on the plane $z = 0.04\,\mathrm{m}$, 14 microphones were located equidistantly on a line along the direction of the y-axis centered at $(0, 0, 0.04)\,\mathrm{m}$ with intervals of $0.0283\,\mathrm{m}$. In **Plane**, on the xy-plane, 30 microphones were located on a 6×5 grid with intervals of $0.05\,\mathrm{m}$. The array configurations projected into the xy-plane are shown in Fig. 1. The target dimension after the dimensionality reduction was set as $K := 8$. The set of the frequencies Ω was set as

$$\Omega := \{2\pi f \mid f = -4000, -3900, \ldots, 3900, 4000\,\mathrm{Hz}\}. \quad (16)$$

The covariance matrices $(\mathbf{V}(\omega))_{\omega\in\Omega}$ were defined on the basis of the diffuse field model [20], i.e.,

$$\mathrm{E}[x_m(\omega)x_n(\omega)^*] := \mathrm{sinc}\left(\frac{\omega}{c}\|\mathbf{r}_m - \mathbf{r}_n\|_2\right) \quad (m, n \in \{1, \ldots, M\}, \omega \in \Omega), \quad (17)$$

where $\mathrm{sinc}(\cdot)$ is the (unnormalized) sinc function, $\mathbf{r}_1, \ldots, \mathbf{r}_M$ denote the sensor positions, and the sound speed c was set as $c := 340\,\mathrm{m/s}$. The noise covariance matrices $(\mathbf{\Sigma}(\omega))_{\omega\in\Omega}$ were defined as

$$\mathbf{\Sigma}(\omega) := \lambda\mathbf{I} \quad (\omega \in \Omega) \quad (18)$$

with $\lambda := 10^{-2}$.

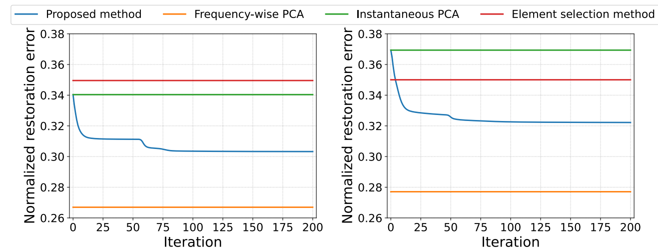
First, to see the convergence of the proposed algorithm, the normalized restoration error defined as

$$\text{Normalized restoration error} := \frac{F(\mathbf{P})}{\sum_{\omega\in\Omega}\mathrm{E}[\|\mathbf{x}(\omega)\|_2^2]} \quad (19)$$

was plotted against the number of iterations in Fig. 2. For comparison, the normalized restoration errors for the other methods were also plotted as horizontal lines. The monotonic nonincreasing property for the objective function of the proposed algorithm can be seen in this figure. Moreover, we can see that the proposed method achieved the lowest restoration error among the three instantaneous linear dimensionality reduction methods. Although the restoration error for the proposed method was still higher than that for the frequency-wise PCA, this result means that the proposed method was able to reduce drastically the computational cost of the dimensionality reduction process compared to the frequency-wise PCA while suppressing degradation of performance to some degree. In this experimental setting, in particular, the proposed method was able to reduce the space complexity (i.e., the amount of required memory) of the dimensionality reduction process to $1/|\Omega| = 1/81$.

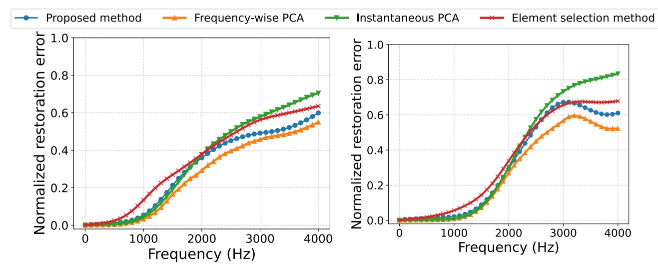
For further investigation, the normalized restoration error at each frequency defined as

$$\text{Normalized restoration error}(\omega) := \frac{R(\mathbf{P}, \omega)}{\mathrm{E}[\|\mathbf{x}(\omega)\|_2^2]} \quad (20)$$



(a) **Circle + Line** (b) **Plane**

Fig. 2: Normalized restoration error plotted against the number of iterations.



(a) **Circle + Line** (b) **Plane**

Fig. 3: Normalized restoration error plotted against frequency.

was also plotted in Fig. 3. Here, the compression matrix derived after 200 iterations in Algorithm 1 was evaluated for the proposed method. This figure indicates that the proposed method achieved lower restoration error than the instantaneous PCA at high frequencies. One possible reason for this tendency is explained as follows. From (17), we can see the ideal component of the observed signal has strong interchannel correlation at low frequencies, which means the covariance matrix has a large eigenvalue for some eigenvector. In contrast, the interchannel correlation becomes weak at high frequencies, which means the covariance matrix does not have a large eigenvalue for any eigenvector. Therefore, eigenvalues and eigenvectors of $\sum_{\omega_\Omega}\mathbf{V}(\omega)$, i.e., the compression matrix of the instantaneous PCA, was dominated by those for low frequencies. On the other hand, the above-mentioned properties of the interchannel correlation also mean that the restoration matrix has some degree of freedom at low frequencies, i.e., the signals can be restored well for almost any compression matrix, whereas this degree of freedom becomes small at high frequencies, i.e., the signal can be restored well only for a certain compression matrix. As a result, the proposed method put importance mainly on high frequencies to minimize the total restoration error for all frequencies.

5. CONCLUSION

We proposed an instantaneous linear dimensionality reduction method for multichannel time-series signals having time-invariant interchannel and intertemporal correlations. To derive the optimal compression matrix, we also proposed an iterative algorithm based on the coordinate descent method. We compared the proposed method with other linear dimensionality reduction methods by numerical simulations and demonstrated the effectiveness of the proposed method. In the future work, we will consider more specific covariance matrices of observed and noise signals, study a further generalization of the proposed method for practical problems, and conduct experiments of practical applications such as beamforming and source localization.

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