OPTIMAL RESOURCE ALLOCATION AND BEAMFORMING FOR TWO-USER MISO WPCNs FOR A NON-LINEAR CIRCUIT-BASED EH MODEL

(Invited Paper)

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ABSTRACT

We study two-user multiple-input single-output (MISO) wireless powered communication networks (WPCNs), where the user devices are equipped with non-linear energy harvesting (EH) circuits. We consider time-division duplex (TDD) transmission, where the users harvest power from the signal received in the downlink phase, and then, utilize this harvested power for information transmission in the uplink phase. In contrast to existing works, we adopt a non-linear model of the harvested power based on a precise analysis of the employed EH circuit. We jointly optimize the beamforming vectors in the downlink and the time allocated for downlink and uplink transmission to minimize the average transmit power in the downlink under per-user data rate constraints in the uplink. We provide conditions for the feasibility of the resource allocation problem and the existence of a trivial solution, respectively. For the case where the resource allocation has a non-trivial solution, we show that it is optimal to employ no more than three beamforming vectors for power transfer in the downlink. To determine these beamforming vectors, we develop an iterative algorithm based on semi-definite relaxation (SDR) and successive convex approximation (SCA). Our simulation results reveal that the proposed resource allocation scheme outperforms two baseline schemes based on linear and sigmoidal EH models, respectively.

1. INTRODUCTION

The growth of the number of low-power Internet-of-Things (IoT) devices has fuelled a significant interest in wireless powered communication networks (WPCNs) that enable energy-sustainable communication with such devices [1–5]. A typical WPCN comprises a base station (BS) that broadcasts a radio frequency (RF) signal to the user devices in the downlink [1,2]. The users are equipped with energy harvesting (EH) circuits that extract the received RF energy. The collected energy is then used for information transmission in the uplink [1, 2].

A WPCN employing multiple antennas at the BS was considered in [3]. The authors optimized the beamforming vectors and transmit powers for maximization of the total throughput in the uplink. Although the results in [3] provide insights for WPCN design, they are based on a linear EH model, whereas

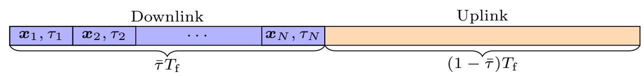

Fig. 1. Structure of a time frame of length T_f.

the experiments in [6] and [7] showed that typical EH circuits are non-linear. In order to take the non-linearities of EH circuits into account, the authors in [8] proposed a practical EH model based on a parametrized sigmoidal function of the average power of the received RF signal. This model was utilized for the analysis of multi-antenna WPCNs in [4], where the authors optimized the covariance matrix of the transmit symbol vectors in the downlink for maximization of the sum information rate in the uplink. Since the EH model in [8] characterizes only the *average* harvested power at the user devices, the WPCN design in [4] does not fully capture the non-linearity of the EH circuits. Therefore, based on an accurate analysis of a typical EH circuit, the authors in [9] developed a precise EH model that maps the *instantaneous* received RF power to the *instantaneous* harvested power. Based on this model, the authors in [10] considered multi-user multi-antenna wireless power transfer (WPT) systems and showed that the optimal transmit strategy that maximizes a weighted sum of the average harvested powers at EH nodes employs multiple beamforming vectors at the BS.

The main contributions of this paper can be summarized as follows. We consider a two-user multiple-input single-output (MISO) WPCN, where the EH at the user devices is characterized by the non-linear circuit-based EH model developed in [9]. We formulate an optimization problem to minimize the average transmit power in the downlink under per-user rate constraints in the uplink. We provide conditions for the feasibility of the optimization problem and the existence of a trivial solution, respectively. Next, when the problem is feasible and has a non-trivial solution, we show that three beamforming vectors are sufficient for optimal transmission in the downlink. To determine these beamforming vectors, we design an iterative algorithm based on semi-definite relaxation (SDR) and successive convex approximation (SCA) [11]. Our simulation results demonstrate that the proposed approach significantly outperforms two baseline schemes that are based on the linear and sigmoidal EH models in [3] and [4], respectively.

Throughout this paper, we use the following notations. Bold upper case letters $\boldsymbol{X}$ represent matrices. Bold lower case letters $\boldsymbol{x}$ stand for vectors and x_i is the i^th element of $\boldsymbol{x}$. $\boldsymbol{X}^H$

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denotes the Hermitian transpose of matrix $\boldsymbol{X}$. $\|\boldsymbol{x}\|_2$ represents the L_2-norm of $\boldsymbol{x}$. The sets of complex and real numbers are denoted by $\mathbb{C}$ and $\mathbb{R}$, respectively. The real part of a complex number is denoted by $\mathcal{R}\{\cdot\}$. $\boldsymbol{I}_K$ stands for the square identity matrix of size K. The imaginary unit is denoted by j.

2. SYSTEM MODEL

We consider a MISO WPCN, where $K = 2$ single-antenna users are equipped with EH circuits [9] and $N_\mathrm{t} \geq K$ antennas are employed at the BS. We adopt time-division duplex (TDD) transmission and assume that each time frame of length T_f is divided into two subframes, as shown in Fig. 1. In the first subframe of length $\bar{\tau}T_\mathrm{f}$, $\bar{\tau} \in [0, 1)$, the BS transmits a power-carrying RF signal to the user devices, which harvest the received power. In the subsequent subframe of length $(1-\bar{\tau})T_\mathrm{f}$, this harvested power is utilized for information transmission in the uplink. We assume that the channels between the BS and the user devices are constant for the duration of a time frame. Furthermore, we denote the channel between the BS and user $k \in \{1, 2\}$ by $\boldsymbol{h}_k \in \mathbb{C}^{N_\mathrm{t}}$ and assume that it is perfectly known at the BS.

2.1. Downlink Phase

In the downlink, the BS broadcasts a pulse-modulated RF signal, whose equivalent complex baseband representation is modelled as $\boldsymbol{x}(t) = \sum_{n=1}^{N} \boldsymbol{x}_n \psi_n(t)$, where $\psi_n(t) = \Pi\big(\frac{t/T_\mathrm{f} - \sum_{k=0}^{n-1}\tau_k}{\tau_n}\big)$ is the transmit pulse, $\tau_0 = 0$, $\Pi(t)$ is a rectangular function that takes value 1 if $t \in [0, 1)$ and 0, otherwise, $\boldsymbol{x}_n \in \mathbb{C}^{N_\mathrm{t}}$ is the symbol vector transmitted in time slot $n \in \{1, 2, \cdots, N\}$, and N is the number of employed symbol vectors, see Fig. 1. Here, τ_n is the portion of the time frame of length T_f utilized for the transmission of symbol vector $\boldsymbol{x}_n$ with $\sum_{n=1}^{N} \tau_n = \bar{\tau}$. Thus, the RF signal received at user $k \in \{1, 2\}$ is given by $z_k^{\mathrm{RF}}(t) = \sqrt{2}\mathcal{R}\Big\{\boldsymbol{h}_k^H \boldsymbol{x}(t) \exp(j2\pi f_c t)\Big\}$, where f_c denotes the carrier frequency. Similar to [9], the noise received at the users is neglected since its contribution to the harvested power is negligible.

To harvest power, the users are equipped with identical non-linear EH circuits. We denote the power harvested at user k in time slot n by $\rho_{k,n}$ and, as in [9] and [12], model it by a non-linear monotonic non-decreasing function $\phi(\cdot)$ of the instantaneous received power. Thus, $\rho_{k,n} = \phi\big(|z_{k,n}|^2\big)$ with $z_{k,n} = \boldsymbol{h}_k^H \boldsymbol{x}_n$ and $\phi(\cdot)$ is given by [9]:

$$\phi\big(|z|^2\big) = \min\big\{\varphi(|z|^2), \varphi(A_s^2)\big\}, \tag{1}$$

$$\varphi(|z|^2) = \lambda\Big[\mu^{-1} W_0\Big(\mu \exp(\mu) I_0\big(\nu\sqrt{2|z|^2}\big)\Big) - 1\Big]^2, \tag{2}$$

where $W_0(\cdot)$ is the principle branch of the Lambert-W function, $I_0(\cdot)$ is the modified Bessel function of the first kind and order zero, and λ, μ, and ν are parameters of the EH circuit that depend on the circuit elements but not on the received signal. Since practical EH circuits are driven into saturation for large input powers [8,9], $\phi(\cdot)$ in (1) is bounded, i.e., $\phi(|z|^2) \leq \phi(|A_s|^2), \ \forall z \in \mathbb{C}$, where A_s is the minimum input signal magnitude level at which the output power starts to saturate. Hence, the average power harvested at user $k \in \{1, 2\}$ in the downlink can be expressed as follows:

$$p_k^\mathrm{d} = \sum_{n=1}^{N} \tau_n \phi\big(|\boldsymbol{h}_k^H \boldsymbol{x}_n|^2\big) \tag{3}$$

Finally, we assume that user k is equipped with a rechargeable built-in battery having initial energy q_k, which is known to the BS. Thus, at the end of the downlink phase, the amount of energy available at user k is given by $E_k = q_k + p_k^\mathrm{d} T_f$.

2.2. Uplink Phase

In the uplink, user $k \in \{1, 2\}$ transmits information symbols s_k with zero mean and unit variance to the BS utilizing a portion of the available energy E_k. Assuming uplink-downlink reciprocity of the channel, the symbol vector $\boldsymbol{r} \in \mathbb{C}^{N_\mathrm{t}}$ received at the BS in the scheduled time slot is given by

$$\boldsymbol{r} = \sum_{k=1}^{K} \boldsymbol{h}_k \sqrt{p_k^\mathrm{u}} s_k + \boldsymbol{n}, \tag{4}$$

where p_k^u is the power utilized by user k for information transmission and $\boldsymbol{n} \in \mathbb{C}^{N_\mathrm{t}}$ is an additive white Gaussian noise (AWGN) vector with zero mean and covariance matrix $\sigma^2 \boldsymbol{I}_{N_\mathrm{t}}$. Since we have $N_\mathrm{t} \geq K$, we can adopt zero forcing (ZF) equalization to suppress the inter-user interference at the BS. Hence, the detected information symbol $\hat{s}_k$ of user k can be expressed as follows:

$$\hat{s}_k = \boldsymbol{f}_k \boldsymbol{r} = \sqrt{p_k^\mathrm{u}} s_k + \tilde{n}_k, \tag{5}$$

where $\tilde{n}_k = \boldsymbol{f}_k \boldsymbol{n}$ is the equivalent AWGN with variance $\tilde{\sigma}_k^2 = \|\boldsymbol{f}_k\|_2^2 \sigma^2$ for user k at the BS. Here, equalization vector $\boldsymbol{f}_k \in \mathbb{C}^{N_\mathrm{t}}$ is the k^{th} row of matrix $\boldsymbol{F} = (\boldsymbol{H}^H \boldsymbol{H})^{-1} \boldsymbol{H}^H$ with $\boldsymbol{H} = [\boldsymbol{h}_1 \ \boldsymbol{h}_2]$. Finally, the data rate of user k is given by $R_k = (1-\bar{\tau}) \log_2(1+\Gamma_k)$, where $\Gamma_k = p_k^\mathrm{u}/\tilde{\sigma}^2$ is the signal-to-noise ratio (SNR).

3. PROBLEM FORMULATION AND SOLUTION

In this section, we develop a resource allocation algorithm for the considered MISO WPCN.

3.1. Problem Formulation

We formulate the following non-convex optimization problem:

$$\underset{\substack{\boldsymbol{\tau}, \bar{\tau} \in [0,1), \\ \boldsymbol{X}, \boldsymbol{p}^\mathrm{u}, N}}{\text{minimize}} \ P_\mathrm{DL} \tag{6a}$$

$$\text{subject to } R_k \geq R_k^{\mathrm{req}}, \forall k \tag{6b}$$

$$(1-\bar{\tau}) p_k^\mathrm{u} T_f \leq E_k, \forall k \tag{6c}$$

$$\sum_{n=1}^{N} \tau_n = \bar{\tau}, \tag{6d}$$

where we jointly optimize $\bar{\tau}$, $\boldsymbol{\tau} = [\tau_1, \tau_2 \cdots \tau_N]$, the uplink transmit powers $\boldsymbol{p}^u = [p_1^u, p_2^u]$, symbol vectors $\boldsymbol{x}_n$ collected in $\boldsymbol{X} = [\boldsymbol{x}_1 \ \boldsymbol{x}_2 \cdots \boldsymbol{x}_N]$, and the number of time slots N. In (6), we minimize the average transmit power in the downlink $P_\mathrm{DL} = \sum_n \tau_n \|\boldsymbol{x}_n\|_2^2$ under per-user rate constraints in the uplink in (6b), where R_k^{req} is the minimum required rate of user k. Here, (6c) and (6d) ensure that the transmit energy consumed by user k in the uplink does not exceed E_k and the obtained resource allocation is feasible, respectively. We note that in contrast to the WPCN design in [3] and [4], where covariance matrix $\tilde{\boldsymbol{X}} = \sum_{n=1}^{N} \frac{\tau_n}{\bar{\tau}} \boldsymbol{x}_n \boldsymbol{x}_n^H$ was optimized, the EH model in (1) characterizes the instantaneous harvested power and enables the optimization of individual transmit symbol vectors $\boldsymbol{x}_n$ in (6).

3.2. Characterization of Optimal Solution

In the following propositions, we provide a feasibility condition and characterize the optimal solution of (6).

Proposition 1. *Problem (6) is feasible if and only if* $\exists \bar{\tau} \in [0, 1)$*, such that the following condition holds* $\forall k \in \{1, 2\}$:

$$f_k(\bar{\tau}) \triangleq \frac{1-\bar{\tau}}{\bar{\tau}} \left[2^{\frac{R_k^{req}}{1-\bar{\tau}}} - 1\right] \tilde{\sigma}^2 - \frac{q_k}{T_f \bar{\tau}} \leq \phi(A_s^2). \quad (7)$$

Proof. The proof is available in a longer version of this paper [13] and is omitted here due to the space constraints. ■

Proposition 2. *If the following conditions hold*:

$$\left[2^{R_k^{\text{req}}} - 1\right] \tilde{\sigma}^2 < \frac{q_k}{T_f}, \forall k \in \{1, 2\}, \quad (8)$$

problem (6) is feasible and the optimal solution is trivial[1]: $\bar{\tau}^* = 0, N^* = 0, \tau^* = 0, \boldsymbol{x}^* = \boldsymbol{0}, p_k^{u^*} = \left[2^{R_k^{\text{req}}} - 1\right] \tilde{\sigma}^2, \forall k \in \{1, 2\}$.

Proof. The proof is available in a longer version of this paper [13] and is omitted here due to the space constraints. ■

Proposition 3. *If problem (6) is feasible and conditions (8) do not hold, the optimal number of transmit symbol vectors is* $N^* \leq 3$ *and* $\bar{\tau}^* \in [\bar{\tau}^{\min}, \bar{\tau}^{\max}]$ *with* $\bar{\tau}^{\max} = \max\{\bar{\tau}_1^{\max}, \bar{\tau}_2^{\max}\}$ *and* $\bar{\tau}^{\min} = \max\{\bar{\tau}_1^{\min}, \bar{\tau}_2^{\min}\}$. *Here, for* $k \in \{1, 2\}$, $\bar{\tau}_k^{\max}$ *is the solution of the following equation*:

$$2^{\frac{R_k^{\text{req}}}{1-\bar{\tau}_k^{\max}}} \ln 2 R_k^{\text{req}} \tilde{\sigma}^2 = f_k(\bar{\tau}_k^{\max}) + \frac{q_k}{T_f} \quad (9)$$

and $\bar{\tau}_k^{\min} = \min\{\bar{\tau} : f(\bar{\tau}) = \phi(A_s^2)\}$.

Proof. The proof is available in a longer version of this paper [13] and is omitted here due to the space constraints. ■

Proposition 1 shows that problem (6) is feasible if and only if the power required for uplink transmission with rate R_k^{req} is not greater than $\phi(A_s^2)$ for all $k \in \{1, 2\}$. Next, Proposition 2 reveals that no power transfer in the downlink is needed if the available power q_k is sufficient for transmission at the data rate required for each user. Finally, as shown in Proposition 3, if the problem is feasible and the solution is non-trivial, the optimal number of time slots satisfies $N^* \leq 3$ and the optimal length of the downlink subframe $\bar{\tau}^* \in [\bar{\tau}^{\min}, \bar{\tau}^{\max}]$. In the following, we consider the case, where problem (6) is feasible and has a non-trivial solution.

We note that the functions that appear in (6) are invariant with respect to a scalar phase rotation of the symbol vectors in $\boldsymbol{X}$. Hence, for the solution of (6), the transmit symbol vectors can be decomposed as $\boldsymbol{x}_n = \boldsymbol{w}_n d_n$, $n \in \{1, 2, 3\}$, where $\boldsymbol{w}_n$ are beamforming vectors rotated by unit-norm symbols $d_n = \exp(j\theta_n)$ with arbitrary phases θ_n.

[1] We note that the trivial solution of problem (6) is not unique. Here, we provide the energy efficient solution with the minimum feasible $p_k^{\text{u}}, \forall k$.

3.3. Reformulation of Optimization Problem

Next, we determine the optimal fraction $\bar{\tau}^*$ using a one-dimensional grid search as $\bar{\tau}^* = \arg\min_{\bar{\tau} \in [\bar{\tau}^{\min}, \bar{\tau}^{\max}]} P_{\text{DL}}^*(\bar{\tau})$, where

$$P_{\text{DL}}^*(\bar{\tau}) = \min_{\mathcal{F}} \{P_{\text{DL}}\} \quad (10)$$

with $\mathcal{F} = \{\bar{\boldsymbol{W}}, \boldsymbol{p}^{\text{u}}, \boldsymbol{\tau} : \text{(6b), (6c), (6d)}\}$ and $\bar{\boldsymbol{W}} = [\boldsymbol{w}_1 \ \boldsymbol{w}_2 \ \boldsymbol{w}_3]$. In order to solve (10), we equivalently reformulate constraints (6b) and (6c) as follows:

$$p_k^{\text{u}} \geq \rho_k^{\text{req}}, \ \forall k, \quad (11)$$

$$p_k^{\text{u}} \leq \bar{q}_k / T_{\text{f}} + p_k^{\text{d}} \bar{\bar{\tau}}, \ \forall k, \quad (12)$$

respectively, where $\rho_k^{\text{req}} = (2^{\gamma_k^{\text{req}}} - 1)\tilde{\sigma}_k^2$, $\bar{q}_k = \frac{q_k}{(1-\bar{\tau})}$, and $\bar{\bar{\tau}} = (1-\bar{\tau})^{-1}$. Here, $\gamma_k^{\text{req}} = \frac{R_k^{\text{req}}}{1-\bar{\tau}}$ is the equivalent rate required by user k. Thus, we can equivalently reformulate optimization problem (10) as follows:

$$\underset{\boldsymbol{\beta}, \boldsymbol{p}^{\text{u}}, \boldsymbol{W}_1, \boldsymbol{W}_2, \boldsymbol{W}_3}{\text{minimize}} \ \bar{\tau} \sum_{n=1}^{3} \beta_n \text{Tr}\{\boldsymbol{W}_n\} \quad (13a)$$

$$\text{subject to} \quad \text{rank}\{\boldsymbol{W}_n\} \leq 1, \boldsymbol{W}_n \in \mathcal{S}_+, \forall n \quad (13b)$$

$$\text{(6d), (11), (12)}$$

where $\boldsymbol{\beta} = [\beta_1, \beta_2, \beta_3]^\top$, $\beta_n = \frac{\tau_n}{\bar{\tau}} \geq 0$, $\boldsymbol{W}_n = \boldsymbol{w}_n \boldsymbol{w}_n^H, n \in \{1, 2, 3\}$, and $\mathcal{S}_+ \subset \mathbb{C}^{N_{\text{t}} \times N_{\text{t}}}$ denotes the set of positive semidefinite matrices. Here, the harvested power is given by $p_k^{\text{d}} = \bar{\tau} \sum_{n=1}^{3} \beta_n \phi(\boldsymbol{h}_k^H \boldsymbol{W}_n \boldsymbol{h}_k)$. Problem (13) is still non-convex due to the non-convexity of constraints (12) and (13b).

3.4. Suboptimal Iterative Solution

In the following, we design an iterative algorithm to solve (13). First, we drop the rank-1 constraint in (13b). Next, we note that the EH circuits of the user devices are not driven into saturation if

$$\boldsymbol{h}_k^H \boldsymbol{W}_n \boldsymbol{h}_k \leq A_s^2, \ \forall n, k. \quad (14)$$

Furthermore, if condition (14) holds, function $\phi(\cdot)$ is convex and, thus, problem (13) can be efficiently solved via SCA [11]. Therefore, in the following, we assume that the EH circuits are not driven into saturation[2], i.e., condition (14) holds. Thus, in iteration t of the algorithm, we linearize convex function $\phi(\cdot)$ in (12) as follows [11]:

$$\begin{aligned} \beta_n \phi(\boldsymbol{h}_k^H \boldsymbol{W}_n \boldsymbol{h}_k) &\geq \phi'(\boldsymbol{h}_k^H \boldsymbol{W}_n^{(t)} \boldsymbol{h}_k) \text{Tr}\{\boldsymbol{H}_k \boldsymbol{V}_n\} + \\ &\beta_n \hat{\psi}(\boldsymbol{W}_n^{(t)}) \triangleq \Psi_k(\beta_n, \boldsymbol{V}_n; \boldsymbol{W}_n^{(t)}), \end{aligned} \quad (15)$$

where $\boldsymbol{V}_n = \beta_n \boldsymbol{W}_n$, $\boldsymbol{W}_n^{(t)} = \boldsymbol{V}_n^{(t)} / \beta_n^{(t)}$, $\boldsymbol{H}_k = \boldsymbol{h}_k \boldsymbol{h}_k^H$, $\hat{\psi}(\boldsymbol{W}_n^{(t)}) = \phi(\boldsymbol{h}_k^H \boldsymbol{W}_n^{(t)} \boldsymbol{h}_k) - \phi'(\boldsymbol{h}_k^H \boldsymbol{W}_n^{(t)} \boldsymbol{h}_k) \text{Tr}\{\boldsymbol{H}_k \boldsymbol{W}_n^{(t)}\}$, and $\beta_n^{(t)}$ and $\boldsymbol{W}_n^{(t)}$ are the value of β_n and matrix $\boldsymbol{W}_n$ obtained in iteration $t-1$ of the algorithm, respectively. Thus, we reformulate constraint (12) as follows:

$$p_k^{\text{u}} - \bar{\bar{\tau}} \bar{\tau} \sum_{n=1}^{3} \Psi_k(\beta_n, \boldsymbol{V}_n; \boldsymbol{W}_n^{(t)}) \leq \bar{q}_k / T_{\text{f}} \quad (16)$$

[2] Alternatively, as in [10], one can adopt an exhaustive search exhibiting high computational complexity to identify the users that are driven into saturation.

Algorithm 1: Algorithm for solving (6)

Initialize: Required rates R_k^{req}, initial energies $q_k, \forall k$, and error tolerances ϵ_{SCA}, ϵ_τ.

1. Set initial values $\bar{\tau} = \bar{\tau}^{\min}$, $i = 1$.

repeat

2. Determine required harvested powers ρ_k^{req}
3. Randomly initialize $\boldsymbol{V}_n^{(0)}, \forall n, \boldsymbol{\beta}^{(0)}$, set $t = 0$.

repeat

a. For given $\boldsymbol{V}_n^{(t)}, \boldsymbol{\beta}^{(t)}$, obtain $\boldsymbol{p}^{\text{u}}, \boldsymbol{V}_n^{(t+1)}, \boldsymbol{\beta}^{(t+1)}$ as the solution of (17)

b. Evaluate $h^{(t+1)} = \bar{\tau} \sum_{n=1}^{3} \text{Tr}\{\boldsymbol{V}_n\}$

c. Determine $\boldsymbol{W}^{(t)} = \boldsymbol{V}_n^{(t)} / \beta_n^{(t)}$

d. Set $t = t + 1$

until $|h^{(t)} - h^{(t-1)}| \leq \epsilon_{\text{SCA}}$;

4. Obtain $\boldsymbol{w}_n^* = \lambda_n \hat{\boldsymbol{w}}_n^*, \forall n$
5. Store $\bar{\boldsymbol{W}}_i = [\boldsymbol{w}_1^* \boldsymbol{w}_2^* \boldsymbol{w}_3^*]$, $\boldsymbol{\beta}_i = \boldsymbol{\beta}^{(t-1)}$, $\boldsymbol{p}_i^{\text{u}} = \boldsymbol{p}^{\text{u}}$, $\xi_i = \bar{\tau} \sum_n \beta_n^{(t-1)} \|\boldsymbol{w}_n^*\|_2^2$
6. Set $\bar{\tau} = \bar{\tau} + \epsilon_\tau$, $i = i + 1$.

until $\tau \geq \bar{\tau}^{\max}$;

7. Find index i^* yielding the minimum $\boldsymbol{\xi}$

Output: $\bar{\tau}^* = \bar{\tau}^{\min} + (i^* - 1)\epsilon_\tau$, $\boldsymbol{\beta}_{i^*}$, $\bar{\boldsymbol{W}}_{i^*}$, $\boldsymbol{p}_{i^*}^{\text{u}}$

Finally, in iteration t of the algorithm, we solve the following optimization problem:

$$\underset{\substack{\boldsymbol{V}_1, \boldsymbol{V}_2, \boldsymbol{V}_3 \\ \boldsymbol{\beta}, \boldsymbol{p}^{\text{u}}}}{\text{minimize}} \quad \bar{\tau} \sum_{n=1}^{3} \text{Tr}\{\boldsymbol{V}_n\} \tag{17a}$$

$$\text{subject to} \quad \boldsymbol{V}_n \in \mathcal{S}_+, \ \forall n \tag{17b}$$

$$(6\text{d}), (11), (14), (16)$$

Optimization problem (17) is convex and, hence, can be solved via a convex solver, such as CVX [14]. Furthermore, it can be shown that the solution of (17) satisfies[3] $\text{rank}\{\boldsymbol{V}_n^*\} \leq 1, \forall n$. Hence, as solution of (13), we obtain $\beta_n^* = \beta_n^{(t)}$ and $\boldsymbol{w}_n^* = \lambda_n \hat{\boldsymbol{w}}_n^*$, where λ_n and $\hat{\boldsymbol{w}}_n^*$ are the non-zero eigenvalue and the corresponding eigenvector of $\boldsymbol{W}_n^* = \boldsymbol{V}_n^{(t)} / \beta_n^{(t)}$, $\forall n$, respectively. The proposed algorithm[4] is summarized in Algorithm 1. Since the channel gains $\boldsymbol{h}_k$, the required rates R_k^{req}, and the initial energies $q_k, \forall k$, are known to the BS, problem (6) can be solved at the BS with Algorithm 1.

4. NUMERICAL RESULTS

In this section, we evaluate the performance of the proposed resource allocation scheme via simulations. In order to enable reliable communication, we assume that the BS and each user device have a line-of-sight link. The noise variance is set to $\sigma^2 = -110\,\text{dBm}$. We compute the path losses as $\left(\frac{c_l}{4\pi d_k f_c}\right)^2$, where $f_c = 868\,\text{MHz}$, and c_l and $d_k = 10\,\text{m}, k \in \{1, 2\}$, are the speed of light and the distance between the BS and user device k, respectively. Furthermore, the channel gains $\boldsymbol{h}_k$ follow Ricean distributions with Ricean factor 1. For the EH model in (1), we adopt parameter values $\mu = 1.85, \nu = 2.2 \cdot 10^3, \lambda = 2.5 \cdot 10^{-7}, A_s^2 = 2 \cdot 10^{-4}$ as in [9]. For Algorithm 1, we adopt error tolerances $\epsilon_{\text{SCA}} = 10^{-4}$ and $\epsilon_\tau = 0.1$ and the initial energy for each user is set to $q_k = 0\,\text{J}$. All simulation results are averaged over 100 channel realizations.

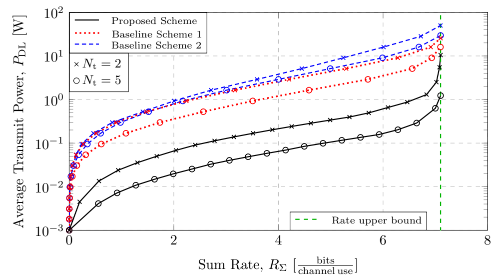

Fig. 2. Average transmit powers for different required rates and numbers of BS antennas.

In Fig. 2, we show the average transmit power P_DL as a function of sum rate R_Σ, where $R_\Sigma = \sum_k R_k^{\text{req}}$ and $R_1^{\text{req}} = R_2^{\text{req}}$. As Baseline Scheme 1 and Baseline Scheme 2, we adopt the WPCN designs in [4] and [3], which are based on the sigmoidal and linear EH models, respectively. Since time fraction $\bar{\tau}$ and covariance matrix $\tilde{\boldsymbol{X}}$ were optimized in [3] and [4], for the baseline schemes, in the downlink, we adopt $\tilde{N} = \text{rank}\{\tilde{\boldsymbol{X}}\}$ symbol vectors with $\tau_n = \bar{\tau}/\tilde{N}, n \in \{1, 2, \cdots, \tilde{N}\}$, and obtain these vectors from the dominant eigenvectors of $\tilde{\boldsymbol{X}}$. First, we observe that for each considered system setup, the sum rates are bounded from above (indicated by the dashed green line) since the EH circuits are driven into saturation for high transmit power levels. Next, we note that for given N_t and R_Σ, the proposed scheme requires a significantly lower transmit power than the baseline schemes. This is due to the more accurate modelling of the EH circuits which enables the optimization of the instantaneous powers harvested at the user devices. Finally, since a higher number of BS antennas leads to beamforming gain and channel hardening, we observe a better system performance in this case.

5. CONCLUSIONS

We considered two-user MISO WPCNs with non-linear EH circuits at the user devices, where the downlink and uplink transmission phases were utilized for power and information transfer, respectively. We formulated an optimization problem for the minimization of the transmit power in the downlink with per-user rate constraints in the uplink. We provided conditions for the feasibility of the formulated problem and the existence of a trivial solution, respectively. Next, for the case when the problem is feasible and the solution is non-trivial, we proved that three beamforming vectors are sufficient for optimal power transfer in the downlink. We obtained suboptimal solutions for these vectors via SDR and SCA. Our simulation results revealed that the proposed WPCN design outperforms baseline schemes based on linear and sigmoidal EH models.

[3]The corresponding proof is available in a longer version of this paper [13] and is omitted here due to the space constraints.

[4]We note that the extension of Algorithm 1 to the multi-user case is possible.

6. REFERENCES

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