SEMIDEFINITE RELAXATION METHOD FOR MOVING OBJECT LOCALIZATION USING A STATIONARY TRANSMITTER AT UNKNOWN POSITION

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ABSTRACT

This paper addresses the multistatic localization of a moving object in position and velocity using time delay (TD) and Doppler frequency shift (DFS) measurements, where the position of the transmitter is unknown and has not yet been synchronized with the receivers. Based on the TD and DFS measurements from the direct and indirect paths, we formulate a non-convex weighted least squares (WLS) minimization problem, and then apply semidefinite relaxation (SDR) to relax the WLS problem into a convex semidefinite program. Simulation results show that the proposed SDR method is able to achieve the Cramer-Rao lower bound accuracy under mild Gaussian noise condition and outperforms the existing method.

Index Terms— Time delay, Doppler frequency shift, unknown transmitter position, unknown offset, semidefinite relaxation.

1. INTRODUCTION

Object localization has been widely applied in both military and civilian fields, and thereby has attracted much interests and attention. To accomplish the task of localization, several kinds of measurements that contain the position information must be collected. Common measurements for localization include time-of-arrival (TOA) [1–4], time-difference-of-arrival (TDOA) [5–10], angle-of-arrival (AOA) [11–13], and received-signal-strength (RSS) [14–17].

We focus on the multistatistic localization problem in this work. Multistatic localization belongs to the paradigm of active positioning, which requires a transmitter to send a signal. The signal reaches the object to be located and its echo is then observed at the receivers, generating the time delay (TD) measurements. When the transmitter position is known, the object position can be determined by finding the intersection point of the ellipses defined by the TDs with the transmitter and receiver positions as the foci. Various methods have been proposed to estimate the object position in this setting [18–20].

In practice, the transmitter position may not be available, especially in harsh environments. For example, the transmitter may be located somewhere in which the GPS is not accessible. It could also happen in the passive coherent location (PCL) systems [21, 22], where an existing facility is used as the transmitter and which facility the signal comes from for positioning is not known. In such a case, although the TD information from the transmitter to the receiver directly is not related to the object, it contains the information about the transmitter position and can also be used for the purpose of jointly estimating the object and transmitter positions. Recently, several methods have been developed for this estimation problem [23, 24]. With relative motion between the object and the sensors (consisting of the transmitter and receivers), the Doppler frequency shift (DFS) measurements can be incorporated to the joint estimation process for locating the object and transmitter. Different two-stage weighted least squares (TSWLS) methods were proposed [25, 26] to handle different cases, where the transmitter can be moving or stationary.

In this paper, we consider the localization scenario of a moving object and a stationary transmitter (MOST) at an unknown position, where the measurements are TD and DFS. The transmitter and receivers are not synchronized and have carrier frequency mismatch as in most practical scenarios. As a result, unknown time and frequency offsets are present in the measurements. We propose a semidefinite relaxation (SDR) method that can improve the performance of the object position and velocity estimation as compared to the TSWLS method in [26]. We first formulate a weighted least squares (WLS) minimization problem to jointly estimate the object position, object velocity, transmitter position and the time and frequency offsets. The WLS problem is a nonlinear constrained optimization problem, which is non-convex and difficult to solve. We propose to relax it into a convex semidefinite program (SDP) by applying SDR. Simulation results show that the relaxed SDP problem is tight such that it globally solves the original WLS problem when the noise is not too large. The proposed SDR method is new. Although the MOST localization scenario has been considered in [25], it does not use SDP to solve the problem. Moreover, compared to [24] that studied only the stationary object and transmitter scenario, the proposed method addresses a moving object situation and a more practical case in which the transmitter and receivers are not synchronized.

We use bold lowercase and uppercase letters to represent column vectors matrices, respectively. $a_{(i)}$, $\boldsymbol{a}_{(i:j)}$, $A_{(i,j)}$, and $\boldsymbol{A}_{(i:j,k:l)}$ are the i-th element of $\boldsymbol{a}$, the subvector containing the i-th to the j-th element of $\boldsymbol{a}$, the (i, j)-th element of $\boldsymbol{A}$, and the submatrix containing the elements from the i-th to the j-th row and the k-th to the l-th column of $\boldsymbol{A}$, respectively. $\boldsymbol{\rho}_{\boldsymbol{a}} = \boldsymbol{a}/\|\boldsymbol{a}\|$ is the unit length vector of $\boldsymbol{a}$. $\mathrm{tr}\{\boldsymbol{A}\}$ and $\mathrm{rank}(\boldsymbol{A})$ are the trace and rank of $\boldsymbol{A}$, respectively. $\mathrm{diag}(\boldsymbol{a})$ is the diagonal matrix formed by having the elements of $\boldsymbol{a}$ on the diagonal. $\mathrm{blkdiag}(\boldsymbol{A}, \boldsymbol{B})$ denotes the block-diagonal matrix with $\boldsymbol{A}$ and $\boldsymbol{B}$ on the diagonal. $\boldsymbol{1}_k$ and $\boldsymbol{0}_k$ are the all-one and all-zero vectors of length k. $\boldsymbol{I}_k$ and $\boldsymbol{O}_k$ are the $k \times k$ identity and zero matrices, respectively. $\otimes$ denotes the Kronecker product.

2. MEASUREMENT MODEL

Consider a moving object in a multistatic system that consists of one transmitter and N receivers. The receivers are placed at the

This work was supported by the Zhejiang Provincial Natural Science Funds for Distinguished Young Scholars under Grant LR20F010001.

known positions denoted by $\boldsymbol{s}_j^o \in \mathbb{R}^k$, for $j = 1, \ldots, N$, and the transmitter is located at $\boldsymbol{t}^o \in \mathbb{R}^k$, which is not known. The position and velocity of the moving object are denoted by $\boldsymbol{u}^o \in \mathbb{R}^k$ and $\dot{\boldsymbol{u}}^o \in \mathbb{R}^k$, respectively, and k is the space dimension. The moving object localization is accomplished by exploiting the TD and DFS measurements observed at the N receivers. Fig. 1 illustrates the localization scenario of MOST. The signal emitted by the transmitter can be reflected by the object and then picked up by the receivers, and can also be observed by the receivers directly. Using the signals of the two paths observed by the receivers, we are able to obtain two types of measurements. One type is the TD, and the other is the DFS measurement due to the motion of the object. Since the TD is equivalent to range after multiplying by the signal propagation speed, and the DFS is equivalent to the range rate after multiplying by the signal propagation speed and dividing by the carrier frequency, we use them interchangeably in the following.

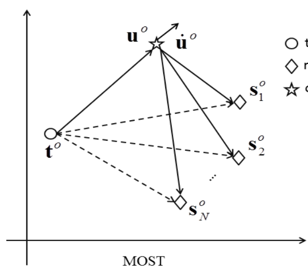

Fig. 1. Illustration of the localization scenario of MOST, solid lines represent the indirect paths and dashed lines denote the direct paths.

The true range in the indirect path is the sum of the range from the transmitter to the object and that from the object to the receiver, which can be expressed as [26]

$$r_j^o = \|\boldsymbol{u}^o - \boldsymbol{s}_j^o\| + \|\boldsymbol{u}^o - \boldsymbol{t}^o\| + b_\tau^o,\ j = 1, \ldots, N, \tag{1}$$

and the true range in the direct path is

$$d_j^o = \|\boldsymbol{s}_j^o - \boldsymbol{t}^o\| + b_\tau^o,\ j = 1, \ldots, N, \tag{2}$$

where b_τ^o is the unknown time offset due to the unsynchronized clocks between the transmitter and receivers.

In the MOST scenario, the DFS information comes from the motion of the object. The true range rates in the indirect and direct paths are [26]:

$$\begin{aligned} \dot{r_j}^o &= \boldsymbol{\rho}_{\boldsymbol{u}^o - \boldsymbol{s}_j^o}^T \dot{\boldsymbol{u}}^o + \boldsymbol{\rho}_{\boldsymbol{u}^o - \boldsymbol{t}^o}^T \dot{\boldsymbol{u}}^o + b_f^o,\ j = 1, \ldots, N, \\ \dot{d_j}^o &= b_f^o,\ j = 1, \ldots, N, \end{aligned} \tag{3}$$

where b_f^o is the unknown frequency offset.

In practice, the measurements observed by the receiver are contaminated by noise, i.e., for $j = 1, \ldots, N$,

$$\begin{aligned} r_j &= r_j^o + \varepsilon_{r,j},\ d_j = d_j^o + \varepsilon_{d,j}, \\ \dot{r}_j &= \dot{r}_j^o + \dot{\varepsilon}_{\dot{r},j},\ \dot{d}_j = \dot{d}_j^o + \dot{\varepsilon}_{\dot{d},j}, \end{aligned} \tag{4}$$

where $\varepsilon_{r,j}$, $\dot{\varepsilon}_{\dot{r},j}$, $\varepsilon_{d,j}$, and $\dot{\varepsilon}_{\dot{d},j}$ are the additive noise. For simplicity, we stack the noise into vectors $\boldsymbol{\varepsilon_r} = [\varepsilon_{r,1}, \varepsilon_{r,2}, \ldots, \varepsilon_{r,N}]^T$, $\dot{\boldsymbol{\varepsilon}}_{\dot{\boldsymbol{r}}} = [\dot{\varepsilon}_{\dot{r},1}, \dot{\varepsilon}_{\dot{r},2}, \ldots, \dot{\varepsilon}_{\dot{r},N}]^T$, $\boldsymbol{\varepsilon_d} = [\varepsilon_{d,1}, \varepsilon_{d,2}, \ldots, \varepsilon_{d,N}]^T$ and $\dot{\boldsymbol{\varepsilon}}_{\dot{\boldsymbol{d}}} = [\dot{\varepsilon}_{\dot{d},1}, \dot{\varepsilon}_{\dot{d},2}, \ldots, \dot{\varepsilon}_{\dot{d},N}]^T$, respectively. Furthermore, we define the composite vector $\boldsymbol{\varepsilon}$ as $\boldsymbol{\varepsilon} = [\boldsymbol{\varepsilon}_{\boldsymbol{r}}^T, \dot{\boldsymbol{\varepsilon}}_{\dot{\boldsymbol{r}}}^T, \boldsymbol{\varepsilon}_{\boldsymbol{d}}^T, \dot{\boldsymbol{\varepsilon}}_{\dot{\boldsymbol{d}}}^T]^T$. Without loss of generality, we assume that $\boldsymbol{\varepsilon}$ follows a Gaussian distribution with zero mean and covariance matrix $\boldsymbol{Q}$.

3. SEMIDEFINITE RELAXATION FOR MOST LOCALIZATION

We will jointly estimate the object position and velocity, the transmitter position, and the time and frequency offsets by using the range and range rate measurements from both the direct and indirect paths. Starting from the model in (4), we will formulate a WLS minimization problem, and then relax it as an SDP.

In the indirect path model (1), let $\xi^o = \|\boldsymbol{u}^o - \boldsymbol{t}^o\|$. Moving ξ^o and b_τ^o to the left-hand side and squaring both sides yield

$$\begin{aligned} &\frac{1}{2}(r_j^{o2} - \|\boldsymbol{s}_j^o\|^2) + \boldsymbol{s}_j^{oT}\boldsymbol{u}^o - \frac{1}{2}\|\boldsymbol{u}^o\|^2 - r_j^o\xi^o - r_j^o b_\tau^o \\ &+ \frac{1}{2}\xi^{o2} + \xi^o b_\tau^o + \frac{1}{2}b_\tau^{o2} = 0. \end{aligned} \tag{5}$$

The true value r_j^o is not available. In terms of the measurements by substituting $r_j^o = r_j - \varepsilon_{r,j}$ in (5) and neglecting the second-order noise terms $\varepsilon_{r,j}^2$, we obtain

$$\frac{1}{2}(r_j^2 - \|\boldsymbol{s}_j^o\|^2) + \boldsymbol{s}_j^{oT}\boldsymbol{u}^o - r_j\xi^o - r_j b_\tau^o - \frac{1}{2}\varphi_1^o \simeq \|\boldsymbol{u}^o - \boldsymbol{s}_j^o\|\varepsilon_{r,j}, \tag{6}$$

where $\varphi_1^o = \|\boldsymbol{u}^o\|^2 - \xi^{o2} - 2\xi^o b_\tau^o - b_\tau^{o2}$.

Eq. (6) is the indirect path TD equation that we will use for the proposed SDP method. The equation for range rate is obtained based on the time-derivative of (6) and incorporating the frequency offset,

$$\begin{aligned} &r_j\dot{r}_j + \boldsymbol{s}_j^{oT}\dot{\boldsymbol{u}}^o - \dot{r}_j\xi^o - r_j\dot{\xi}^o - \dot{r}_j b_\tau^o - r_j b_f^o - \varphi_2^o \\ &\simeq \boldsymbol{\rho}_{\boldsymbol{u}^o - \boldsymbol{s}_j^o}^T \dot{\boldsymbol{u}}^o \varepsilon_{r,j} + \|\boldsymbol{u}^o - \boldsymbol{s}_j^o\|\dot{\varepsilon}_{\dot{r},j}, \end{aligned} \tag{7}$$

where $\dot{\xi}^o = \boldsymbol{\rho}_{\boldsymbol{u}^o - \boldsymbol{t}^o}^T \dot{\boldsymbol{u}}^o$ and $\varphi_2^o = \boldsymbol{u}^{oT}\dot{\boldsymbol{u}}^o - \xi^o\dot{\xi}^o - \dot{\xi}^o b_\tau^o - \xi^o b_f^o - b_\tau^o b_f^o$.

For the direct path model, the range model (2) can be manipulated similarly and approximated by

$$\frac{1}{2}(d_j^2 - \|\boldsymbol{s}_j^o\|^2) + \boldsymbol{s}_j^{oT}\boldsymbol{t}^o - d_j b_\tau^o - \frac{1}{2}\varphi_3^o \simeq \|\boldsymbol{t}^o - \boldsymbol{s}_j^o\|\varepsilon_{d,j}, \tag{8}$$

where $\varphi_3^o = \|\boldsymbol{t}^o\|^2 - b_\tau^{o2}$. Owing to the fact that both the transmitter and receivers are stationary, there is no Doppler effect in the direct path. The frequency offset, however, is present in the DFS observations [26]:

$$\dot{d}_j = b_f^o + \dot{\varepsilon}_{\dot{d},j},\ j = 1, \ldots, N. \tag{9}$$

To ease the following derivations, we define the unknown vector $\boldsymbol{y}^o$:

$$\boldsymbol{y}^o = [\underbrace{\boldsymbol{u}^{oT}}_{1\times k}, \underbrace{\dot{\boldsymbol{u}}^{oT}}_{1\times k}, \underbrace{\boldsymbol{t}^{oT}}_{1\times k}, \underbrace{b_\tau^o}_{1}, \underbrace{b_f^o}_{1}, \underbrace{\xi^o}_{1}, \underbrace{\dot{\xi}^o}_{1}, \underbrace{\varphi_1^o}_{1}, \underbrace{\varphi_2^o}_{1}, \underbrace{\varphi_3^o}_{1}]^T.$$

Using $\boldsymbol{y}^o$, we can express the pseudo-linear equations (6)-(9) as the following matrix forms:

$$\boldsymbol{b_r} - \boldsymbol{A_r}\boldsymbol{y}^o \simeq \boldsymbol{B_r}\boldsymbol{\varepsilon_r}, \tag{10a}$$

$$\boldsymbol{b_{\dot{r}}} - \boldsymbol{A_{\dot{r}}}\boldsymbol{y}^o \simeq \boldsymbol{B_{\dot{r}}}\boldsymbol{\varepsilon_r} + \boldsymbol{B_r}\dot{\boldsymbol{\varepsilon}}_{\dot{\boldsymbol{r}}}, \tag{10b}$$

$$\boldsymbol{b_d} - \boldsymbol{A_d}\boldsymbol{y}^o \simeq \boldsymbol{B_d}\boldsymbol{\varepsilon_d}, \tag{10c}$$

$$\boldsymbol{b_{\dot{d}}} - \boldsymbol{A_{\dot{d}}}\boldsymbol{y}^o = \dot{\boldsymbol{\varepsilon}}_{\dot{\boldsymbol{d}}}, \tag{10d}$$

where

$$\begin{aligned} \boldsymbol{B_r} &= \mathrm{diag}(\|\boldsymbol{u}^o - \boldsymbol{s}_1^o\|, \|\boldsymbol{u}^o - \boldsymbol{s}_2^o\|, \ldots, \|\boldsymbol{u}^o - \boldsymbol{s}_N^o\|), \\ \boldsymbol{B_{\dot{r}}} &= \mathrm{diag}(\boldsymbol{\rho}_{\boldsymbol{u}^o - \boldsymbol{s}_1^o}^T\dot{\boldsymbol{u}}^o, \boldsymbol{\rho}_{\boldsymbol{u}^o - \boldsymbol{s}_2^o}^T\dot{\boldsymbol{u}}^o, \ldots, \boldsymbol{\rho}_{\boldsymbol{u}^o - \boldsymbol{s}_N^o}^T\dot{\boldsymbol{u}}^o), \end{aligned}$$

$$\boldsymbol{B_d} = \mathrm{diag}(\|\boldsymbol{t}^o - \boldsymbol{s}_1^o\|, \|\boldsymbol{t}^o - \boldsymbol{s}_2^o\|, \ldots, \|\boldsymbol{t}^o - \boldsymbol{s}_N^o\|),$$
$$\boldsymbol{b_r} = \frac{1}{2}\left[(r_1^2 - \|\boldsymbol{s}_1^o\|^2), \ldots, (r_N^2 - \|\boldsymbol{s}_N^o\|^2)\right]^T,$$
$$\boldsymbol{b_d} = \frac{1}{2}\left[(d_1^2 - \|\boldsymbol{s}_1^o\|^2), \ldots, (d_N^2 - \|\boldsymbol{s}_N^o\|^2)\right]^T,$$
$$\boldsymbol{b_{\dot r}} = [r_1\dot r_1, r_2\dot r_2, \ldots, r_N\dot r_N]^T, \boldsymbol{b_{\dot d}} = [\dot d_1, \dot d_2, \ldots, \dot d_N]^T, \quad (11)$$

and

$$\boldsymbol{A_r} = [\boldsymbol{a}_{r_1}, \boldsymbol{a}_{r_2}, \ldots, \boldsymbol{a}_{r_N}]^T, \boldsymbol{A_{\dot r}} = [\boldsymbol{a}_{\dot r_1}, \boldsymbol{a}_{\dot r_2}, \ldots, \boldsymbol{a}_{\dot r_N}]^T,$$
$$\boldsymbol{A_d} = [\boldsymbol{a}_{d_1}, \boldsymbol{a}_{d_2}, \ldots, \boldsymbol{a}_{d_N}]^T, \boldsymbol{A_{\dot d}} = [\boldsymbol{a}_{\dot d_1}, \boldsymbol{a}_{\dot d_2}, \ldots, \boldsymbol{a}_{\dot d_N}]^T, \quad (12)$$

with

$$\boldsymbol{a}_{r_j} = [-\boldsymbol{s}_j^{oT}, \boldsymbol{0}_k^T, \boldsymbol{0}_k^T, r_j, 0, r_j, 0, \frac{1}{2}, \boldsymbol{0}_2^T]^T,$$
$$\boldsymbol{a}_{\dot r_j} = [\boldsymbol{0}_k^T, -\boldsymbol{s}_j^{oT}, \boldsymbol{0}_k^T, \dot r_j, r_j, \dot r_j, r_j, 0, 1, 0]^T,$$
$$\boldsymbol{a}_{d_j} = [\boldsymbol{0}_k^T, \boldsymbol{0}_k^T, -\boldsymbol{s}_j^{oT}, d_j, 0, \boldsymbol{0}_4^T, \frac{1}{2}]^T,$$
$$\boldsymbol{a}_{\dot d_j} = [\boldsymbol{0}_k^T, \boldsymbol{0}_k^T, \boldsymbol{0}_k^T, 0, 1, \boldsymbol{0}_5^T]^T. \quad (13)$$

Combining the approximate equations in (10a)-(10d) yields

$$\boldsymbol{b} - \boldsymbol{A}\boldsymbol{y}^o \simeq \boldsymbol{B}\boldsymbol{\varepsilon}, \quad (14)$$

where

$$\boldsymbol{B} = \mathrm{blkdiag}\left(\begin{bmatrix}\boldsymbol{B_r} & \boldsymbol{O}_N \\ \boldsymbol{B_{\dot r}} & \boldsymbol{B_r}\end{bmatrix}, \begin{bmatrix}\boldsymbol{B_d} & \boldsymbol{O}_N \\ \boldsymbol{O}_N & \boldsymbol{I}_N\end{bmatrix}\right),$$
$$\boldsymbol{b} = [\boldsymbol{b}_{\boldsymbol{r}}^T, \boldsymbol{b}_{\boldsymbol{\dot r}}^T, \boldsymbol{b}_{\boldsymbol{d}}^T, \boldsymbol{b}_{\boldsymbol{\dot d}}^T]^T,\ \boldsymbol{A} = [\boldsymbol{A}_{\boldsymbol{r}}^T, \boldsymbol{A}_{\boldsymbol{\dot r}}^T, \boldsymbol{A}_{\boldsymbol{d}}^T, \boldsymbol{A}_{\boldsymbol{\dot d}}^T]^T. \quad (15)$$

Based on (14), we can formulate a WLS problem to estimate $\boldsymbol{y}^o$:

$$\min_{\boldsymbol{y}}\ (\boldsymbol{b} - \boldsymbol{A}\boldsymbol{y})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{b} - \boldsymbol{A}\boldsymbol{y}) \quad (16a)$$
$$\text{s.t. } y_{(3k+3)}^2 = \|\boldsymbol{y}_{(1:k)} - \boldsymbol{y}_{((2k+1):3k)}\|^2, \quad (16b)$$
$$y_{(3k+4)} = \frac{(\boldsymbol{y}_{(1:k)} - \boldsymbol{y}_{((2k+1):3k)})^T}{\|\boldsymbol{y}_{(1:k)} - \boldsymbol{y}_{((2k+1):3k)}\|}\boldsymbol{y}_{((k+1):2k)}, \quad (16c)$$
$$y_{(3k+3)}y_{(3k+4)} = (\boldsymbol{y}_{(1:k)} - \boldsymbol{y}_{((2k+1):3k)})^T\boldsymbol{y}_{((k+1):2k)}, \quad (16d)$$
$$y_{(3k+5)} = \|\boldsymbol{y}_{(1:k)}\|^2 - (y_{(3k+3)} + y_{(3k+1)})^2, \quad (16e)$$
$$y_{(3k+6)} = -(y_{(3k+1)} + y_{(3k+3)})(y_{(3k+2)} + y_{(3k+4)}) + \boldsymbol{y}_{(1:k)}^T\boldsymbol{y}_{(k+1:2k)}, \quad (16f)$$
$$y_{(3k+7)} = \|\boldsymbol{y}_{((2k+1):3k)}\|^2 - y_{(3k+1)}^2, \quad (16g)$$

where $\boldsymbol{y} = [\boldsymbol{u}^T, \dot{\boldsymbol{u}}^T, \boldsymbol{t}^T, b_\tau, b_f, \xi, \dot\xi, \varphi_1, \varphi_2, \varphi_3]^T$ is the optimization variable corresponding to $\boldsymbol{y}^o$ and $\boldsymbol{\Sigma} = \boldsymbol{BQB}^T$. The relationships among the elements of $\boldsymbol{y}$ have been written into the constraints in (16). It is clear that problem (16) is non-convex, implying that the globally optimal solution is difficult to obtain. To address this difficulty, we further transform the problem by introducing an auxiliary variable $\boldsymbol{Y} = \boldsymbol{y}\boldsymbol{y}^T$ and then apply SDR to relax it into a convex SDP problem.

By using the auxiliary variable $\boldsymbol{Y}$, the objective function in problem (16) can be written as

$$(\boldsymbol{b} - \boldsymbol{A}\boldsymbol{y})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{b} - \boldsymbol{A}\boldsymbol{y}) = \mathrm{tr}\left\{\begin{bmatrix}\boldsymbol{Y} & \boldsymbol{y} \\ \boldsymbol{y}^T & 1\end{bmatrix}\boldsymbol{\Phi}\right\}, \quad (17)$$

where $\boldsymbol{\Phi} = \begin{bmatrix}\boldsymbol{A}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{A} & -\boldsymbol{A}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{b} \\ -\boldsymbol{b}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{A} & \boldsymbol{b}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{b}\end{bmatrix}$.

Similarly, the constraints (16b) and (16d)-(16g) can be expressed in terms of the elements in $\boldsymbol{Y}$ as

$$Y_{(3k+3,3k+3)} = \mathrm{tr}\{\boldsymbol{Y}_{(1:k,1:k)}\} - 2\mathrm{tr}\{\boldsymbol{Y}_{(1:k,(2k+1):3k)}\} + \mathrm{tr}\{\boldsymbol{Y}_{((2k+1):3k,(2k+1):3k)}\}, \quad (18a)$$
$$Y_{(3k+3,3k+4)} = \mathrm{tr}\{\boldsymbol{Y}_{(1:k,(k+1):2k)}\} - \mathrm{tr}\{\boldsymbol{Y}_{((2k+1):3k,(k+1):2k)}\}, \quad (18b)$$
$$y_{(3k+5)} = \mathrm{tr}\{\boldsymbol{Y}_{(1:k,1:k)}\} - Y_{(3k+3,3k+3)} - 2Y_{(3k+1,3k+3)} - Y_{(3k+1,3k+1)}, \quad (18c)$$
$$y_{(3k+6)} = \mathrm{tr}\{\boldsymbol{Y}_{(1:k,k+1:2k)}\} - Y_{(3k+3,3k+4)} - Y_{(3k+1,3k+4)} - Y_{(3k+3,3k+2)} - Y_{(3k+1,3k+2)}, \quad (18d)$$
$$y_{(3k+7)} = \mathrm{tr}\{\boldsymbol{Y}_{(2k+1:3k,2k+1:3k)}\} - Y_{(3k+1,3k+1)}, \quad (18e)$$

In addition, using the Cauchy-Schwartz inequality in (16c) gives

$$y_{(3k+4)} = \dot\xi = \boldsymbol{\rho}_{\boldsymbol{u}-\boldsymbol{t}}^T\dot{\boldsymbol{u}} \le \|\dot{\boldsymbol{u}}\|. \quad (19)$$

By squaring both sides of (19) and writing it in terms of the elements of $\boldsymbol{Y}$ lead to the following constraint

$$Y_{(3k+4,3k+4)} \le \mathrm{tr}\{\boldsymbol{Y}_{((k+1):2k,(k+1):2k)}\}. \quad (20)$$

Now, problem (16) can be equivalently written as the following problem:

$$\min_{\boldsymbol{Y},\boldsymbol{y}}\ \mathrm{tr}\left\{\begin{bmatrix}\boldsymbol{Y} & \boldsymbol{y} \\ \boldsymbol{y}^T & 1\end{bmatrix}\boldsymbol{\Phi}\right\}$$
$$\text{s.t. } \boldsymbol{Y} = \boldsymbol{y}\boldsymbol{y}^T,\ \text{(18a)-(18e), (20)}. \quad (21)$$

Problem (21) is ready for performing SDR. Before that, we first invoke the following equivalence for the constraint $\boldsymbol{Y} = \boldsymbol{y}\boldsymbol{y}^T$:

$$\boldsymbol{Y} = \boldsymbol{y}\boldsymbol{y}^T \Leftrightarrow \begin{bmatrix}\boldsymbol{Y} & \boldsymbol{y} \\ \boldsymbol{y}^T & 1\end{bmatrix} \succeq \boldsymbol{0},\ \mathrm{rank}\left(\begin{bmatrix}\boldsymbol{Y} & \boldsymbol{y} \\ \boldsymbol{y}^T & 1\end{bmatrix}\right) = 1. \quad (22)$$

By dropping the non-convex rank-1 constraint, problem (21) can be relaxed to the following convex SDP:

$$\min_{\boldsymbol{y}}\ \mathrm{tr}\left\{\begin{bmatrix}\boldsymbol{Y} & \boldsymbol{y} \\ \boldsymbol{y}^T & 1\end{bmatrix}\boldsymbol{\Phi}\right\}$$
$$\text{s.t. (18a)-(18e), (20), } \begin{bmatrix}\boldsymbol{Y} & \boldsymbol{y} \\ \boldsymbol{y}^T & 1\end{bmatrix} \succeq \boldsymbol{0}. \quad (23)$$

After solving (23), the estimates of the object position and velocity and the transmitter position can be extracted through

$$\hat{\boldsymbol{u}} = \boldsymbol{y}_{(1:k)},\ \hat{\dot{\boldsymbol{u}}} = \boldsymbol{y}_{((k+1):2k)},\ \hat{\boldsymbol{t}} = \boldsymbol{y}_{((2k+1):3k)}. \quad (24)$$

Solving the above SDP problem requires us to know $\boldsymbol{\Sigma}$, which is not available since it depends on the unknown position and velocity of the object and transmitter position. To handle this issue, we shall first set $\boldsymbol{B}$ as the identity matrix and solve problem (23) to obtain initial estimates of the object position, object velocity, and transmitter position. Using the initial estimates, we form an approximate weighting matrix $\boldsymbol{\Sigma}$ and solve problem (23) again to produce the final estimates. Such an approximation is commonly applied in the literature and the performance degradation caused by the approximation is minor [25–27].

4. SIMULATIONS

In this section, we evaluate the performance of the proposed method by performing Monte Carlo (MC) simulations for 3-D localization. To highlight the superior performance of the proposed method, we include the performance of the TSWLS method in [26] for comparison. Moreover, we use the CRLB and Maximum-Likelihood Estimator (MLE) as performance benchmarks. The MLE is solved by the MATLAB function “fminunc”, having the true values of the unknown parameters as the initial guess for iteration. The SDP problem (23) is solved by the MATLAB toolbox CVX [28] with SeDuMi [29] as the solver.

The mean square error (MSE) is employed to evaluate the localization performance of the proposed method, defined by $\mathrm{MSE}(\hat{\zeta}) = \frac{1}{KL}\sum_{j=1}^{K}\sum_{i=1}^{L}\|\hat{\zeta}_{ji} - \zeta_j^o\|^2$, where $\hat{\zeta}_{ji}$ is the estimate of the true value ζ_j^o in the i-th MC run for the j-th configuration. K and L are the numbers of configurations and MC runs, respectively. In this paper, 10 configurations are randomly selected, and for each configuration, we perform 1000 MC runs to compute the MSE, i.e., $K = 10$ and $L = 1000$. The CRLB shown in each figure is the average of the CRLBs for the generated ten configurations. Moreover, we assume that the noise terms in (4) are uncorrelated such that $\boldsymbol{Q}$ can be written as $\boldsymbol{Q} = \mathrm{blkdiag}(\boldsymbol{Q_r}, \boldsymbol{Q_{\dot{r}}}, \boldsymbol{Q_d}, \boldsymbol{Q_{\dot{d}}})$. The settings of $\boldsymbol{Q_r}$ and $\boldsymbol{Q_d}$ are the same as those in [24]. The covariance matrices for the DFS measurements are set to $\boldsymbol{Q_{\dot{r}}} = \mu\boldsymbol{Q_r}$ and $\boldsymbol{Q_{\dot{d}}} = \mu\boldsymbol{Q_d}$, where $\mu = 0.1$ [30].

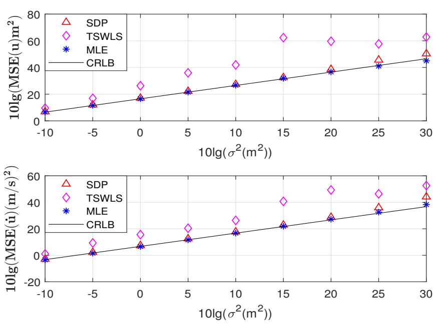

Fig. 2. MSE comparison for 3-D localization: ten randomly generated configurations each with one transmitter and five receivers.

Table 1. Proportion of Rank-1 Solutions (10000 MC Runs in Total for Each Value of σ^2)

$10\lg(\sigma^2)$	-10	-5	0	5	10	15	20	25	30
Prop.(%)	100	100	100	99.2	96.7	93.4	87.8	77.8	67.7

Scenario 1: In this scenario, one transmitter and five receivers are used, whose positions are randomly selected in the 3-D space of size $(-4000, 4000) \times (-4000, 4000) \times (1000, 3000)$ m^3. We fix the object position and velocity at $\boldsymbol{u}^o = [-1000, 500, 1500]^T$ m and $\dot{\boldsymbol{u}}^o = [15, 15, 30]^T$ m/s, respectively. The unknown time and frequency offsets are set to $b_\tau^o = 200$ m and $b_f^o = 5$ m/s, respectively. Fig. 2 shows the simulation results. The proposed SDR method achieves the CRLB accuracy at low noise levels, and performs slightly worse than MLE at high noise levels. By contrast, the TSWLS method performs poorly, even when the noise is small.

Fig. 3 illustrates the results when six receivers are used. The performance of the TSWLS method improves considerably by adding

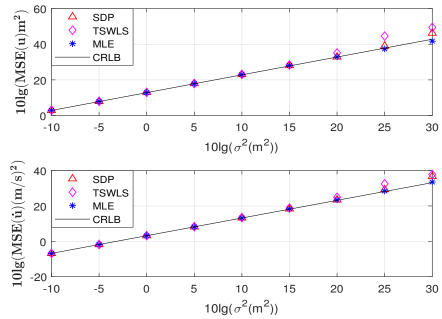

Fig. 3. MSE comparison for 3-D localization: ten randomly generated configurations each with one transmitter and six receivers.

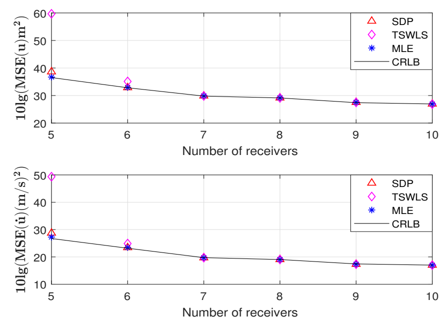

Fig. 4. MSE comparison for 3-D localization at $\sigma = 10$ m as the number of receivers varies: ten randomly generated configurations.

one more receiver. However, the proposed SDR method still outperforms the TSWLS method at large noise levels. In addition, we list in Table 1 the proportion of rank-1[1] solutions of the proposed method in this simulation. As can be seen, the solutions in all the MC runs for the proposed SDP method are of rank 1 when the noise is relatively small, and even when the noise power is as high as $\sigma^2 = 1000$ m^2, 67.7% of them are rank-1 solutions.

Scenario 2: In this scenario, we aim to test the performance of the proposed method as the number of receivers varies from 5 to 10. The unknown parameters are set the same as in Scenario 1. Fig. 4 shows the MSE of the object position and velocity estimation with the noise level fixed at $\sigma = 10$ m. Clearly, the estimation accuracy improves as the number of receivers increases. Moreover, the proposed method outperforms the TSWLS method when using fewer receivers.

5. CONCLUSION

In this paper, we have investigated the multistatic object localization problem in the MOST scenario using TD and DFS measurements, where only the receiver positions are available. We have formulated a WLS problem and solved it approximately using SDP by employing the SDR technique. Simulation results show that the proposed SDR method is able to achieve the CRLB accuracy for the estimation of object position and velocity under mild Gaussian noise.

[1]We regard that the SDP is of rank 1 if the ratio of the largest eigenvalue of $\boldsymbol{Y}$ to the second largest eigenvalue is greater than 10^5.

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