PHASE-ONLY RECONFIGURABLE SPARSE ARRAY BEAMFORMING USING DEEP LEARNING

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ABSTRACT

The paper considers phase-only reconfigurable sparse arrays (RSAs) for receive beamforming to maximize signal-to-interference plus noise ratio (MaxSINR). We develop a design approach based on supervised deep neural network (DNN) to learn and mimic a phase-only sparse MaxSINR beamformer. The proposed approach strives to match the SINR performance of data driven sparse Capon beamformer. The problem is posed as a multi-label classification problem, where the received antenna correlations is the input to the fully connected neural network (FCNN) which outputs the optimum sensor locations for effective interference mitigation. We evaluate the performance of DNN based optimization of RSAs in terms of the ability of the classified sparse array to mitigate interference and maximize signal power using phase-only beamforming. The phase-only DNN-based sparse sensor placement reduces hardware requirements, shifts optimization algorithm complexity to satisfying training data sufficiency, and is amenable to real-time implementation.

***Index Terms*—** Reconfigurable sparse arrays, DNN, Hybrid Beamforming, Analogue beamforming.

1. INTRODUCTION

Multi-sensor systems have become ubiquitous across many sensing modalities. Transmit/receive platforms are found in many applications ranging from radio astronomy arrays, modern radars, 5G and beyond wireless, indoor localization, spatial audio, and UAVs to miniaturized arrays for augmented reality (AR/VR) headsets and ultrasound arrays [1–6]. Non-uniform linear arrays can efficiently use hardware resources for improved system performance as compared to uniform sensor topologies. This is made possible by the additional degrees of freedom (DOF) that are provided by incorporating sensor locations as part of the design objective. Depending on the application and the performance criteria, array design can be either environment-independent or strongly guided by the operating environment. While the former results in an array structure that is invariant with the spatial spectrum [7, 8], the latter design varies as targets and emitters in the array field of view (FOV) change in locations and signal strengths [9, 10].

The environment-sensitive array design objectives, such as maximizing signal to interference plus noise ratio (MaxSINR), have recently become more realizable due to advances in efficient sensor switching technologies that readily activate a subset of sensors on predefined grid points. Hence, the system cost can significantly be reduced by limiting the number of expensive RF transceivers chains [11]. In this paper, we consider MaxSINR sparse array design for receive beamforming. The beamformer performance depends mainly on the selected positions of the sensors as well as the locations of the desired source and interferers in FOV [12–15]. It is important to note that with sparse arrays, MaxSINR beamformer must not only find the optimum beamformer weights, as commonly used in uniform arrays, but also the optimum array configuration [16, 17]. This is clearly an entwined optimization problem and requires attaining maximum SINR considering all possible sparse array configurations.

To further simplify the cost of RSAs for adaptive beamforming, it is preferable to control the beampattern characteristics by enforcing a unit modulus phase-only control instead of utilizing the optimum beamformer weights [18, 19]. Reconfigurable sparse arrays implemented through phase shifters is a key technology that aims at trading digital processing with antenna switching. This would not only reduce the cost of expensive transceiver hardware but also pay significant dividends in terms of developing solutions with extremely low latency. RSAs relying exclusively on analogue processing can lead to low-cost high resolution real-time source isolation, identification, and localization for spectrum monitoring and surveillance. The proposed analogue design employing a sparse configuration strives to outperform the performance of a fully digital system implementing a uniform linear array (ULA) which is commonly used for receive beamformer.

The sparse array configuration utilizes the same number of transceiver RF chains as used by the ULA. However, it would require the adjustment of sensor locations which is determined by the present environment and through the use of fast antenna switching technology. Several iterative algorithms have been proposed to optimize the sparse array beamformer design. Although, convex based optimization algorithms, such as semidefinite relaxation (SDR) and successive convex approximation (SCA) have been developed to yield sparse configurations with desirable beamforming performances [15], real time implementations of these algorithms remain limited due to the relatively high computation cost. The problem becomes more pronounced in rapidly changing environments which result from temporal and spatial non-stationary behaviors of the sources in the FOV.

In this paper, the DNN is trained to learn and mimic a data-driven phase-only sparse MaxSINR beamformer striving to match the SINR performance of optimum sparse Capon beamformer. In essence, DNN is used in the underlying problem to approximate the unknown mapping from the receiver data spatial correlations to the output sparse array configuration. Training data is generated by enumeration technique yielding the training labels for any given sensor correlation function. In this case, MaxSINR array configuration is obtained by sifting through all possible sparse configurations employing phase-only beamformer and choosing the best performing array topology. It is shown that DNN effectively learns the optimum phase-only sparse arrays which makes DNN, in requiring a few simple operations, suitable for real-time implementation.

The paper is structured as follows. In section 2, we state the problem formulation for maximizing the output SINR. Section 3 deals with the DNN based sparse array design. In section 4, with the aid of design examples, we demonstrate the usefulness of proposed algorithms in achieving MaxSINR sparse array design. Concluding remarks follow at the end.

2. PROBLEM FORMULATION

Consider a desired source and L independent interfering sources whose signals impinge on a linear array with N uniformly placed sensors. The baseband data received at the array at sampling instant n is then given by,

$$\mathbf{x}(n) = (\alpha(n))\mathbf{s}(\theta) + \sum_{l=1}^{L}(\beta_l(n))\mathbf{i}(\theta_l) + \mathbf{v}(n), \qquad (1)$$

where, $\mathbf{s}(\theta)$ and $\mathbf{i}(\theta_l) \in \mathbb{C}^N$ are the steering vectors corresponding to the direction of arrival, θ or θ_l of the desired source and lth interference, respectively, and are defined as follows,

$$\mathbf{s}(\theta) = [1 \;\; e^{j(2\pi/\lambda)dcos(\theta)} \ldots e^{j(2\pi/\lambda)d(N-1)cos(\theta)}]^T. \qquad (2)$$

where d is the inter-element spacing and $(\alpha(n), \beta_l(n)) \in \mathbb{C}$ are the complex amplitudes of the incoming baseband signals [20]. The additive Gaussian noise $\mathbf{v}(n) \in \mathbb{C}^N$ has variance σ_v^2. The elements of the received data vector $\mathbf{x}(n)$ are combined linearly by the N-sensor beamformer that strives to maximize the output SINR. The output signal $y(n)$ of the optimum beamformer for maximum SINR is given by [21],

$$y(n) = \mathbf{w}_o^H \mathbf{x}(n), \qquad (3)$$

where $\mathbf{w}_o$ is the optimum weight vector resulting in the optimum output SINR$_o$,

$$\text{SINR}_o = \frac{\mathbf{w}_o^H \mathbf{R}_s \mathbf{w}_o}{\mathbf{w}_o^H \mathbf{R}_{s'} \mathbf{w}_o}. \qquad (4)$$

For statistically independent signals, the desired source correlation matrix is $\mathbf{R}_s = \sigma^2 \mathbf{s}(\theta)\mathbf{s}^H(\theta)$, where $\sigma^2 = E\{\alpha(n)\alpha^H(n)\}$. Likewise, the interference and noise correlation matrix, $\mathbf{R}_{s'} = \sum_{l=1}^{L}(\sigma_l^2 \mathbf{i}(\theta_l)\mathbf{i}^H(\theta_l)) + \sigma_t^2 \mathbf{I}_{N\times N}$, with $\sigma_l^2 = E\{\beta_l(n)\beta_l^H(n)\}$ being the power of the lth interfering source. In order to maximize the SINR expression in (4), we constraint the numerator and minimize the denominator term as follows,

$$\begin{aligned} \underset{\mathbf{w}\in\mathbb{C}^N}{\text{minimize}} \quad & \mathbf{w}^H \mathbf{R}_{s'} \mathbf{w}, \\ \text{s.t.} \quad & \mathbf{w}^H \mathbf{R}_s \mathbf{w} = 1. \end{aligned} \qquad (5)$$

The problem in (5) can be written equivalently by replacing $\mathbf{R}_{s'}$ with the received data covariance matrix, $\mathbf{R_{xx}} = \mathbf{R}_s + \mathbf{R}_{s'}$ as follows [21],

$$\begin{aligned} \underset{\mathbf{w}\in\mathbb{C}^N}{\text{minimize}} \quad & \mathbf{w}^H \mathbf{R_{xx}} \mathbf{w}, \\ \text{s.t.} \quad & \mathbf{w}^H \mathbf{R}_s \mathbf{w} \geq 1. \end{aligned} \qquad (6)$$

To bring aperture sparsity and phase-only formulation into optimum beamformer design, the constraint optimization (6) can be re-formulated by incorporating sparsity enhancing l_1-norm and modulus constraint on the weight vector as follows;

$$\begin{aligned} \underset{\mathbf{w}\in\mathcal{U}^N}{\text{minimize}} \quad & \mathbf{w}^H \mathbf{R_{xx}} \mathbf{w}, \\ \text{s.t.} \quad & \mathbf{w}^H \mathbf{R}_s \mathbf{w} \geq 1, \\ & ||\mathbf{w}||_1 = P, \\ & \mathbf{w}(n) \in \{0, \mathcal{U}\} \quad \forall \; n. \end{aligned} \qquad (7)$$

The beamformer weights $\mathbf{w} \in \mathcal{U}^N$ are confined to $\mathcal{U}^N$, which is the space of unit-modulus complex values. Here, $||.||_1$ is the l_1 norm, which coupled with modulus constraint on each element of beamformer $\mathbf{w}(n) \in \{0, \mathcal{U}\}$, ensures phase-only beamforming with the cardinality of the weight vector $\mathbf{w}$ equal to the number of available receiver chains P. This is a combinatorial optimization problem and can be solved by enumerating over all possible sensor locations or employing data-dependent convex relaxation algorithms realized through the SDR and SCA algorithms [12, 15]. These algorithms, however, have high computational costs, impeding real time implementations, especially in applications involving rapidly changing environments.

3. DNN BASED PHASE-ONLY RSA

DNNs are universal approximators for arbitrary continuous functions and have the required capacity to accurately model the behaviour of optimization algorithms. For effective learning, it is important that the DNN can generalize to a broader class than that represented by the finite number of drawn training examples [10]. From the Capon beamforming perspective, a given arrangement of a desired source direction, interference DOAs and respective SNR/INRs constitute just one particular example. A class, in this case, is defined by any arbitrary permutation of the interference DOAs and respective powers while keeping the desired source DOA fixed. The DNN task is, therefore, to learn from a data-set, characterized by a set of different training examples and corresponding optimum sparse array predictions. For DNN based Capon beamforming formulation, it is imperative to incorporate the knowledge of the desired source which can either be incorporated by exclusively training the DNN for each desired source DOA or the desired source DOA can be incorporated as an additional input feature to DNN. In this paper, we adopt the former approach.

For DNN, we use a fully connected neural network (FCNN). Assuming a stationary environment, the input comprise of the correlation values of the received data. The received data is generated in the following manner. For a given desired source location, the ith training realization is simulated by randomly selecting L interfering signals from a DOA grid spanning the range of 0^0 to 180^0. The interferers are allocated random powers uniformly distributed with INR from 0 dB to 20 dB. For this given scenario, the received correlation function, which includes the desired source signal, is calculated corresponding to the full sensor configuration. The input layer is of size $2N-1$ and the output layer is of size N. Although there are N unique correlation lags, extracted from the first row of the correlation matrix ($\mathbf{r_x}(n-1) = \mathbf{R_{xx}}(1, n)$), the dimensionality of the input layer is $2N-1$ owing to concatenating the real and imaginary entries of the generally complex valued correlation lags, except the zeroth lag. The corresponding configuration resulting in MaxSINR by employing phase shifts is found through enumeration involving singular value decomposition (SVD) requiring $\mathcal{O}(N^3)$ operations for each iteration. The network output is a binary vector such that 1 indicates sensor selection and 0 indicates the absence of the corresponding sensor location.

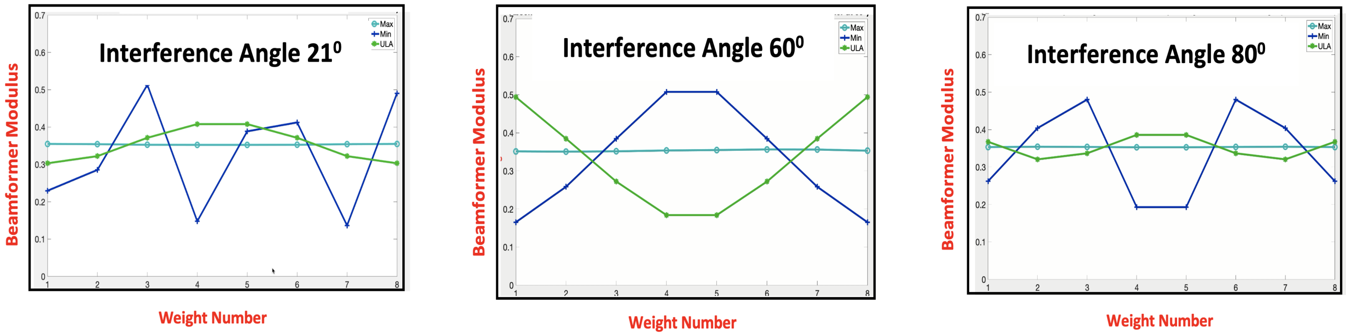
For the training stage, the weights of the neural network are optimized to minimize the mean squared error between the label and the output of the network using the ADAM algorithm. The learning rate is set to 0.001 and dropout regularization with the keep probability of 0.9 is used. The weights are initialized using the Xavier initialization [22]. We chose FCNN for its simplicity. In essence, FCNN presents the baseline with other networks, such as Convolutional Neural Networks (CNN), are slated to give better performance with sufficiently large training data [23].

The robustness of the learned models is demonstrated by generating the test data that is different from the training stage, by assuming the DOA of the interfering signals off grid. This is simulated by adding unit variance Gaussian noise to the interference DOAs on the grid. We present the results under limited data snapshots. The sparse array design can only have few active sensors at a time, in essence, making it difficult to furnish the correlation values corresponding to the inactive sensor locations. However, for the scope of this paper, we assume that the estimate of all the correlation lags corresponding to the full aperture array are available to input for prediction. This can typically be achieved by employing a low rank matrix completion strategy that permits the interpolation of the missing correlation lags [15]. Additionally, to ensure the selection of P antenna locations at the output of DNN, we declare the P highest values in the output as the selected sensor locations.

4. SIMULATIONS

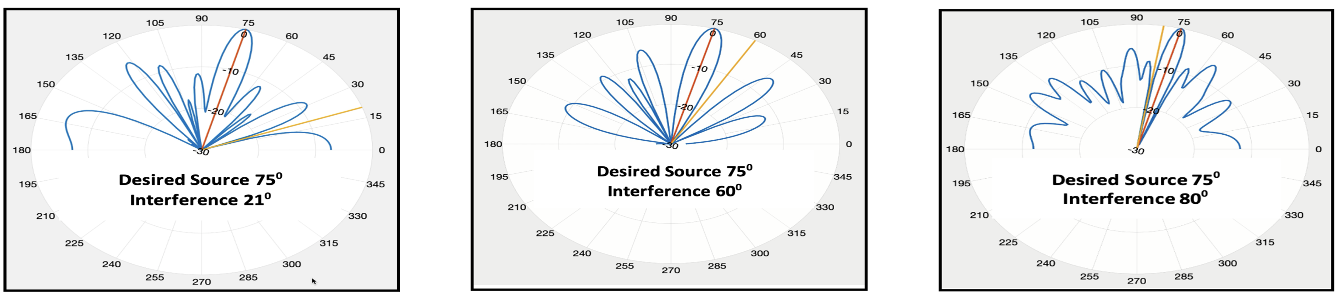
To demonstrate the potential of phase-only Capon beamformer for interference mitigation, a desired emitter signal and a single unwanted interference is simulated at 75^0 and 21^0, respectively. SINR performance is evaluated for various array configurations generated by selecting 8 sensor locations from 16 equally spaced possible locations that are half wavelength apart. Among all possible topologies, the modulus of the optimum beamformer weights are compared in Fig. 1a for three different array configurations, namely an 8 element MaxSINR sparse array, an eight element compact ULA, and an eight element sparse array offering minimum SINR performance. We observe that the modulus of optimum sparse array beamformer is constant for all eight sensors. Whereas, ULA has greater variation in the modulus of the weight vector, this variation is even greater for the configuration with the worst SINR performance. This observation is consistent for different interference directions while keeping the same desired source DOA as shown in Figs. 1b and 1c. Figs. 2a through 2c show that appropriately optimizing antenna locations can place deep nulls in the direction of interference while maintaining maximum response towards isolated source. All three beampatterns were generated with constant modulus beamforming weights with phases pointing towards the desired emitter direction. This shows that the interference nulling in this case can exclusively be achieved by controlling the array configuration. We show the effectiveness of the DNN approach for sparse array design achieving MaxSINR under modulus constraint on beamformer weights. We pose the problem as selecting $P = 6$ antennas from $N = 12$ possible equally spaced locations with inter-element spacing of $\lambda/2$. Figure 3 shows the output SINR performance comparisons for four scenarios; optimum weights applied to optimum array configuration found through enumeration referred to as EN-OP design, phase shifters applied to sparse array configuration optimized through DNN referred to as DNN-PH design, ULA with optimum weights and ULA with phase shifters referred to as ULA-OP and ULA-PH, respectively. Each point on the horizontal axis represents a Monte Carlo scenario generated by simulating a desired source DOA at 75^0 with 0dB SNR, whereas the interference DOA and INR are randomly chosen.

The performance of DNN-PH design, presented in Fig. 3, shows the first two hundred testing scenarios (total 900 testing scenarios). Training data is obtained by simulating 500 data snapshots for each testing scenario, and employing enumerated labels to train a fully connected DNN which has three hidden layers, one input layer, and one output layer. The input layer is of size $2N-1$ and the output layer is of size N, making the input vector of size 23 and output vector length is equal to the perspective sensor locations i.e. 12 in this case.



(a) (b) (c)

Fig. 1: Modulus of the optimum beamformer isolating emitter at 75^0 with Interference at a) 21^0 b) 60^0 c) 80^0



(a) (b) (c)

Fig. 2: Unit modulus beampattern corresponding to 3 optimum arrays; desired source at 75^0; interference a) 21^0 b) 60^0 c) 80^0

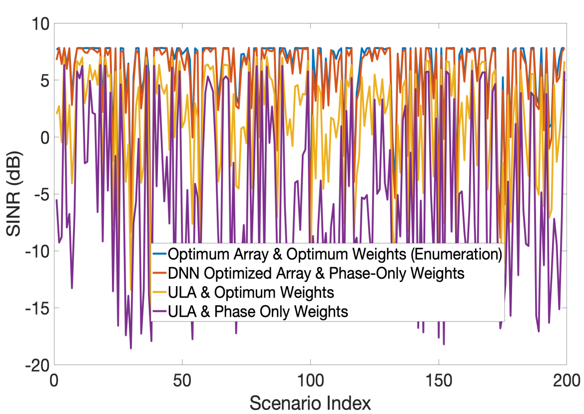


Fig. 3: Performance comparison of array topology and beamformer weights for different array configurations

The network performance is reported by averaging over 900 testing scenarios for each desired source DOA. It is evident that the DNN-PH approach performs close (0.1 dB trade off) to the performance of EN-OP design. The latter amounts to trying all possible configurations and choosing the one with the highest SINR. It not only involves singular value decomposition (SVD) for each enumeration making it un-scalable with the problem size (facing the curse of dimensionality) but also it requires digital beamforming circuitry as opposed to DNN-PH design which requires only phase shifters. It is noted that the phase progression applied to sparse array is not necessarily equal to the steering vector of desired source DOA. Therefore, we apply a local search over a 5 degree grid around the desired source to optimize the phase shift required for maximizing the SINR. It is clear from Fig. 3 that the proposed design yields significant gains over ULA-OP and ULA-PH designs.

5. CONCLUSION

This paper considered reconfigurable sparse array design for maximizing the beamformer output SINR for a desired source in an interference active scenario. A DNN based phase-only approach was developed to configure a data-driven sparse beamformer by learning the enumerated design. We employed FCNN for its simplicity and limited training data requirements. Other networks with complex structures, like CNN, can yield better performance with larger training data set, especially in complicated interference environments. It was shown through design examples that the performance of the proposed DNN sparse array design approach based on phase only beamformer is very close to the optimum enumerated array design employing non-constant modulus beamformer weights. The proposed approach is robust against limited data snapshots and promise high performance with reduced computational complexity.

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