BLIND SEPARATION OF LINEAR-QUADRATIC MIXTURES OF MUTUALLY INDEPENDENT AND AUTOCORRELATED SOURCES

Shahram HOSSEINI, Yannick DEVILLE

Université de Toulouse, UPS, CNRS, CNES,
IRAP (Institut de Recherche en Astrophysique et Planétologie),
14 Av. Edouard Belin, 31400 Toulouse, France

ABSTRACT

In this paper, we are interested in the blind separation of linear-quadratic mixtures of mutually independent sources when successive samples of each source are correlated. When a linear source separation method based on second-order statistics, like the well-known AMUSE method, is applied to this type of mixture, it provides subclasses of the initial mixture where each source can remain mixed with its square. We propose a new approach to then separate these two components. Simulations show the very good performance of our method, as compared with two other methods.

***Index Terms*—** Blind source separation, Linear-quadratic mixing model, Second-order statistics

1. INTRODUCTION

Blind source separation consists in restoring unknown source signals from their observed mixtures when the mixture parameters are unknown [1, 2, 3]. Whereas linear mixtures have widely been studied, less work has been done on nonlinear mixtures, which are much more complex [4, 5, 6, 7, 8]. In order to reduce the complexity of nonlinear mixtures, the structure of the mixing model can be constrained to obtain exploitable but realistic models. Among these models, the Linear-Quadratic (LQ) mixture [9, 10] is one of the most interesting because it can accurately model several mixtures encountered in applications such as Earth observation [11, 12, 13], analysis of chemical and gas sensor data [14, 15] and processing of scanned documents [16, 17]. Moreover, it can be considered as a truncated Taylor series expansion of a general nonlinear mixture. The LQ mixture is described by the following equation for $i = 1, \cdots, K$:

$$x_i(n) = \sum_{j=1}^{L} a_j(i)s_j(n) + \sum_{j=1}^{L}\sum_{k=j}^{L} a_{j,k}(i)s_j(n)s_k(n), \quad (1)$$

where $x_i(n)$ and $s_j(n)$ are the i-th observed and the j-th source signals, and $a_j(i)$ and $a_{j,k}(i)$ are the coefficients of the linear and quadratic parts of the mixture. L and K respectively represent the number of sources and observations.

Several methods, exploiting the non-negativity [11, 12], sparseness [18, 19] or mutual independence [9, 20] of the source signals have been proposed to separate their LQ mixtures. Independent component analysis (ICA) methods, based on the assumption of statistical independence of the source signals, are often constrained by other properties of mixtures and/or sources. For example, the ICA-LQ methods proposed in [21, 22] can only be applied to complex-valued and circular sources, whereas the method proposed in [23] is only applicable to binary sources. The methods described in [17, 20] are only suited to determined mixtures (i.e. $L = K$) so that some of the available observations cannot be used. In addition, the latter methods often require the inversion of a nonlinear system of equations, which is not trivial.

In this paper, we propose a new method for separating LQ mixtures of mutually independent sources when successive samples of each source are correlated. It may be considered as an extension of the well-known AMUSE method [24] for linear mixtures, where unlike [21, 22], we consider the case of real-valued sources. We have already proposed such an extension for the special case of bilinear mixtures [25] where the squared term coefficients $a_{j,j}(i)$ in (1) are equal to zero. We will see in the following that general LQ mixtures are more problematic. Indeed, by applying an LQ version of the AMUSE algorithm we obtain subclasses of the initial mixture where each source can remain mixed with its square. We propose a solution to this problem by developing a method to separate each source from its square. For the sake of simplicity, we consider in this paper only the special case of $L = 2$ and $K = 5$, i.e. when we have 5 LQ mixtures of 2 sources. However, the method can be generalized to the case of L sources and K observations provided $K \geq L(L + 3)/2$.

2. APPLYING AMUSE TO LQ MIXTURES

In the special case of $L = 2$ and $K = 5$, the LQ model (1) reads for $i = 1, \cdots, 5$:

$$\begin{aligned} x_i(n) &= a_1(i)s_1(n) + a_2(i)s_2(n) + a_{1,2}(i)s_1(n)s_2(n) \\ &+ a_{1,1}(i)s_1^2(n) + a_{2,2}(i)s_2^2(n). \end{aligned} \quad (2)$$

In the following, we assume that s_1 and s_2 are statistically independent and stationary. Denoting by $\bar{s}_i = E[s_i(n)]$ the mean of $s_i(n)$ and by $\tilde{s}_i(n) = s_i(n) - \bar{s}_i$ the centered version of $s_i(n)$, (2) can be rewritten as

$$\begin{aligned} x_i(n) &= a_1(i)(\tilde{s}_1(n)+\bar{s}_1) + a_2(i)(\tilde{s}_2(n)+\bar{s}_2) \\ &+ a_{1,2}(i)(\tilde{s}_1(n)+\bar{s}_1)(\tilde{s}_2(n)+\bar{s}_2) \\ &+ a_{1,1}(i)(\tilde{s}_1(n)+\bar{s}_1)^2 + a_{2,2}(i)(\tilde{s}_2(n)+\bar{s}_2)^2, \end{aligned}$$

or, after development and some computation, as

$$\begin{aligned} x_i(n) &= \tilde{a}_1(i)\tilde{s}_1(n) + \tilde{a}_2(i)\tilde{s}_2(n) \\ &+ a_{1,2}(i)\tilde{s}_1(n)\tilde{s}_2(n) \\ &+ a_{1,1}(i)\tilde{s}_1^2(n) + a_{2,2}(i)\tilde{s}_2^2(n) + c_i, \end{aligned} \quad (3)$$

with

$$\tilde{a}_1(i) = a_1(i) + a_{1,2}(i)\bar{s}_2 + 2a_{1,1}(i)\bar{s}_1,$$

$$\tilde{a}_2(i) = a_2(i) + a_{1,2}(i)\bar{s}_1 + 2a_{2,2}(i)\bar{s}_2,$$

$$c_i = a_1(i)\bar{s}_1 + a_2(i)\bar{s}_2 + a_{1,2}(i)\bar{s}_1\bar{s}_2 + a_{1,1}(i)\bar{s}_1^2 + a_{2,2}(i)\bar{s}_2^2.$$

From (3), since $s_1(n)$ and $s_2(n)$ are zero-mean and independent, the mean value of each observation reads

$$\bar{x}_i = a_{1,1}(i)\overline{\tilde{s}_1^2} + a_{2,2}(i)\overline{\tilde{s}_2^2} + c_i,$$

where $\overline{\tilde{s}_i^2}$ stands for the mean of $\tilde{s}_i^2(n)$. Thus, the centered version of each observation, $\tilde{x}_i(n) = x_i(n) - \bar{x}_i$, reads

$$\begin{aligned} \tilde{x}_i(n) &= \tilde{a}_1(i)\tilde{s}_1(n) + \tilde{a}_2(i)\tilde{s}_2(n) \\ &+ a_{1,2}(i)\tilde{s}_1(n)\tilde{s}_2(n) \\ &+ a_{1,1}(i)(\tilde{s}_1^2(n) - \overline{\tilde{s}_1^2}) + a_{2,2}(i)(\tilde{s}_2^2(n) - \overline{\tilde{s}_2^2}). \end{aligned} \quad (4)$$

According to (4), the 5 centered observations $\tilde{x}_i(n)$ may be considered as linear mixtures[1] of the 5 zero-mean extended sources $\tilde{s}_1(n)$, $\tilde{s}_2(n)$, $\tilde{s}_3(n) = \tilde{s}_1(n)\tilde{s}_2(n)$, $\tilde{s}_4(n) = \tilde{s}_1^2(n) - \overline{\tilde{s}_1^2}$ and $\tilde{s}_5(n) = \tilde{s}_2^2(n) - \overline{\tilde{s}_2^2}$. Thus, this model can be written in the following matrix form: $\tilde{\mathbf{x}}(n) = \tilde{\mathbf{A}}\tilde{\mathbf{s}}(n)$, where $\tilde{\mathbf{x}}(n)$ and $\tilde{\mathbf{s}}(n)$ are column vectors respectively containing $x_i(n)$ and $\tilde{s}_i(n)$, and $\tilde{\mathbf{A}}$ is a 5×5 matrix whose i-th row contains the mixing coefficients of Eq. (4) for a given value of i. Assuming that the extended sources are autocorrelated and have different normalized autocorrelation profiles (or equivalently different normalized power spectra), and disregarding their possible cross-correlation at this stage, we may try and apply the AMUSE algorithm [24], initially proposed for separating linear mixtures, to the centered observation vector $\tilde{\mathbf{x}}(n)$ in the following manner:

- Estimate the zero-lag correlation matrix of $\tilde{\mathbf{x}}(n)$, denoted by $\mathbf{R}_{\tilde{\mathbf{x}}}(0)$.
- Compute the whitened observations $\mathbf{v}(n) = \mathbf{D}^{-1/2}\mathbf{E}^T\tilde{\mathbf{x}}(n)$, where $\mathbf{E}$ is a matrix containing eigenvectors of $\mathbf{R}_{\tilde{\mathbf{x}}}(0)$ and $\mathbf{D}$ is a diagonal matrix containing eigenvalues of $\mathbf{R}_{\tilde{\mathbf{x}}}(0)$ on its diagonal.
- Estimate the correlation matrix of $\mathbf{v}(n)$ for a lag $\tau \neq 0$, denoted by $\mathbf{R}_{\mathbf{v}}(\tau)$.
- Compute the output signals using $\mathbf{z}(n) = \mathbf{H}^T\mathbf{v}(n)$ where $\mathbf{H}$ is a matrix containing eigenvectors of $(\mathbf{R}_{\mathbf{v}}(\tau) + \mathbf{R}_{\mathbf{v}}^T(\tau))/2$.

If all the extended sources $\tilde{s}_i(n)$ were mutually uncorrelated (and autocorrelated with different normalized autocorrelation profiles for the chosen lag τ), the output signals, i.e. the entries of $\mathbf{z}(n)$, would be equal to these source signals up to a permutation and scale factors [24]. In our problem, since $s_1(n)$ and $s_2(n)$ are supposed to be independent, it is clear that $\tilde{s}_1(n)$ is uncorrelated with $\tilde{s}_2(n)$ and $\tilde{s}_5(n)$. It is also uncorrelated with $\tilde{s}_3(n)$ because

$$\begin{aligned} E[\tilde{s}_1(n+m)\tilde{s}_3(n)] &= E[\tilde{s}_1(n+m)\tilde{s}_1(n)\tilde{s}_2(n)] \\ &= E[\tilde{s}_1(n+m)\tilde{s}_1(n)]E[\tilde{s}_2(n)] = 0. \end{aligned} \quad (5)$$

However, in general $\tilde{s}_1(n)$ and $\tilde{s}_4(n)$ are correlated. In the same manner, $\tilde{s}_2(n)$ is uncorrelated with all the extended sources except $\tilde{s}_5(n)$. $\tilde{s}_3(n)$ is uncorrelated with all the extended sources. $\tilde{s}_4(n)$ is uncorrelated with all the extended sources except $\tilde{s}_1(n)$. Finally, $\tilde{s}_5(n)$ is uncorrelated with all the extended sources except $\tilde{s}_2(n)$. As a result, when applying AMUSE to the centered observations, it is expected to find at its output $\tilde{s}_3(n)$ (up to a scale factor), two different mixtures of $\tilde{s}_1(n)$ and $\tilde{s}_4(n)$, and two different mixtures of $\tilde{s}_2(n)$ and $\tilde{s}_5(n)$.

In other words, these 5 outputs are[2]: $z_1(n) = k_{11}\tilde{s}_1(n) + k_{12}\tilde{s}_4(n)$, $z_2(n) = k_{21}\tilde{s}_1(n) + k_{22}\tilde{s}_4(n)$, $z_3(n) = k'_{11}\tilde{s}_2(n) + k'_{12}\tilde{s}_5(n)$, $z_4(n) = k'_{21}\tilde{s}_2(n) + k'_{22}\tilde{s}_5(n)$ and $z_5(n) = k''\tilde{s}_3(n)$. Our objective is to estimate $\tilde{s}_1(n)$ and $\tilde{s}_2(n)$ from these outputs. In the following section, we introduce a new method for estimating $\tilde{s}_1(n)$ from $z_1(n)$ and $z_2(n)$. The same approach may be used to estimate $\tilde{s}_2(n)$.

3. SEPARATING A SOURCE FROM ITS SQUARE

Given the following two mixtures

$$\begin{aligned} z_1(n) &= k_{11}\tilde{s}_1(n) + k_{12}(\tilde{s}_1^2(n) - \overline{\tilde{s}_1^2}), \\ z_2(n) &= k_{21}\tilde{s}_1(n) + k_{22}(\tilde{s}_1^2(n) - \overline{\tilde{s}_1^2}), \end{aligned} \quad (6)$$

we aim at estimating $\tilde{s}_1(n)$. To this end, we linearly combine the above mixtures in the following manner:

$$y_1(n) = w_{11}z_1(n) + w_{12}z_2(n) \quad (7)$$

[1] This is possible because the LQ mixture is linear with respect to its parameters (see also [11, 10]).

[2] In practice, because of the permutation indeterminacy, $\mathbf{z}(n)$ contains these 5 components in a random order.

$$y_2(n) = z_1(n) + w_{22}z_2(n), \quad (8)$$

and want to estimate the parameters w_{11}, w_{12}, w_{22} so that $y_2(n) = \alpha\tilde{s}_1(n)$ and $y_1(n) = \alpha^2(\tilde{s}_1^2(n) - \overline{\tilde{s}_1^2})$. Note that in this case

$$y_1(n) = y_2^2(n) - \overline{y_2^2}. \quad (9)$$

In the appendix, we show that these constraints permit to find the parameters w_{11}, w_{12}, w_{22} in a unique manner as functions of k_{11}, k_{12}, k_{21} and k_{22}. Here, we propose an algorithm to blindly estimate the parameters w_{ij}. Considering z_1, z_2, y_1 and y_2 as random variables, (9), (7) and (8) yield

$$w_{11}z_1 + w_{12}z_2 = (z_1 + w_{22}z_2)^2 - \overline{(z_1 + w_{22}z_2)^2}.$$

The least mean-square estimate of the parameters w_{11}, w_{12}, w_{22} is obtained by minimizing the cost function $C = E[\epsilon^2]$, with

$$\epsilon = w_{11}z_1 + w_{12}z_2 - (z_1 + w_{22}z_2)^2 + \overline{(z_1 + w_{22}z_2)^2}. \quad (10)$$

This cost function can be minimized using a gradient descent algorithm:

$$w_{ij} = w_{ij} - \mu\frac{\partial C}{\partial w_{ij}}. \quad (11)$$

The derivative of the cost function with respect to each parameter w_{ij} reads

$$\frac{\partial C}{\partial w_{ij}} = 2E[\epsilon\frac{\partial \epsilon}{\partial w_{ij}}], \quad (12)$$

where, from (10): $\frac{\partial\epsilon}{\partial w_{11}} = z_1$, $\frac{\partial\epsilon}{\partial w_{12}} = z_2$, and

$$\frac{\partial\epsilon}{\partial w_{22}} = -2(z_1 + w_{22}z_2)z_2 + 2\overline{(z_1 + w_{22}z_2)z_2}.$$

Inserting these derivatives and the expression of ϵ, defined by (10), in (12), then developing the results, and noting that the AMUSE algorithm provides centered and zero-lag non-correlated signals such that $E[z_1] = 0$, $E[z_2] = 0$ and $E[z_1z_2] = 0$, after simplifications, we get

$$\frac{\partial C}{\partial w_{11}} = 2\big(-E[z_1^3] + E[z_1^2]w_{11} -2E[z_1^2z_2]w_{22} - E[z_1z_2^2]w_{22}^2\big), \quad (13)$$

$$\frac{\partial C}{\partial w_{12}} = 2\big(-E[z_1^2z_2] + E[z_2^2]w_{12} -2E[z_1z_2^2]w_{22} - E[z_2^3]w_{22}^2\big), \quad (14)$$

$$\frac{\partial C}{\partial w_{22}} = -4\big(-E[z_1^3z_2] + E[z_1^2z_2]w_{11} + E[z_1z_2^2]w_{12} +(-3E[z_1^2z_2^2] + E[z_1^2]E[z_2^2])w_{22} + E[z_1z_2^2]w_{11}w_{22} +E[z_2^3]w_{12}w_{22} - 3E[z_1z_2^3]w_{22}^2 +(-E[z_2^4] + E[z_2^2]^2)w_{22}^3\big). \quad (15)$$

In practice, the expected values are estimated from data.

4. SOME PRACTICAL CONSIDERATIONS

In the approach explained in Section 3, we assumed that the first two signals $z_1(n)$ and $z_2(n)$ at the output of AMUSE corresponded to the 2 mixtures containing $\tilde{s}_1(n)$ and $\tilde{s}_1^2(n) - \overline{\tilde{s}_1^2}$. In practice, however, because of the permutation indeterminacy, AMUSE provides its outputs in a random order. A possible solution, which we used in our tests, consists in repeating the above gradient algorithm for all the signal pairs obtained at the output of the AMUSE algorithm, then measuring the correlation coefficient between signals $y_1(n)$ and $y_2^2(n) - \overline{y_2^2}$ where $y_1(n)$ and $y_2(n)$ are the outputs of the gradient algorithm after convergence. According to (9), if the pair is well chosen this correlation coefficient should have a large value, otherwise its value should be small. In our tests, the pair is retained if the correlation coefficient is greater than 0.9.

The gradient algorithm may theoretically converge towards a local minimum. A solution consists in initializing it with several different initial values, and choosing the solution which corresponds to the lowest value of the cost function.

5. SIMULATION RESULTS

Two independent signals $e_1(n)$ and $e_2(n)$, distributed following a log-normal distribution with parameters $\mu = 0$ and $\sigma = 0.5$, were generated. This choice provides non-negative signals with asymmetric probability density functions[3]. These signals were then filtered by two first-order autoregressive filters to get $N = 100000$ samples of two autocorrelated sources following the model $s_i(n) = e_i(n) + \rho_i s_i(n-1)$. We chose $\rho_1 = 0.5$ and $\rho_2 = 0.7$. Then, these source signals were mixed using the LQ model (2) to obtain 5 observed signals $x_i(n)$. The random mixing coefficients were uniformly distributed over $[0, 1]$.

To separate the sources, at first the observed signals were centered by subtracting their mean values. Then, the standard AMUSE algorithm (with a time lag $\tau = 1$) was applied, which provides 5 zero-mean, unit-variance signals $z_i(n)$. To better study these signals, we compute the coefficients of the least-square fits of each signal on the centered extended sources $\tilde{s}_i(n)$ defined in Section 2, i.e. the coefficients $b_j(i)$ in the following model: $z_i(n) = \sum_{j=1}^{5} b_j(i)\tilde{s}_j(n)$.

Table 1 shows the value of these coefficients in one of the simulations. As can be seen, in this simulation the second output corresponds to $\tilde{s}_3$, i.e. the product of the centered original sources. The first and fourth outputs correspond to mixtures of $\tilde{s}_1$ and $\tilde{s}_4$, i.e. the first centered original source and the centered version of its square. Finally, the third and fifth outputs correspond to mixtures of $\tilde{s}_2$ and $\tilde{s}_5$, i.e. the second centered original source and the centered version of its square. Thus, the algorithm proposed in Section 3 may be applied to the first

[3]Note, however, that these properties are not necessary for our method.

Table 1. Coefficients $b_j(i)$ of the least-square fits at the output of AMUSE for $j = 1, \cdots, 5$.

Output 1	0.357	-0.004	-0.072	-0.506	-0.004
Output 2	-0.023	-0.004	-1.003	0.027	0.024
Output 3	-0.036	-0.278	0.049	0.000	0.543
Output 4	-1.153	0.025	0.004	0.153	-0.021
Output 5	0.019	1.108	0.009	-0.005	-0.144

and forth outputs to estimate the centered version of the first original source, and to the third and fifth outputs to estimate the centered version of the second original source. Note however that AMUSE provides its outputs in a random order, so that the approach explained in Section 4 will be used in our tests, presented below.

100 Monte Carlo simulations, corresponding to 100 different random generator seeds used to generate sources and mixing coefficients, were performed. For each simulation, our whole algorithm was applied to estimate the centered versions of the original sources. The gradient learning rate μ was fixed to 0.002. To avoid local minima, for each simulation, the gradient algorithm was executed 10 times with different random initial values of w_{ij}, and the solution corresponding to the lowest value of the cost function was chosen. To evaluate the performance of each simulation, the Signal to Interference Ratio (SIR) was calculated using the following equation

$$SIR = \frac{1}{2}\sum_{i=1}^{2} 10\log_{10}\frac{\text{mean}(\tilde{s}_i^2)}{\text{mean}((\tilde{s}_i - \hat{s}_i)^2)}, \qquad (16)$$

where $\hat{s}_i$ is the estimated centered source after removing the permutation and scale factor indeterminacies.

Data used in our simulation being non-negative, we also tested the NMF-Mult-LQ algorithm [11], which is an NMF-based multiplicative method proposed for the LQ mixtures. The physical constraints of the original method, i.e. the sum-to-one constraint of the linear coefficients and the upper-bound constraint of the quadratic coefficients, which have been used in [11] to adapt the original algorithm to remote sensing applications, were omitted in our tests. We also tested the SOBI method [26] which performs source separation by exploiting the autocorrelation of sources, but assuming that the mixing model is linear. As can be seen in Table 2, the proposed method outperforms the other two methods.

6. CONCLUSION

We proposed a new method for separating LQ mixtures of mutually independent and autocorrelated sources. At a first step, the AMUSE algorithm was applied to the original mixture, providing subclasses containing mixtures of each source with its square. Then, we developed a gradient-based algorithm to separate each source from its square. The simulations led to very good performance for the proposed method.

Table 2. The mean, standard deviation, maximum and minimum of SIR (in dB), and the computation time of each simulation (in seconds), for the proposed method, the NMF-Mult-LQ method and the SOBI method.

	New method	NMF-Mult-LQ	SOBI
Mean(SIR)	34.02	15.02	11.19
Std(SIR)	4.62	3.09	0.67
Max(SIR)	45.37	22.68	11.69
Min(SIR)	22.88	-0.79	4.89
time	3.92	1194.87	0.06

In this paper, only the case of mixtures of two sources was developed. An extension of this method to any number of sources will be proposed in future works. It is also possible to replace AMUSE by more efficient second order source separation algorithms.

Appendix: Uniqueness of solution

Given Equations (7) and (8), we want to show that there is a unique solution for the parameters w_{ij} so that

$$y_2(n) = \alpha \tilde{s}_1(n), \qquad (17)$$

$$y_1(n) = \alpha^2(\tilde{s}_1^2(n) - \overline{\tilde{s}_1^2}). \qquad (18)$$

Inserting Equations (6) in (7) and (8) yields

$$\begin{aligned} y_1(n) &= (w_{11}k_{11} + w_{12}k_{21})\tilde{s}_1(n) \\ &+ (w_{11}k_{12} + w_{12}k_{22})(\tilde{s}_1^2(n) - \overline{\tilde{s}_1^2}), \end{aligned} \qquad (19)$$

$$y_2(n) = (k_{11} + w_{22}k_{21})\tilde{s}_1(n) + (k_{12} + w_{22}k_{22})(\tilde{s}_1^2(n) - \overline{\tilde{s}_1^2}). \qquad (20)$$

From (20), it is clear that (17) holds if and only if

$$w_{22} = -k_{12}/k_{22}, \qquad (21)$$

$$\alpha = k_{11} + w_{22}k_{21}. \qquad (22)$$

From (19), it is clear that the (18) holds if ind only if

$$w_{11}k_{11} + w_{12}k_{21} = 0, \qquad (23)$$

$$\alpha^2 = w_{11}k_{12} + w_{12}k_{22}. \qquad (24)$$

Using (22) and (24), and replacing w_{22} by its value defined in (21), we get

$$w_{11}k_{12} + w_{12}k_{22} = (\frac{k_{11}k_{22} - k_{12}k_{21}}{k_{22}})^2. \qquad (25)$$

(23) and (25) constitute a linear system of equations with respect to w_{11} and w_{12}. Solving this system yields

$$w_{11} = (k_{12}k_{21} - k_{11}k_{22})k_{21}/k_{22}^2, \qquad (26)$$

$$w_{12} = (k_{11}k_{22} - k_{12}k_{21})k_{11}/k_{22}^2. \qquad (27)$$

Thus, there is a unique solution for parameters w_{ij} defined by (21), (26) and (27), provided $k_{22} \neq 0$.

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