MULTIVARIATE MULTISCALE COSINE SIMILARITY ENTROPY

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ABSTRACT

The rapid development in sensor technology has made it convenient to acquire data from multi-channel systems but has also highlighted the need for the analysis of nonlinear dynamical properties at a higher level - the so-called structural complexity. Traditional single-scale entropy measures, such as the amplitude based Sample Entropy (SampEn), are designed to give a quantification of irregularity and randomness. Its enhanced versions, Multiscale Sample Entropy (MSampEn) and Multivariate Multiscale Sample Entropy (MMSE), are capable of detecting the structure within a signal at high scales and for multivariate data, however, the scaling process comes at a cost of the reduction of the number of sample points that results in reduced stability and limitations regarding the selection of the embedding dimension. In addition, the analyses of structure on the basis of MSampEn and MMSE require relatively high scales, yet without prior-knowledge of the scale degree. To this end, we propose a new multivariate entropy method based on the recently introduced Cosine Similarity Entropy (CSE). The proposed Multivariate Multiscale Cosine Similarity Entropy (MMCSE) is based on angular distance which makes it possible to assess long-term correlation within a system at both a low and large scales, and thus assess the true structural complexity in a more physically meaningful way. Both synthetic and real world signals are utilized to examine the performance of the proposed approach, with the resulting simulations supporting the approach.

Index Terms— Multivariate multiscale analysis, cosine similarity entropy, long-term correlation, structural complexity.

1. INTRODUCTION

Complex and nonlinear dynamic behaviours are ubiquitous in real world phenomena, whereby the characteristics of complex systems manifest themselves through non-linearity [1], non-stationarity [2], multiscale variability [3] and time irreversibility [4]. When it comes to the quantification of these characteristics, entropy-based assessment is one of the most useful tools [5–8], which combines Taken's embedding theory [9] with the knowledge of data statistics [1].

Real-world data are far from Gaussian distributed, and usually do not obey any standard probability distribution function (pdf). In other words, for real data, we cannot employ standard closed form pdfs; to this end, empirical entropy methods have been introduced. These include the well-known Sample Entropy (SampEn) measure [10], an amplitude based method which has been widely utilized in quantifying the degree of randomness of signals [11]. Within the single-variate single-scale SampEn framework, a system with a completely irregular behaviour is associated with highest entropy. However, structural complexity is different from entropy as it reflects long range correlations, as e.g. in self-similar infinitely repeating patterns in fractals, such as the 1/f "pink" noise. It is therefore rather counter-intuitive that a system with a completely uncorrelated structure should be given the highest complexity score, as with SampEn. Indeed, both complete order and extreme disorder (e.g., periodic and random signals) should exhibit lowest degrees of structural complexity, while long-range correlation behaviour should be associated with highest degrees of complexity [12].

To resolve these issues with SampEn, the Multiscale Entropy method [3] was proposed which is based on single-scale entropy algorithms but evaluated over multiple temporal scales, to reveal the long-range structure in complex signals. To this end, the Multiscale Sample Entropy (MSampEn) method employs time scales in the form of coarse graining of a time series so that the long-range correlations within the signal can be revealed at high scales. Namely, by averaging over the temporal scales through coarse graining, the components of "uncorrelated structure" in time series are filtered out so that only true complexity remains present at high scales [3]. The MSampEn has been successfully applied in the analysis of health conditions of humans, for example, based on heart rate variability [13], gait dynamics [14], brain electrical activity [15], respiratory intervals [13], and dynamics of eye gaze [16].

The widely acknowledged Complexity Loss Theory (CLT) has established initial connections between structural complexity of physical signals and the health condition of an individual, whereby the pathology will result in a decrease of randomness in physiological data [17]. The more recent theory introduced in [18] has established that the effects of pathology within physiological signals are not only manifested by a decrease in irregularity but also through an increase in self-correlation, which highlights the importance of robust quantification of structural complexity. This is particularly important to achieve over short time segments in order to provide high temporal resolution in complexity measures.

In line with the MSampEn, the Multivariate Multiscale Sample Entropy (MMSE) method was introduced for complexity analysis of multi-channel systems [19], to enable conjoint structural complexity analysis over various data channels and temporal scales [20, 21]. Nevertheless, the process of exploring long-term correlation based on amplitude-based entropy methods at high scale is difficult to define, since the scale of interest is unknown in advance and is varying under different conditions; in addition, such methods are "data hungry" and are thus limited by the available data size which is repeatedly reduced by the scale factor as the scale increases.

To further explore the property of self-correlation at low scales, the angular distance based Cosine Similarity Entropy (CSE) was recently introduced in [18]. Unlike the amplitude based MSampEn methods, the angular distance within CSE has been shown to exhibit advantages, such as less dependence on data size and higher stability at large scales. In this work, we extend the CSE to the multivariate case, and introduce Multivariate Multiscale Cosine Similarity Entropy (MMCSE). The proposed MMCSE is first evaluated over a range of its parameters and over benchmark synthetic signals. Finally, real world data are utilized to assess the performance of the proposed approach against standard MMSE and univariate CSE.

2. MULTIVARIATE MULTISCALE COSINE SIMILARITY ENTROPY

Prior to introducing the Multivariate Multiscale Cosine Similarity Entropy (MMCSE) algorithm, we first outline the procedure of CSE [18] in Algorithm 1. By employing angular distance in place of amplitude distance, the CSE estimation those exhibits less sensitivity to outliers than MSE, such as coming from sharp changes in amplitude. Indeed, the upper bound of angular distance of 2π leverages CSE to generate a well-defined measurement with a range between 0 and 1, regardless of the variance of the time series, thus providing enhanced stability when dealing with highly dynamical signals. We also show that the stable estimate provided by CSE is of critical importance for multi-variate analysis when the data sets from different channels show substantial discrepancies in variance, a case where MMSE is inadequate.

To extend CSE to multi-variate multi-scale entropy, Coarse Graining Process (CGP) is applied in the initial step for each data channel through a non-overlapping window with a scale factor, τ [3]. Next, the scaled signal was reconstructed in the form of a Composite Delay Vector (CDV). Finally, instead of an embedding vector as in MSampEn, CDV is applied in Algorithm 1 following Steps 2-6 to compute the Multivariate Multiscale Cosine Similarity Entropy (MMCSE). Note that MMCSE is sensitive to DC offset as it requires the origin-coordination to project the similarity within tolerance angle. Therefore, the DC offset or a long-term trend needs to be removed by subtracting their median or by filtering of MMCSE. The proposed multivariate entropy is presented in Algorithm 2.

Algorithm 1. Cosine Similarity Entropy (CSE)

Assume a univariate data set $\{x(i)\}_{i=1}^{N}$ is of length, N, and the given parameters are the embedding dimension, m, tolerance, r, and time delay, l.

1. (Optional) Remove the offset and form the embedding vectors, $\mathbf{x}_m(i)$, derived from the zero-median signal $\{u(i)\}_{i=1}^{N}$ by $u(i) = x(i) - median(\{x(i)\}_{i=1}^{N})$, where $\mathbf{x}_m(i) = (u(i), u(i+l), \ldots, u(i+(m-1)l))$.
2. Calculate the angular distance between pairwise embedding vectors $\mathbf{x}_m(i)$ and $\mathbf{x}_m(j)$ based on Cosine Similarity, that is, $d_m(i,j) = \frac{1}{\pi}\cos^{-1}(\frac{\mathbf{x}_m(i)\cdot\mathbf{x}_m(j)}{|\mathbf{x}_m(i)||\mathbf{x}_m(j)|})$, $i \neq j$.
3. Compute the number of similar patterns defined as similar pair $B_m^r(i)$ that satisfy the criterion $d_m(i,j) \leq r$.
4. Compute the local probability of $B_m^r(i)$ by $C_m^r(i) = \frac{B_m^r(i)}{N-n-1}$, where $n = (m-1)l$.
5. Calculate the global probability as $\Phi_m^r = \frac{\sum_{i=1}^{N-n} C_m^r(i)}{N-n}$.
6. Cosine Similarity Entropy is defined as $CSE(m,l,r,N) = -[\Phi_m^r \log_2 \Phi_m^r + (1-\Phi_m^r)\log_2(1-\Phi_m^r)]$.

2.1. Selection of Tolerance and Embedding Dimension

There are several parameters in the MMCSE method that need to be manually selected, including the tolerance, r, and embedding dimension, m.

Algorithm 2. Multivariate Multiscale Cosine Similarity Entropy

Assume that a multi-variate data set $\{x_{k,i}\}_{i=1}^{N}$, $1 \leq k \leq p$ has p channels. The manually selected parameters are the embedding dimension, $M = [m_1, m_2, \ldots, m_p]$, tolerance, r, time delay, $L = [l_1, l_2, \ldots, l_p]$ and scale factor, τ.

1. Normalize the original multi-variate data sets by subtracting the median.
2. Perform the Coarse Graining process to obtain the scaled multi-channel time series $\{y_{k,i}^{(\tau)}\}_{j=1}^{N/\tau}$, according to $y_{k,i}^{(\tau)}(j) = \frac{1}{\tau}\sum_{i=j-\tau/2-1}^{j+\tau/2-1} x_k(i)$, $1 \leq j \leq \frac{N}{\tau}$, $k = 1, 2, \ldots, P$.
3. Form the Composite Delay Vectors, $\mathbf{Y}_M(i)$, according to M and L, in the form

$$\begin{aligned}\mathbf{Y}_M(i) = [&y_{1,i}, y_{1,i+l_1}, \ldots, y_{1,i+(m_1-1)l_1},\\ &y_{2,i}, y_{2,i+l_2}, \ldots, y_{2,i+(m_1-1)l_2},\\ &\vdots\\ &y_{p,i}, y_{p,i+l_p}, \ldots, y_{p,i+(m_1-1)l_p}]\end{aligned}$$

4. Apply the scaled Composite Delay Vectors, $\mathbf{Y}_M(i)$, instead of embedding vector into Step 2-6 of Cosine Similarity Entropy presented in Algorithm 1 to obtain the result of Multivariate Multiscale Cosine Similarity Entropy.

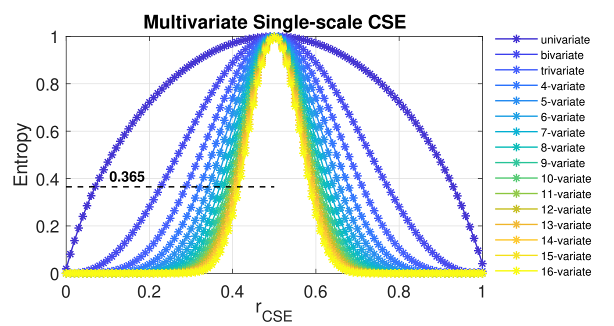


Fig. 1: Behaviour of the Multivariate Single-scale CSE on the estimation of White Gaussian Noise (WGN), as a function of the tolerance, r. The default parameters were set as N = 10000, m = 2, and l = 1. The error-bars designate the average over 10 realizations.

Tolerance. In terms of the tolerance, different from SampEn, the selection of tolerance, r, for angular distance within CSE and MCSE is independent of the variation in time series. Fig.1 shows the outcomes of multivariate single-scale CSE applied to White Gaussian Noise (WGN) as a function of tolerance for a varying number of data channels from univariate to 16-variate cases. The tolerance was initially set to vary from 0.01 to 1 at 0.05 intervals and was then interpolated using values ranging from 0 to 1 at 0.01 intervals. Observe, in all curves, a rise of entropy with an increase in tolerance from 0 to 0.5 and a decrease of entropy with tolerance moving from 0.5 to 1. Due to the symmetry property, only the range of 0 to 0.5 was considered to give the common relationship that the larger the tolerance, the smaller the similarity present as higher complexity. As shown in [18], the tolerance for the single-variate MCSE was empirically set as 0.07 for White Gaussian Noise to yield 0.365. To this end, the mathematical relationship between the number of variates and the tolerance, r, was estimated by setting the entropy to 0.365 as a standard criterion (the black dashed line shown in Fig. 1), as revealed in Fig. 2 using a piece-wise cubic interpolation fitting curve. It was found that the best fit was for

$$r_{cse} = -0.4(p^{-0.71}) + 0.47, \tag{1}$$

where p denotes the number of variates.

Therefore, the tolerance in MMCSE is a function of the number of variates, and for a common situation when the channel number is 2 (bivariate data), the typical setting for the value of tolerance is 0.225, while for the tri-variate case is 0.287.

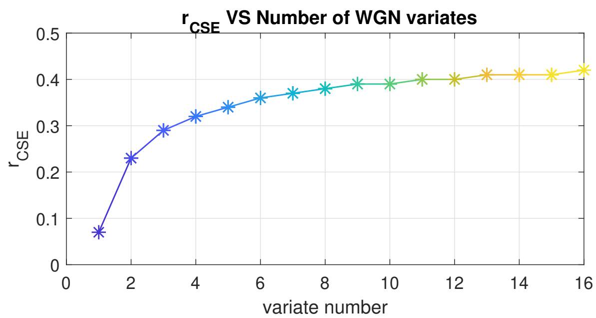


Fig. 2: The tolerance, r, as a function of the number of variates.

Embedding dimension. To investigate the effect of the embedding dimension on Multivariate CSE, four synthetic signals were generated: White Gaussian Noise (WGN); 1/f fractal "pink" noise; auto-regressive process of order one, AR(1), generated from $x(t) = 0.9x(t-1) + \varepsilon(t)$, and auto-regressive process of order two, AR(2), generated from $x(t) = 0.85x(t-1) + 0.1x(t-2) + \varepsilon(t)$, where $\varepsilon \sim \mathcal{N}(0,1)$. The suggested value of the embedding dimension for standard MMSE is usually restricted to $m < 5$. In contrast, as shown in Fig. 3, as the scale increases, the Multivariate CSE exhibits a decreased value, yet it keeps the expected performance, with a well-defined separation among the four considered models. A highly stable estimation is exhibited by Multivariate CSE at high scales, which is found to be difficult to achieve in the standard MMSE method.

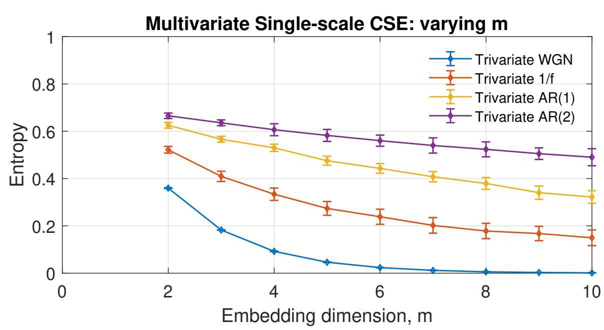


Fig. 3: Multivariate Single-scale CSE as a function of the embedding dimension, m, over 30 realizations with 1000 samples. White Gaussian Noise (WGN), 1/f noise, AR(1) and AR(2) processes were considered, with $N = 1000$, $r = 0.287$, and $l = 1$.

2.2. Effect of Data Length

In real world signal processing, the size of obtained sample points is a general limitation for all the existing entropy methods, as illustrated in Fig. 4, where the synthetic signals were generated from the same four models as in Fig. 3. Observe that the uncorrelated WGN and truly complex 1/f noise exhibit a clear separation after as few as $N = 60$ sample points, while the minimal data length for a consistent estimation of WGN and 1/f noise is suggested as $N = 300$ in standard MMSE in [22]. With the proposed MCSE, well-defined entropy values can be found for quite a short data length of $N = 10$, even for relatively complex signals such as the two AR processes considered. When data length is longer than $N = 600$, the two AR processes with different orders can also be separated, while the shortest data length for the univariate CSE in this case is $N = 700$, as stated in [18]. Therefore, generally speaking, the angular-based CSE requires much less sample points than the amplitude-based SampEn, and this property is further highlighted and improved in the multivariate case.

Remark 1: The angular distance based Cosine Similarity Entropy exhibits an order of magnitude higher temporal resolution in complexity estimation compared to standard Sample Entropy.

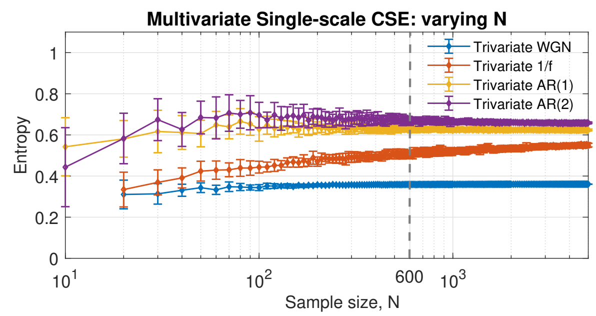


Fig. 4: Multivariate Single-scale CSE as a function of the number of sample points, N, evaluated for White Gaussian Noise (WGN), 1/f noise, and AR processes. Error-bars represent an average over 10 independent realizations of trivariate systems. The default parameters are set as $m = 2$, $r = 0.287$, and $l = 1$.

2.3. Complexity Profile

Based on the analysis of parameters in Multivariate Single-scale CSE in the previous sections, the complexity profile of Multivariate Multiscale CSE is shown in the right panel of Fig. 5, against the standard MMSE in the left panel, where the manually selected parameters were set to $m = 2$, $r_{SE} = 0.15 * tr(S)$ for MMSE, $r_{CSE} = 0.287$ for MMCSE, and $l = 1$. The error-bars were generated based on an average of 10 independent realizations with $N = 10000$.

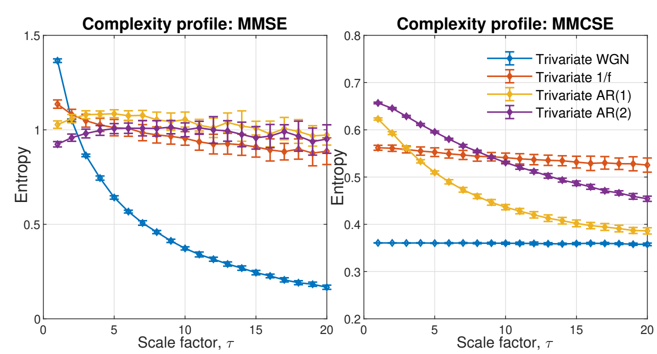


Fig. 5: Complexity profiles of the MMSE and MMCSE algorithm for White Gaussian Noise (WGN), 1/f noise, and AR processes.

Observe from the left panel in Fig. 5 that with the standard MMSE, WGN was wrongly assigned to have a correlated structure at low scales, while the three synthetic signals, 1/f noise, AR(1) and AR(2), were overlapped across all the multiscale cases.

For the MMCSE on the right hand side of Fig. 5, at low scales, the MMCSE achieved a well defined separation of the structural complexity among the four stimulated signals based on self-correlation. Across all scales, AR(2) exhibited higher self-correlation with more degrees of freedom compared to AR(1). As the scale increased, the overall structural complexity of both AR processes decreased due to the low pass filter behaviour of the coarse gaining process, and hence the complexity approached the curve of WGN. In terms of the complexity of 1/f noise and WGN, there is no intersection between the curves for the two signals, with WGN showing a completely uncorrelated structure while the highly complex fractal 1/f noise exhibited a truly complex structure. Unlike the standard MMSE, that gave a correct relationship of structural complexity between 1/f and WGN only under large scales, MMCSE yielded a consistent estimate across all scales.

Remark 2: The proposed Multivariate Multiscale Cosine Similarity Entropy yields stable estimates at high temporal scales, as shown in Fig. 5, thus making it possible to examine structural complexity of physical processes with long-range correlations, such as those coming from physiological and behavioural measurements.

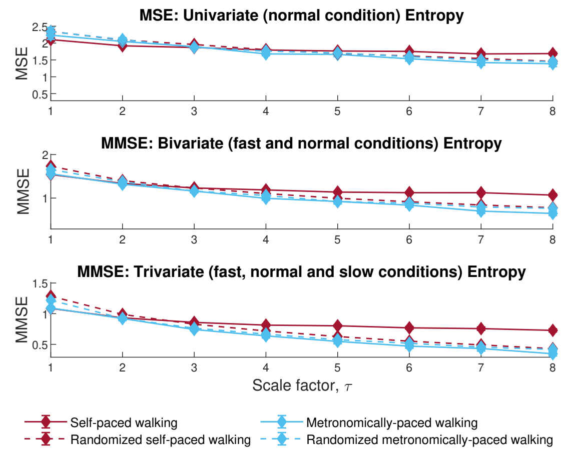
3. PERFORMANCE ON PHYSICAL DATABASE

We next examined the performance of MMCSE in analysis of real world gait dynamics using data from [23]. The experiment recorded stride intervals of 10 subjects walking for one hour at their own pace and then walking following a metronome (self-paced vs metronomically-paced). There, three recordings set-ups (in slow, normal, fast pace) were implemented for each condition and were considered as the three sub-channels of a tri-variate stride signal.

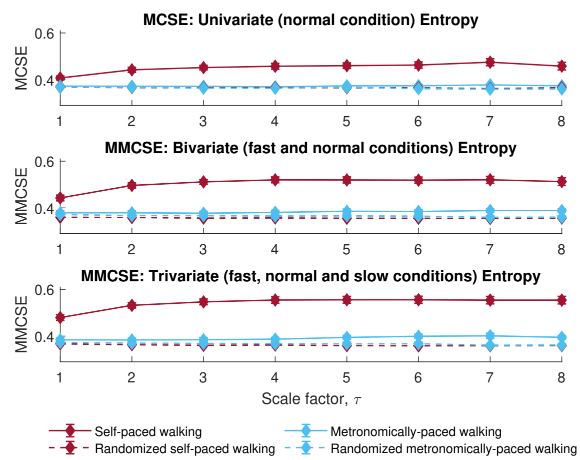
To assess the structural complexity in the two conditions, we compared the results of the behavioural signals with randomized versions of the recorded signals, produced by shuffling the order of the real time series to destroy the structure but maintain the statistical features. The shuffled signals are expected to have uncorrelated behaviours analogous to WGN, owing to their temporal relationships being destroyed. The manually selected parameters in the MMSE and MMCSE were both set as $m = 2$, $l = 1$, $N = 1200$, and the tolerance r for MMSE was selected as 0.15 of the total variation, $tr(S)$, as suggested in [19], while the tolerance, r, for MMCSE was chosen following (1). Due to the limitation of the sample size, the achieved maximal scale was $\tau = 8$, as in Fig. 6.

Fig. 6(a) illustrates the performance of uni-variate amplitude based MSE, which yielded a non-significant discrimination between the two conditions, as evidenced by the blue line (metronomically-paced walking) being below the broken line which corresponds to the noise-like reshuffled original data. The second and third panels show the estimation performance given by MMSE with bivariate and trivariate channels, respectively, that were able to separate the conditions of self-paced from metronomically-paced only at large scales, whereas the constrained condition (in blue solid line) wrongly suggested lower complexity than the uncorrelated randomized signals (in dotted lines).

In contrast, the results of the proposed angle-based MMCSE in Fig. 6(b) were as desired, whereby the shuffled signals exhibited lowest complexity due the broken correlations by a randomized sample order. The constrained (metronomically-paced) condition exhibits less structure than the unconstrained (self-paced) condition, which was in a good agreement with the CSE on a global scale. However, as the constrained condition with metronomically-paced walking lowers the structural complexity, the metronomically-paced curve failed to be separated from the shuffled cases in univariate MCSE. In contrast, the bivariate and trivariate MCSE achieved a good structure separation of all real signals from their randomized versions, which guarantees physically meaningful estimation for quantifying structural complexity real world signals.



(a) Multivariate Multiscale Sample Entropy



(b) Multivariate Multiscale Cosine Similarity Entropy

Fig. 6: Behaviour of a) Standard MMSE, and b) The proposed MMCSE, applied to real world gait dynamics.

4. CONCLUSION

This work has extended the recently introduced univariate Cosine Similarity Entropy (CSE) method to the multivariate case, to provide an efficient quantification of structural complexity of real world data. The proposed Multivariate Multiscale Cosine Similarity Entropy (MMCSE) method has been shown to exhibit a valid estimation of the structure and long-term correlation present in multichannel signals, with higher stability at large scales and an order of magnitude lower requirement on data length than the existing amplitude-based Sample Entropy methods. The performance of MMCSE has been examined on four synthetic benchmark signals as well as on real world stride dynamical signals, with MMCSE showing an improved separation of constrained walking conditions from unconstrained conditions and uncorrelated randomized time series. This opens new avenues for a wide range of high resolution applications in complexity science for real-world data.

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