# CRAMÉR-RAO BOUND AND ANTENNA SELECTION OPTIMIZATION FOR DUAL RADAR-COMMUNICATION DESIGN

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## ABSTRACT

We consider multi-input multi-output (MIMO) dual function radar communication (DFRC) systems, and design a transmit beamforming matrix that optimizes a weighted combination of the radar estimate Cramér-Rao bound (CRB) and the communication rate. A hybrid beamforming structure is considered, with fewer RF chains than antennas, to achieve the benefits of MIMO systems while maintaining low cost. However, such a structure may have a rank-deficient beamforming matrix, resulting in degraded estimation performance. We propose antenna selection as means to ensure a full-rank beamforming matrix, and also select the communication channels so that high communication rate can be achieved. A learning approach is employed to optimally select antennas and design the corresponding beamforming matrix. By leveraging a combination of softmax neural networks, the proposed solution is able to optimize the joint performance metric for a DFRC system.

***Index Terms—*** MIMO, DFRC, antenna selection, CRB

## 1. INTRODUCTION

Incorporating a sensing functionality in communication systems is an emerging trend in next-generation wireless systems [1], such as vehicular networks, WLAN indoor positioning, and unmanned aerial vehicle (UAV) networks [2]. In all those scenarios, sensing and communication are a pair of intertwined functionalities, often required to be carried out simultaneously for the purpose of increasing the spectral efficiency and reducing costs.

The demand for low cost, light weight, energy and hardware efficiency gave rise to Dual Function Radar Communication (DFRC) systems, which can perform sensing and communication from the same platform and using the same waveform [3]. There are various ways via which information can be conveyed in the sensing process. It can be embedded directly in the radar waveforms [4–8],or in the way the waveforms are paired with transmit antennas [8–10], or in the phase of the sidelobes in the array beampattern [11], or in the antenna activation pattern [7, 12]. Most of the aforementioned designs of DFRC signaling strategies consider the transmitter side only. However, the key performance metric for a radar system is the target estimation error at the receiver side.

Recently, [3] considered multi-input multi-output (MIMO) DFRC system design based on a criterion that combines the Cramér-Rao bound (CRB) of the target estimate with the signal to interference and noise ratio (SINR) at a number of single-antenna receivers. In particular, [3] considers the problem of detecting a near-field target that encompasses multiple point-like scatters, in which case, depending on the number of communication streams as compared to the number of available antennas, the beamforming matrix may be rank-deficient and the resulting target estimation performance degraded. To tackle this issue, [3] proposed to transmit extra probing streams, which do not contain any information and are to be used for sensing only, and correspondingly add beamformers to the beamforming matrix to increase its rank. However, this introduction of extra probing streams generates interference to the communication receivers and increases the complexity of the system. Further, more radio-frequency (RF) chains are required to generate those streams, which would increase the hardware cost.

Here, we propose an alternative method for ensuring a full-rank beamforming matrix, that does not require additional probing streams, or equivalently, additional RF chains. In particular, we propose to use antenna selection to reduce the size of the beamforming matrix, so that the matrix naturally becomes full-rank. Antenna selection has been widely used in recent radar works [13–15] to improve the system performance. In this paper, in addition to helping with the rank of the beamforming matrix, antenna selection also results in selecting the communication channels between the communication transmitter and receiver, thus bad channels can be avoided for achieving high communication rate. The optimization problem is solved by modifying the softmax learning approach *learn to select* (L2S) in [16] where the selection of antennas is modeled by softmax neural networks. While machine learning for antenna selection has been investigated in [17, 18], those works treat antenna selection as a classification problem. However, the combinatorial explosion problem renders those methods impractical even in cases with a moderate number of antennas. On the other hand,

Work supported in part by NSF under grant ECCS-2033433.

L2S in [16] can be efficiently scaled to larger problems as it avoids the combinatorial explosion of the selection problem. It also offers a flexible array design framework as the selection problem can be easily formulated for any metric.

## 2. PROBLEM FORMULATION

We consider a hybrid beamforming MIMO DFRC system equipped with $N_t$ transmit and $N_r$ receive antennas and $N_s$ RF chains. The antennas formulate a uniform linear array, with spacing between adjacent antennas denoted by $d$. As the cost of RF chains is high, the transmitter comprises a small number of RF chains, which are connected to a large number of antennas through a network of phase shifters [19]. Such a hybrid beamforming structure enjoys the benefits of an fully digital MIMO system in terms of high-throughput communication and high-accuracy sensing performance while it has reduced cost.

The DFRC system is tracking a distributed target, e.g., nearby pedestrians or vehicles, and also communicates with a single user with $N_s$ antennas. The transmitter performs hybrid beamforming, with beamforming matrix $\mathbf{F} \in \mathbb{C}^{N_t \times N_s}$, by transmitting the signal

$$\mathbf{V} = \mathbf{F}\mathbf{X}, \tag{1}$$

where $\mathbf{X} \in \mathbb{C}^{N_s \times L}$ consists of $N_s$ unit power data streams of length $L$, which are orthogonal to each other so that $\frac{1}{L}\mathbb{E}\{\mathbf{X}\mathbf{X}^H\} = \mathbf{I}_{N_s}$.

At the communication receiver, the received signal can be expressed as

$$\mathbf{Y}_c = \mathbf{H}\mathbf{V} + \mathbf{N}_c, \tag{2}$$

where $\mathbf{H} \in \mathbb{C}^{N_s \times N_t}$ denotes the channel matrix and $\mathbf{N}_c$ is additive white Gaussian noise matrix with zero mean and covariance $\sigma_c^2 \mathbf{I}_{N_c}$. Assuming Gaussian signalling, the achievable rate equals

$$G = \log_2 |\mathbf{I}_{N_c} + \frac{1}{\sigma_c^2}\mathbf{H}\mathbf{R}\mathbf{H}^H|, \tag{3}$$

where $\mathbf{R} = \frac{1}{L}\mathbf{V}\mathbf{V}^H = \mathbf{F}\mathbf{F}^H$ is the sample covariance matrix of $\mathbf{V}$. The upper bound of communication rate subject to a trasnmit power constraint, i.e., $tr(\mathbf{R}) \leq P$, equals [20]

$$G_{max} = \sum_{i}^{N_t} (\log \mu\lambda_i)^+, \tag{4}$$

where $(a)^+ = \max\{a, 0\}$, the $\lambda_i$'s are the eigenvalues of $\mathbf{H}^H\mathbf{H}$, and $\mu$ satisfies $\mathrm{tr}(\mathbf{R}) = \sum_i^{N_t} (\mu - \lambda_i^{-1})^+ \leq P$.

For an extended target encompassing $M$ distributed, point-like scatters, the radar echoes can be written as

$$\begin{aligned} \mathbf{Y}_r &= \sum_{m=0}^{M} \alpha_m \mathbf{b}(\theta_m)\mathbf{a}^H(\theta_m)\mathbf{V} + \mathbf{N}_r && (5) \\ &= \mathbf{C}\mathbf{V} + \mathbf{N}_r, && (6) \end{aligned}$$

where complex $\alpha_m$ represents the combined effect of path attenuation and radar cross section (RCS) of the $m$-th scatter at direction $\theta_m$; $\mathbf{N}_r$ has similar definition to $\mathbf{N}_c$ with each entry having variance $\sigma_r^2$; $\mathbf{a}(\theta) \in \mathbb{C}^{N_t \times 1}$, $\mathbf{b}(\theta) \in \mathbb{C}^{N_r \times 1}$ are the transmit and receive steering vectors towards direction $\theta$; $\mathbf{C} = \sum_{m=0}^{M} \alpha_m \mathbf{b}(\theta_m)\mathbf{a}^H(\theta_m)$ is the target response matrix.

For target estimation, $\mathbf{C}$ is estimated first, and then used to extract the parameters of the scatters via various approaches such as MUSIC and APES algorithms [21]. The CRB corresponding to matrix $\mathbf{C}$ is given as [22]

$$\mathrm{CRB}(\mathbf{C}) = \frac{\sigma_r^2 N_r}{L}\mathrm{tr}(\mathbf{R}^{-1}). \tag{7}$$

Let us for simplicity assume that $N_t \leq N_r$. As the rank of $\mathbf{R}$ is $N_s$, one can see that if $N_s < N_t$, $\mathbf{R}$ is singular and the CRB of $\mathbf{C}$ is not defined. In that case, the estimate of the rank $N_t$ matrix $\mathbf{C}$ will be degraded due to the rank-deficient matrix $\mathbf{F}$. To avoid the problem, [3] proposed the use of extra probing streams $\mathbf{X}_r \in \mathbb{C}^{(N_t - N_s) \times L}$, which are orthogonal to $\mathbf{X}$, along with the corresponding beamformers $\mathbf{F}_r \in \mathbb{C}^{N_t \times (N_t - N_s)}$, such that $\tilde{\mathbf{F}} = [\mathbf{F}, \mathbf{F}_r] \in \mathbb{C}^{N_t \times N_t}$ is a full rank matrix. However, the use of dedicated sensing signals would cause interference to the communication user and further, it would require additional RF chains, thus increasing the system cost.

In our proposed solution to the rank deficiency problem, we ensure that the $\mathbf{F}$ is full rank by performing transmit antennas selection. Let $\mathbf{S} \in \mathbb{R}^{N_s \times N_t}$ be a selection matrix where all the elements in $\mathbf{S}$ are 0, except for exactly one element per row which is equal to 1. Correspondingly, one can see that $\mathbf{S}^H\mathbf{S}$ is a diagonal matrix where the diagonal entries are 1 if the corresponding antennas are active, and 0 otherwise. The selection matrix $\mathbf{S}$ satisfies $\mathbf{S}\mathbf{S}^H = \mathbf{I}_{N_s}$. Then, the selected channel matrix is

$$\mathbf{H}_s = \mathbf{H}\mathbf{S}^H. \tag{8}$$

The CRB of the target response matrix can be written as

$$\mathrm{CRB}(\mathbf{C}) = \frac{\sigma_r^2 N_r}{L}\mathrm{tr}((\mathbf{S}\mathbf{F}\mathbf{F}^H\mathbf{S}^H)^{-1}), \tag{9}$$

and the communication rate becomes

$$G_s = \log_2 |\mathbf{I}_{N_r} + \frac{1}{\sigma_c^2 N_s}\mathbf{H}_s\mathbf{S}\mathbf{F}\mathbf{F}^H\mathbf{S}^H\mathbf{H}_s^H|. \tag{10}$$

One can see that in (10), the selection matrix basically selects the channels along which the signal will propagate to the communication receiver, therefore, its choice will affect the communicate rate.

We propose a DFRC system design by optimally selecting matrices $\mathbf{F}$ and $\mathbf{S}$, so that the weighted sum of the communication rate and the inverse CRB is maximized, subject to transmit power constraints and a special structure for $\mathbf{S}$, i.e.,

$$\begin{aligned} \max_{\mathbf{F},\mathbf{S}} \quad & G_s + \lambda(1/\mathrm{tr}((\mathbf{S}\mathbf{F}\mathbf{F}^H\mathbf{S}^H)^{-1})) \\ \text{s.t.} \quad & \mathrm{tr}(\mathbf{S}\mathbf{F}\mathbf{F}^H\mathbf{S}^H) \leq P \\ & \mathbf{S} \text{ is a valid selection matrix.} \end{aligned} \tag{11}$$

where $\lambda$ is the coefficient to trade-off between communication and radar. A larger coefficient means focusing more on minimizing the CRB.

## 3. SOFTMAX CO-DESIGN

Given the selection matrix, one can easily find the optimal matrix $\mathbf{F}$. However, finding the selection matrix is not easy. Here, we propose to use the learning approach in [16] to co-design $\mathbf{F}$ and the antenna selection matrix $\mathbf{S}$.

Each row of selection matrices $\mathbf{S}$ can be modeled by a separate softmax neural network [23]. Taking the antenna selection matrix $\mathbf{S}$ as an example, the outputs of the $m$-th network will be

$$s_{m,i} = \frac{\exp(\mathbf{w}_i^T \mathbf{x} + b_{m,i})}{\sum_{j=1}^{N_s} \exp(\mathbf{w}_j^T \mathbf{x} + b_{m,j})}, \quad i = 1, \ldots, N_t \tag{12}$$

where $\mathbf{w}_i$, $b_{m,i}$ are respectively the weights and biases, and $\mathbf{x}$ is the input. Note that $0 \leq s_{m,j} \leq 1$ and

$$\sum_{i=1}^{N_t} s_{m,i} = 1. \tag{13}$$

Essentially, $s_{m,i}$ represents the probability that antenna $i$ will be our $m$-th selected antenna.

Since the selection matrix does not depend on time $t$, the input $\mathbf{x}$ should be constant, and thus, the constant value $b_i' = \mathbf{w}_i^T \mathbf{x}$ can be merged into the bias term $b_i$. Without loss of generality, such a model is equivalent to a softmax model with $\mathbf{x} = 0$, where the only trainable parameters are the biases.

The approximate selection matrix, $\hat{\mathbf{S}}$, is formed based on the outputs $\mathbf{s}_i = [s_{m,1}, \ldots, s_{m,N_t}]$ of all the softmax models as its rows. Clearly, $\hat{\mathbf{S}}$ will be a soft selection matrix since the values $s_{m,i}$ range between 0 and 1. By the end of the training, the matrix should converge very close to hard binary values so the approximation will be successful.

In order to achieve a realistic solution, the softmax models must produce hard binary values. The following constraint enforces this requirement:

$$\sum_{i=1} s_{m,i}^2 = 1, \forall m. \tag{14}$$

Indeed, (14) holds iff $s_{mi} \in \{0, 1\}$. The 'if' part of this statement is obvious. The 'only if' part comes readily from (13) since

$$\Big[\sum_{i=1} s_{mi}\Big]^2 - \sum_{i=1} s_{mi}^2 = 0 \Rightarrow 2 \sum_{i \neq j} s_{mi} s_{mj} = 0$$

implying that at most one element of $\mathbf{s}_m$ can be equal to 1 and all other elements must be equal to 0. Combined with (13) this means that *exactly* one element of $\mathbf{s}_m$ is equal to 1 and all other elements are equal to 0.

We also need to impose another constraint since the same antenna can not be selected more than once, i.e.

$$s_{m,i} = 1 \Rightarrow s_{n,i} = 0, \forall n \neq m$$

If $s_{m,i} \in \{0, 1\}$ then the above constraint is equivalent to

$$\mathbf{s}_m^H \mathbf{s}_n = 0. \tag{15}$$

Combining (14) and (15) it follows that $\hat{\mathbf{S}}$ must be a valid selection matrix.

Using Lagrange multiplier, we can define the total loss function as

$$\begin{aligned}\mathcal{L}(\mathbf{S}, \mathbf{F}) = &-G_s - \lambda_1/(\text{tr}((\mathbf{SFF}^H\mathbf{S}^H)^{-1})) \\ &+ \lambda_2(\text{tr}(\mathbf{SFF}^H\mathbf{S}^H) - P)^2 + \lambda_3||\mathbf{SS}^H - \mathbf{I}_{N_s}||_F.\end{aligned} \tag{16}$$

where $\lambda_1$ $\lambda_2$ and $\lambda_3$ are the Lagrange multipliers, $\|\cdot\|_F$ denotes the matrix Frobenius norm and $\text{tr}((\mathbf{SFF}^H\mathbf{S}^H)^{-1})$ is replaced by $-1/\text{tr}((\mathbf{SFF}^H\mathbf{S}^H)^{-1})$ to better balance the terms in the total loss function. Note that for the third term of (16), to ensure that the difference between power budget and transmit power is positive, and to have a differentiable loss function, square is taken of the difference.

The trainable sets are the biases, $\mathbf{b}_m = [b_{m,1}, \ldots, b_{m,N_t}]$ in the softmax neural networks, and matrix $\mathbf{F}$. Both, $\mathbf{b}_m, \mathbf{F}$ are randomly generated in the initial stage. We propose a two-stage optimization approach, by alternating between optimizing over one set of parameters while fixing others. The algorithm runs for $N_{epoch}$ learning epochs and each alternating stage runs for a small number of steps $N_{step}$. The proposed scheme is shown in Algorithm 1. One can improve the speed of convergence by using other optimizers instead of gradient descent. In the simulations shown next we used the Adam optimizer [24].

**Algorithm 1:** Learn to select.

```
for epoch=1 to N_epochs do
    Fix R and optimize L w.r.t. b:
    for step=1 to N_steps do
        Update b_m,   m = 1, ..., N_s
    Fix b and optimize L w.r.t. F:
    for step=1 to N_steps do
        Update F
```

## 4. NUMERICAL RESULTS

Here we demonstrate the performance of the proposed method. In all experiments, the transmitting power constraint is set to 20dBm and we take $\sigma_c = \sigma_r = 1$. The length of DFRC symbols $L = 30$. The transmit array has $N_t = 16$ antennas spaced apart by half wavelength, and the communication receive array is equipped with $N_r = 24$ antennas, spaced apart by half wavelength. In the training process, the Adam

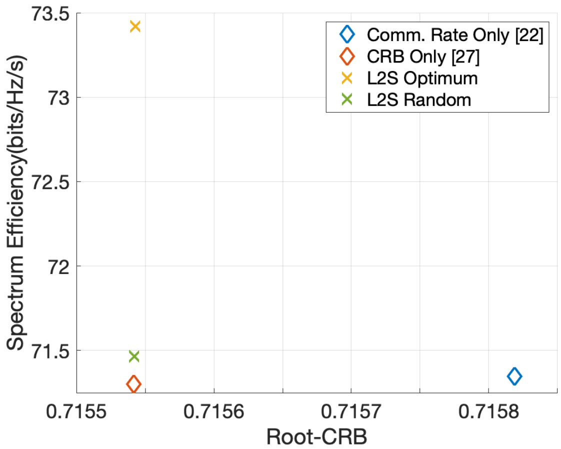


**Fig. 1**. Comparison of communication rate and CRB between schemes that (i) only optimize communication rate [20], (ii) only optimize CRB [25], (iii) apply L2S with optimum antenna selection, and (iv) apply L2S with random antenna selection.

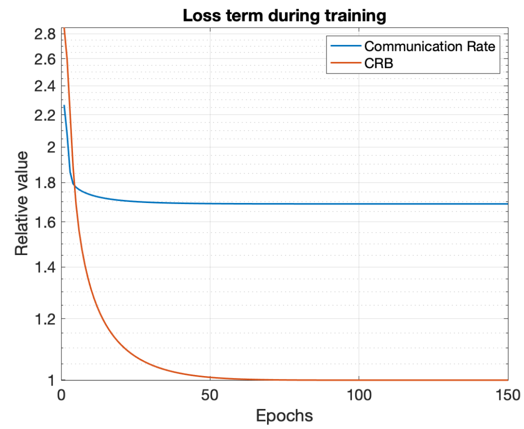


**Fig. 2**. Communication and CRB loss terms during training

stochastic optimization procedure of learning rate $0.1$ is used with a total of $N_{epoch} = 50$ epochs. In each epoch $N_{step} = 2$ steps are executed. The weights of CRB, power constraint and selection error are $\lambda_1 = 0.75$, $\lambda_2 = 0.05$ and $\lambda_3 = 0.1$, respectively. These weights indicate the importance of each loss term during the learning process.

In the first experiment, $N_s = 8$ antennas are selected to be active. In order to validate the performance of L2S, comparisons are made between three schemes: (i) use Eq.(4) to find the mean of maximum communication rate of all possible antenna combinations and the corresponding CRBs, i.e., optimizing only w.r.t. communication rate; (ii) use the closed-form solution from [25] to compute the mean of minimal CRBs of all antenna selection patterns and the corresponding communication rates, i.e., optimizing only w.r.t. CRB; (iii) randomly select antennas and only optimize the beamforming matrix $\mathbf{F}$ using L2S. In the third scheme, $10$ trials are repeated with same weights, where the X-axis is the square root of CRB which equals to the mean square error in angle estimation with an unbiased estimator, and Y-axis shows the spectrum efficiency of the communication system. As shown in Fig.1, L2S optimally selects the antenna activation pattern and designs the corresponding beamforming matrix which has the largest spectrum efficiency and a very small CRB. The root-CRB for L2S is only $1 \times 10^{-6}$ larger than the minimum CRB value while L2S is 2.12 more in communication rate than considering CRB only.

In the second experiment, in order to show that L2S can be efficiently scaled to larger problems as it avoids the combinatorial explosion of the classification problem, $32$ out of $64$ antennas are selected and same configuration is used except that the number of epochs is increased to $150$. Fig.2 shows the values of loss terms during the training where the orange line is the maximum communication rate from closed form solution divided by the computed communication rate and the blue line is the computed CRB divided by the closed form solution, respectively. For both loss terms, a value close to $1$ means a smaller difference towards its extremum. From Fig.2 one can see that in the first $10$ epochs both communication rate and CRB are jointly and efficiently optimized as both curved lines dropping fast. Then after the first $40$ epochs, communication rate first reaches convergence since we assign a larger weight to communication rate as compared with CRB. Last, CRB reaches convergence after the first $50$ epochs.

In a classification problem, such a selection would result in selecting one from $1.8 \times 10^{18}$ possible combinations, and for each choice we would need to optimize with respect to the beamforming matrix. This would take an unacceptably long time to compute. On the other hand, running on a $2019$ MacBook pro with $2.3$ GHz $8$-Core Intel Core i9, the first experiment takes $3$ minutes to select $8$ out of $16$ antennas in $50$ epochs while the second experiment takes $18$ minutes to select $32$ out of $64$ antennas in $150$ epochs.

## 5. CONCLUSION

In this paper, we have proposed a joint design of MIMO DFRC system with antenna selection. In the scenario where the radar detects a distributed target, the use of antenna selection leads to a beamforming matrix which is naturally full-rank and avoids bad communication channels. The proposed L2S method leverages softmax neural networks to approximate a valid selection matrix and optimizes the trainable parameters in an alternating fashion. Compared with the classification method, the complexity of the softmax selection does not grow exponentially. Numerical results have been provided to validate the performance of the proposed approach, showing that the L2S method can achieve the desired joint radar and communication system design via selecting a limited number of antennas and optimizing the beamforming matrix with respect to the joint performance metric.

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