A NOVEL ANGULAR ESTIMATION METHOD IN THE PRESENCE OF NONUNIFORM NOISE

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ABSTRACT

A novel algorithm for direction-of-arrival (DOA) estimation in nonuniform sensor noise is developed. The diagonal nonuniform sensor noise covariance matrix is estimated by an iterative procedure, which only requires a few iterations. Using the generalized eigendecomposition of two matrices and the least squares, the noise subspace is refined and the noise covariance matrix is estimated iteratively. Since there is no need for knowledge of true DOAs when estimating the noise covariance matrix, our method is superior to most existing approaches. For the proposed noise covariance estimator, we also derive the asymptotic variance of one iteration. Numerical simulations are carried out to demonstrate the advantages of the proposed algorithm over existing state-of-the-art methods.

Index Terms— DOA estimation, subspace methods, nonuniform noise.

1. INTRODUCTION

The problem of finding direction-of-arrivals (DOAs) of desired signals arises in diverse practical applications such as wireless communication, automotive radar and sonar [1–6], to name a few. Although multiple rigorous approaches have been developed to tackle the DOA estimation problem in the presence of white uniform sensor noise [7–13], the often more practical assumptions of white nonuniform sensor noise and unknown noise field have drawn attention rather recently [14, 15]. It is worth mentioning that most of the methods designed for the former case are not applicable readily to the later cases. Therefore, devising proper methods which take into consideration the presence of nonuniform sensor noise is vital for many applications.

Several methods designed for the case of white nonuniform sensor noise have been presented recently in the literature. Particularly, the deterministic maximum likelihood (ML) estimator and corresponding Cramer-Rao bound (CRB) for both deterministic and stochastic signals have been derived in [14], while the stochastic ML estimator has been developed in [16]. A simple method has been devised in [17] that requires less computational cost than ML estimator and improves the DOA estimation accuracy. Moreover, via exploiting the ML and least squares (LS) criteria, two iterative methods referred to as iterative ML subspace estimation (IMLSE) and iterative least squares subspace estimation (ILSSE) have been developed in [18]. The aim of both the IMLSE and ILSSE is to estimate the signal subspace and noise covariance matrices first, followed by finding DOAs by identifying peaks of the multiple signal classification (MUSIC) pseudo-spectrum. In addition, it has been shown in [19] that the signal and noise subspaces can be separated by applying the eigendecomposition (ED) of the reduced covariance matrix (RCM) when sources are uncorrelated. In [19], the authors have developed also a rank minimization based approach for coping with the case of correlated sources. An efficient method referred to as non-iterative subspace-based (NISB) method has been developed recently in [20]. It achieves both high performance and low computational complexity. The essence of the NISB method is to find a proper estimate of the noise covariance matrix by employing the ED of the RCM [19], followed by identifying the noise subspace via applying the generalized eigendecomposition (GED) of the matrix pair of the sample covariance matrix (SCM) and the estimated noise covariance matrix.

In this paper, we develop a new method to address the problem of DOA estimation in the presence of white nonuniform sensor noise. The core of our method is to estimate the noise subspace and noise covariance matrix iteratively. Specifically, the noise subspace and noise covariance matrix are refined by exploiting the GED of two matrices and using the LS criteria, respectively, in iterative manner. A study of the asymptotic variance of such estimator is also conducted. Since we do not need to know the actual DOAs to estimate the noise covariance matrix, our approach can be considered superior to the majority of existing methods. The proposed noise covariance matrix estimator works efficiently for sensor arrays with arbitrary geometries and also in the presence of correlated sources. Our method is also appealing computationally because it only requires a few iterations to obtain a proper noise covariance matrix estimate. Following the computation of the noise subspace, the MUSIC framework is employed to identify DOAs. Simulation results are included to demonstrate the effectiveness of the proposed method.

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2. SIGNAL MODEL

Consider a uniform linear array (ULA) with M omni-directional sensors receiving L ($L < M$) independent narrowband signals emitted by L sources. The sources are located in the far-field at distinguished directions, denoted by θ_l, $l = 1, \cdots, L$. Then, the signal received by the sensor array at the time instant t is given as

$$\mathbf{x}(t) = \mathbf{A}(\boldsymbol{\theta})\mathbf{s}(t) + \mathbf{n}(t) \tag{1}$$

where $\mathbf{s}(t) \triangleq [s_1(t) \cdots s_L(t)]^T \in \mathbb{C}^L$ is the vector of source signals, $\mathbf{n}(t) \in \mathbb{C}^M$ denotes the sensor noise vector, $\boldsymbol{\theta} \triangleq [\theta_1 \cdots \theta_L]^T$ contains the source DOAs, and $\mathbf{A}(\boldsymbol{\theta}) \triangleq [\mathbf{a}(\theta_1) \cdots \mathbf{a}(\theta_L)] \in \mathbb{C}^{M\times L}$ with the steering vector $\mathbf{a}(\theta_l) = [1\ e^{-j2\pi \sin(\theta_l)d/\lambda}\ \cdots\ e^{-j2\pi(M-1)\sin(\theta_l)d/\lambda}]^T \in \mathbb{C}^M$ corresponding to lth DOA. Also here λ stands for the carrier wavelength and $d = \lambda/2$. For notation simplicity, $\mathbf{A}$ is used instead of $\mathbf{A}(\boldsymbol{\theta})$ hereafter.

The array covariance matrix can be written as

$$\mathbf{R} \triangleq E\{\mathbf{x}(t)\mathbf{x}^H(t)\} = \mathbf{A}\mathbf{P}\mathbf{A}^H + \mathbf{Q} \tag{2}$$

where $\mathbf{P} \in \mathbb{C}^{L\times L}$ and $\mathbf{Q} \in \mathbb{R}^{M\times M}$, respectively, denote the signal and noise covariance matrices defined as

$$\mathbf{P} \triangleq E\{\mathbf{s}(t)\mathbf{s}^H(t)\},\ \ \mathbf{Q} \triangleq E\{\mathbf{n}(t)\mathbf{n}^H(t)\}. \tag{3}$$

Considering the case of spatially and temporally uncorrelated nonuniform sensor noise that is zero-mean Gaussian, the noise covariance matrix can be written as

$$\mathbf{Q} = \mathrm{diag}\{[\sigma_1^2, \cdots, \sigma_M^2]\} \tag{4}$$

where $\mathrm{diag}\{\cdot\}$ stands for a diagonal matrix generated by plugging the entries of the bracketed argument into its main diagonal. In (4), σ_m^2, $m = 1, \cdots, M$ are the noise variances, which are considered to be nonidentical, i.e, $\sigma_i^2 \neq \sigma_j^2$ for $i \neq j$. When $\sigma_1^2 = \sigma_2^2 = \cdots = \sigma_M^2 = \sigma^2$, the noise covariance matrix is just a scaled identity matrix $\mathbf{Q} = \sigma^2\mathbf{I}_M$, i.e., the sensor noise is uniform. Whereas the latter case has been probed to a great extent in the literature, the former has been received more attention only in the recent years.

Because $\mathbf{R}$ is unknown in practice, the SCM is typically used, and it is given by

$$\hat{\mathbf{R}} \triangleq \frac{1}{N}\sum_{t=1}^{N} \mathbf{x}(t)\mathbf{x}^H(t) = \frac{1}{N}\mathbf{X}\mathbf{X}^H. \tag{5}$$

Here, the matrix signal notation is also used, that is,

$$\mathbf{X} = \mathbf{A}\mathbf{S} + \mathbf{N} \tag{6}$$

with $\mathbf{X} \triangleq [\mathbf{x}(1) \cdots \mathbf{x}(N)]$, $\mathbf{S} \triangleq [\mathbf{s}(1) \cdots \mathbf{s}(N)]$, $\mathbf{N} \triangleq [\mathbf{n}(1) \cdots \mathbf{n}(N)]$, and N being the number of snapshots.

3. NOISE COVARIANCE MATRIX ESTIMATION

It is desirable for an algorithm estimating $\mathbf{Q}$ that it would not require any knowledge of the true DOAs while providing an acceptable accuracy with an affordable computational cost. Furthermore, such algorithm should provide reliable results in extreme scenarios. The presence of closely located sources and small sample size are two examples of extreme scenarios. To develop such an algorithm, both sides of (2) are multiplied by $\mathbf{U} \in \mathbb{C}^{M\times(M-L)}$ which satisfies the following condition

$$\mathbf{A}^H\mathbf{U} = \mathbf{0}_{L\times(M-L)} \tag{7}$$

where the constraint $\mathbf{U}^H\mathbf{U} = \mathbf{I}_{M-L}$ is also imposed to avoid ambiguities in obtaining $\mathbf{U}$. It can be seen from (7) that the columns of $\mathbf{U}$ span the noise subspace. For the case of nonuniform noise, finding $\mathbf{U}$ is not as simple as for the uniform noise case where $\mathbf{U}$ is obtained by just calculating the eigenvectors of $\hat{\mathbf{R}}$.

Multiplying (2) by $\mathbf{U}$ and using (7), we find that [20]

$$\hat{\mathbf{R}}\mathbf{U} = \mathbf{Q}\mathbf{U} \tag{8}$$

where $\mathbf{R}$ is replaced by $\hat{\mathbf{R}}$. Knowing $\mathbf{Q}$, it can be observed from (8) that the columns of the best estimate of $\mathbf{U}$, denoted as $\hat{\mathbf{U}}$, are calculated as the $M-L$ eigenvectors corresponding to the $M-L$ smallest eigenvalues obtained after applying the GED to the pair of matrices $\left\{\hat{\mathbf{R}}, \mathbf{Q}\right\}$.

Moreover, according to (8), $\mathbf{Q}$ can be estimated by the LS approach. Towards this end, we formulate the following LS minimization problem with respect to $\mathbf{Q}$

$$\hat{\mathbf{Q}} = \arg\min_{\mathbf{Q}} \|(\hat{\mathbf{R}} - \mathbf{Q})\hat{\mathbf{U}}\|_{\mathrm{F}}^2 \tag{9}$$

where $\mathbf{U}$ is replaced by $\hat{\mathbf{U}}$ and $\|\cdot\|_{\mathrm{F}}$ denotes the the Frobenius norm of a matrix. Problem (9) needs to be solved subject to the constraint of $\mathbf{Q}$ being a diagonal matrix. The objective function of (9) can be rewritten as

$$\begin{aligned} f(\mathbf{Q}) &\triangleq \left\|(\hat{\mathbf{R}} - \mathbf{Q})\hat{\mathbf{U}}\right\|_{\mathrm{F}}^2 \\ &= \mathrm{trace}\left\{\left((\hat{\mathbf{R}} - \mathbf{Q})\hat{\mathbf{U}}\right)\left((\hat{\mathbf{R}} - \mathbf{Q})\hat{\mathbf{U}}\right)^H\right\} \\ &= \mathrm{trace}\left\{\hat{\mathbf{U}}\hat{\mathbf{U}}^H\hat{\mathbf{R}}^2\right\} - \mathrm{trace}\left\{\hat{\mathbf{R}}\hat{\mathbf{U}}\hat{\mathbf{U}}^H\mathbf{Q}\right\} \\ &\quad - \mathrm{trace}\left\{\hat{\mathbf{U}}\hat{\mathbf{U}}^H\hat{\mathbf{R}}\mathbf{Q}\right\} + \mathrm{trace}\left\{\hat{\mathbf{U}}\hat{\mathbf{U}}^H\mathbf{Q}^2\right\} \end{aligned} \tag{10}$$

where $\mathrm{trace}\{\cdot\}$ stands for the trace of a square matrix, and the properties $\|\mathbf{X}\|_{\mathrm{F}}^2 = \mathrm{trace}\left\{\mathbf{X}\mathbf{X}^H\right\}$, $\mathrm{trace}\left\{\mathbf{X}\mathbf{Y}\right\} = \mathrm{trace}\left\{\mathbf{Y}\mathbf{X}\right\}$, $\hat{\mathbf{R}} = \hat{\mathbf{R}}^H$, and $\mathbf{Q} = \mathbf{Q}^H$ are used. The partial derivative of (10) with respect to $\mathbf{Q}$ is obtained as

$$\frac{\partial f(\mathbf{Q})}{\partial \mathbf{Q}} = 2\mathcal{D}\left\{\hat{\mathbf{U}}\hat{\mathbf{U}}^H\right\}\mathbf{Q} - \mathcal{D}\left\{\hat{\mathbf{R}}\hat{\mathbf{U}}\hat{\mathbf{U}}^H + \hat{\mathbf{U}}\hat{\mathbf{U}}^H\hat{\mathbf{R}}\right\} \tag{11}$$

where the operator $\mathcal{D}\{\cdot\}$ generates a diagonal matrix by preserving the main diagonal of the bracketed matrix and setting all other entries to zero (see details in [21]). Equating (11) to zero, the optimal estimate of $\mathbf{Q}$ is obtained as

$$\hat{\mathbf{Q}} = \frac{1}{2}\mathcal{D}\left\{\hat{\mathbf{R}}\hat{\mathbf{U}}\hat{\mathbf{U}}^H + \hat{\mathbf{U}}\hat{\mathbf{U}}^H\hat{\mathbf{R}}\right\}\mathcal{D}\left\{\hat{\mathbf{U}}\hat{\mathbf{U}}^H\right\}^{-1}. \quad (12)$$

Because of the dependencies of calculating $\hat{\mathbf{Q}}$ and $\hat{\mathbf{U}}$ in (8) and (12), it is natural to use an iterative scheme for estimating $\hat{\mathbf{Q}}$ and $\hat{\mathbf{U}}$. It starts with properly initializing $\hat{\mathbf{Q}}$, denoted as $\hat{\mathbf{Q}}^{(0)}$. Then $\hat{\mathbf{U}}^{(0)}$ is estimated as the $M-L$ generalized eigenvectors of the pair $\{\hat{\mathbf{R}},\ \hat{\mathbf{Q}}^{(0)}\}$ corresponding to the $M-L$ smallest eigenvalues. Next, $\hat{\mathbf{Q}}^{(1)}$ is obtained via (12) after replacing $\hat{\mathbf{U}}$ with $\hat{\mathbf{U}}^{(0)}$. The alternations carry on until a predefined stopping criterion is satisfied. It is worth noting that any diagonal matrix with positive diagonal entries can be used as $\hat{\mathbf{Q}}^{(0)}$, however, we suggest to employ $\hat{\mathbf{Q}}^{(0)} = \mathcal{D}\{\hat{\mathbf{R}}\}$.

Algorithm 1: Noise Covariance Matrix Estimation

1: Compute $\hat{\mathbf{R}} = 1/N\sum_{t=1}^{N}\mathbf{x}(t)\mathbf{x}^H(t)$.
2: Set $i=0$, $\hat{\mathbf{Q}}^{(0)} = \mathcal{D}\{\hat{\mathbf{R}}\}$, and the maximum number of iterations $i_{max}=5$.
while $i \leq i_{max}$
3: Carry out the GED of the pair $\{\hat{\mathbf{R}},\ \hat{\mathbf{Q}}^{(i)})\}$ to obtain $\hat{\mathbf{U}}^{(i)}$ as the $M-L$ eigenvectors corresponding to the $M-L$ smallest eigenvalues.
4: Calculate $\hat{\mathbf{Q}}^{(i+1)}$ using (12).
5: set $i = i+1$.
end

The steps of the proposed algorithm for the noise covariance matrix estimation are outlined in Algorithm 1. Carrying out the GED of the pair $\{\hat{\mathbf{R}},\ \hat{\mathbf{Q}})\}$,[1] the noise subspace is obtained as the $M-L$ eigenvectors associated with the $M-L$ smallest eigenvalues. Although any subspace-based method can be adopted for DOA estimation, we use the spectral MUSIC method via finding the locations of L peaks in the following pseudo-spectrum

$$\mathbf{S}_{MU}(\theta) = \frac{1}{\mathbf{a}^H(\theta)\hat{\mathbf{U}}\hat{\mathbf{U}}^H\mathbf{a}(\theta)}. \quad (13)$$

The asymptotic mean square error (MSE) of the proposed noise covariance estimation method in (12) is given in the following proposition.

Proposition 1: The asymptotic variance of estimating each diagonal element of $\hat{\mathbf{Q}}$ in (12), given a particular $\hat{\mathbf{U}}$, is

$$\mathbb{E}\left\{\left(\Delta\sigma_m^2\right)^2\right\} = \frac{1}{2N\tau_m^2}\Big(\Re\left\{[\mathbf{R}]_{mm}\mathbf{v}_m^H\mathbf{R}\mathbf{v}_m\right\} + \Re\left\{\mathbf{v}_m^H(\mathbf{r}_m^T\otimes\mathbf{R})\mathbf{K}(\mathbf{d}_m\otimes\mathbf{I}_M)\mathbf{v}_m^*\right\}\Big) \quad (14)$$

where $\tau_m \triangleq [\hat{\mathbf{U}}\hat{\mathbf{U}}^H]_{mm}$ is the entry in the intersection of the ith row and jth column of the matrix $\hat{\mathbf{U}}\hat{\mathbf{U}}^H$, $\mathbf{v}_m \triangleq [\hat{\mathbf{U}}\hat{\mathbf{U}}^H]_{:,m}$ is the mth column of the aforementioned matrix, $\Re\{\cdot\}$ returns the real part of the bracketed argument, $\mathbf{r}_m$ denotes the mth column of $\mathbf{R}$, $\otimes$ denotes the Kronecker product, $\mathbf{K}$ is the commutation matrix and $\mathbf{d}_m \in \mathbb{R}^M$ is a vector with 1 on the mth position and 0 elsewhere.

Proof: Using (12), the mth diagonal entry of $\hat{\mathbf{Q}}$, denoted by $\hat{\sigma}_m^2$, can be written as

$$\hat{\sigma}_m^2 = \frac{\left(\mathbf{v}_m^H\hat{\mathbf{r}}_m + (\mathbf{v}_m^H\hat{\mathbf{r}}_m)^H\right)}{2\tau_m} = \frac{\Re\{\mathbf{v}_m^H\hat{\mathbf{r}}_m\}}{\tau_m} \quad (15)$$

where $\hat{\mathbf{r}}_m$ denotes the mth column of $\hat{\mathbf{R}}$. Expressing $\hat{\mathbf{r}}_m$ as $\hat{\mathbf{r}}_m = \mathbf{r}_m + \mathbf{\Delta r}_m$, where $\mathbf{\Delta r}_m$ is the estimation error of the mth column of the SCM, it can be found that

$$\Delta\sigma_m^2 = \frac{\Re\{\mathbf{v}_m^H\mathbf{\Delta r}_m\}}{\tau_m} \quad (16)$$

where $\Delta\sigma_m^2$ is the difference between the actual σ_m^2 and the estimate $\hat{\sigma}_m^2$, i.e., $\Delta\sigma_m^2 = \hat{\sigma}_m^2 - \sigma_m^2$. Consequently, the variance of $\Delta\sigma_m^2$ can be expressed as

$$\begin{aligned}\mathbb{E}\left\{\left(\Delta\sigma_m^2\right)^2\right\} &= \frac{1}{4\tau_m^2}\mathbb{E}\Big\{\left(\mathbf{v}_m^H\mathbf{\Delta r}_m + \mathbf{v}_m^T\mathbf{\Delta r}_m^*\right) \\ &\times\left(\mathbf{\Delta r}_m^H\mathbf{v}_m + \mathbf{\Delta r}_m^T\mathbf{v}_m^*\right)\Big\} \\ &= \frac{1}{4\tau_m^2}\Big(\mathbf{v}_m^H\mathbb{E}\left\{\mathbf{\Delta r}_m\mathbf{\Delta r}_m^H\right\}\mathbf{v}_m + \mathbf{v}_m^H\mathbb{E}\left\{\mathbf{\Delta r}_m\mathbf{\Delta r}_m^T\right\}\mathbf{v}_m^* \\ &+ \mathbf{v}_m^T\mathbb{E}\left\{\mathbf{\Delta r}_m^*\mathbf{\Delta r}_m^H\right\}\mathbf{v}_m + \mathbf{v}_m^T\mathbb{E}\left\{\mathbf{\Delta r}_m^*\mathbf{\Delta r}_m^T\right\}\mathbf{v}_m^*\Big).\end{aligned} \quad (17)$$

According to [22], the asymptotic covariance and pseudo-covariance matrices of the vector $\mathbf{\Delta r} \triangleq \mathrm{vec}\{(\hat{\mathbf{R}} - \mathbf{R})\} \in \mathbb{C}^{M^2}$ are

$$\mathbb{E}\left\{\mathbf{\Delta r}\mathbf{\Delta r}^H\right\} = \frac{1}{N}(\mathbf{R}^T\otimes\mathbf{R}) \quad (18)$$

$$\mathbb{E}\left\{\mathbf{\Delta r}\mathbf{\Delta r}^T\right\} = \frac{1}{N}(\mathbf{R}^T\otimes\mathbf{R})\mathbf{K}. \quad (19)$$

Using (18) and (19), it is straightforward to show that [23]

$$\mathbb{E}\left\{\mathbf{\Delta r}_m\mathbf{\Delta r}_m^H\right\} = \left(\frac{[\mathbf{R}]_{mm}}{N}\right)\mathbf{R} \quad (20)$$

$$\mathbb{E}\left\{\mathbf{\Delta r}_m\mathbf{\Delta r}_m^T\right\} = \frac{1}{N}(\mathbf{r}_m^T\otimes\mathbf{R})\mathbf{K}(\mathbf{d}_m\otimes\mathbf{I}_M). \quad (21)$$

Plugging (20) and (21) into (17) yields (14), which completes the proof. ∎

Remark 1: It is worth noting that (12) represents the power domain (PD) method [17] in an alternative way. The PD method estimates σ_m^2's as

$$\sigma_m^2 = \frac{\left(\mathbf{d}_m^T\mathbf{P}_{\mathbf{A}}^{\perp}\hat{\mathbf{r}}_m + \hat{\mathbf{r}}_m^H\mathbf{P}_{\mathbf{A}}^{\perp}\mathbf{d}_m\right)}{2\mathbf{d}_m^T\mathbf{P}_{\mathbf{A}}^{\perp}\mathbf{d}_m}, \quad m = 1,\cdots,M \quad (22)$$

[1] If there exist correlated sources, $\hat{\mathbf{R}}_{\mathrm{FB}} = \frac{1}{2}\left(\hat{\mathbf{R}} + \mathbf{J}_M\hat{\mathbf{R}}^*\mathbf{J}_M\right)$ is preferred over $\hat{\mathbf{R}}$ where $\mathbf{J}_M$ is the exchange matrix.

where $\mathbf{P}_{\mathbf{A}}^{\perp} \in \mathbb{C}^{M\times M}$ is the orthogonal projection matrix of the signal subspace, i.e., $\mathbf{P}_{\mathbf{A}}^{\perp} = \mathbf{I}_M - \mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H$. The primary difference between (12) and (22) is the use of $\hat{\mathbf{U}}\hat{\mathbf{U}}^H$ as an estimate of $\mathbf{P}_{\mathbf{A}}^{\perp}$ rather than conducting a multidimensional search like in [17] for an ML estimate. In general, such multidimensional search is known to be very computationally demanding. Due to its reduced computational requirements, the proposed method has significant advantage over the PD technique.

Remark 2: The sufficient number of iterations for achieving a precise estimate of the noise covariance matrix for $\hat{\mathbf{Q}}^{(0)} = \mathcal{D}\{\hat{\mathbf{R}}\}$ is 3–5 as will be shown in the next section.

4. SIMULATION RESULTS

We evaluate the performance of the proposed method and compare it to that of the other state-of-the-art algorithms. The methods used for comparison are the "NISB+MUSIC" method of [20], "IMLSE+MUSIC" method of [18], and "RTM+MUSIC" method of [19]. The nonuniform stochastic CRB [14] is used as the benchmark. A ULA with $M = 16$ sensors separated by half wavelength collecting $N = 8$ snapshots is considered, and 2000 Monte Carlo runs are conducted to calculate the root mean square error (RMSE) defined as

$$\mathrm{RMSE} = 10\log_{10}\sqrt{\frac{1}{2000L}\sum_{l=1}^{L}\sum_{i=1}^{2000}(\hat{\theta}_{l,i} - \theta_l)^2}.$$

The SNR is computed as $\mathrm{SNR} = \frac{\sigma_s^2}{M}\sum_{m=1}^{M}\frac{1}{\sigma_m^2}$ where σ_s^2 represents the identical powers of different sources. The sensor noise covariance matrix is set to $\mathbf{Q} = \mathrm{diag}\{[6,\ 2,\ 0.5,\ 2.5,\ 3,\ 10,\ 5.5,\ 30,\ 11,\ 1.2,\ 3.5,\ 18,\ 2,\ 8.5,\ 36,\ 6.5]\}$. As a result, the worst noise power ratio (WNPR) in these examples is

$$\mathrm{WNPR} = \frac{\sigma_{max}^2}{\sigma_{min}^2} = \frac{36}{0.5} = 72.$$

Fig. 1 shows the RMSE performance of the methods tested versus SNR for the setup of two uncorrelated sources with $\boldsymbol{\theta} = [-29^\circ, 18^\circ]$. We observe that the proposed method achieves a higher threshold performance than other methods tested. In addition, Fig. 2 shows the strengths of the methods tested against the presence of closely located sources for the case of uncorrelated sources. The setup used for producing Fig. 2 is $\boldsymbol{\theta} = [18^\circ, (18 + \Delta\theta)^\circ]$ and SNR = 10 dB with $\Delta\theta$ varying from 4° to 10°. Compared to the other methods tested, the performance of the proposed method is better as illustrated in Fig. 2.

5. CONCLUSION

A novel algorithm is presented to estimate DOAs when the sensor noise is nonuniform. Our algorithm iteratively estimates the nonuniform noise covariance matrix. Each iteration involves estimation of the noise subspace using GED first, followed by updating the noise covariance matrix using LS. In addition to being applicable to a wide variety of array geometries, the proposed noise covariance matrix estimator is also fast and easy to implement since it only requires a few iterations to provide accurate estimates. Simulation examples show that the proposed algorithm is superior to the existing approaches.

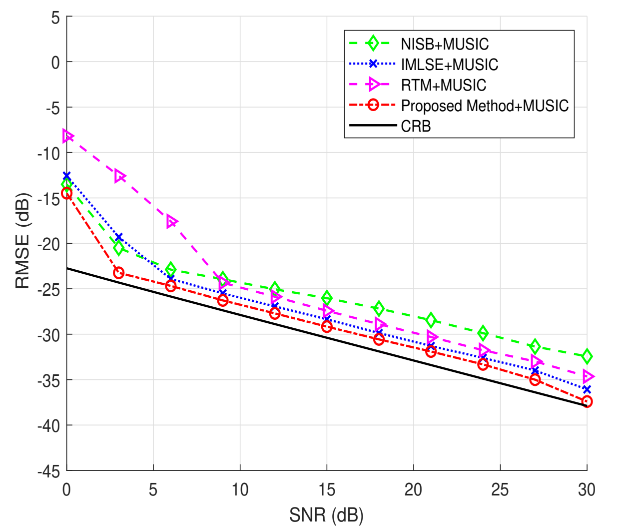

Fig. 1. RMSE vs. SNR for $L = 2$ uncorrelated sources with $\boldsymbol{\theta} = [-29^\circ, 18^\circ]$, $M = 16$, and $N = 8$.

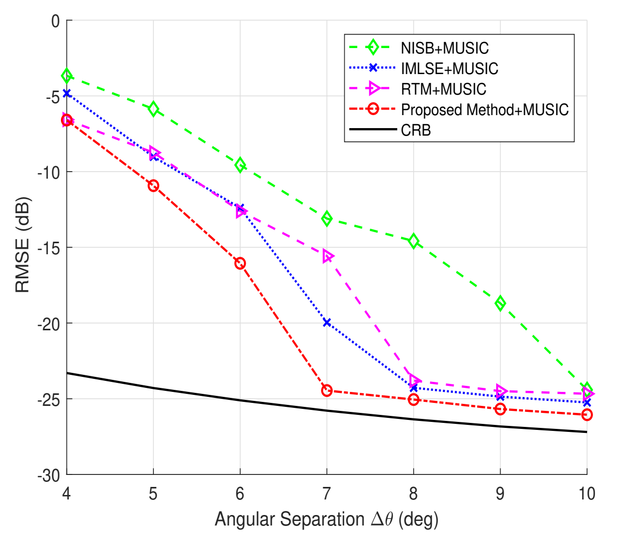

Fig. 2. RMSE vs. the angular separation for $L = 2$ uncorrelated sources with $\boldsymbol{\theta} = [18^\circ, (18 + \Delta\theta)^\circ]$, SNR = 10 dB, $M = 16$, and $N = 8$.

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