UNDERDETERMINED TWO-DIMENSIONAL LOCALIZATION FOR WIDEBAND SOURCES BASED ON DISTRIBUTED SENSOR ARRAY NETWORKS

Hantian Wu[1], *Qing Shen*[1], *Wei Liu*[2], *Yibao Liang*[1]

[1]School of Information and Electronics, Beijing Institute of Technology, 100081, China
[2]Department of Electronic and Electrical Engineering, University of Sheffield, S1 3JD, United Kingdom

ABSTRACT

In this paper, we consider the underdetermined two dimensional (2-D) source localization problem for wideband sources based on a distributed sensor array network, where a sparse sub-array is placed on each observation platform and the source number is larger than the sensor number of each sub-array. The received signals are first decomposed into different frequency bins via discrete Fourier transform (DFT), followed by the vectorization process to obtain the virtual array model with a larger aperture. Then, focusing is applied to the virtual array instead of the physical array for performance improvement, and a group sparsity based 2-D localization method exploiting the difference co-array is proposed, with increased DOFs for localization. Improved performance is achieved as demonstrated by computer simulations.

Index Terms— Distributed sensor array networks, group sparsity, focusing, two-dimensional localization, difference co-array.

1. INTRODUCTION

Recently, two-dimensional (2-D) localization based on distributed sensor array networks has received considerable attentions given its wide range of applications in radar, sonar, and wireless communications [1, 2]. Typically, a localization task can be achieved by analyzing the signal metrics such as receive signal strength (RSS) [3], time of arrival (TOA) [4–6], angle of arrival (AOA) [7, 8], and time difference of arrival (TDOA) [9, 10]. Compared with the others, the AOA based method is an attractive candidate since it is available for both active and passive sensing networks, and no synchronization is required among the distributed platforms [8].

Direction of arrival (DOA) estimation is an essential part of AOA based localization, and various high-resolution algorithms have been proposed, such as MUSIC [11], ESPRIT [12], and their extensions [13]. In the underdetermined case, the spatial smoothing based MUSIC (SS-MUSIC) [14, 15] and the compressive sensing (CS) based methods [16–18] can be employed to exploit the difference co-array (DCA) concept. For wideband DOA estimation, group sparsity based methods [19, 20] within the CS framework and the focusing algorithm [21,22] have been introduced, with the Cramér-Rao bound derived in [23, 24].

For 2-D localization based on distributed sensor array networks with multiple sub-arrays employed in widely separated spatial positions, the locations of near-field sources (near-field compared to the entire sensor array network, but still far-field compared with each sub-array) are usually estimated by combining all the DOA estimates obtained by each sub-array [2, 25, 26]. In [27, 28], the maximum likelihood estimator (MLE) is adopted for localization by minimizing the total DOA measurement errors among all distributed platforms under the least square sense, with the DOAs estimated at the first step using traditional methods. For complexity reduction, various low-complexity iterative methods [29] and closed-form location estimators [8, 30] are proposed. To further improve the estimation performance, a general group sparsity based 2-D localization method (GS-Localization) [31] is proposed, processing the collected information across all the observation platforms jointly instead of fusing the separately measured angle results. However, localization for the underdetermined case where the number of sources exceeds the sensor number of each sub-array is not feasible in the methods mentioned above, even if uncorrelated sources are employed.

As indicated in [23], the *a priori* knowledge of uncorrelated sources improves the resolution capacity of the array. In this paper, we focus on the 2-D source localization problem for wideband uncorrelated signals based on a distributed sensor array network, and present a general flexible approach capable of localizing more sources than the maximum sensor number of each sunarray with potential frequency and spatial diversity in one step. After decomposing the received signals into different frequency bins via discrete Fourier transform (DFT), the virtual array model corresponding to the DCA of each sub-array across all frequency bins is obtained by vectorizing the correlation matrix. Next, the focusing algorithm is applied to the virtual array instead of the physical array with reduced focusing model error achieved. Then, a group sparsity based 2-D localization method exploiting the DCA concept is proposed under the CS framework, referred to as DCA-GS-Localization, where the information of all sub-

This work was supported in part by the National Natural Science Foundation of China under Grant 61801028, and in part by the UK Engineering and Physical Sciences Research Council (EPSRC) under grants EP/T517215/1 and EP/V009419/1.

arrays is processed simultaneously to directly localize the positions of near-field sources, leading to improved estimation performance. It is noted that the increased DOFs associated with the *a priori* knowledge of uncorrelated sources can be fully exploited, and thus more sources can be localized than the existing method with the same array network.

2. SIGNAL MODEL

Consider a distributed sensor array network (as shown in Fig. 1(a)) composed of M sparse sub-arrays, each of which contains L_m sensors. There are K mutually uncorrelated sources impinging on this distributed sensor array network. Here, the sources standing apart in sufficient distances are near-field compared to the entire sensor array network, but still far-field compared to each sub-array (the distances between the subarrays and the sources are much larger than the subarray aperture). The locations of the k-th source ($k = 1, 2, \ldots, K$) and the m-th sub-array ($m = 1, 2, \ldots, M$) are represented by $\mathrm{T}_k(x_{T_k}, y_{T_k})$ and $\mathrm{U}_m(x_m, y_m)$, respectively.

The sensor position set of the m-th sub-array is given as

$$\mathbb{S}_m = \left\{\hbar_l^m d, 0 \leq l_m \leq L_m - 1\right\} , \tag{1}$$

where $\hbar_l^m$ denotes the position of the l-th sensor in the m-th sub-array, and d is the unit spacing.

For the m-th sub-array shown in Fig. 1(b), φ_m is the rotation angle, measured between the end-fire direction of the sub-array and the x-axis. $\phi_{m,k}$ is the incident angle of the k-th source measured between the direction of the impinging signal on the m-th sub-array and the y-axis, expressed as [31]

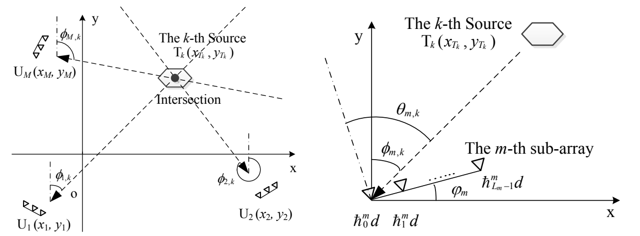
$$\begin{aligned}\phi_{m,k} &= \arctan 2(\Delta x_{m,k}, \Delta y_{m,k}) , \\ &= \begin{cases} \arctan(\frac{\Delta x_{m,k}}{\Delta y_{m,k}}), & \Delta y_{m,k} > 0, \\ \arctan(\frac{\Delta x_{m,k}}{\Delta y_{m,k}}) + \pi, & \Delta x_{m,k} \geq 0, \Delta y_{m,k} < 0, \\ \arctan(\frac{\Delta x_{m,k}}{\Delta y_{m,k}}) - \pi, & \Delta x_{m,k} < 0, \Delta y_{m,k} < 0, \\ +\pi/2, & \Delta x_{m,k} > 0, \Delta y_{m,k} = 0, \\ -\pi/2, & \Delta x_{m,k} < 0, \Delta y_{m,k} = 0, \\ \text{undefined}, & \Delta x_{m,k} = 0, \Delta y_{m,k} = 0, \end{cases}\end{aligned} \tag{2}$$

where $\arctan 2(\Delta x_{m,k}, \Delta y_{m,k}) \in (-\pi, \pi]$ represents the four-quadrant inverse tangent of $\Delta x_{m,k}$ and $\Delta y_{m,k}$, with $\Delta x_{m,k} = x_{T_k} - x_m$, and $\Delta y_{m,k} = y_{T_k} - y_m$.

Then, $\theta_{m,k} = \phi_{m,k} + \varphi_{m,k}$ represents the incident angle of the k-th source relative to the m-th sub-array.

The discrete source signal vector of the m-th sub-array is expressed as $\mathbf{s}_m[i] = \left[s_{m,1}[i], \ldots, s_{m,K}[i]\right]^T$, and denote $\mathbf{x}_m[i]$ as the $L_m \times 1$ column vector consisting of the observed signals in discrete form. The noise at different sensors are assumed to be white Gaussian and uncorrelated.

For the wideband case, each received signal is divided into non-overlapping groups with length L, followed by an



(a) A typical localization geometry. (b) A general array structure for the m-th sub-array.

Fig. 1. The localization geometry of a distributed sensor array network.

L-point DFT to obtain the output array model at the l-th frequency bin and the p-th group, given by

$$\mathbf{X}_m[l, p] = \mathbf{A}_m(l, \boldsymbol{\theta}_m)\mathbf{S}_m[l, p] + \overline{\mathbf{N}}_m[l, p] , \tag{3}$$

where $\mathbf{X}_m[l, p]$, $\mathbf{S}_m[l, p]$, and $\overline{\mathbf{N}}_m[l, p]$ are the DFTs of $\mathbf{x}_m[i]$, $\mathbf{s}_m[i]$, and the noise vector $\overline{\mathbf{n}}_m[i]$, respectively, where $l = 0, 1, \ldots, L-1$ and $p = 1, 2, \ldots, P$. $\mathbf{A}_m(l, \boldsymbol{\theta}_m) = \left[\mathbf{a}_m(l, \theta_{m,1}), \mathbf{a}_m(l, \theta_{m,2}), \ldots, \mathbf{a}_m(l, \theta_{m,K})\right]$ is the $L_m \times K$ steering matrix at $f_l = \frac{l}{L} f_s$ (f_s is the sampling frequency), with its column vector $\mathbf{a}_m(l, \theta_{m,k})$ representing the steering vector corresponding to the k-th source at frequency f_l, expressed as

$$\mathbf{a}_m(l, \theta_{m,k}) = \left[e^{-j\frac{2\pi\hbar_0^m d \sin(\theta_{m,k})}{\lambda_f}}, \cdots, e^{-j\frac{2\pi\hbar_{L_m-1}^m d \sin(\theta_{m,k})}{\lambda_f}}\right]^T , \tag{4}$$

where $\lambda_f = c/f_l$ represents the wavelength corresponding to f_l, and c is the wave propagation speed.

3. UNDERDETERMINED 2-D LOCALIZATION BASED ON THE DIFFERENCE CO-ARRAY

In this section, based on the *a priori* knowledge of uncorrelated sources, the 2-D localization method for wideband near-field sources exploiting the difference co-array is proposed, leading to improved performance in terms of both resolution capacity and estimation accuracy.

3.1. Focusing on the Difference Co-Array

As evaluated in [21], focusing on the virtual array instead of the physical array improves the DOA estimation performance for a single linear array. This idea is further extended to the 2-D localization scenario in this subsection.

For the l-th frequency bin, the correlation matrix of the m-th sub-array is calculated by

$$\begin{aligned}\mathbf{R}_{\mathbf{X}_m}[l] &= \mathrm{E}\left\{\mathbf{X}_m[l, p] \cdot \mathbf{X}_m^H[l, p]\right\} \\ &= \sum_{k=1}^{K} \sigma_{m,k}^2[l]\mathbf{a}_m(l, \theta_{m,k})\mathbf{a}_m^H(l, \theta_{m,k}) + \sigma_{m,\bar{n}}^2[l]\mathbf{I}_{L_m} ,\end{aligned} \tag{5}$$

where $\sigma_{m,k}^2[l]$ and $\sigma_{m,\bar{n}}^2[l]$ are the power of the k-th signal and the noise received by the m-th sub-array, respectively, and $\mathbf{I}_{L_m}$ is an $L_m \times L_m$ identity matrix.

Then, we obtain a virtual array model corresponding to the difference co-array of the m-th sub-array by vectorizing $\mathbf{R}_{\mathbf{X}_m}[l]$, given as

$$\begin{aligned}\mathbf{z}_m[l] &= \text{vec}\left\{\mathbf{R}_{\mathbf{X}_m}[l]\right\} \\ &= \tilde{\mathbf{A}}_m(l,\boldsymbol{\theta}_m)\tilde{\mathbf{s}}_m[l] + \sigma_{m,\bar{n}}^2[l]\tilde{\mathbf{I}}_{L_m^2},\end{aligned} \tag{6}$$

where the equivalent steering matrix of the virtual array $\tilde{\mathbf{A}}_m(l,\boldsymbol{\theta}_m) = \left[\tilde{\mathbf{a}}_m(l,\theta_{m,1}),\ldots,\tilde{\mathbf{a}}_m(l,\theta_{m,K})\right]$ is composed of equivalent steering vectors $\tilde{\mathbf{a}}_m(l,\theta_{m,k}) = \mathbf{a}_m^*(l,\theta_{m,k}) \otimes \mathbf{a}_m(l,\theta_{m,k})$ with $\otimes$ as the Kronecker product, and $\tilde{\mathbf{s}}_m[l] = \left[\sigma_{m,1}^2[l],\ldots,\sigma_{m,K}^2[l]\right]^T$ is the $K \times 1$ equivalent source signal vector. $\tilde{\mathbf{I}}_{L_m^2}$ is an $L_m^2 \times 1$ column vector obtained by vectorizing $\mathbf{I}_{L_m}$.

Denote f_r as the reference frequency for focusing which corresponds to the l_r-th frequency bin. The rotational signal-subspace (RSS) focusing matrix of the m-th sub-array is given by $\mathbf{T}_m[l] = \mathbf{V}_m[l]\mathbf{U}_m^H[l]$, where the column vectors in $\mathbf{U}_m^H[l]$ and $\mathbf{V}_m^H[l]$ are the left and right singular vectors of the matrix $\tilde{\mathbf{A}}_m(l,\boldsymbol{\theta_F})\tilde{\mathbf{A}}_m^H(l_r,\boldsymbol{\theta_F})$, respectively, with $\boldsymbol{\theta_F}$ holding the angles involved for focusing.

Then, for the m-th sub-array, we have

$$\begin{aligned}\mathbf{y}_m[l] &= \mathbf{T}_m[l]\mathbf{z}_m[l] \\ &= \mathbf{T}_m[l]\tilde{\mathbf{A}}_m(l,\boldsymbol{\theta}_m)\tilde{\mathbf{s}}_m[l] + \mathbf{T}_m[l]\sigma_{m,\bar{n}}^2[l]\tilde{\mathbf{I}}_{L_m^2} \\ &\approx \tilde{\mathbf{A}}_m(l_r,\boldsymbol{\theta}_m)\tilde{\mathbf{s}}_m[l] + \mathbf{T}_m[l]\sigma_{m,\bar{n}}^2[l]\tilde{\mathbf{I}}_{L_m^2}\ .\end{aligned} \tag{7}$$

Assume that there are J frequency bins of interest indexed by l_j, $j = 0,1,\ldots,J-1$. The focused virtual array model is obtained by averaging array models at different frequencies of interest, expressed as

$$\begin{aligned}\bar{\mathbf{y}}_m &= \sum\nolimits_{j=0}^{J-1}\mathbf{y}_m[l] \\ &= \tilde{\mathbf{A}}_m(l_r,\boldsymbol{\theta}_m)\tilde{\mathbf{p}}_m + \sum\nolimits_{j=0}^{J-1}\mathbf{T}_m[l_j]\sigma_{m,\bar{n}}^2[l_j]\tilde{\mathbf{I}}_{L_m^2}\ ,\end{aligned} \tag{8}$$

where the $L_m \times 1$ column vector $\tilde{\mathbf{p}}_m = \sum_{j=0}^{J-1}\tilde{\mathbf{s}}_m[l_j]$ is the equivalent signal vector after focusing. Under the white Gaussian noise assumption, we have $\sigma_{m,\bar{n}}^2[l_j] = \sigma_{m,\bar{n}}^2$, $\forall j = 0,1,\ldots,J-1$.

Denote $\overline{\mathbf{T}}_m = \sum_{j=0}^{J-1}\mathbf{T}_m[l_j]\tilde{\mathbf{I}}_{L_m^2}$, (8) can be rewritten as

$$\bar{\mathbf{y}}_m = \tilde{\mathbf{A}}_m(l_r,\boldsymbol{\theta}_m)\tilde{\mathbf{p}}_m + \sigma_{m,\bar{n}}^2\overline{\mathbf{T}}_m. \tag{9}$$

3.2. 2-D Localization Exploiting the Difference Co-Array

In this subsection, a 2-D localization method under the CS framework working on the difference co-array with the focused virtual array model, referred to as DCA-GS-Localization, is proposed, exploiting the information acquired by all sub-arrays jointly to form the final localization results.

The incident angles of the same source for different sub-arrays are distinct, i.e., $\phi_{m_1,k} \neq \phi_{m_2,k}$ for $m_1 \neq m_2$, whereas all the sources are still far-field compared to the sub-array aperture.

Assume that the square shaped area of interest in the Cartesian coordinate system is divided into K_xK_y grids, where K_x and K_y represent the number of grids along the x-axis and y-axis, respectively. Denote $G(x_{k_x}, y_{k_y})$, $k_x = 0,1,\ldots,K_x-1$ and $k_y = 0,1,\ldots,K_y-1$, as the position of the (k_x,k_y)-th search grid. Then, its corresponding incident angle $\theta_{g,m}(k_x,k_y)$ is given by

$$\theta_{g,m}(k_x,k_y) = \arctan 2(\Delta x_{m,k_x}, \Delta y_{m,k_y}) + \varphi_m\ , \tag{10}$$

with $\Delta x_{m,k_x} = x_{k_x} - x_m$, and $\Delta y_{m,k_y} = y_{k_y} - y_m$.

By stacking all potential incident angles, we construct a $K_xK_y \times 1$ vector $\tilde{\boldsymbol{\theta}}_{\boldsymbol{g},m}$

$$\begin{aligned}\tilde{\boldsymbol{\theta}}_{\boldsymbol{g},m} = \big[&\theta_{g,m}(0,0), \theta_{g,m}(0,1), \cdots, \theta_{g,m}(0,K_y-1), \\ &\cdots, \theta_{g,m}(K_x-1,0), \cdots, \theta_{g,m}(K_x-1,K_y-1)\big]^T\ .\end{aligned} \tag{11}$$

Then, for each sub-array, an overcomplete representation of the equivalent steering matrix is constructed by $\tilde{\mathbf{A}}_{\mathbf{g},m}(l,\tilde{\boldsymbol{\theta}}_{\boldsymbol{g},m}) = \left[\tilde{\mathbf{a}}(l,\theta_{g,m}(0,0)),\ldots,\tilde{\mathbf{a}}(l,\theta_{g,m}(K_x-1,K_y-1))\right]$. The column vector $\tilde{\mathbf{s}}_{\mathbf{g},m}[l]$ is composed of K_xK_y elements, with each element representing a potential source signal at the corresponding incident angle associated with the grid position.

The right side of (9) can be represented under the CS framework, denoted by

$$\tilde{\mathbf{b}}_m = \tilde{\mathbf{A}}_{\mathbf{g},m}(l,\tilde{\boldsymbol{\theta}}_{\boldsymbol{g},m})\tilde{\mathbf{p}}_{\mathbf{g},m} + \sigma_{m,\bar{n}}^2\overline{\mathbf{T}}_m\ , \tag{12}$$

where the search grid employed in $\tilde{\mathbf{A}}_{\mathbf{g},m}(l,\tilde{\boldsymbol{\theta}}_{\boldsymbol{g},m})$ and $\tilde{\mathbf{p}}_{\mathbf{g},m}$ is replaced by $\tilde{\boldsymbol{\theta}}_{\boldsymbol{g},m}$.

Then, column vectors $\tilde{\mathbf{y}}$ and $\tilde{\mathbf{b}}$ with the size of $\sum_{m=1}^{M} {L_m}^2 \times 1$ as well as a $K_xK_y \times M$ matrix $\tilde{\mathbf{U}}_{\mathbf{g}}$ are generated by

$$\begin{aligned}\tilde{\mathbf{y}} &= \left[\bar{\mathbf{y}}_1^T, \bar{\mathbf{y}}_2^T, \ldots, \bar{\mathbf{y}}_M^T\right]^T\ , \\ \tilde{\mathbf{b}} &= \left[\tilde{\mathbf{b}}_1^T, \tilde{\mathbf{b}}_2^T, \ldots, \tilde{\mathbf{b}}_M^T\right]^T\ , \\ \tilde{\mathbf{U}}_{\mathbf{g}} &= \left[\tilde{\mathbf{p}}_{\mathbf{g},1}, \tilde{\mathbf{p}}_{\mathbf{g},2}, \ldots, \tilde{\mathbf{p}}_{\mathbf{g},M}\right]\ ,\end{aligned} \tag{13}$$

with row vector $\tilde{\mathbf{u}}_{\mathbf{g}}^{k_g}, 0 \leq k_g \leq K_xK_y - 1$, as the k_g-th row of the matrix $\tilde{\mathbf{U}}_{\mathbf{g}}$.

Based on the generated grids, the entries in each row vector $\tilde{\mathbf{U}}_{\mathbf{g}}$ share the same two-dimensional support since they are associated with the same location $G(x_{k_x}, y_{k_y})$, where $k_g = k_xK_y + k_y$. Then, a $(K_xK_y+1) \times 1$ column vector $\tilde{\mathbf{u}}_{\mathbf{g}}^{\circ}$ is constructed by applying ℓ_2 norm to $\tilde{\mathbf{u}}_{\mathbf{g}}^{k_g}$ and the noise terms across all the sub-arrays, given by

$$\tilde{\mathbf{u}}_{\mathbf{g}}^{\circ} = \left[\|\tilde{\mathbf{u}}_{\mathbf{g}}^{0}\|_2, \|\tilde{\mathbf{u}}_{\mathbf{g}}^{1}\|_2, \cdots, \|\tilde{\mathbf{u}}_{\mathbf{g}}^{K_xK_y-1}\|_2, \sigma_{\bar{n}}^2\right]\ , \tag{14}$$

where $\sigma_{\bar{n}}^2 = \|[\sigma_{1,\bar{n}}^2, \sigma_{2,\bar{n}}^2, \ldots, \sigma_{M,\bar{n}}^2]\|_2$ is also considered as an unknown variable to be estimated.

Due to the shared 2-D support, the locations of the sources can be estimated jointly, and the proposed group sparsity based 2-D localization method exploiting the difference co-array concept (DCA-GS-Localization) can be formulated as

$$\begin{aligned} &\min_{\tilde{\mathbf{u}}_{\mathbf{g}}^{\circ}} \ \|\tilde{\mathbf{u}}_{\mathbf{g}}^{\circ}\|_1 \\ &\text{subject to} \ \|\tilde{\mathbf{y}} - \tilde{\mathbf{b}}\|_2 \leq \varepsilon \ , \end{aligned} \tag{15}$$

where ε is the allowable error bound. The first $K_x K_y$ elements in $\tilde{\mathbf{u}}_{\mathbf{g}}^{\circ}$ are the corresponding localization results over the $K_x K_y$ search grids, which are finally translated into the 2-D source positions in the Cartesian coordinate system.

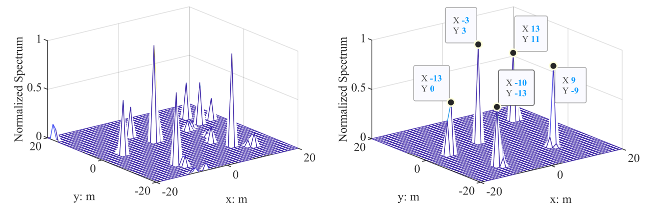
Remark 1: The DCA-GS-Localization method is proposed for underdetermined source localization from the perspective of virtual array aperture extension by adopting the *a priori* knowledge of uncorrelated sources. By forming a larger difference co-array and performing focusing on the co-array, increased estimation performance and DOFs (exceeding the sensor number in each sub-array) can be achieved.

4. SIMULATION RESULTS

Consider a distributed sensor array network with $M = 6$ sub-arrays located at positions $U_1(0, -30)$, $U_2(25, 10)$, $U_3(-25, 30)$, $U_4(-25, 0)$, $U_5(0, 30)$, and $U_6(25, -25)$, with rotation angles being $0°$, $110°$, $-135°$, $-90°$, $180°$, and $45°$, respectively. Each sub-array has $L_m = 4$ sensors with positions given by $\mathbb{S}_m = \{0, 1, 4, 6\}d$, $\forall m = 1, 2, \ldots, 6$. The normalized frequency of interest ranges from 0.75π to π with the center frequency $f_r = 0.875\pi$ chosen as the reference. The unit spacing $d = \lambda_r/2$, where λ_r represents the wavelength corresponding to f_r. The number of DFT points is $L = 64$. There are $K = 5$ targets at positions $T_1(-10, -13)$, $T_2(-3, -3)$, $T_3(13, 11)$, $T_4(9, -9)$, and $T_5(-13, 0)$, and the square area of interest in the Cartesian coordinate system is represented by (x, y), with $-20 \leq x \leq 20$ m and $20 \leq y \leq 20$ m.

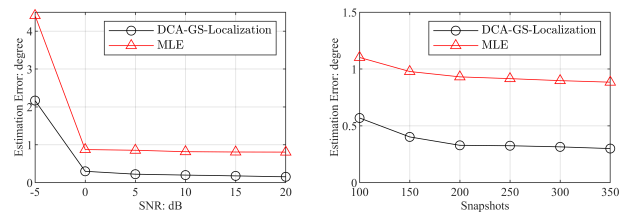
For the first set of simulations, the input signal-to-noise ratio (SNR) is fixed at 0 dB, and the number of snapshots for each frequency bin is 500. A large step size of 1 m is utilized for localization, and the localization results obtained by different methods are displayed in Fig. 2, where the results of the existing GS-Localization [31] is shown in Fig. 2(a), while the results obtained by the proposed DCA-GS-Localization is given in Fig. 2(b). Clearly, the existing method has failed, while the proposed one has successfully identified the five sources with only four sensors at each sub-array.

Then, the root mean square error (RMSE) results with respect to the input SNR and the number of snapshots are provided in Fig. 3(a) (500 snapshots) and Fig. 3(b) (0 dB SNR), respectively, where each point is an averaged result via Monte carlo simulations of 200 trials and a small step size of 0.05 m



(a) Results by GS-Localization. (b) Results by the proposed DCA-GS-Localization.

Fig. 2. Localization results obtained by different methods.



(a) RMSE versus input SNR. (b) RMSE versus the number of snapshots.

Fig. 3. RMSE results of different methods.

is adopted. Here, the MLE refers to the commonly used AOA-fusion method with AOAs estimated by applying SS-MUSIC to the focused virtual array model (9) for each sub-array. It is obvious that a better performance has been achieved by the proposed DCA-GS-Localization than the MLE since MLE is sensitive to the accuracy of individual AOA estimation results.

5. CONCLUSION

The underdetermined 2-D localization problem for wideband uncorrelated sources based on a distributed sensor array network was studied, where the number of sources exceeds the sensor number of each sub-array. The virtual array model at each frequency bin was obtained by vectorizing the corresponding subband correlation matrix after DFT decomposition, followed by focusing on the difference co-array instead of the physical array as a pre-processing step. Under the CS framework, the group sparsity based 2-D localization method exploiting the difference co-array (DCA-GS-Localization) was proposed, exploiting the *a priori* knowledge of uncorrelated sources. It has been shown by simulations that the proposed DCA-GS-Localization is capable of resolving more sources than the number of physical sensors of each sub-array, while the existing one fails. It has also shown by simulations that a better performance can be achieved by DCA-GS-Localization compared with the MLE.

6. REFERENCES

[1] M. Gavish and A. J. Weiss, "Performance analysis of bearing-only target location algorithms," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 28, no. 3, pp. 817–828, Jul. 1992.

[2] G. Mao, B. Fidan, and B. D. Anderson, "Wireless sensor network localization techniques," *Computer networks*, vol. 51, no. 10, pp. 2529–2553, 2007.

[3] C. Liu, D. Fang, Z. Yang, H. Jiang, X. Chen, W. Wang, T. Xing, and L. Cai, "RSS distribution-based passive localization and its application in sensor networks," *IEEE Trans. Wireless Commun.*, vol. 15, no. 4, pp. 2883–2895, Apr. 2016.

[4] D. B. Jourdan, D. Dardari, and M. Z. Win, "Position error bound for uwb localization in dense cluttered environments," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 44, no. 2, pp. 613–628, 2008.

[5] I. Guvenc and C.-C. Chong, "A survey on TOA based wireless localization and nlos mitigation techniques," *IEEE Communications Surveys & Tutorials*, vol. 11, no. 3, Aug. 2009.

[6] A. Conti, M. Guerra, D. Dardari, N. Decarli, and M. Z. Win, "Network experimentation for cooperative localization," *IEEE Journal on Selected Areas in Communications*, vol. 30, no. 2, pp. 467–475, 2012.

[7] J. Xu, M. Ma, and C. L. Law, "AOA cooperative position localization," in *Proc. IEEE Global Telecommunications Conference (GLOBECOM)*, 2008, pp. 1–5.

[8] Y. Wang and K. Ho, "An asymptotically efficient estimator in closed-form for 3-D AOA localization using a sensor network," *IEEE Trans. Wireless Commun.*, vol. 14, no. 12, pp. 6524–6535, Dec. 2015.

[9] T. Rappaport, J. Reed, and B. Woerner, "Position location using wireless communications on highways of the future," *IEEE Communications Magazine*, vol. 34, no. 10, pp. 33–41, 1996.

[10] J. Huang and Q. Wan, "Analysis of TDOA and TDOA/SS based geolocation techniques in a non-line-of-sight environment," *Journal of Communications and Networks*, vol. 14, no. 5, pp. 533–539, 2012.

[11] R. O. Schmidt, "Multiple emitter location and signal parameter estimation," *IEEE Trans. Antennas Propag.*, vol. 34, no. 3, pp. 276–280, Mar. 1986.

[12] R. Roy and T. Kailath, "ESPRIT-estimation of signal parameters via rotational invariance techniques," *IEEE Trans. Acoust., Speech, Signal Process.*, vol. 37, no. 7, pp. 984–995, Jul. 1989.

[13] W. Du and R. L. Kirlin, "Improved spatial smoothing techniques for DOA estimation of coherent signals," *IEEE Trans. Signal Process.*, vol. 39, no. 5, pp. 1208–1210, May 1991.

[14] P. Pal and P. P. Vaidyanathan, "Nested arrays: a novel approach to array processing with enhanced degrees of freedom," *IEEE Trans. Signal Process.*, vol. 58, no. 8, pp. 4167–4181, Aug. 2010.

[15] A. Raza, W. Liu, and Q. Shen, "Thinned coprime array for second-order difference co-array generation with reduced mutual coupling," *IEEE Trans. Signal Process.*, vol. 67, no. 8, pp. 2052–2065, 2019.

[16] Y. D. Zhang, M. G. Amin, and B. Himed, "Sparsity-based DOA estimation using co-prime arrays," in *Proc. IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*, Vancouver, Canada, May 2013, pp. 3967–3971.

[17] Q. Shen, W. Liu, W. Cui, and S. Wu, "Extension of co-prime arrays based on the fourth-order difference co-array concept," *IEEE Signal Process. Lett.*, vol. 23, no. 5, pp. 615–619, May 2016.

[18] Q. Shen, W. Liu, W. Cui, S. Wu, and P. Pal, "Simplified and enhanced multiple level nested arrays exploiting high-order difference co-arrays," *IEEE Trans. Signal Process.*, vol. 67, no. 13, pp. 3502–3515, Jul. 2019.

[19] Q. Shen, W. Liu, W. Cui, S. Wu, Y. D. Zhang, and M. G. Amin, "Low-complexity direction-of-arrival estimation based on wideband co-prime arrays," *IEEE/ACM Trans. Audio, Speech, Language Process.*, vol. 23, no. 9, pp. 1445–1456, Sep. 2015.

[20] Q. Shen, W. Cui, W. Liu, S. Wu, Y. D. Zhang, and M. G. Amin, "Underdetermined wideband DOA estimation of off-grid sources employing the difference co-array concept," *Signal Processing*, vol. 130, pp. 299–304, 2017.

[21] Q. Shen, W. Liu, W. Cui, S. Wu, Y. D. Zhang, and M. G. Amin, "Focused compressive sensing for underdetermined wideband DOA estimation exploiting high-order difference coarrays," *IEEE Signal Process. Lett.*, vol. 24, no. 1, pp. 86–90, Jan. 2017.

[22] W. Cui, Q. Shen, W. Liu, and S. Wu, "Low complexity DOA estimation for wideband off-grid sources based on re-focused compressive sensing with dynamic dictionary," *IEEE Journal of Selected Topics in Signal Processing*, vol. 13, no. 5, pp. 918–930, 2019.

[23] Y. Liang, W. Cui, Q. Shen, W. Liu, and S. Wu, "Cramér-Rao bound analysis of underdetermined wideband DOA estimation under the subband model via frequency decomposition," *IEEE Trans. Signal Process.*, 2021.

[24] Y. Liang, W. Cui, Q. Shen, W. Liu, and H. Wu, "Cramér-Rao bound for DOA estimation exploiting multiple frequency pairs," *IEEE Signal Process. Lett.*, 2021.

[25] N. Garcia, H. Wymeersch, E. G. Larsson, A. M. Haimovich, and M. Coulon, "Direct localization for massive mimo," *IEEE Trans. Signal Process.*, vol. 65, no. 10, pp. 2475–2487, May 2017.

[26] Z. Wang, J.-A. Luo, and X.-P. Zhang, "A novel location-penalized maximum likelihood estimator for bearing-only target localization," *IEEE Trans. Signal Process.*, vol. 60, no. 12, pp. 6166–6181, Dec. 2012.

[27] L. M. Kaplan, Q. Le, and N. Molnar, "Maximum likelihood methods for bearings-only target localization," in *IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP)*, vol. 5. IEEE, 2001, pp. 3001–3004.

[28] C. Wang, F. Qi, G. Shi, and X. Wang, "Convex combination based target localization with noisy angle of arrival measurements," *IEEE Wireless Communication Letters*, vol. 3, no. 1, pp. 14–17, Feb. 2014.

[29] J.-P. Le Cadre and C. Jaetffret, "On the convergence of iterative methods for bearings-only tracking," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 35, no. 3, pp. 801–818, Jul. 1999.

[30] Y. Wang and K. Ho, "Unified near-field and far-field localization for AOA and hybrid AOA-TDOA positionings," *IEEE Trans. Wireless Commun.*, vol. 17, no. 2, pp. 1242–1254, Feb. 2018.

[31] Q. Shen, W. Liu, L. Wang, and Y. Liu, "Group sparsity based localization for far-field and near-field sources based on distributed sensor array networks," *IEEE Trans. Signal Process.*, vol. 68, pp. 6493–6508, 2020.