FAST FAULT DIAGNOSIS METHOD OF ROLLING BEARINGS IN MULTI-SENSOR MEASUREMENT ENVIROMENT

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ABSTRACT

In this paper, a fast bearing state detection method based on multi-sensor signal fusion and compression feature extraction is proposed. The best estimation in the random weighted fusion algorithm is adaptively adjusted by the fluctuation factor to realize the high-precision fusion of variable signals and reduce the noise component in the signals. In the compressed sensing framework, a partial Hadamard matrix is selected as the measurement matrix, and the signal reconstruction is abandoned, leading to reduced average sampling rate and less data for signal acquisition, transmission, and extraction of fault features. The proposed method for diagnosis of rolling bearing fault is fast, effective, and accurate, as verified by experimental results.

Index Terms—Rolling bearings, multi-sensor signal fusion, compressed sensing, compression feature extraction, fault diagnosis

1. INTRODUCTION

As a key component of rotating machinery, rolling bearings must maintain good condition and realize real-time monitoring in order to avoid accidents [1 - 3]. Since the monitoring system and the acquisition device are usually located at different places, it is necessary to transmit a large number of bearings signals over a long distance, which will cause great pressure on the signal acquisition and transmission device [4, 5]. In order to ensure accuracy and integrity of the target signals, multiple sensors of the same type are usually used for synchronous monitoring, leading to the increase of the number of bearings signals collected and transmitted [6].

In the process of signal acquisition by single sensor, due to the influence of noise and other external conditions, it will inevitably cause the loss of key information, and the interference noise is usually large [7, 8]. In order to ensure that the target information will not be easily lost, multi-sensor acquisition methods are adopted for signal acquisition. Since the observation noise of each sensor is independent of each other, the signals acquired by multi-sensor can be fused to reduce system uncertainty and expand system observation range, leading to the improvement of system accuracy and enhancement of system reliability [9]. In addition, with the establishment of redundant observations, the system can still work normally even if one or more sensors fail [10]. At present, many signal fusion methods have been proposed, such as Kalman filter [11,12], fuzzy threshold theory [13,14] and random weighted fusion algorithm [15]. As a new type of signal fusion algorithm, random weighted fusion algorithm can effectively realize the ultra-high precision fusion of constant signals, and greatly suppress the noise component while retaining the target signal.

In the process of bearings vibration signal acquisition, the traditional Nyquist-Shannon sampling theorem limits the sampling frequency, which greatly increases the pressure on the collection and transmission equipment. As a new type of signal sampling theory, compressed sensing theory breaks through the limitation of Nyquist-Shannon sampling theorem and realizes synchronous compression in the process of signal sampling [16]. Li et al. [17] improved the sparse basis exploiting the features of bearings fault signal to realize a fast and efficient sparse decomposition of fault signal, and then extract the features. Ramakrishnan et al. [18] classified the data loss caused by noise or subject movement in sleep research as information loss caused by under-sampling, and the compressed sensing theory was used to reconstruct it. However, in engineering practice, the fault signal of mechanical system is often complex, with poor sparsity and difficult to reconstruct, so the fault signal reconstruction will take a lot of time and increase software cost [19].

With the above observations, this paper proposes a bearings fault diagnosis method based on multi-sensor signal fusion and compression feature extraction. Firstly, a random weighted fusion algorithm which can fuse variable signals with high precision and suppress noise is proposed. Secondly, under the framework of compressed sensing, the Hadamard measurement matrix is used to collect signals at a low average sampling rate. Combining it with signal fusion theory, the sampled and transmitted bearings signals can be greatly reduced. Finally, in order to improve the efficiency of fault diagnosis, the step of signal reconstruction is abandoned, and the discriminative feature extraction is directly performed on the compressed signal. With the integration of the above three aspects, the workload of bearings fault diagnosis will be greatly reduced, and in the process of signal fusion, compressed sampling and fault diagnosis, all the parameters involved in the proposed method can be adjusted adaptively.

2. BACKGROUND

2.1. Multi-sensor fusion technology

Traditional multi-sensor fusion technology can reduce noise and ensure the accuracy of fault diagnosis effectively in the process of constant signal weighted fusion. Take the random weighted fusion algorithm as an example:

Suppose there are P sensors simultaneously observing the same target, where the true value of the target signal is x , the observed values of the sensors are $\{x_1, x_2, ..., x_P\}$ and the weighting factors are $\{v_1, v_2, ..., v_P\}$ respectively. Each sensor contains different degrees of noise. If the collected value $\{x_1, x_2, ..., x_P\}$ of each sensor is independent of each other and is an unbiased estimate of x , then

$$\hat{x} = \sum_{i=1}^{P} v_i x_i \quad (\sum_{i=1}^{P} v_i = 1) \tag{1}$$

$$\sigma^2 = E[(x-\hat{x})^2] = E[\sum_{i=1}^{P} v_i^2 (x-x_i)^2 + 2\sum_{\substack{i=1,j=1 \\ i\neq j}}^{P} v_i v_j (x-x_i)(x-x_j)] = \sum_{i=1}^{P} v_i^2 \sigma_i^2 \tag{2}$$

As a classic fusion algorithm, the random weighted fusion algorithm mainly includes two parts, one is the determination of the best estimate, and the other is the calculation of the best weight. The best estimated value at k-th signal point in the random weighted fusion algorithm is solved as follows:

$$x_i^{'}(k) = \frac{1}{k}\sum_{j=1}^{k} x_i(j) \quad (i = 1,2,..., P) \tag{3}$$

When processing constant signals, the total mean square error of the system is as follows:

$$\hat{\sigma}^2 = E[\sum_{i=1}^{P} v_i^2 (x - x_i^{'}(k))^2] = \frac{1}{k}\sum_{i=1}^{P} v_i^2 \sigma_i^2 = \frac{1}{k}\sigma^2 \tag{4}$$

After being processed by the random weighted fusion algorithm, the total mean square error of the system can be controlled at $1/k$ before processing, and the variance value will decrease with the increase of k value. However, for variable signals, the best estimation value obtained in Eq. (4) is not applicable (especially when the estimated trues are derived from the bearings vibration signal with fast change speed), and the direct weighted fusion of the collected values of sensors will reduce the suppression effect of the algorithm on system error [20].

2.2. Compressed sensing theory

The mathematical expression of the compressed sensing algorithm can be expressed as

$$y = \Phi x \tag{5}$$

In Eq. (5), y is the observed M-dimensional vector, Φ is the measurement matrix and x is the original signal with dimension N ($M<<N$). The actual signal x is generally not sparse and can be expressed by sparse dictionary Ψ, the correlation can be expressed as

$$x = \Psi\theta \tag{6}$$

In Eq. (6), Ψ is the sparse dictionary. θ is a sparse representation vector of K sparse, that is, θ contains K non-zero items, and $K<<M<<N$ in general. Substitute Eq. (6) into Eq. (5) to get

$$y = \Phi\Psi\theta \tag{7}$$

It is an underdetermined problem to reconstruct the original signal x by using the observation matrix y, which cannot be solved directly. However, when x is a sparse signal, it can be transformed into a problem of solving the minimum l_0 norm, which is expressed by

$$\min\|\theta\|_0, \text{ s.t. } y = \Phi x \tag{8}$$

Solving Eq. (8) is a NP-hard problem, thus the restricted isometry property (RIP) is introduced:

$$(1-\delta_k)\|\theta\|_2^2 \le \|\Phi\theta\|_2^2 \le (1+\delta_k)\|\theta\|_2^2 \quad (0<\delta_k<1) \tag{9}$$

where δ_k is the RIP parameter. When the observation matrix satisfies RIP condition, the problem of minimizing l_0 norm can be transformed into solving the problem of minimizing l_1 norm.

$$\min\|\theta\|_1, \text{ s.t. } y = \Phi x \tag{10}$$

Sparse signals can be recovered by solving Eq. (10). It is worth noting that Eqs. (5) - (10) refer to the literature [21, 22]. According to the RIP theorem, the higher the degree of incoherence between the observation matrix and the transform base, the lower the requirement for signal sparsity.

3. PROPOSED METHOD

3.1. Random weighted fusion method based on signal fluctuation

In order to address the limitations of the random weighted fusion algorithm, the best estimation value is improved to make it suitable for variable signal, and the application range of the algorithm is extended. The best estimation is proposed as follows

$$x_i^{'}(k) = \begin{cases} x_i(k) & (k=1) \\ \tau x_i(k) + (1-\tau)x_i^{'}(k\text{-}1) & (\tau\in[0,1], k\ge 2) \end{cases} \tag{11}$$

The estimate of the true values from P sensors can be written as

$$x^{'} = \sum_{i=1}^{P} v_i x_i^{'} \tag{12}$$

where τ is the adaptive equalization factor, which can realize the adaptive adjustment of the proportional relationship between the sample value $x_i(k)$ at k-th point and the sample value $x_i(k-1)$ at (k-1)-th point, and then obtain the best estimated value $x_i^{'}(k)$. The system error of the sensor and the random error of the measurement can be further reduced by fully considering the change of the k-th signal point and the correlation between the k-th signal point and the pre-k-th signal point. The total mean square error $\sigma^{'2}$ is lower than the σ^2.

When τ value is larger, the weight of the current time acquisition value is larger, the size of the current acquisition value is more dominant , and the system error of the sensor cannot be controlled effectively, but it can adapt to the change of the signal. When the value of τ is smaller, the weight of the previous time acquisition value is larger, and the value of the previous acquisition value is more dominant. Hence, the system error can be controlled at a lower level, but cannot adapt to the signal changes. Therefore, in order to get better fusion results, the relationship between the current measurement value and the previous measurement value should be considered comprehensively.

Since the adaptive equalization factor $\tau(k)$ has a great correlation with the instantaneous change of signal, a method for the calculation of the equalization factor based on signal fluctuation is designed in this paper.

$$\tau(k) = \begin{cases} |\eta_i(k)|, & |\eta_i(k)| \le \lambda \\ \lambda, & |\eta_i(k)| > \lambda \end{cases} \quad (k=1,..., N) \tag{13}$$

$$\begin{cases} \eta_i(k) = (\xi_i(k)\text{-}\xi_i(k\text{-}1)) \\ \xi_i(k) = \dfrac{x_i(k) - (\sum_{j=1}^{k} x_i(j))/k}{|(\sum_{j=1}^{k} x_i(j))/k|} \\ 0 \le \lambda \le 1 \end{cases} \tag{14}$$

where $\xi_i(k)$ represents the signal fluctuation value at k-th point, The difference between the signal fluctuation value at the k-th signal point and the (k-1)-th signal point is defined as the relative fluctuation value $\eta_i(k)$ of the signal. λ is the relative fluctuation value threshold, and 0≤ λ ≤1. If the relative fluctuation value $\eta_i(k)$ at the k-th point exceeds 1, it is still used as the adaptive equalization factor for the k-th point, which will amplify the current signal value and interfere with the normal acquisition of the signal. In addition, when the relative fluctuation of the signal is less than 1, the relative error of the reconstructed signal can be suppressed.

3.2. Rolling bearings fault diagnosis method based on compression feature extraction

3.2.1. Construction of partial Hadamard measurement matrix

Definition 1 A discrete time signal x with N points, and N=2q (q is a positive integer), the discrete Hadamard change is defined as

$$X_H(k)=\sum_{n=1}^{N} x(n)(-1)^{\sum_{i=0}^{q-1} b_i(n)b_i(k)} \quad (k=1,...,N) \tag{15}$$

where X_H is the signal of x after Hadamard transform (X_H is also a discrete time signal with N points), $b_i(x)$ is the i-th bit in the binary form of a nonnegative integer. For example, the binary form of 8 can be expressed as 1000, then $b_1(8)=0$, $b_2(8)=0$, $b_3(8)=0$, $b_4(8)=1$. Discrete Hadamard transform is more intuitive to express with Hadamard matrix.

Definition 2 Suppose a matrix is an orthogonal square matrix composed of +1 and -1 elements. If it is satisfied:

$$HH^T = n\mathrm{I} \tag{16}$$

where I is an identity matrix, then H is a Hadamard matrix of n-th order. The second order canonical Hadamard matrix is defined as

$$H_2=\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \tag{17}$$

The fourth order canonical Hadamard matrix can be expressed as

$$H_4 = H_2 \otimes H_2 = \begin{bmatrix}H_2 & H_2\\ H_2 & -H_2\end{bmatrix}=\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\end{bmatrix} \tag{18}$$

It has been proved that the order of Hadamard matrix is a multiple of four except for the second-order standard Hadamard matrix.

The discrete Hadamard transform can be expressed by Hadamard matrix as

$$X_H = Hx \tag{19}$$

According to the principle of compressed sensing, the Hadamard matrix can be used to design and construct commonly used and effective measurement matrices—part of Hadamard measurement matrices. The acquisition of signals by Hadamard measurement matrix is equivalent to the discrete Hadamard transform of signals. It is worth noting that Eqs. (16) - (19) refer to the literature [23, 24].

To construct a partial Hadamard measurement matrix, First, it is necessary to construct an N×N Hadamard matrix, and then extract the first M rows in it to obtain part of the M×N Hadamard measurement matrix. The original signal with length N can be mapped to M-dimensional feature space by using the measurement matrix to collect signals. In addition, the measurement matrix is a deterministic measurement matrix, which is simple to construct and easy to implement.

Therefore, the partial Hadamard matrix is selected as the measurement matrix in this paper. Since Hadamard matrix is orthogonal matrix and discrete Hadamard transformation is orthogonal transformation, according to the property that the inner product of orthogonal transformation is invariant in Euclidean space, it can be proved that the energy of x sequence remains unchanged after discrete Hadamard transformation. The following expression can be obtained by [24]

$$\sum_{n=1}^{N} |x(n)|^2 = \sum_{k=1}^{N} |X_H(k)|^2 \tag{20}$$

It can be seen from Eq. (20) that the trend feature parameters of the original data can still be well preserved after Hadamard transformation. Combined with the root mean square formula, the root mean square value of the signal collected under the Hadamard matrix is directly proportional to the root mean square value of the original signal. The data sampled under the measurement matrix have little change to some statistical feature parameters, and is effective for specific statistical inference tasks. Therefore, the compressed sampling signal can be directly used to extract fault feature information.

3.2.2. Rolling bearings fault diagnosis

In the process of data reconstruction, along with reconstruction errors and software costs, the complexity and non-sparseness of faulty signals bring great challenges to signal reconstruction. In this paper, a fault diagnosis method based on compressed feature extraction is proposed under the framework of compressed sensing. According to the properties of measurement matrix and fault sensitive feature parameters, the process of signal reconstruction is omitted, and fault sensitive feature parameters are extracted directly from a small number of observations for fault diagnosis, which greatly reduces the time required for fault diagnosis. The specific implementation consists of the following three parts:

(1) Data collection. The compressed sensing theory is used to determine the measurement matrix as part of the Hadamard observation matrix, determine the compression ratio, and use the measurement matrix to realize data compression collection.

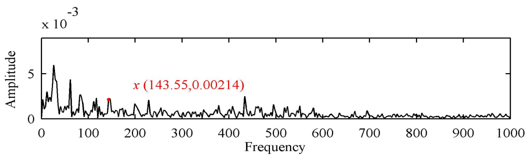
(2) Feature extraction. Select the appropriate time domain feature parameters as the fault sensitive feature parameters, and calculate the fault sensitive feature parameters of each group of data obtained by sampling.

(3) Fault diagnosis. The selected fault sensitive feature parameters is used to train support vector machine (SVM), and the fault type of impulse component is determined by using it.

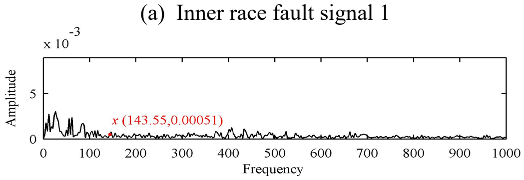
4. EXPERIMENT AND RESULT ANALYSIS

In order to verify the effectiveness of the fault diagnosis method proposed in this paper, the bearings vibration signals under different states (four states : normal, inner race fault (IF), outer race fault (OF) and ball fault (BF)) collected by the multi-sensor system are used as input to extract fault features, and the diagnosis accuracy is given through the SVM classifier. The speed of the motor in the experimental platform is steadily maintained at 1800 r/min, and the fundamental frequency of the rotating shaft is f_0 = 30 Hz. The acquisition system consists of 4 acceleration sensors and an NI 9234 acceleration acquisition card. The design and arrangement of the experimental setup conform to the requirements of the international standard ISO 10816t.

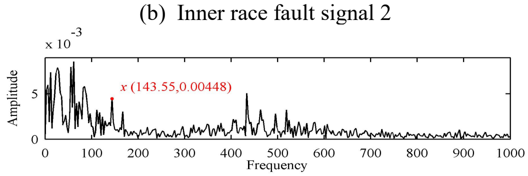
The improved signal fusion method is used to fuse four groups of sampled signals in the same fault state, and the envelope spectrum of the collected signals and the fused signals are obtained respectively, as shown in Fig. 1 (This paper only gives the envelope spectrum of the IF signals). From the envelope spectrum, it can be seen that the fusion signal obtained has obvious impact peak at fault frequency (143.55 Hz), and the noise component has also been suppressed, which helps to reduce the difficulty of the subsequent fault diagnosis process.



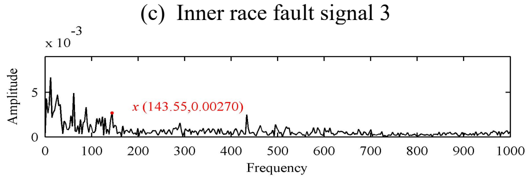
(a) Inner race fault signal 1



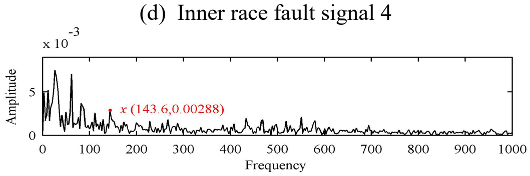
(b) Inner race fault signal 2



(c) Inner race fault signal 3



(d) Inner race fault signal 4

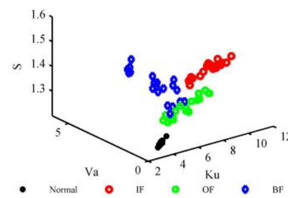


(e) Inner race fault fusion signal

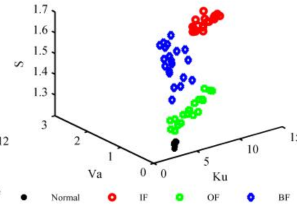
Fig. 1. Envelope spectrum of collected signals and fusion signal.

A partial Hadamard measurement matrix with a dimension of 4096×8192 is constructed to compress and collect the signal, the compression ratio Cr = 0.5, and the data length after compression is 4096. The sensitive feature parameters of 20 groups of data samples under four states are calculated respectively, and the three-dimensional fault feature vector (Variance: Va; Kurtosis: Ku; Waveform factor: S) distribution graph is obtained (in Fig. 2 (a)). Then the fusion signal is compressed and sampled, the compression ratio Cr =0.5, and the compressed data length is 4096, 20 groups of data are taken in total, and the sensitive feature parameters of each group of compressed data are calculated. The three-dimensional fault feature parameters distribution of the bearings in four states is obtained as shown in Fig. 2 (b). At the same time, SVM classifier is used to verify the fault recognition rate of the method proposed in this paper. The confusion matrix and fault recognition rate are shown in Fig. 3 and Table 1.

It can be seen from Fig. 3 and Table 1 that the recognition rate of fault diagnosis process using compressed features is higher than that using reconstructed features (The compressed signal is reconstructed based on FBP [25] and IAFBP algorithm [26]), and the signal fusion operation can further improve its fault recognition rate. It is noted that the fault diagnosis based on the fused compression features needs less time.

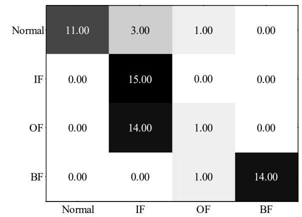


(a)

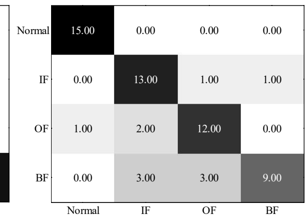


(b)

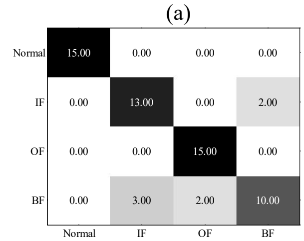
Fig. 2. Cr =0.5 , three-dimensional bearings fault feature parameters distribution diagram under four states. (a) Before fusion. (b) After fusion.



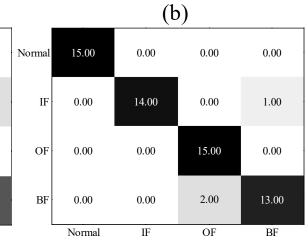
(a)



(b)



(c)



(d)

Fig. 3. Confusion matrix corresponding to the four fault recognition methods. (a) FBP reconstruction feature recognition method. (b) IAFBP reconstruction feature recognition method. (c) Compressed feature recognition method. (d) Fusion compressed feature recognition method.

Table 1. Fault recognition rates and time for four fault recognition methods.

Recognition method	Fault recognition rate	Time (s)
FBP Reconstruction feature recognition method	0.8210	42.5069
IAFBP Reconstruction feature recognition method	0.8705	26.8874
Compressed feature recognition method	0.9200	11.2564
Fusion compressed feature recognition method	0.9669	15.5987

5. CONCLUSION

In this paper, a fast diagnosis method of bearings fault is proposed. By combining multi-sensor fusion technology and compressed feature extraction technology, the amount and processing complexity of signal are reduced, and the real-time diagnosis accuracy of bearings fault can be ensured. The experimental results show that variable signals can be fused with high precision by the improved random weighted algorithm based on adaptive adjustment of relative fluctuation, which expands the application range of the traditional random weighting algorithm.

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