JOINT SOURCE LOCALIZATION AND ASSOCIATION THROUGH OVERCOMPLETE REPRESENTATION UNDER MULTIPATH PROPAGATION ENVIRONMENT

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ABSTRACT

This work addresses the source localization and association problem in a multipath propagation environment. By focusing on the limitation of the prior information in practical applications, we propose a target localization and association method based on iterative optimization with semi-unitary constraint and eigen-decomposition techniques. In contrast to the previous works, the proposed method can localize spatial sources and associate the incident paths to each source without prior knowledge pertaining to the propagation environment. Moreover, the proposed approach can be applied to an arbitrary array geometry without reducing the effective array aperture. Both simulations and real data experiments validate the effectiveness and robustness of the proposed method.

Index Terms— Multipath propagation, overcomplete representation, semi-unitary constraint, source localization and association.

1. INTRODUCTION

Direction-of-arrival (DOA) source localization is a fundamental problem in array signal processing fields [1–5]. High-resolution subspace-based methods such as multiple signal classification (MUSIC) [6, 7] have been developed to achieve good DOA estimates for uncorrelated signals. However, the performance of these methods reduce in a multipath propagation environment, where multiple coherent signals are induced at a receiving array.

In general, a preprocessing technique (referred to as spatial smoothing (SS) [8]) is employed to decorrelate coherent signals so that conventional subspace-based methods can deal with the coherent signal. However, such decorrelation process reduces the resolution and the degree of freedom of an array system. While maximum likelihood (ML) methods (which are less sensitive to multipath effects) can be applied in the presence of multipath propagation [9, 10], these approaches require nonlinear optimization resulting in high computational complexity [11]. In addition, sparse representation-based methods such as orthogonal matching pursuit (OMP) [12] and sparse Bayesian learning (SBL) [13] have been proposed for source localization.

Existing source localization methods mainly focus on estimating DOAs of all incident paths without considering any source association information, i.e., which source a detected path is associated with. Such information is useful for identifying, locating, and tracking sources. In [14], source localization accuracy is improved by assuming that the number of sources and multipaths from each source are known. A sources association method [15] was proposed to group spatial paths according to each source. The technique proposed in [16] can achieve satisfactory source association performance by utilizing a coarse DOA estimated by the existing algorithm. However, the performance of these methods degrades when prior information is unavailable, such as when source signals are corrupted by unknown multipath reflections. In such a scenario, the ability to detect and classify incident paths is required to achieve accurate source localization and association performance.

We propose a joint source localization and association (JSLA) algorithm in the presence of multipath propagation. Compared to the previous works, JSLA can effectively mitigate the detrimental effects of coherent multipath without reducing the effective array aperture. The main advantage of JSLA includes avoiding the need for selecting hyperparameters, sparsity level of incident paths, or any prior information pertaining to the propagation scenario. Moreover, it can be applied with an arbitrarily defined array geometry. We demonstrate the efficacy of JSLA via both simulated and experiment results.

2. SIGNAL MODEL

A multipath propagation scenario is depicted in Fig. 1, where an array system comprises M isotropic sensors that is impinged by D_K distinct signals from K far-field narrowband sources $s_k(t)$ with $k = 1, 2, \dots, K$. Due to multipath propagation, we also assume the presence of $P_k \geq 1$ coherent propagation paths corresponding to the propagation of signal from $s_k(t)$ such that the pth path originates from direction

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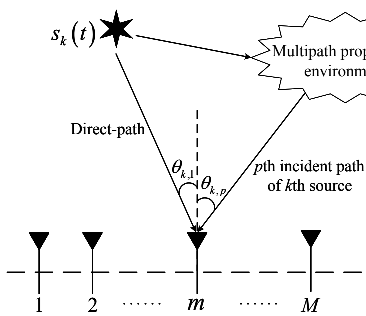

Fig. 1: Multipath propagation geometry model.

$\theta_{k,p}$, where path index $p = 1, 2, \ldots, P_k$. In addition, we assume that signals corresponding to different sources are uncorrelated with each other.

The $M \times 1$ array output at time t can be represented by

$$\mathbf{x}(t) = \sum_{k=1}^{K} \sum_{p=1}^{P_k} \mathbf{a}(\theta_{k,p})\, c_{k,p} s_k(t) + \mathbf{n}(t), \tag{1}$$

where $\mathbf{a}(\theta_{k,p})$ and $c_{k,p}$ are, respectively, the array steering vectors toward direction $\theta_{k,p}$ and the attenuation coefficient of the pth incident path corresponding to the kth source. With $t \in \{t_1, t_2, \ldots, t_L\}$, the variable L denotes the number of snapshots, and $\mathbf{n}(t) \in \mathbb{C}^{M\times 1}$ denotes the noise vector. We further assume $\mathbf{n}(t)$ is a white Gaussian noise with mean zero and covariance $E\left\{\mathbf{n}(t)\,\mathbf{n}^H(t)\right\} = \sigma_n^2 \mathbf{I}_M$, where $E\{\cdot\}$, $(\cdot)^H$, σ_n^2, and $\mathbf{I}_M$ denote the expectation operator, the conjugate transpose, the noise power, and the $M \times M$ identity matrix, respectively.

Equation (1) can further be expressed compactly via

$$\mathbf{x}(t) = \mathbf{B}\mathbf{s}(t) + \mathbf{n}(t), \tag{2}$$

where $\mathbf{B} = [\mathbf{A}_{\boldsymbol{\theta}_1}\mathbf{c}_1, \ldots, \mathbf{A}_{\boldsymbol{\theta}_k}\mathbf{c}_k, \ldots, \mathbf{A}_{\boldsymbol{\theta}_K}\mathbf{c}_K]$ and $\mathbf{s}(t) = [s_1(t), \ldots, s_k(t), \ldots, s_K(t)]^T$. The $M \times P_k$ matrix $\mathbf{A}_{\boldsymbol{\theta}_k} = [\mathbf{a}(\theta_{k,1}), \ldots, \mathbf{a}(\theta_{k,p}), \ldots, \mathbf{a}(\theta_{k,P_k})]$ is formed by concatenating steering vectors corresponding to the kth source given that $\boldsymbol{\theta}_k = [\theta_{k,1}, \ldots, \theta_{k,p}, \ldots, \theta_{k,P_k}]^T$ and $\mathbf{c}_k = [c_{k,1}, \ldots, c_{k,p}, \ldots, c_{k,P_k}]^T$ with $c_{k,1} = 1$ for the direct-path component of the kth source. Given multichannel observations $\{\mathbf{x}(t)\}_{t=t_1}^{t_L}$, our objective is to achieve accurate source locations, as well as their association information.

3. THE PROPOSED JSLA ALGORITHM

3.1. Overcomplete Representation for Multiple Time Samples

By exploiting the sparseness characteristic of incident signals in the spatial domain, we formulate the received signal model in (2) as a sparse representation. This process transforms a location parameter estimation problem into sparse spectrum estimations [17, 18].

Let Ω denote the set of possible locations of paths of interest and $\{\theta_n\}_{n=1}^{N}$ denote a sampling grid set that covers Ω with N being the number of potential path directions. An overcomplete dictionary is first constructed as an $M \times N$ matrix $\mathbf{D}_\Omega = [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_n), \ldots, \mathbf{a}(\theta_N)]$. Using an overcomplete representation, (2) can then be reformulated as

$$\mathbf{x}(t) = \mathbf{D}_\Omega \mathbf{r}(t) + \mathbf{n}(t), \tag{3}$$

where $\mathbf{r}(t) \in \mathbb{C}^{N\times 1}$ represents the sparse (parameterized) coefficient vector with the nth element being

$$r_n(t) = \begin{cases} c_{k,p} s_k(t), & \theta_n = \theta_{k,p}, k = 1, 2, \ldots, K, \\ & p = 1, 2, \ldots, P_k; \\ 0, & \text{otherwise}. \end{cases} \tag{4}$$

With sufficient quantization of sampling grids, we infer from (2) and (3) that $\mathbf{r}(t)$ will, in theory, consist of all zeros except for D_K non-zero elements corresponding to K sources.

Since source localization with multiple snapshots in a multipath propagation scenario is of practical importance, we then consider the overcomplete representation employing multiple time samples. Let $\mathbf{X} = [\mathbf{x}(t_1), \ldots, \mathbf{x}(t_L)] \in \mathbb{C}^{M\times L}$ and define $\mathbf{R} \in \mathbb{C}^{N\times L}$ and $\mathbf{N} \in \mathbb{C}^{M\times L}$ similarly. The single snapshot representation (3) can then be extended as

$$\mathbf{X} = \mathbf{D}_\Omega \mathbf{R} + \mathbf{N}. \tag{5}$$

In fact, the overcomplete representation in (5) makes it possible to reformulate the source localization problem as estimating a series of $\{\mathbf{r}(t)\}_{t=t_1}^{t_L}$.

3.2. Source Localization Based on Iterative Implementation with Semi-unitary Constraint

Based on the observed data matrix $\mathbf{X}$, the proposed multi-source localization technique aims to estimate the $M \times N$ adaptive filter bank matrix $\mathbf{W}$ and the $N \times L$ parameterized matrix $\mathbf{R}$ via the cost function

$$\min_{\mathbf{R},\mathbf{W}} f(\mathbf{R}, \mathbf{W}) \quad \text{s.t.} \mathbf{W}\mathbf{W}^H = \mathbf{I}_M, \tag{6}$$

where $f(\mathbf{R}, \mathbf{W}) = \left\|\mathbf{R} - \mathbf{W}^H\mathbf{X}\right\|_2^2$ with $\|\cdot\|_2$ denoting the ℓ_2-norm. Here, the semi-unitary constraint is employed to avoid trivial dual-zero solutions and maintain a constant modulus of the filter bank.

To solve (6), we introduce an iterative optimization strategy [19] by alternating between updating the estimate of $\mathbf{W}$ and the estimate of $\mathbf{R}$. The optimization strategy starts with the use of a matched filter bank [20] such that the initial estimate $\widehat{\mathbf{R}}_0 = \mathbf{D}_\Omega^H \mathbf{X}$. Defining $\widehat{\mathbf{R}}_i$ and $\widehat{\mathbf{W}}_i$, respectively, as the estimated parameterized matrix and adaptive filter bank in the ith iteration, the estimate $\widehat{\mathbf{W}}_{i+1}$ can then be achieved via

$$\widehat{\mathbf{W}}_{i+1} = \arg\min_{\mathbf{W}} \left\|\widehat{\mathbf{R}}_i - \mathbf{W}^H\mathbf{X}\right\|_2^2 \quad \text{s.t.} \mathbf{W}\mathbf{W}^H = \mathbf{I}_M. \tag{7}$$

It is important to note that the above optimization problem is non-convex. Instead of determining a general unitary matrix, we employ a re-parameterization method [21] to determine a full-row rank matrix $\mathbf{F}$ subject to $\mathbf{W} = \left(\mathbf{F}\mathbf{F}^H\right)^{-\frac{1}{2}}\mathbf{F}$ that minimizes the cost function in (7). The Euclidean gradient of the cost function w.r.t. $\mathbf{F}^*$ is given by

$$\nabla_{\mathbf{F}^*} f\left(\widehat{\mathbf{R}}_i, \left(\mathbf{F}\mathbf{F}^H\right)^{-\frac{1}{2}}\mathbf{F}\right) = \mathbf{U}\left(\mathbf{G}^H + \mathbf{G}\right)\boldsymbol{\Sigma}\mathbf{V}^H + \mathbf{U}\boldsymbol{\Sigma}^{-1} \times \mathbf{U}^H \nabla_{\mathbf{W}^*} f\left(\widehat{\mathbf{R}}_i, \mathbf{W}\right). \tag{8}$$

In (8), the matrix $\mathbf{F}=\mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H$ is decomposed using economy-sized singular value decomposition (eSVD), where $\mathbf{U} \in \mathbb{C}^{M\times M}$ is a unitary matrix whose columns are the left-singular vectors of $\mathbf{F}$, $\boldsymbol{\Sigma} \in \mathbb{R}^{M\times M}$ is a diagonal matrix with entries being the singular values of $\mathbf{F}$, and $\mathbf{V} \in \mathbb{C}^{N\times M}$ is a semi-unitary matrix with columns being the right-singular vectors of $\mathbf{F}$. The function $\nabla_{\mathbf{W}^*} f\left(\widehat{\mathbf{R}}_i, \mathbf{W}\right)$ denotes the gradient of $f\left(\widehat{\mathbf{R}}_i, \mathbf{W}\right)$ w.r.t. $\mathbf{W}^*$ with $(\cdot)^*$ being the conjugation operator, and

$$\mathbf{G}=-\left(\boldsymbol{\Sigma}^{-1}\mathbf{U}^H\nabla_{\mathbf{W}^*} f\left(\widehat{\mathbf{R}}_i, \mathbf{W}\right)\mathbf{V}\right)\oslash\left(\mathbf{1}_M\boldsymbol{\sigma}^T+\boldsymbol{\sigma}\mathbf{1}_M^T\right), \tag{9}$$

where $\boldsymbol{\sigma}^T$, $\mathbf{1}_M$, and $\oslash$ are the diagonal vector of $\boldsymbol{\Sigma}$, the all-one column vector, and the element-wise division operator, respectively. With reference to (8), the Nesterov accelerated gradient technique [22] can be employed to solve (7) to obtain the $(i+1)$th estimate $\widehat{\mathbf{W}}_{i+1}$.

To update the estimate of $\mathbf{R}$ in the $(i+1)$th iteration, we then minimize $f\left(\mathbf{R}, \widehat{\mathbf{W}}_{i+1}\right)$ w.r.t. $\mathbf{R}$ resulting in

$$\widehat{\mathbf{R}}_{i+1} = \widehat{\mathbf{W}}_{i+1}^H\mathbf{X}. \tag{10}$$

The above process is repeated until a stop criterion is satisfied and the final estimate $\widehat{\mathbf{R}}_{\text{opt}} \in \mathbb{C}^{N\times L}$ can then be obtained. Elements within each column of $\widehat{\mathbf{R}}_{\text{opt}}$, therefore, describe the estimated spatial spectrum corresponding to the spatial signals impinging on the array. Defining $\mathbb{N}$ as the set of indices n corresponding to peaks of the function

$$P_{\text{spe}}(n) = 10\log_{10}\left(\left|\frac{1}{L}\sum_{l=1}^{L}\widehat{\mathbf{R}}_{\text{opt}}^{(l)}(n)\right|^2\right) \tag{11}$$

and $\widehat{\mathbf{R}}_{\text{opt}}^{(l)}$ denoting the nth element of the lth column of $\widehat{\mathbf{R}}_{\text{opt}}$, the DOA estimation set is then given by $\widehat{\Theta}_s = \{\theta_n \mid n \in \mathbb{N}\}$.

3.3. Source Association via Eigen-decomposition

To gain information of the propagation field, it is necessary to associate all the incident paths to each source. With reference to $\mathbf{x}(t)$ in (2), the array covariance matrix is given by

$$\mathbf{R}_{\text{x}} = E\left\{\mathbf{x}(t)\mathbf{x}^H(t)\right\} = \mathbf{B}\mathbf{R}_s\mathbf{B}^H + \sigma_n^2\mathbf{I}_M, \tag{12}$$

where the signal covariance matrix $\mathbf{R}_s = E\left\{\mathbf{s}(t)\mathbf{s}^H(t)\right\} \in \mathbb{R}^{K\times K}$ is positive-definite since the source signals are statistically independent with each other. Performing eigenvalue decomposition (EVD) on $\mathbf{R}_{\text{x}}$ results in

$$\mathbf{R}_{\text{x}} = \mathbf{U}_s\boldsymbol{\Lambda}_s\mathbf{U}_s^H + \mathbf{U}_n\boldsymbol{\Lambda}_n\mathbf{U}_n^H, \tag{13}$$

where $\boldsymbol{\Lambda}_s \in \mathbb{R}^{K\times K}$ and $\boldsymbol{\Lambda}_n \in \mathbb{R}^{(M-K)\times(M-K)}$ are two diagonal matrices with diagonal elements corresponding, respectively, to the eigenvalues of the signal and the noise. Matrices $\mathbf{U}_s \in \mathbb{C}^{M\times K}$ and $\mathbf{U}_n \in \mathbb{C}^{M\times(M-K)}$ span the signal-subspace (with columns being the eigenvectors corresponding to the K largest eigenvalues) and the noise-subspace

Algorithm 1 Proposed JSLA algorithm

Input: Observed data matrix $\mathbf{X}$, overcomplete dictionary $\mathbf{D}_\Omega$, the maximum iteration number I, and number index vectors $\{\mathbf{z}_g\}_{g=1}^G$;

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Θ_1, ..., Θ_g, ..., Θ_G ← ∅;
for i = 1, 2, ..., I do
    Update Ŵ_{i+1} by (7);
    Update R̂_{i+1} by (10);
end for
Achieve Θ̂_s by (11) and U_n by (13), respectively;
for g = 1, 2, ..., G do
    θ̂^q_res ← Θ̂_s;
    for h = 1, 2, ..., D_K − K + 1 do
        q ← 1;
        while q ≤ z_{g,h} do
            θ̂^q_{g,h} = argmin_{θ^q_{g,h} ⊂ θ̂^q_res} f_1^{-1}(θ^q_{g,h});
            Θ_g ← Θ_g ∪ θ̂^q_{g,h}, θ̂^{q+1}_res ← θ̂^q_res \ θ̂^q_{g,h};
        end while
    end for
end for
Identify the final source association set by {ĝ, Θ̂_SA} = argmin_{θ∈Θ_g} F_{Θ_g}(θ);
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Output: Estimation of source location set $\widehat{\Theta}_s$ and association set $\widehat{\Theta}_{\text{SA}}$.

(with columns being the eigenvectors associated with the remaining small eigenvalues), respectively. Since the signal-subspace and the noise-subspace are orthogonal to each other, $\text{span}(\mathbf{B}) = \text{span}(\mathbf{U}_s) = \text{null}\left(\mathbf{U}_n^H\right)$ and

$$\left\|\mathbf{U}_n^H\mathbf{A}_{\boldsymbol{\theta}_k}\mathbf{c}_k\right\|_2 = 0, \quad k = 1, 2, \ldots, K. \tag{14}$$

It can be observed from (14) that the matrix $\mathbf{U}_n^H\mathbf{A}_{\boldsymbol{\theta}_k}$ is not full column rank if $M \geq \max\{P_1, P_2, \ldots, P_K\} + K$. Therefore, a true combinatorial group $\boldsymbol{\theta}$, i.e., $\boldsymbol{\theta} \in \{\boldsymbol{\theta}_k\}_{k=1}^K$ will result in a peak for the cost function

$$f_1(\boldsymbol{\theta}) = \frac{\det\left(\mathbf{A}_{\boldsymbol{\theta}}^H\mathbf{A}_{\boldsymbol{\theta}}\right)}{\det\left(\mathbf{A}_{\boldsymbol{\theta}}^H\mathbf{U}_n\mathbf{U}_n^H\mathbf{A}_{\boldsymbol{\theta}}\right)}, \tag{15}$$

where $\det(\cdot)$ denotes the matrix determinant. On the contary, for a wrong source association set Θ_w, we have

$$F_{\Theta_w}(\boldsymbol{\theta}) = \sum_{\boldsymbol{\theta}\in\Theta_w} f_1^{-1}(\boldsymbol{\theta}) > 0 \tag{16}$$

when $M \geq \max\{P_1, P_2, \ldots, P_K\} + D_K$.

We define Θ_g and $\mathbf{z}_g = [z_{g,1}, \ldots, z_{g,h}, \ldots, z_{g,D_K-K+1}]$ as the gth source association set and the corresponding number index vector, where $g = 1, 2, \ldots, G$ with G being the number of possible combinations and the subscript D_K-K+1 denotes the maximum path number of one source. In addition, the hth element $z_{g,h}$ in $\mathbf{z}_g$ denotes the number of sources that have h propagation paths in the gth association set with $h=1, 2, \ldots, D_K-K+1$. An estimate of the source association can be achieved by selecting one that minimizes $F_{\Theta_w}(\boldsymbol{\theta})$

in (16) among all the G combinations. For clarity, the procedure of the proposed JSLA algorithm is summarized in Algorithm 1, where the gth source association set Θ_g is identified from Steps 8 to 14. It is worth highlighting that $\{\mathbf{z}_g\}_{g=1}^{G}$ can be achieved by the potential combination $\{P_1, P_2, \ldots, P_K\}$ in the gth association set. In Step 11, $\widehat{\boldsymbol{\theta}}_{res}^{q}$ and $\widehat{\boldsymbol{\theta}}_{g,h}^{q}$ denote the corresponding residual DOA set and the classified qth DOA group including h propagation paths in the gth association set, respectively.

4. RESULTS

4.1. Results on Simulated Data

We conduct simulation to evaluate the performance of the proposed JSLA algorithm, where a 16-sensor uniform linear array (ULA) with half-wavelength element spacing is employed. Estimation performance associated with source DOA is evaluated in terms of root-mean-square error (RMSE)

$$\mathrm{RMSE} = \sqrt{\frac{1}{U_{\mathrm{MC}} K} \sum_{u=1}^{U_{\mathrm{MC}}} \sum_{k=1}^{K} \left\| \widehat{\boldsymbol{\theta}}_k^{(u)} - \boldsymbol{\theta}_k \right\|_2^2}, \tag{17}$$

where $\widehat{\boldsymbol{\theta}}_k^{(u)}$ and U_{MC} are the estimate of $\boldsymbol{\theta}_k$ in the uth trial and the number of Monte Carlo trials, respectively. We compare the source localization performance of the proposed JSLA method with baseline algorithms including SS-MUSIC [8], the relax method [10], the OMP method [12], and the SBL method [13]. In addition, JSLA is compared with the baseline algorithm proposed in [15]. The angular search grid and the number of snapshots are $0.1°$ and 128, respectively.

Five incident paths emitted from three far-field sources is considered in the simulation, where the DOA groups are set as $\boldsymbol{\theta}_1 = [-25°, -16°]^T$, $\boldsymbol{\theta}_2 = 8°$, and $\boldsymbol{\theta}_3 = [-10°, 25°]^T$. The corresponding attenuation coefficient groups are set as $\mathbf{c}_1 = [1, 0.8\exp(j1.8\pi)]^T$, $\mathbf{c}_2 = 1$, and $\mathbf{c}_3 = [1, 0.7\exp(j1.2\pi)]^T$, respectively. Variations of classification probability and RMSE with SNR are illustrated in Fig. 2 and Fig. 3, respectively. We note from Fig. 2 that JSLA, LASSO, and SBL methods achieve almost similar performance. Results in Fig. 3 show that satisfactory source association performance can be achieved by JSLA even under low SNR conditions. It is useful to note that the SS-MUSIC method and the relax method may suffer from degradation in localization performance under practical scenarios since they lack path association ability. The proposed JSLA method, on the other hand, can achieve satisfactory source association and localization performance under the multipath propagation scenario without using any prior information associated with the multipath propagation environment.

4.2. Results on Measured Data

We present results obtained from a real dataset collected by an array radar system with 8 channels to validate the effectiveness of JSLA in a practical scenario, where a target is tracked by the radar system. It is worth highlighting that the target is involved in a multipath propagation environment due to ground reflections.

Estimated target location parameter achieved by different methods is shown in Fig. 4(a), where the dashed lines denote the real target direction recorded by the GPS sensor on the target. It can be noted that the baseline methods are adversely affected by the complex multipath propagation during almost the observation time. On the contrary, localization performance of JSLA outperforms that of the other three methods. The time variation of the DOA estimation error shown in Fig. 4(b) highlights that JSLA achieves a relatively consistent estimation performance during the entire observation time. These results indicate that JSLA outperforms the baseline methods in terms of estimation accuracy and robustness.

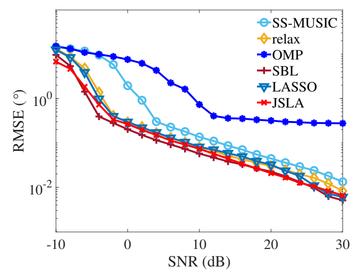


Fig. 2: Variation of RMSE in terms of DOA estimate.

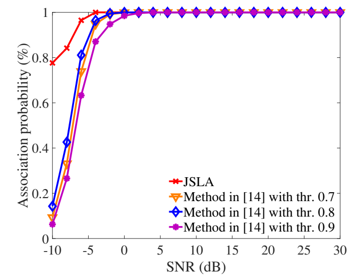
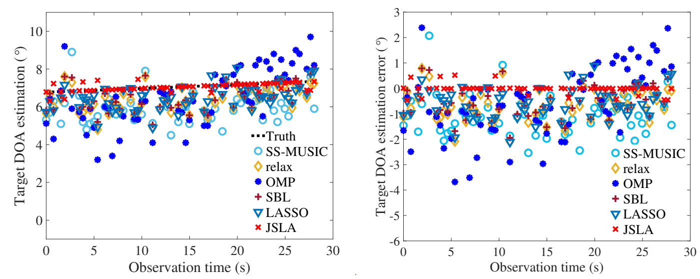


Fig. 3: The correct association probability versus SNR.



(a) (b)

Fig. 4: Location parameter estimation result and the estimation error versus the observation time via measured data. (a) Estimation result of target DOA produced by different methods. (b) Estimation error of target DOA produced by different methods.

5. CONCLUSIONS

We investigated the target localization and association problem under a complex multipath propagation environment. The proposed algorithm can localize the spatial sources and associate the incident paths to each source without requiring additional decorrelation preprocessing nor prior information pertaining to the propagation environment. Simulation and experiment results based on synthetic and measured data demonstrate the effectiveness of the proposed algorithm.

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