# DESIGNING A QAM SIGNAL DETECTOR FOR MASSIVE MIMO SYSTEMS VIA PS-ADMM APPROACH

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## ABSTRACT

This paper presents an efficient quadrature amplitude modulation (QAM) signal detector for massive multiple-input multiple-output (MIMO) communication systems via the penalty-sharing alternating direction method of multipliers (PS-ADMM). The content of the paper is summarized as follows: first, we formulate QAM-MIMO detection as a maximum-likelihood optimization problem with bound relaxation constraints. Decomposing QAM signals into a sum of multiple binary variables and exploiting introduced binary variables as penalty functions, we transform the detection optimization model to a non-convex sharing problem; second, a customized ADMM algorithm is presented to solve the formulated non-convex optimization problem. In the implementation, all variables can be solved analytically and in parallel; third, it is proved that the proposed PS-ADMM algorithm converges under mild conditions. Simulation results demonstrate the effectiveness of the proposed approach.

***Index Terms***— Massive MIMO, maximum-likelihood detection, penalty method, sharing-ADMM, nonconvex optimization

## 1. INTRODUCTION

Massive multiple-input multiple-output (MIMO) technology is considered to be one of the disruptive technologies for the fifth-generation (5G) communication systems [1], [2]. The foremost benefit of massive MIMO is significant increase in the spatial degrees of freedom which can improve throughput and energy efficiency significantly in comparison with conventional MIMO systems [3]. However, numerous practical challenges arise in implementing massive MIMO technology in order to achieve such improvements. One such challenge is signal detection lying in the uplink massive systems since it is difficult to achieve an effective compromise among good detecting performance, low computational complexity, and high processing parallelism [4].

In recent years, the alternating direction method of multipliers (ADMM) technique is widely used to solve convex and non-convex problems due to its simplicity, operator splitting capability, and guaranteed-convergence under mild conditions [5]. The ADMM detection algorithm was applied in various scenarios for MIMO systems [6]-[8] and improved for massive MIMO systems [9]-[12]. Although the existing ADMM-based methods provide better BER performance than conventional detectors, the BER performance could be improved further since constraints set of optimization problem is over-relaxed.

In this paper, we focus on designing a new ADMM-based quadrature amplitude modulation (QAM) signal detector for massive MIMO systems. By exploiting ideas of penalized bound relaxation for the ML detection formulation [13], binary transformation for high-order QAM signals [14], and sharing-ADMM technique [5], [15], we obtain a new detector for massive MIMO systems called PS-ADMM, which has favorable BER performance and cheap computational cost.

## 2. SYSTEM MODEL AND PROBLEM FORMULATION

The considered signal detection problem lies in uplink multiuser massive MIMO systems, where BS equipped with $B$ antennas serves $U$ single-antenna users. Here, we assume $B \geq U$. Typically, the received signal vector at BS can be characterized by the following model

$$\mathbf{r} = \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{1}$$

where $\mathbf{x} \in \mathbb{X}^U$ is the transmitted signal vector from $U$ users and $\mathbb{X}$ refers to the signal constellation set, $\mathbf{r} \in \mathbb{C}^B$ is the BS received signal vector, $\mathbf{H} \in \mathbb{C}^{B\times U}$ denotes the MIMO channel matrix, and $\mathbf{n} \in \mathbb{C}^B$ denotes additive white Gaussian noise. The entries of $\mathbf{H}$ and $\mathbf{n}$ are assumed to be independent and identically distributed (i.i.d.) complex Gaussian variables with zero mean.

The optimal MIMO ML detector for $4^Q$-QAM signals, i.e., achieving minimum error probability of detecting $\mathbf{x}$ from the received signal $\mathbf{r}$ can be formulated as the following discrete least square problem [16]

$$\min_{\mathbf{x}\in\mathbb{X}^U} \|\mathbf{r} - \mathbf{H}\mathbf{x}\|_2^2, \tag{2}$$

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where $\mathbb{X} = \{x = x_\mathrm{R} + jx_\mathrm{I} | x_\mathrm{R}, x_\mathrm{I} \in \{\pm 1, \pm 3, \cdots, \pm(2^Q - 1)\}\}$ and $Q$ is some positive integer. Obtaining its global optimal solution is prohibitive in practice since the corresponding computational complexity grows exponentially with the users' number $U$, BS's antenna number $B$, and set $\mathbb{X}$'s size [17]. In the following, we propose a *relaxation-tighten* technique and transform it to the well-known sharing problem. The main challenge comes from how to handle multiple quantization levels of QAM signals, which cannot be treated by state-of-the-art $\{1, -1\}$ binary processing techniques directly. To tackle this issue, we decompose high-order QAM symbols into a sum of multiple binary variables. Specifically, since any transmitted signal vector $\mathbf{x} \in \mathbb{X}^U$ can be expressed by

$$\mathbf{x} = \sum\nolimits_{q=1}^{Q} 2^{q-1}\mathbf{x}_q, \; q = 1, \cdots, Q, \tag{3}$$

where $\mathbf{x}_q \in \mathbb{X}_q^U$ and $\mathbb{X}_q = \{x_q = x_{q\mathrm{R}} + jx_{q\mathrm{I}} | x_{q\mathrm{R}}, x_{q\mathrm{I}} \in \{1, -1\}\}$. Then, model (2) can be equivalent to

$$\begin{aligned} \min_{\mathbf{x}_q} \;\; & \frac{1}{2}\|\mathbf{r} - \mathbf{H}(\sum\nolimits_{q=1}^{Q} 2^{q-1}\mathbf{x}_q)\|_2^2, \\ \text{s.t.} \;\; & \mathbf{x}_q \in \mathbb{X}_q^U, \; q = 1, \cdots, Q. \end{aligned} \tag{4}$$

Relaxing the binary constraints in model (4) and then adding a sum of quadratic penalty functions into the objective of the model (4), model (4) is changed to

$$\begin{aligned} \min_{\mathbf{x}_q} \; & \frac{1}{2}\|\mathbf{r} - \mathbf{H}(\sum\nolimits_{q=1}^{Q} 2^{q-1}\mathbf{x}_q)\|_2^2 - \sum\nolimits_{q=1}^{Q} \frac{\alpha_q}{2}\|\mathbf{x}_q\|_2^2, \\ \text{s.t.} \;\; & \mathbf{x}_q \in \tilde{\mathbb{X}}_q^U, \; q = 1, \cdots, Q, \end{aligned} \tag{5}$$

where penalty parameters $\alpha_q \geq 0$ and $\tilde{\mathbb{X}}_q = \{\mathbf{x}_{q\mathrm{R}} + j\mathbf{x}_{q\mathrm{I}} | \mathbf{x}_{q\mathrm{R}}, \mathbf{x}_{q\mathrm{I}} \in [-1\ 1]\}$. The problem (5) is called as sharing problem [5, Section 7.3]. It can be seen that the introduced penalty functions make the real/imaginary part of the optimal solutions close to $+1$ or $-1$.

## 3. PS-ADMM SOLVING ALGORITHM

In this section, we develop an efficient ADMM algorithm, named PS-ADMM, to solve the model (5). We transform it equivalently to the following linearly constrained problem by introducing auxiliary variable $\mathbf{x}_0 \in \mathbb{C}^U$

$$\begin{aligned} \min_{\mathbf{x}_0, \mathbf{x}_q} \;\; & \frac{1}{2}\|\mathbf{r} - \mathbf{H}\mathbf{x}_0\|_2^2 - \sum\nolimits_{q=1}^{Q} \frac{\alpha_q}{2}\|\mathbf{x}_q\|_2^2, \\ \text{s.t.} \;\; & \mathbf{x}_0 = \sum\nolimits_{q=1}^{Q} 2^{q-1}\mathbf{x}_q, \; \mathbf{x}_q \in \tilde{\mathbb{X}}_q^U, q = 1, \cdots, Q. \end{aligned} \tag{6}$$

The augmented Lagrangian function of the model (6) can be expressed as

$$\begin{aligned} L_\rho(\{\mathbf{x}_q\}_{q=1}^Q, \mathbf{x}_0, \mathbf{y}) = & \frac{1}{2}\|\mathbf{r} - \mathbf{H}\mathbf{x}_0\|_2^2 - \sum\nolimits_{q=1}^{Q} \frac{\alpha_q}{2}\|\mathbf{x}_q\|_2^2 \\ + \mathrm{Re}\langle \mathbf{x}_0 - & \sum\nolimits_{q=1}^{Q} 2^{q-1}\mathbf{x}_q, \mathbf{y}\rangle + \frac{\rho}{2}\left\|\mathbf{x}_0 - \sum\nolimits_{q=1}^{Q} 2^{q-1}\mathbf{x}_q\right\|_2^2, \end{aligned} \tag{7}$$

where $\mathbf{y} \in \mathbb{C}^U$ and $\rho > 0$ are the Lagrangian multiplier and penalty parameter respectively. Based on the above augmented Lagrangian, the proposed PS-ADMM solving algorithm framework can be described as

$$\mathbf{x}_q^{k+1} = \arg\min_{\mathbf{x}_q \in \tilde{\mathbb{X}}_q^U} L_\rho(\mathbf{x}_1^{k+1}, \cdots, \mathbf{x}_{q-1}^{k+1}, \mathbf{x}_q, \mathbf{x}_{q+1}^k, \cdots, \mathbf{x}_Q^k, \mathbf{x}_0^k, \mathbf{y}^k), \quad q = 1, \cdots, Q, \tag{8a}$$

$$\mathbf{x}_0^{k+1} = \arg\min_{\mathbf{x}_0} \;\; L_\rho(\{\mathbf{x}_q^{k+1}\}_{q=1}^Q, \mathbf{x}_0, \mathbf{y}^k), \tag{8b}$$

$$\mathbf{y}^{k+1} = \mathbf{y}^k + \rho\Big(\mathbf{x}_0^{k+1} - \sum\nolimits_{q=1}^{Q} 2^{q-1}\mathbf{x}_q^{k+1}\Big), \tag{8c}$$

where $k$ denotes iteration number.

The main challenge of implementing (8) lies in how to solve optimization subproblems (8a) and (8b) efficiently. For (8a), it can be observed that $L_\rho(\{\mathbf{x}_q\}_{q=1}^Q, \mathbf{x}_0^k, \mathbf{y}^k)$ is a strongly convex quadratic function with respect to (w.r.t.) $\mathbf{x}_q$ when $4^{q-1}\rho - \alpha_q > 0$. It means that, if we set $4^{q-2}\rho > \alpha_q$, the optimal solution of the subproblems (8a) can be obtained through the following procedure: Set the gradient of the augmented Lagrangian function w.r.t. $\mathbf{x}_q$ to be zero and solving the corresponding linear equation, its optimal solutions can also be determined analytically as follows

$$\begin{aligned} \mathbf{x}_q^{k+1} = \prod_{[-1,1]} \Bigg( & \frac{2^{q-1}}{4^{q-1}\rho - \alpha_q}\Big(\rho\mathbf{x}_0^k - \rho\sum_{i<q} 2^{i-1}\mathbf{x}_i^{k+1} \\ & - \rho\sum_{i>q} 2^{i-1}\mathbf{x}_i^k + \mathbf{y}^k\Big)\Bigg), \quad q = 1, \cdots, Q, \end{aligned} \tag{9}$$

where operator $\prod_{[-1,1]}(\cdot)$ projects every entry's real part and imaginary part of the input vector onto [-1,1] respectively.

Moreover, $L_\rho(\{\mathbf{x}_q^{k+1}\}_{q=1}^Q, \mathbf{x}_0, \mathbf{y}^k)$ is a strongly convex quadratic function w.r.t. $\mathbf{x}_0$ since $\rho > 0$ and matrix $\mathbf{H}^H\mathbf{H}$ is positive semidefinite, hence the optimal solution of the subproblem (8b) can also be obtained by setting $\nabla_{\mathbf{x}_0} L_\rho(\{\mathbf{x}_q^{k+1}\}_{q=1}^Q, \mathbf{x}_0, \mathbf{y}^k)$ to be zero and solving the corresponding linear equation, which results in

$$\mathbf{x}_0^{k+1} = (\mathbf{H}^H\mathbf{H} + \rho\mathbf{I})^{-1}\Bigg(\mathbf{H}^H\mathbf{r} + \rho\sum_{q=1}^{Q} 2^{q-1}\mathbf{x}_q^{k+1} - \mathbf{y}^k\Bigg). \tag{10}$$

To be clear, we summarize the proposed PS-ADMM algorithm for solving model (6) in *Algorithm 1*.

**Algorithm 1** The proposed PS-ADMM algorithm

**Require:** $\mathbf{H}$, $\mathbf{r}$, $Q$, $\rho$, $\{\alpha_q\}_{q=1}^Q$
**Ensure:** $\mathbf{x}_0^k$
1: Initialize $\{\mathbf{x}_q^1\}_{q=1}^Q, \mathbf{x}_0^1, \mathbf{y}^1$ as the all-zeros vectors.
2: **For** $k = 1, 2, \cdots$
3: Step 1: Update $\{\mathbf{x}_q^{k+1}\}_{q=1}^Q$ sequentially via (9).
4: Step 2: Update $\mathbf{x}_0^{k+1}$ via (10).
5: Step 3: Update $\mathbf{y}^{k+1}$ via (8c).
6: **Until** some preset condition is satisfied.

## 4. PERFORMANCE ANALYSIS

In this section, a brief analysis of the proposed PS-ADMM algorithm on convergence, iteration complexity, and computational cost are provided. Due to the limited space, the details of the proof are not included here. Please refer to [18].

### 4.1. Convergence property

We have the following theorem to show convergence properties of the proposed PS-ADMM algorithm.

***Theorem 1***: Assume $4^{q-1}\rho > \alpha_q$ and $\rho > \sqrt{2}\lambda_{\max}(\mathbf{H}^H\mathbf{H})$ are satisfied. Then, sequence $\{\{\mathbf{x}_q^k\}_{q=1}^Q, \mathbf{x}_0^k, \mathbf{y}^k\}$ generated by *Algorithm 1* is convergent, i.e.,

$$\begin{aligned} &\lim_{k\to+\infty} \mathbf{x}_q^k = \mathbf{x}_q^*, \ \lim_{k\to+\infty} \mathbf{x}_0^k = \mathbf{x}_0^*, \ \lim_{k\to+\infty} \mathbf{y}^k = \mathbf{y}^*, \\ &\forall\, \mathbf{x}_q \in \tilde{\mathbb{X}}_q^U, \ q = 1, \cdots, Q. \end{aligned} \tag{11}$$

Moreover, $\{\mathbf{x}_q^*\}_{q=1}^Q$ is a stationary point of problem (5), i.e., $\forall\, \mathbf{x}_q \in \tilde{\mathbb{X}}_q^U, \ q = 1, \cdots, Q$, which satisfies

$$\begin{aligned} &\mathrm{Re}\Big\langle \nabla_{\mathbf{x}_q}\Big(\frac{1}{2}\|\mathbf{r} - \mathbf{H}(\textstyle\sum_{q=1}^Q 2^{q-1}\mathbf{x}_q^*)\|_2^2 \\ &- \textstyle\sum_{q=1}^Q \dfrac{\alpha_q}{2}\|\mathbf{x}_q^*\|_2^2\Big), \mathbf{x}_q - \mathbf{x}_q^*\Big\rangle \geq 0. \end{aligned} \tag{12}$$

### 4.2. Iteration complexity

We use the following residual

$$\sum_{q=1}^Q \|\mathbf{x}_q^{k+1} - \mathbf{x}_q^k\|_2^2 + \|\mathbf{x}_0^{k+1} - \mathbf{x}_0^k\|_2^2 \tag{13}$$

to measure the convergence progress of the PS-ADMM algorithm since it converges to zero as $k \to +\infty$. Then, we have Theorem 2 about its convergence progress.

***Theorem 2***: Let $t$ be the minimum iteration index such that the residual in (13) is less than $\epsilon$, where $\epsilon$ is the desired precise parameter for the solution. Then, we have the following iteration complexity result

$$\begin{aligned} t \leq \frac{1}{C\epsilon}\Bigg(& L_\rho(\{\mathbf{x}_q^1\}_{q=1}^Q, \mathbf{x}_0^1, \mathbf{y}^1) \\ &-\frac{1}{2}\|\mathbf{r} - \mathbf{H}(\textstyle\sum_{q=1}^Q 2^{q-1}\mathbf{x}_q^*)\|_2^2 - \textstyle\sum_{q=1}^Q \dfrac{\alpha_q}{2}\|\mathbf{x}_q^*\|_2^2\Big)\Bigg), \end{aligned} \tag{14}$$

where $C = \min\Big\{\{\frac{\gamma_q(\rho)}{2}\}_{q=1}^Q, \Big(\frac{\gamma_0(\rho)}{2} - \frac{\lambda_{\max}^2(\mathbf{H}^H\mathbf{H})}{\rho}\Big)\Big\}$, and $\gamma_q(\rho) = 4^{q-1}\rho - \alpha_q$, $\gamma_0(\rho) = \rho + \lambda_{\min}(\mathbf{H}^H\mathbf{H})$ and $\lambda(\cdot)$ denotes the eigenvalue of a matrix.

### 4.3. Computational cost

The overall computational complexity of the PS-ADMM detection algorithm consists of two parts: the first part, which is independent of the number of iterations, is required to compute the $\mathbf{x}_0$ update of PS-ADMM in (10); then, it needs to be calculated only once when detecting each transmitted symbol vector. The first part of the calculations is performed in three steps: first, the multiplication of the $U \times B$ matrix $\mathbf{H}^H$ by the $B \times U$ matrix $\mathbf{H}$; second, the computation of the inversion of the regularized Gramian matrix $\mathbf{H}^H\mathbf{H} + \rho\mathbf{I}$; and third, the computation of the $U \times B$ matrix $\mathbf{H}^H$ by the $B \times 1$ vector $\mathbf{y}$ to obtain the matched-filter vector $\mathbf{H}^H\mathbf{y}$. These steps require $BU^2$, $U^3$ and $BU$ complex multiplications, respectively. The second part, which is iteration dependent, needs to be repeated every iteration in two steps: first, the $Q$ scalar multiplications by the $U$ vectors in (9) for $Q$ $\mathbf{x}_q$ are implemented sequentially; second, a multiplication of the $U \times U$ matrix by the $U \times 1$ vector and the $Q$ scalar multiplications by the $U \times 1$ vectors in (10). These steps require $\frac{1}{2}Q^2U$, $U^2 + \frac{1}{2}QU$ complex multiplications, respectively. Combining this result with Theorem 2, we conclude that the total computational cost to attain an $\epsilon$-optimal solution is $U^3 + BU^2 + BU + K(U^2 + \frac{1}{2}Q(Q+1)U)$, where $K$ is the maximum number of iterations. Since $K \ll B$ for massive MIMO detection, the computational complexity of the PS-ADMM mainly lies in matrix multiplication and inversion computations, which is comparable to the linear detector.

## 5. SIMULATION RESULTS

In this section, numerical results related to the proposed PS-ADMM detector are presented. Throughout this section, we show simulation results for uncoded and hard decision-based signal detection with the i.i.d. Rayleigh fading channel in different $B \times U$ $(B \geq U)$ multiuser massive MIMO system. We assume that perfect knowledge of the channel state information is known at the receiver side.

### 5.1. BER performance and computational complexity

In this subsection, BER performance and computational complexity of the proposed PS-ADMM detector are evaluated and compared with conventional and state-of-the-art MIMO detectors by numerical simulations, which are the classical MMSE detector, Neumann detector [19], GS detector [20], SD detector, K-best detector, SDR detector, TASER detector [21], OCD-BOX detector [22], and two ADMM-based detectors ADMM and ADMIN in [6] and [9] respectively. The termination criterium is that iteration number $K$ reaches 30 or the residual in (13) is less than $10^{-5}$. The data points plotted in all BER curves are averaged over 1000 Monte-Carlo trials.

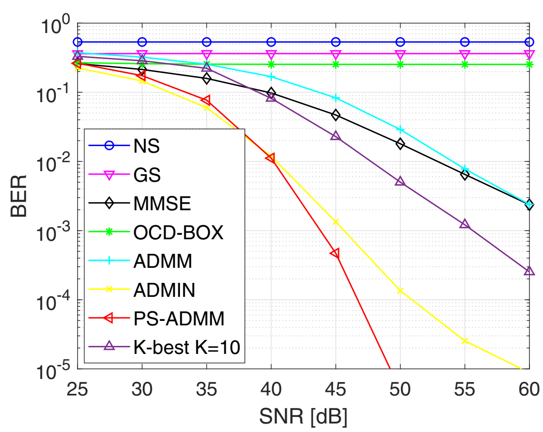


**Fig. 1**. Comparisons of BER performance using various massive MIMO detectors.

Fig. 1 shows BER performance of considered detectors for 256-QAM modulation with 128 $\times$ 128 massive MIMO systems. In Fig. 1, we observe that that BER curves of all detectors have a similar changing trend at low SNRs that continues to drop in a waterfall manner in relatively high SNR regions. We can see clearly that the BER performance of proposed PS-ADMM detector outperforms other detectors when the ratio of the BS's antenna number to the user number approaches one.

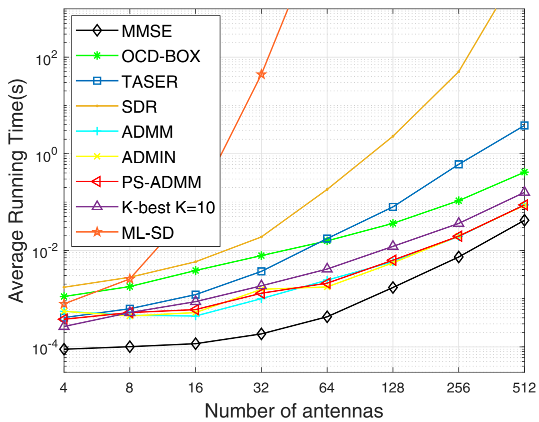


**Fig. 2**. Complexity comparison of massive MIMO detectors with $U = B$ for 256-QAM, SNR=12dB.

In Fig. 2, one can see that the running time of the proposed PS-ADMM detector is higher than MMSE, similar to ADMM, ADMIN, and K-best (K=10), and lower than other ones. However, we should note that they have much better detection performance than MMSE and K-best (K=10). Therefore, one can conclude that the proposed PS-ADMM detector can delivers an attractive tradeoff between BER performance and computational complexity.

### 5.2. Choice of Parameters

In this subsection, we show that the proper parameters $\rho$ and $\alpha$ can achieve lower BER performance and speed up convergence of the proposed PS-ADMM detector. The considered modulation scheme is 4-QAM and the simulation parameters are $B = 128$, $U = 128$, and SNR $= 10$dB.

In Fig. 3, it shows the effects on BER performance when the different values of the penalty parameters $\rho$ and $\alpha$ are chosen. From the figure, one can have the following observations: first, both $\rho$ and $\alpha$ can affect BER performance of the proposed PS-ADMM decoder; second, too large or too small values of $\alpha$ and $\rho$ can worsen BER performance of the detector. For the case of the presented simulation, one can see the proper $\rho \in [120\ 200]$ and $\alpha \in [60\ 80]$ respectively.

In Fig. 4, not only can the impact of $\rho$ and $\alpha$ on convergence performance be observed, it can also see that the proposed PS-ADMM algorithm can converge within a few tens of iterations to converge to modest accuracy solutions, which is promising for large-scale MIMO detection scenarios.

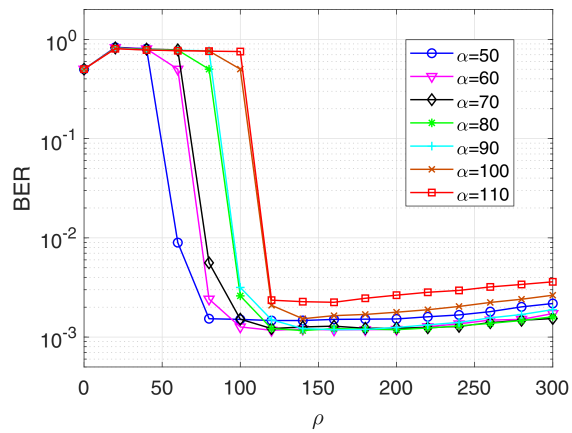


**Fig. 3**. The impact of $\rho$ and $\alpha$ on BER performance of the PS-ADMM detector.

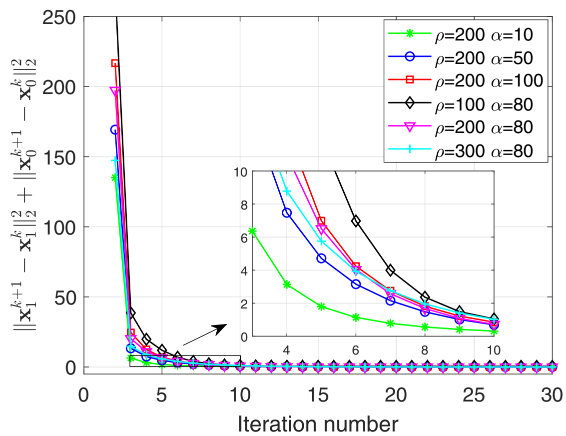


**Fig. 4**. The impact of the maximum iteration number $K$ on convergence performance of the PS-ADMM detector.

## 6. CONCLUSION

In this paper, we proposed a new MIMO detector for high-order QAM modulation signals via the PS-ADMM approach. We show that the proposed PS-ADMM detector is theoretically-guaranteed convergent under some mild conditions. The numerical results demonstrate the proposed one has competitive BER performance and cheap computational complexity in comparison with state-of-the-art works. Since synchronization, CSI availability, channel coding, etc. are essentially used in practical systems, it would be meaningful to study how the proposed PS-ADMM approach can be applied to such scenarios in furture.

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