S2 REDUCER: HIGH-PERFORMANCE SPARSE COMMUNICATION TO ACCELERATE DISTRIBUTED DEEP LEARNING

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ABSTRACT

Distributed stochastic gradient descent (SGD) approach has been widely used in large-scale deep learning, and the gradient collective method is vital to ensure the training scalability of the distributed deep learning system. Collective communication such as AllReduce has been widely adopted for the distributed SGD process to reduce the communication time. However, AllReduce incurs large bandwidth resources while most gradients are sparse in many cases since many gradient values are zeros and should be efficiently compressed for bandwidth saving. To reduce the sparse gradient communication overhead, we propose Sparse-Sketch Reducer (S2 Reducer), a novel sketch-based sparse gradient aggregation method with convergence guarantees. S2 Reducer reduces the communication cost by only compressing the non-zero gradients with count-sketch and bitmap, and enables the efficient AllReduce operators for parallel SGD training. We perform extensive evaluation against four state-of-the-art methods over five training models. Our results show that S2 Reducer converges to the same accuracy, reduces 81% sparse communication overhead, and achieves 1.8× distributed training speedup compared to state-of-the-art approaches.

Index Terms— Distributed training, Deep learning, Sparse, Communication, Sketch.

1. INTRODUCTION

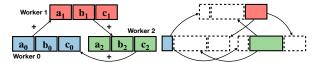
Deep neural networks (DNN) have been widely used in computer vision [1–3], speech recognition [4, 5], and natural language processing (NLP) tasks [6,7], A recent trend is that the state of the art (SOTA) results in these areas are achieved by large DNN models. Training large models needs long training periods and high computing costs. Distributed deep learning process uses many computing devices to reduce the training time [8], which parallelizes the computation of the model. Data-parallel Stochastic Gradient Descent (SGD) [9, 10] is the most popular approach to implement the distributed deep learning paradigm, because of its flexibility and well support

by the deep learning toolkit, such as TensorFlow [11], Py-Torch [12] and DeepSpeed [13]. The data-parallel SGD iterates over two steps as follows: (1) Each device conducts the forward and backward computation of the DNN model in parallel with sampled mini-batch data, and produces the local gradient; (2) Each device collects all the gradients from other devices and calculates the average of these gradients to update the model parameters.

In distributed SGD, iterative gradient aggregation is conducted among devices, which costs heavy network bandwidth resources. The bandwidth costs increase fast with larger models and even delay the training process [14, 15]. To reduce the communication time, AllReduce, an efficient and widelysupported collective communication operation [16], aggregates the gradients on GPU directly [17, 18]. However, these implementations ignore the fact that AllReduce is not suitable for sparse gradient, in which most of the elements are zero. Sparse gradient are quite prevalent, e.g., the gradient sparsity of embedding layers in some NLP models exceed 94% [19]. Some communication-efficient SGD, such as block-wise gradient sparsification [19, 20] also yield sparse gradient. Thus, a large volume of zeros is transmitted [21]. What's worse, the compressed sparse gradient, which typically contains index and value of non-zeros of sparse gradient, cannot be added up directly, unlike what the dense gradients do in AllReduce, because the indices of non-zero values among GPUs are maybe different. The current implementation of sparse collective communication in deep learning frameworks, such as Py-Torch, requires two All-Gather operations to synchronize the indices and values of sparse gradient separately. However, each device has to collect all the other sparse gradients before summing up, which has poor communication efficiency. Some other sparse-oriented systems, such as SparCML [22] and Parallax [21] introduce dynamic sparse collective communication methods to spend less transmission time, yet they have to use the inefficient AllGather or convert the sparse input to dense. Although OmniReducer [19] devises a communication library that also identifies and aggregates the non-zero blocks, it still wastes bandwidth to transfer zero blocks. While recent work [23] also enables AllReduce on a compressed gradient using count-sketch, it can only approximately find the Top-K items from the dense gradient, which

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(a) AllReduce w.r.t dense gradient. (b) AllGather w.r.t sparse gradient.

Fig. 1: Gradient transmission in AllReduce and AllGather. (a) A step of ring AllReduce. (b) Sparse gradients from other workers are gathered to concatenate.

does not return estimate values from the sketch directly.

In this paper, we propose Sparse-Sketch Reducer(S2 Reducer), an aggregation method to collect sparse gradients efficiently. The key idea of our approach is compressing the non-zeros with a novel sparse-sketch, which supports AllReduce operation and guarantees the convergence of data-parallel SGD. The sparse-sketch compresses the non-zeros with a count-sketch and a bitmap, which can be synchronized with AllReduce. S2 Reducer produces an unbiased estimation when querying the gradient from the aggregated sketch, which guarantees the convergence theoretically and experimentally.

We have realized S2 Reducer¹ and conducted extensive experiments on various CNN and NLP models in a cluster with up to 16 GPUs. The results demonstrate that our method achieves the same level of model accuracy against the baseline approaches while achieving up to $1.8\times$ speedup compared with the SOTA method and reducing 81% sparse communication time.

2. BACKGROUND AND MOTIVATION

2.1. AllReduce Opertaion

In distributed SGD, worker node $i \in \{1,\ldots,W\}$ samples a mini-batch data, computes local gradient g with backpropagation. Typically, they synchronize these gradients with $ring\ AllReduce$ to update the model parameters. Most high-performance distributed DNN training systems deploy AllReduce to achieve the speed records [8, 10, 17]. Fig. 1 (a) illustrates an example of a reduce step in the ring AllReduce for three devices. Each GPU worker partitions its gradient into W parts, such as $\{a_0,b_0,c_0\}$, then each part are reduced across all workers by W-1 summation steps. Thus, only the data structure that can be added up supports AllReduce.

2.2. Saprse Gradient & AllGather

Given a gradient vector of size d, block-wise sparsification transform it to the matrix $g^{b \times \frac{d}{b}}$, where $\frac{d}{b}$ is the block size. Block-wise Top-K computes the ℓ_2 norm of each block and finds the largest K blocks. The elements in these blocks are selected to communicate.

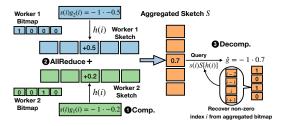


Fig. 2: An example of the workflow of S2 Reducer.

The sparse gradients from the embedding layers are also block-wise. Each embedding vector encodes a word. In one SGD iteration, only a subset of word embedding vectors are selected as mini-batch and produce corresponding gradient vectors, each vector is a block.

These sparse gradients are typically compressed with the indices and values of the non-zeros. However, The difference among the indices in different workers prevents the aggregation method from using the efficient AllReduce. A compromising approach is AllGather. As shown in Fig. 1 (b), workers collect non-zeros from others to synchronize. However, it is much slower than AllReduce, because it has to send extra traffic [19].

2.3. Sketch Compression

Count-sketch is one of the most popular communication traffic statistical algorithms in distributed systems, thanks to its mergeability property [24].

The block-wise sparsity of gradient provides an opportunity to compress the sparse data with such a sketch structure that can be merged in AllReduce. The small volume of nonzero values can be stored with a fixed-sized count-sketch. Elements in the same block share the same row index. Thus, the indices of the sparse gradient can be encoded with a mergeable bitmap. Then, the irregular sparse gradients, like these in Fig. 1 (b), are transformed to a data structure that can be added to another in a way that is shown in Fig. 1 (a).

3. SKETCH BASED SPARSE COMMUNICATION

In this section, we devise Sparse-Sketch Reducer. The proposed sparse-sketch addresses two challenges in sparse gradient communication: (1) Enabling All-Reduce operation while compressing the non-zeros; (2) Guaranteeing the convergence of data-parallel SGD.

3.1. Sparse-Sketch Reducer

Communication Workflow. The novel sparse-sketch only compresses the non-zeros and can be collected with efficient AllReduce. We illustrate the workflow of S2 Reducer as Fig. 2. Firstly, the indices are encoded with a bitmap, and non-zero values are inserted into a count-sketch. Then, both the bitmap and count-sketch are aggregated by AllReduce.

¹https://github.com/GeKeShi/Sparse_Sketch_Reducer

Finally, the aggregated sparse gradient is decompressed to update the model parameters.

Compressing Sparse Gradient. Sparse-sketch consists of a bitmap and a count-sketch. The indices of non-zeros can be encoded with the bitmap. The size of the bitmap is the number of word vectors or gradient blocks, and each element of the bitmap indicate whether the block is non-zero (contains non-zero gradients). When several bitmaps aggregate in All-Reduce, the non-zero elements of the result bitmap indicate the non-zero blocks among them. We compress the value of sparse gradient with the count-sketch, which reduces the space cost by storing a large set of items over a small number of bucket arrays. Here we take such a sketch structure that contains one bucket array as an example. Let $q \in \mathbb{R}^d$ denote the vectors of gradients (including the zeros) with size dand let m denote the number of buckets in sketch S, respectively. Let $s \in \{-1, +1\}^d$ denote a vector of random sign. In the compression phase, only the non-zero gradient q(i) are randomly mapped to a bucket by a hash function h, adding a weighted value s(i)q(i) upon the current value of the bucket. This procedure is illustrated on the left side of Fig. 2. The zeros are recorded with the bitmap and have no effect on the bucket value. Thus, our method ignores the zeros, the result is identical to a sketch that compresses the dense gradient.

Collecting Sparse-Sketch with AllReduce. Here, we prove that the compressed sparse gradient values can be added up directly, which is a prerequisite for AllReduce. Let $\vec{A} \in \{-1,0,+1\}^{d \times m}$ denotes the mapping matrix where A(i,j) = +1 or -1 iff the i_{th} item g_i is mapped to the j_{th} bucket where $j \in [1, m]$, and to zero otherwise. Let the j_{th} row vector of A^{\top} denote as $A^{\top}(j,:)$. Then, the value of j_{th} bucket is $A^{\top}(j,:)g$. Consequently, the insertion phase of compressing gradient by count sketch can be mathematically represented as a stochastic projection model: $A^{\top}g$. Similarly, we query g(i) with on i_{th} row vector of A in the decompression phase: $A(i,:)A^{\top}g$. Therefore, the estimation values of sketch can be represented as $AA^{\top}X$, which is a linear operation. When we have W compressed gradient $A^{\top}g_0, A^{\top}g_1, \dots, A^{\top}g_{W-1}$, the aggregated gradient is $A^{\top} \sum g_k$. Therefore, the aggregated sparse sketch is equivalent to that produced on the sum of sparse gradient.

Decompressing Sparse Gradient. The decompression phase is illustrated in the right part of Fig. 2. As mentioned above, we precisely record the indices of non-zeros with the aggregated bitmap, it is straightforward to recover the indices of non-zeros from the aggregated bitmap. Then, the non-zero gradient g(i) is queried as the median of the weighted values s(i)S[h(i)] in the indexed buckets. Although the queried value might not be the original value due to hash collision, we show that the estimated non-zero value is an unbiased estimation for the original gradient in Section 3.2. Let α denote the percentage of non-zero elements, our method can compress the sparse data and consequently reduce the communication

Algorithm 1 Sparse-Sketch Reducer

```
Input: Sparse(g) - the indices of non-zeros; r, c - size of sketch.
Output: Aggregated sparse gradient \hat{g}.
Initialize: sign hashes \{s_{ij}\} and bucket hashes \{h_{ij}\}, r \times c table of count-
    sketch S, bitmap C
 1: for all Worker w \in [0, W-1] do
        C[x] \leftarrow 1 if x_{th} block is non-zero
 2.
 3:
        for all i \in Sparse(g) do
 4.
            for all j \in [0, r-1] do
 5:
                S[j, h_j(i)] + = s_j(i)g(i) // Compression
 6:
 7:
        end for
        AllReduce(S, C)
9:
        Recover Sparse(g) from aggregated bitmap C
10:
        for all i \in Sparse(g) do
            for all j \in [0, r-1] do
11:
                estimates[j,i] = s_j(i)S[j,h_j(i)] \\
12:
13:
14:
            \hat{g}(i) \leftarrow median(estimates) // Decompression
15:
        end for
16:
        return \hat{g}
17: end for
```

cost when the size of sketch $\lambda = \frac{m}{\alpha d} < 1$. The detail of our method is summarized in Algorithm 1.

3.2. The Convergence of S2 Reducer Based SGD

The other challenge of applying S2 Reducer in distributed SGD is ensuring convergence. There are two origins of sparse gradient, one is the embedding matrix, the other is the gradient sparsification. We theoretically prove that our method maintains the convergence rate in both settings. We state the detailed proof in Appendix A.1 and A.2 of [25].

Firstly, we prove that the gradient queried from sparsesketch is an unbiased estimation of the sparse gradient. This result immediately derives in Theorem 1 that our method has the same order of convergence rate as vanilla distributed SGD when applying to the embedding layer gradients.

Theorem 1. Let \hat{g} denote the queried gradient. The expectation of the estimation error is zero, i.e., $\mathbb{E}\left[\hat{g}\right] = g$. S2 Reducer Based SGD updates model parameters $\omega^t \in \mathbb{R}^d$ with the gradients of loss function $f(\omega_t)$ at iteration t, the convergence rate is $\min_{0 \le t \le T} \|\nabla f(\omega^t)\|^2 = O(1/\sqrt{T})$.

Next, we prove the convergence of the S2 Reducer-based SGD matches the convergence rate of vanilla block Top-K method in Theorem 2. We derive that our scheme is a δ -approximate compressor. Then the convergence result is given based on the proof of [26].

Definition 1 (δ -approximate compressor). An operator $\mathcal{U}(\cdot)$: $\mathbb{R}^d \to \mathbb{R}^d$ is a δ -approximate compressor if $\exists \delta \in (0,1)$ such that $\mathbb{E} \|g - \mathcal{U}(g)\|_2^2 \leq (1-\delta) \|g\|_2^2$, $\forall g \in \mathbb{R}^d$.

Theorem 2. Let b denote the total number of blocks. The sketch based block Top-K are δ -approximate compressors

with $\delta = \frac{K}{b}$, i.e., $\mathbb{E} \|g - \mathcal{U}(sparse(g))\|_2^2 = \left(1 - \frac{K}{b}\right) \|g\|_2^2$. The convergence rate is $\min_{0 \le t \le T} \|\nabla f(\boldsymbol{\omega}^t)\|^2 = O(1/\sqrt{T})$.

4. EVALUATION

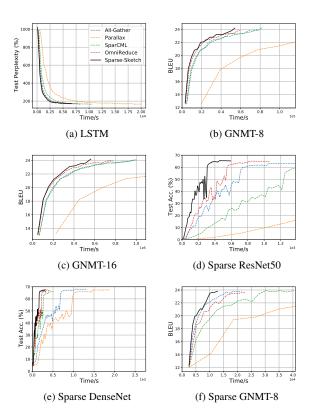


Fig. 3: Training curves of (a)-(c): NLP models, where the embedding layers gradients are sparse. (d)-(f): block-wise sparsification.

In this section, we show the end-to-end training speedup of the S2 Reducer compared with four baselines: All-Gather, Parallax [21], SparCML [22], OmniReduce [19]. We have implemented these methods based on PyTorch, and deployed them on a 16 RTX-3090 GPU cluster that equips a 10Gb Ethernet. The communication library is NCCL [16] except Parallax, which can only use Gloo.

We conduct two kinds of experiments. Firstly, we demonstrate the speedup when training LSTM for language models, GNMT-8 and GNMT-16 [27], which yield sparse gradients from embedding layers. Then we show the performance of accelerating the training of ResNet-50 [1] DenseNet [2] and GNMT-8 when sparsifying the whole gradients. The details of models and datasets are shown in Table 1.

Sparse Gradient from Embedding Layers. As illustrated in Fig. 3 (a) & (b), our method achieves $1.6\times$ speedup compared with SparCML when training LSTM, and also $1.25\times$ speedup compared with AllGather implementation on GNMT-8. Although our method has compressed the gradient, the model accuracy not only is the same as other lossless methods when

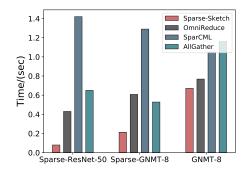


Fig. 4: The communication overhead of sparse gradient. The first two models are ResNet-50 and GNMT-8 with block-wise sparsification, and the last one yields parse gradient from embedding layers.

ſ	Model	Dataset	Embed./Dense	Sparsity
Ī	LSTM	WikiText-2	75/115 MB	99.1%
ĺ	GNMT-8	WMT-16	265/195 MB	95.17%
ſ	GNMT-16	WMT-16	265/245 MB	95.17%
Ì	DenseNet	CIFAR-100	4.04 MB	
ĺ	ResNet-50	CIFAR-100	94.8 MB	

Table 1: The details of training models and datasets. The first three models contain embedding layers, the table lists the size of embedding and other dense weights. We represent the sparsity by the average percentage of non-zeros in embedding layer gradient. The last two are dense models, we apply block-wise sparsification on them.

training the same epochs in LSTM and GNMT-8 but also converges faster to the target BLEU on GNMT-16 than others. Fig. 3 (c) show that our method achieves $1.4\times$ speedup compares with OmniReduce, the state-of-the-art method, with only 11 epochs, while others require 16 epochs.

Block-wise Sparse Gradient. In this section, we apply block-wise sparsification on DenseNet, ResNet-50, and GNMT-8 with a 1/32 compression rate. Due to the small size of DenseNet and ResNet-50 after sparsification, we configure the bandwidth to 1GbE in their training. As shown in Fig. 3 (d), our method reduces 81% sparse communication time compared with OmniReduce, and achieves $4.1\times$ speedup over SparCML and $2.3\times$ over AllGather on ResNet-50 (block-wise sparsification), while maintaining the model accuracy. As illustrated in Fig. 3 (f), our method achieves $1.8\times$ speedup compared with OmniReduce, which is the SOTA approach, on the large NLP model GNMT-8, and reduce 65% sparse communication time. The detailed results of communication overhead and sensitivity of hyper-parameter λ are stated in Appendix B.2 and B.3 of [25] respectively.

5. CONCLUSION

This paper proposes Sparse-Sketch Reducer, a novel sketch-based sparse gradient collective communication method. Our method reduces the communication overhead by enabling AllReduce on the compressed sparse-sketch. Experimental results show that it effectively reduces communication costs and accelerates distributed DNN training.

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