Human Decision Making with Bounded Rationality

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***Abstract*—In critical environments that require a high accuracy of decisions, utilizing human cognitive strengths and expertise in addition to machine observations is advantageous to improve decision quality and enhance situational awareness. While the current literature on human decision making is primarily based on the paradigm of perfect rationality, humans are subject to decision noise and employ stochastic choice rules. Human decision making under such realistic environments needs to be further studied. In this paper, instead of assuming that a human selects the optimal action with probability one, we employ a bounded rationality choice model where all the actions are candidates for selection, but better options are chosen with higher probabilities. In a Bayesian hypothesis testing framework, we evaluate the individual decision making performance when humans have different degrees of bounded rationality. Furthermore, we analyze the decision fusion rule for a team of two human agents and characterize the asymptotic performance of collaborative decision making as the number of human participants becomes large.**

***Index Terms*—Human decision making, bounded rationality, stochastic decision choice, decision fusion in multi-agent systems.**

I. Introduction

The modeling of decision-making and control systems that include human agents along with sensing devices is an important research direction in many applications including military surveillance, natural disaster forecasting, and healthcare. In these environments, decision making must be very accurate. A combination of human expertise and sensor-based observations often improves decision quality and enhances situational awareness. For instance, in electronic warfare (EW) systems, the detection of an adversary radar is required before designing appropriate countermeasures. In such scenarios where the success of the campaign depends on a small detail being observed or missed, automatic sensor-only decision making may not be sufficient and it is necessary to incorporate humans in the loop of decision making, command and control.

Collaborative decision making by (only) machines or sensors has been analyzed extensively in both centralized and distributed settings [1], [2]. However, the choices made by these rational decision making systems differ from those made by humans because humans exhibit cognitive limitations, imperfect rationality, and behavioral uncertainties. In the signal processing literature, behavior of human agents has been studied in different contexts. For example, the authors in [3], [4] have analyzed group decision making by assuming that each human agent makes the local decision using a random threshold. Cognitive biases in sequential belief updating, e.g., conservatism and over-responsiveness, are considered in [5] to study the problem of optimal ordering of information presented to a human. A semi-autonomous human assisted decision making framework was proposed when the human and the machine make correlated observations [6]. In the framework of binary hypothesis testing, the Nobel prize wining prospect theory has been employed to model human cognitive biases in decision making [7]–[10]. Moreover, the decision making behavior of cognitive memory constrained humans has been studied when the decision is made based on multiple sources of information [11].

Despite these efforts that incorporate cognitive biases and limitations of humans in signal processing tasks, there is a lack of a theoretical framework to describe decision noise and uncertainties in human decision making. In experimental psychology, trial-to-trial variability has been observed as a prominent feature of human behavior, i.e., differences of responses of the same human are noticeable even when the external conditions, such as the sensory signal and the task environment, are kept the same [12]. In this paper, we employ the bounded rationality model to quantify human uncertainty such as trial-to-trial variability. In contrast to perfect rationality where the decision maker always chooses the action that has the maximum utility, it is possible for all the actions in the action space to be selected under bounded rationality. Although the optimal option is no longer selected with probability one, actions that have larger utilities are still chosen with higher probabilities. By incorporating stochastic elements into decision making, the property of *better options are selected more often* accurately reflects humans' cognitive and learning process. Such a bounded rationality model is consistent with the concepts of random utility and evolutionary adjustment in the decision process, and has been widely adopted in the behavioral economics literature [13], [14].

This work investigates binary decision making by humans under bounded rationality. We focus on the logistic regression choice model [13], [15] in which the probability of selecting action i is proportional to e^{u_i}, where u_i is the utility produced by action i. First, we study the individual behavioral difference between humans as it pertains to decision making when they have different degrees of bounded rationality. Next, we analyze the performance of decision fusion with two agents who have different bounded rationality parameters. The asymptotic performance of the collaborative decision making system composed of multiple human agents is also derived.

II. Impact of Bounded Rationality on Human Decision Making

It is well documented that humans make decisions in the framework of hypothesis testing, where the decision is made by selecting the hypothesis that best supports the given set of observations [8]. In this section, we investigate the decision making behavior of bounded rational humans under the framework of binary hypothesis testing.

A. The bounded rationality model

Signal detection problems have been extensively studied in centralized as well as distributed settings in different contexts. The standard approach is to assume perfect rationality of the decision makers. Specifically, to select an action among different alternatives $i \in \mathcal{I}$, perfect rational decision makers always choose the alternative that yields the maximum utility $i^* = \arg\max_{i \in \mathcal{I}} u_i$, where u_i represents the utility that action i produces. In contrast to perfect rational decision makers, humans have bounded rationality because of two reasons: i) humans might not either know the entire action space or completely understand how the alternatives differ from each other, and ii) the utility of choosing a particular alternative might not be correctly characterized. The concept of bounded rationality recognizes that there are cognitive biases, limited thinking capacity, lack of available information, time constraints, etc., in human decision making.

To capture bounded rationality, we employ the Logistic (regression) choice model and consider that the human selects actions $i \in \mathcal{I}$ with probability

$$P(i) = \frac{e^{u_i/\beta}}{\sum_{i \in \mathcal{I}} e^{u_i/\beta}} \tag{1}$$

and if the action space $\mathcal{A}$ is continuous, the probability density of choosing action a is given by

$$p(a) = \frac{e^{u(a)/\beta}}{\int_{a \in \mathcal{A}} e^{u(a)/\beta}} \tag{2}$$

In both discrete and continuous action spaces, the human's choice of action is not deterministic but rather a random variable. As human behavior is inherently stochastic (i.e., the comparisons and evaluations of alternatives vary in a probabilistic manner), it has been shown that the Logistic regression model provides a psychologically accurate method to model bounded rationality [13], [15]. In such a probabilistic approach, options that yield higher utilities are selected more often. The Logistic choice model is also known as the log-linear model since the log probability of choosing one action over another is proportional to the utility difference between the two actions.

In the Logistic models (1) and (2), β represents the degree of cognitive and computational limitations exhibited by the human. Note that if $\beta \to \infty$, the choice distribution in (1) becomes the uniform distribution over all possible actions in $\mathcal{I}$. In this extreme case, the decision-maker does not employ any reasoning to make intelligent decisions and instead randomly selects the actions with equal probabilities. On the other hand, if $\beta \to 0$, the probability of selecting the action that has the maximum utility goes to 1, and this is the same as the optimal decision rule of a rational decision maker. Hence, we employ β to describe the degree of bounded rationality: β going from 0 to ∞ corresponds to the state of human going from perfectly rational to totally irrational.

B. Bounded rationality in hypothesis testing

In this subsection, we employ the Logistic choice model (1) to characterize human behavior while decision making using the hypothesis testing framework. Consider a binary hypothesis testing problem where a human decides which of the hypotheses H_0 or H_1 is true, based on an observation $r \in \mathcal{R}$ regarding the phenomenon of interest (PoI). We assume that the prior probabilities are denoted as $P(H_0) = \pi_0$ and $P(H_1) = \pi_1$, and the observation r under the two hypotheses has the probability density functions (pdfs) $f_0(r)$ and $f_1(r)$, respectively. We denote $u_{i,j}$ as the utility of deciding in favor of H_i when the true hypothesis is H_j for $i, j \in \{0, 1\}$.

Given an observation r, the expected utility of declaring H_0 or H_1 is given by

$$\begin{aligned} \boldsymbol{u}_0 &= P(H_0|r)u_{0,0} + P(H_1|r)u_{0,1} \\ \boldsymbol{u}_1 &= P(H_0|r)u_{1,0} + P(H_1|r)u_{1,1}, \end{aligned} \tag{3}$$

where $P(H_i|r)$ denotes the conditional probability that H_i is true given that the observation is r, with $P(H_i|r) = \frac{f(r|H_i)\pi_i}{f(r)} = \frac{f_i(r)\pi_i}{f(r)}$ for $i = 0, 1$, respectively, where $f(r) = \pi_0 f_0(r) + \pi_1 f_1(r)$ denotes the probability density of r.

Under the Logistic choice model given in (1), if the observation is r, the human chooses H_0 and H_1 with probabilities $\phi_0 = e^{\boldsymbol{u}_0/\beta}/(e^{\boldsymbol{u}_0/\beta} + e^{\boldsymbol{u}_1/\beta})$, and $\phi_1 = e^{\boldsymbol{u}_1/\beta}/(e^{\boldsymbol{u}_0/\beta} + e^{\boldsymbol{u}_1/\beta})$, respectively. Substituting $P(H_i|r)$ into (3), and simplifying, we obtain

$$\begin{aligned} \phi_0(r) &= \frac{1}{e^{\{(u_{1,1}-u_{0,1})f_1(r)\pi_1-(u_{0,0}-u_{1,0})f_0(r)\pi_0\}/\{f(r)\beta\}}+1} \\ \phi_1(r) &= \frac{e^{\{(u_{1,1}-u_{0,1})f_1(r)\pi_1-(u_{0,0}-u_{1,0})f_0(r)\pi_0\}/\{f(r)\beta\}}}{e^{\{(u_{1,1}-u_{0,1})f_1(r)\pi_1-(u_{0,0}-u_{1,0})f_0(r)\pi_0\}/\{f(r)\beta\}}+1} \end{aligned} \tag{4}$$

Note that when $\beta \to 0$, the decision probabilities in (4) reduce to those obtained from the optimal likelihood ratio test (LRT) employed by perfect rational decision makers:

$$\frac{f_1(r)}{f_0(r)} \underset{H_0}{\overset{H_1}{\gtrless}} \frac{\pi_0(u_{0,0} - u_{1,0})}{\pi_1(u_{1,1} - u_{0,1})} \triangleq \eta. \tag{5}$$

When $\beta \to \infty$, the human chooses H_0 or H_1 with equal probabilities for all values of the observation r.

Next, we analyze the impact of bounded rationality quantified in terms of β on human decision making performance. For a human that employs the decision probabilities given in (4), the probabilities of false alarm and detection are given by

$$\phi_f^h(\beta) = \int_{\mathcal{R}} \phi_1(r) f_0(r) dr, \quad \phi_d^h(\beta) = \int_{\mathcal{R}} \phi_1(r) f_1(r) dr, \tag{6}$$

respectively. Then, the expected utility of the human is

$$O_h(\beta) = \pi_0(1-\phi_f^h)u_{0,0} + \pi_0\phi_f^h u_{1,0} + \pi_1(1-\phi_d^h)u_{0,1} + \pi_1\phi_d^h u_{1,1} \tag{7}$$

Proposition 1. *For a given hypothesis testing problem, as β goes from 0 to ∞, the human's expected utility $O_h(\beta)$ decreases.*

Heuristic Proof. When the value of β becomes larger, the human has a lower probability of making a correct decision for each value of $r \in \mathcal{R}$. Hence, it is expected that $O_h(\beta)$ will decrease as β increases from 0 to ∞.

$\square$

Next, we provide an example to illustrate the human decision making performance. Consider the observations under the two hypotheses given by:

$$H_0 : r = -m + n, \quad H_1 : r = m + n \tag{8}$$

where $-m$ and m are signal amplitudes under H_0 and H_1 respectively. n represents the additive noise where we assume that its magnitude follows the uniform distribution $U(-v, v)$ where $v > m > 0$. Furthermore, we assume that $\pi_0 = \pi_1 = 1/2$, $u_{1,0} = u_{0,1} = 0$ and $u_{0,0} = u_{1,1} = 1$. Following previous analysis, we have the following result.

Corollary 1. *In the above problem setting where $n \sim U(-v, v)$, the probabilities of false alarm and detection for a human with bounded rationality parameter β are given by ϕ_f^u and ϕ_d^u, respectively, where*

$$\phi_f^u = 1 - \phi_d^u = \frac{1}{2v}\left(\frac{2m}{e^{1/\beta} + 1} + v - m\right) \tag{9}$$

The expected utility of the human is given by

$$O_h^u = \frac{1}{2} + \frac{1}{2v}\left(m - \frac{2m}{e^{1/\beta} + 1}\right) \tag{10}$$

We can observe that O_h^u decreases as the parameters β and v increase. Intuitively, as the human becomes more irrational, and as the noise interval increases, the human has less expected utility while decision making. On the other hand, when the noise n in (8) follows Gaussian distribution, there is no closed form expression for O_h. In this scenario, we rely on simulations to illustrate the performance in Section IV.

In this section, we studied the impact of bounded rational humans in binary hypothesis testing problems. Note that the analysis can be easily extended to M-ary hypothesis testing problems by following the procedures presented in (3)-(7).

III. Collaborative Human Decision Making Under Bounded Rationality

Next, we consider the decision fusion scheme shown in Fig. 1, where human agents A and B make observations r_a and r_b regarding the PoI through two independent channels. The local decisions d_a and d_b made by the two humans are transmitted to a fusion center (FC) to make a final decision d_o. Assuming that A and B have bounded rationality parameters β_a and β_b, the probabilities of false alarm and detection of the two humans, denoted by ϕ_f^a, ϕ_d^a and ϕ_f^b, ϕ_d^b, respectively, can be obtained using (6).

Based on the local decisions d_a and d_b, the FC makes the final decision d_o using the optimal LRT, i.e.,

$$P(d_0 = 1 | d_a = i, d_b = j) = I\left(\frac{P(d_a = i, d_b = j | H_1)}{P(d_a = i, d_b = j | H_0)} \geq \eta\right)$$

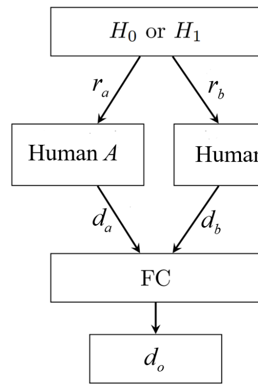

Fig. 1. Collaborative decision making by a 2-human team.

for $i, j \in \{0, 1\}$, where $I(\cdot)$ is an indicator function that equals 1 if the statement inside the parentheses is true, and it equals 0 otherwise. Note that $P(d_a = i, d_b = j | H_k)$ for $i, j, k \in \{0, 1\}$ are functions of $\phi_f^a, \phi_d^a, \phi_f^b, \phi_d^b$: for instance, $P(d_a = 1, d_b = 1 | H_1) = \phi_d^a \phi_d^b$, and $P(d_a = 0, d_b = 0 | H_0) = (1 - \phi_f^a)(1 - \phi_f^b)$. Moreover, the probabilities of false alarm and detection at the FC are:

$$\phi_f^o = \sum_{i=0}^{1} \sum_{j=0}^{1} P(d_0 = 1 | d_a = i, d_b = j) P(d_a = i, d_b = j | H_0)$$

$$\phi_d^o = \sum_{i=0}^{1} \sum_{j=0}^{1} P(d_0 = 1 | d_a = i, d_b = j) P(d_a = i, d_b = j | H_1).$$

Finally, the expected utility of the FC can be calculated as

$$O = \pi_0 (1 - \phi_f^o) u_{0,0} + \pi_0 \phi_f^o u_{1,0} + \pi_1 (1 - \phi_d^o) u_{0,1} + \pi_1 \phi_d^o u_{1,1} \tag{11}$$

Furthermore, we investigate the decision fusion scenario where there are multiple human agents (n>2). Again, it is very unlikely that humans in the group have the same degree of bounded rationality. To model the behavior uncertainty in the group, we assume that the parameter β_i of each human is a random variable that follows pdf $t(\beta)$. Finally, the local decisions made by each human agent, d_i for $i = \{1, \ldots, n\}$, are aggregated at the FC for decision fusion.

As the number of humans in the decision making system becomes larger, the analysis of the LRT based decision rule becomes tedious and intractable. In this case, we evaluate the asymptotic detection performance of the system via the Chernoff Distance [16]:

$$\lim_{n \to \infty} P_e \approx e^{-n \times \mathcal{C}} \tag{12}$$

where P_e is the average probability of error for the optimal Bayesian detector, n is the number of humans, and $\mathcal{C}$ represents the normalized Chernoff Distance. Intuitively, P_e decreases exponentially as the number of agents n increases and $\mathcal{C}$ is the best achievable error exponent. It is desired to have a large value of $\mathcal{C}$ so that the probability of error decreases faster. In the following, we derive the Chernoff Distance that characterizes the asymptotic performance of the collaborative decision making system without providing a detailed proof due to page limits.

Proposition 2. *When the humans' bounded rationality parameter β follows pdf $t(\beta)$, the normalized Chernoff Distance of the collaborative decision making system is given by*

$$\mathcal{C} = -\min_{0\le\lambda\le1} log\left(\left(\frac{\bar{\theta}}{\bar{\alpha}}\right)^{\lambda}\bar{\alpha} + \left(\frac{1-\bar{\theta}}{1-\bar{\alpha}}\right)^{\lambda}(1-\bar{\alpha})\right) \quad (13)$$

with

$$\bar{\theta} = \int_{\beta,\mathcal{R}} \phi_1(r) f_1(r) t(\beta) dr d\beta,$$
$$\bar{\alpha} = \int_{\beta,\mathcal{R}} \phi_1(r) f_0(r) t(\beta) dr d\beta$$

where $\phi_0(r), \phi_1(r)$ are given in (4).

IV. Simulation Results

For illustration, we conduct experiments when a human with bounded rationality parameter β solves the binary hypothesis testing problem in (8). We assume that $\pi_0 = 0.7$, $\pi_1 = 0.3$, $u_{0,1} = -10$, $u_{1,0} = -5$, and $u_{0,0} = u_{1,1} = 5$. We assume the signal magnitude $m = 4$ and let $n \sim \mathcal{N}(0, \sigma)$ be a Gaussian noise with mean 0 and standard deviation σ. In Fig. 2, we plot the expected utility of human decision making O_h as a function of β for different values of σ. It is observed that O_h decreases when a) β increases, i.e., when the human is less rational; and b) σ takes a larger value, i.e., when the signal to noise ratio (SNR) becomes smaller.

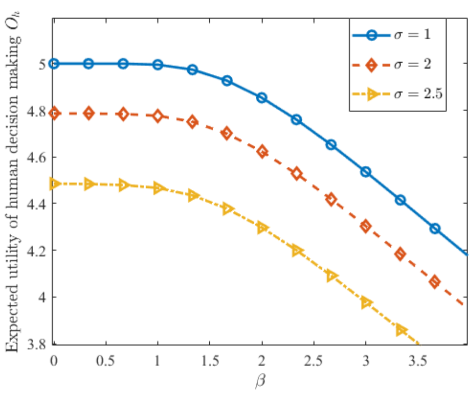

Fig. 2. Expected utility of human with respect to β.

Next, we provide simulation results for the decision making scheme of Fig. 1 under the same parameters with $\sigma = 2$. In Fig. 3, we plot the expected utility of the FC with respect to the bounded rationality parameters of A and B, i.e., β_a and β_b. We can see that the utility of the FC is symmetric, while achieving the maximum utility at $\beta_a = \beta_b = 0$, and achieving the minimum utility at $\beta_a = \beta_b \to \infty$. Moreover, one can note that when the bounded rationality parameter of one human is small, the FC's expected utility in decision fusion does not decrease very fast even if the rationality the other human

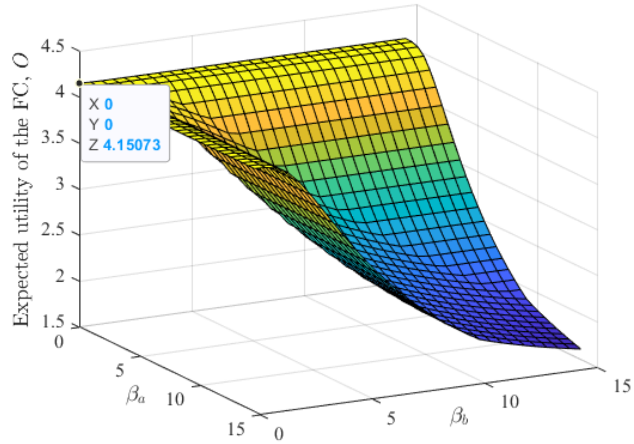

Fig. 3. Expected utility of the FC with respect to β_a and β_b.

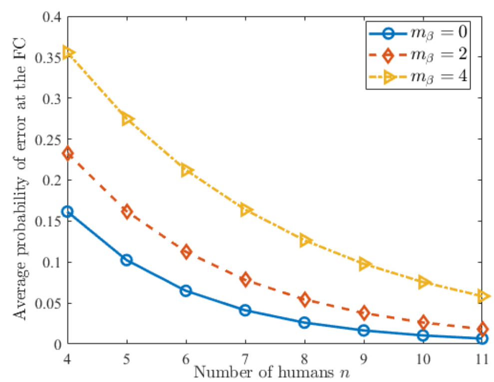

Fig. 4. Average probability of error with respect to n

deteriorates. On the other hand, the FC's performance degrades significantly when both humans become irrational.

In Fig. 4, we plot the FC's average probability of error P_e when the number of humans increases. Let us assume that the bounded rationality parameter of each human follows a truncated Gaussian distribution on the support $\beta \in [0, \infty)$, where the standard deviation is $\sigma_\beta = 2$, and the mean takes different values $m_\beta \in \{0, 2, 4\}$. It is observed that P_e decreases exponentially when n increases. As the mean of the Gaussian distribution m_β becomes large, i.e., when the group of humans is less rational, the probability of error increases.

V. Conclusion

This paper incorporated the bounded rationality decision model for humans into binary hypothesis testing. We investigated the decision fusion of two human agents and derived the asymptotic performance when the number of human agents is larger than two. In future work, we will extend the bounded rationality model to study adaptive strategy design and resource management in human-machine teaming and interactions.

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