IMPROVED BEAMFORMING ENCODING FOR JOINT RADAR AND COMMUNICATION

Tuomas Aittomäki and Visa Koivunen

Department of Signal Processing and Acoustics,
Aalto University
PO Box 13000, FI-00076 Aalto, Finland
Email: tuomas.aittomaki@aalto.fi, visa.koivunen@aalto.fi

ABSTRACT

Integrated Sensing and Communication Systems (ISAC) that are capable of functioning both as radars and communication systems have a tremendous potential to provide significant performance gains and cost savings and facilitate the sharing of the same energy, spectral, and hardware resources. Managing interference in frequency and spatial domains is a crucial task in ISAC. We consider a radar-centric scenario where a radar system is able to do beamforming and transmit communications data on the side. We propose an improved method allowing good control of the transmit beampattern power resulting in lower communication error level. Furthermore, we also propose straightforward method for phase coding of information in radar signals.

Index Terms— Integrated sensing and communications, co-design, radar, communication, coexistence, beamforming, encoding

1. INTRODUCTION

With the ever-increasing demand for both communication and radar capabilities, the usable electromagnetic spectrum is becoming more and more congested [1]. As a solution to this, Integrated Sensing and Communications, where radars and communications systems are operating in shared spectrum have been proposed [2, 3]. The radar and communication systems can co-operate or be even co-designed for mutual benefit [4, 5].

A radar with a beamforming capability can synthesize a beampattern that also transmit payload data for communications users. For transmit beamforming, beams could be steered separately towards the communication receivers, but this would take resources away from the crucial radar tasks. A more efficient approach is to transmit the communication signal on the side while maintaining the performance of the radar. In this type of radar-centric approach, the beampattern is optimized so that the communication signal can transmitted while incurring minimal losses to the performance of the radar. The communication data can be transmitted using amplitude, phase, or both [6].

Transmit beamforming for joint radar–communication systems have been studied previously in several papers. A beamforming approach using the sidelobe level (e.g. the power) of the transmit beampattern was proposed in [7]. However, this method results in narrow notches in the beampattern for the communication signal which can result in errors under uncertainty on the direction of the communication receiver. A constraint on the derivative of the beampattern was introduced in [8], to address this problem. Unfortunately the impact of this type of constraint is limited, and significant errors are still possible. In this paper, we propose a new design approach allowing one to control the beampattern sidelobe levels over a region of uncertainty in the direction of the communication receiver.

Encoding the communication by phase modulation of the beampattern of system with waveform diversity has been described in e.g. [6]. This method proposes using different weights for the transmitted waveforms. In this paper, we propose much simpler and more effective method for encoding the communication signal in the phase of the beampattern.

The benefit of the communication by beampattern modulation is that the communication can be carried out with little extra cost. However, the main drawback is that the data rate for the communication is largely determined by the PRT of the radar, which can also change during the operation.

This paper is organized as follows: The signal model is presented in Section 2. The beampattern design for power modulation is presented in Section 3, whereas the phase modulation method is introduced in Section 4. Numerical examples are provided in Section 5, and final conclusions are made in Section 6.

2. SIGNAL MODEL

In ISAC systems, the radar and communication operations can be designed to take place on the same frequency band, possibly sharing the same hardware and antenna resources. The beamforming capability of the radar can also be used to transmit communication signals. With appropriate design of the transmit beampattern, there is minimal effect on the main operation of the radar. The transmit beampattern of the radar can be written as [9, 10]

$$B(\theta) = \mathbf{a}_T^H(\theta)\mathbf{R}_\mathbf{x}\mathbf{a}_T(\theta), \tag{1}$$

where $\mathbf{a}_T(\theta)$ is the transmit steering vector and $\mathbf{R}_\mathbf{x}$ is the covariance matrix of the transmitted waveforms. Typically, the maximum square error or the integrated square error between the desired beampattern B_0 and the actual beampattern is minimized on the angular region of interest Θ_M. The sidelobes need to be constrained to a tolerable level. In addition, constraints are needed for the total power of the array and the power of each antenna element. This beampattern design problem can be written as

$$\underset{\alpha,\mathbf{R}_\mathbf{x}}{\text{minimize}}\ \max\left|\alpha B_0(\theta) - \mathbf{a}_T^H(\theta)\mathbf{R}_\mathbf{x}\mathbf{a}_T(\theta)\right|^2,\ \theta\in\Theta_M \tag{2a}$$

$$\text{subject to } \mathbf{a}_T^H(\theta)\mathbf{R}_\mathbf{x}\mathbf{a}_T(\theta) \le c_{\text{SL}},\ \theta\in\Theta_{\text{SL}} \tag{2b}$$

$$\operatorname{tr}(\mathbf{R}_\mathbf{x}) \le P_{\text{tot}} \tag{2c}$$

$$(\mathbf{R}_\mathbf{x})_{ii} \le P_{\text{el}} \tag{2d}$$

$$\mathbf{R}_\mathbf{x} \succeq 0, \tag{2e}$$

where α is a scaling constant that is used for match the desired shape of the beampattern with the available power [9], c_{SL} is the allowed sidelobe level, Θ_{SL} is the sidelobe region, P_{tot} is the total transmit power, and P_{el} is the maximum transmit power of each element. Eq. (2e) denotes $\mathbf{R}_\mathbf{x}$ being constrained to be positive-semidefinite.

Suppose that the available waveforms $\mathbf{s}$ are coded with a matrix $\mathbf{W}$ such that $\mathbf{x} = \mathbf{W}\mathbf{s}$. assuming the waveforms in $\mathbf{s}$ are uncorrelated, the transmit covariance matrix is $\mathbf{R}_\mathbf{x} = \mathbf{W}\mathbf{W}^H$. If there are as many waveforms as there are transmit elements, $\mathbf{R}_\mathbf{x}$ has full rank and the beampattern design problem can be solved initially for $\mathbf{R}_\mathbf{x}$, and then the encoding matrix $\mathbf{W}$ can be obtained using a matrix square root or Cholesky factorization.

For fewer waveforms than the number of antenna elements, the covariance matrix becomes rank-deficient. Particularly for a single waveform, we get a rank-one covariance matrix $\mathbf{R}_\mathbf{x} = \mathbf{w}\mathbf{w}^H$. With a rank contraint, (2) becomes a difficult problem to solve. Common approaches include trying to solve $\mathbf{w}$ directly using nonconvex methods, or obtaining the full-rank solution first and using it to get the rank-deficient solution. One such approach is semidefinite relaxation in which the full rank $\mathbf{R}_\mathbf{x}$ is solved first, and the then the low-rank or rank-1 solution is obtained by e.g. randomization [11].

3. POWER MODULATION

With beamforming capability, the radar transmitter is able to change the power radiated towards the communication receiver while maintaining the other properties of the transmit beampattern. In [6], this approach is referred to as amplitude-shift keying.

A certain power level of the trasmit beampattern corresponds to a communication symbol. With this kind of encoding, the number of symbols that can be transmitted depend of the the permitted power level, the channel quality, and the dynamic range of the communication receiver.

Denote the communication symbols by $\{\gamma_1, \gamma_2, \ldots, \gamma_{N_\gamma}\}$ and the direction of the communication receiver by θ_c. In order to have the desired symbol transmitted to the communication receiver while the beampattern shape is maintained elsewhere, we would need an additional constraint

$$\mathbf{a}_T^H(\theta_c)\mathbf{R}_\mathbf{x}\mathbf{a}_T(\theta_c) = \gamma_i.$$

The beampattern design problem needs thus to be solved separately for each communication symbol γ_i.

In [6], a method is proposed a method for the single-waveform case that minimizes the absolute error between the ideal array response b_0 such that $|b_0(\theta)|^2 = B_0(\theta)$, and the actual one $\mathbf{w}^H\mathbf{a}_T(\theta)$. This optimization problem can then be written as

$$\underset{\alpha,\mathbf{w}}{\text{minimize}}\ \max\left|\alpha b_0(\theta) - \mathbf{w}^H\mathbf{a}_T(\theta)\right|,\ \theta\in\Theta_M \tag{3a}$$

$$\text{subject to } |\mathbf{a}_T^H(\theta)\mathbf{w}|^2 \le c_{\text{SL}},\ \theta\in\Theta_{\text{SL}} \tag{3b}$$

$$\|\mathbf{w}\|^2 \le P_{\text{tot}} \tag{3c}$$

$$|(\mathbf{w})_{ii}|^2 \le P_{\text{el}} \tag{3d}$$

$$\mathbf{w}^H\mathbf{a}_T(\theta_c) = \gamma_i^{1/2}, \tag{3e}$$

where the linear constraint (3e) is used to get the desired symbol. This problem is convex but one has to solve a nonconvex problem to obtain $b_0(\theta)$ needed initially.

There are also drawbacks to using the linear constraint (3e). Firstly, it fixes the phase, which not required here. Furthermore, any error in the direction of the communication receiver θ_c could lead to large change in the power of the transmit beampattern. This would result in wrong symbol being decoded and thus, a high error rate. A constraint on the derivative of the beampattern was proposed in [8] to alleviate this problem. In this approach, the constraint

$$\mathbf{w}^H\frac{d\,\mathbf{a}_T(\theta_c)}{d\,\theta} = 0 \tag{4}$$

is used limiting the change of the beampattern in the vicinity of θ_c by generating a local extremum point in the beampattern. However, this approach can only guarantee an extremum at θ_c, but not control the rate of change of the beampattern for $\theta \neq \theta_c$.

We propose an improved approach for the beampattern design that can constrain the deviation of the beampattern from the desired symbol γ_i in the vicinity of θ_c. The optimization problem for this proposed approach can be written as minimization of the maximum absolute error ϵ_M as follows

$$\underset{\alpha,\mathbf{R_x}}{\text{minimize}}\ \epsilon_M \tag{5a}$$

$$\text{subject to}\ \max\left|\alpha B_0(\theta)-\mathbf{a}_T^H(\theta)\mathbf{R_x}\mathbf{a}_T(\theta)\right|^2<\epsilon_M,\ \theta\in\Theta_M \tag{5b}$$

$$\mathbf{a}_T^H(\theta)\mathbf{R_x}\mathbf{a}_T(\theta)\le c_{\mathrm{SL}},\ \theta\in\Theta_{\mathrm{SL}} \tag{5c}$$

$$\mathrm{tr}(\mathbf{R_x})\le P_{\mathrm{tot}} \tag{5d}$$

$$(\mathbf{R_x})_{ii}\le P_{\mathrm{el}} \tag{5e}$$

$$\left|\gamma_i-\mathbf{a}_T^H(\theta)\mathbf{R_x}\mathbf{a}_T(\theta)\right|<\epsilon_\gamma,\ \theta\in\Theta_c \tag{5f}$$

$$\mathbf{R_x}\succeq 0, \tag{5g}$$

where Θ_c is the set of possible directions of the communication receiver and ϵ_γ controls the error in the vicinity of the communication direction. In case of a MIMO radar with full-rank $\mathbf{R_x}$, the direction θ can be discretized to obtain convex optimization problem solvable using interior-point methods. The solution can be then used as the starting point for the low-rank solutions as stated before.

For communication with multiple communication receivers, multiple constraints in the form

$$|\gamma_{i_k}-\mathbf{a}_T^H(\theta)\mathbf{R_x}\mathbf{a}_T(\theta)|<\epsilon_{\gamma,k},\quad \theta\in\Theta_{c,k}$$

are required for each user k. In this case, the beampattern design problem has to be solved for each combination of the transmitted symbols $\{\gamma_{i_1},\gamma_{i_2},\ldots\}$.

4. PHASE MODULATION

A problem in the power modulation approach is that communication is not possible in the direction of the mainlobe without affecting the operation of the radar. This issue can be avoided using phase modulation.

In phase modulation, the data is coded into the phase of the transmitted signal. The set of symbols is typically assumed to be $\gamma_i=e^{j2\pi i/N_\gamma}$, $i=0,\ldots,N_\gamma-1$. This corresponds to phase-shift keying. As the phase of the transmitted signal has no significance for the operation of the radar, only the phase difference of the transmitted and the received signal is meaningful, the phase can be freely used in the transmission to encode the communication symbols.

In joint radar–communication systems, using the phase modulation is simpler than the power modulation as the phase of the transmitted signal has no significance for the operation of the radar, only the phase-difference of the transmitted and the received signal is meaningful. Thus, the phase can be selected for the communication purpose. As the transmit power does not need to be adjusted according to the transmitted communication symbols, communicating to a user within the mainlobe of the radar is also possible.

Let $\mathbf{x}(t)$ be the signal transmitted by the radar. The received signal at the communication receiver is then

$$y_c(t)=h_c\mathbf{a}_T^H(\theta_c)\sqrt{\frac{P_{\mathrm{tot}}}{M}}\mathbf{x}(t-\tau)+n_c(t), \tag{6}$$

where h_c is the communication channel coefficient, and $n_c(t)$ is the communication receiver noise. We see that introducing an additional phase shift to the transmitted signal $\tilde{\mathbf{x}}=\gamma_i\mathbf{x}$ creates an equal phase shift in the signal y_c of the communication receiver. In order to use this method, the the communication channel coefficient h_c has to be taken into account by using pilot signals, for example.

If the communication receiver is in the sidelobe region, it might be necessary to impose an additional constraint to the beampattern design in order to guarantee sufficient power to the direction of the communication receiver. To this end, (5f) can be modified to

$$|P_c-\mathbf{a}_T^H(\theta)\mathbf{R_x}\mathbf{a}_T(\theta)|<\epsilon_\gamma\quad \theta\in\Theta_c, \tag{7}$$

where P_c is desired power level for the communication.

In case the channel coherence time is short and the channel coefficient h_c cannot be reliably estimated, the symbol may be coded in the phase of an additional waveform, provided that such a waveform with sufficient ambiguity properties is available [6]. A complex scheme was proposed in [12] with different weights for each of the waveforms. Here, we propose a simple approach in which the extra waveform with a phase shift is added to one of the existing waveforms.

Let $s_{r,0}(t)$ be the additional waveform orthogonal to the other waveforms. Using encoding

$$\tilde{s}_{r,k}(t)=\frac{1}{\sqrt{2}}s_{r,k}(t)+\frac{\gamma_i}{\sqrt{2}}s_{r,0}(t) \tag{8}$$

will result in the same beampattern. However, the communication receiver can retrieve the symbol γ_i by applying matched filters to the received signal and the comparing the phase of the outputs of the filters matched to $s_{r,0}$ and $s_{r,k}$.

When there are multiple communication receivers using phase-shift keying in a single-waveform case, the first symbol can be encoded as before, but it is necessary to fix the phase difference of the transmitted symbols for the remaining communication users. This can be done either using a linear constraint

$$\mathbf{a}_T^H(\theta_{c,2})\mathbf{w}=\gamma_{i_1}^{-1}\gamma_{i_2}\sqrt{P_c} \tag{9}$$

or a quadratic constraint

$$\left|\gamma_{i_1}^{-1}\gamma_{i_2}\sqrt{P_c}-\mathbf{a}_T^H(\theta_{c,2})\mathbf{w}\right|^2<\epsilon_{\gamma,2},\quad \theta\in\Theta_{c,2}. \tag{10}$$

The latter one is better in the sense of enabling control over the error in the vicinity of $\theta_{c,2}$. These constraints are convex, but it is necessary obtain the desired response $b_0(\theta)$ beforehand as in (3).

For the MIMO radar with multiple waveforms using phase coding for multiple communication receivers, we propose the method similar to (8), but an additional waveform for each communication user. This way, there is no need to solve the beampattern design for every combination of the transmitted symbols.

5. NUMERICAL EXAMPLES

In this section, we show an example of designing the beampattern for communication with power modulation. The radar in this eample is a MIMO radar with a ten-element uniform linear array employing ten waveforms. The mainlobe is centered at $-15°$ with a width of $30°$. The direction of the communication receiver is $45°$. Two symbols are used used for communication with the corresponding power levels $\gamma_1 = -20\text{dB}$ and $\gamma_2 = -40\text{dB}$ relative to the mainlobe power. Given that transmit power in the mainlobe is typically very high, the low relative power will suffice for communications and not pose a problem for the communications receiver. The sidelobe constraint is chosen to be $c_{\text{SL}} = -15\text{dB}$.

Three different design approaces are used in this example. The first one using an linear equality constraint described in e.g. [6]. The second one uses the derivative constraint as in [8]. The third one is the method proposed here constraining the absolute error over an angular region. This angle angular Θ_c for the communication symbol was $10°$ and the allowed error within the region $\epsilon_\gamma = 10^{-4}$. We used CVX package for solving the resulting convex problems [13, 14].

Figure 1 shows resulting transmit beampatterns. It can be seen that all the three approaches have a similar mainlobe. However, there are large changes in the power for small changes in the angle with the equality constraint, which could lead to increased bit error rate. The derivative constraint is more successful with the first symbol, but suffers from the same problem with the second symbol. On the other hand, the proposed method can be used to achieve relatively flat beampattern over the desired range of angles, leading to robust communication.

Figure 2 shows the bit error rate (BER) for the two-level power modulation scheme as the function of the receiver SNR when there is a normally distributed error in the direction of the communication receiver with a standard deviation of $2°$. The proposed method achieves significantly lower BER.

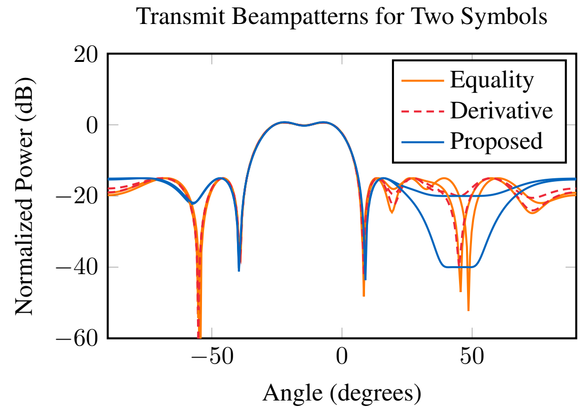

Fig. 1: Joint radar–communication transmit beampattern design with power modulation. The equality constraint approach can set the power only for some angles. The derivative constraint can be used in addition to limit the rate of change of the beampattern near the communication direction. However, these two methods do not enable control over an angular region. Using the proposed constraint for the absolute value, a sufficient control of the transmit power over a range of angles can be achieved, leading to more robust communication.

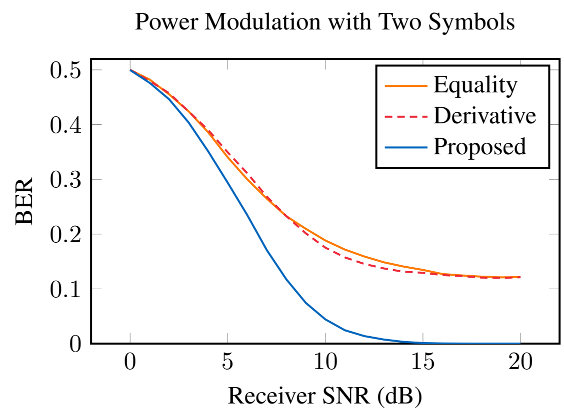

Fig. 2: Bit error rate for using the joint radar–communication transmit beampattern design with power modulation. The proposed method achieves significantly lower BER.

6. CONCLUSIONS

With the increasing demand for spectrum, radars and communication systems have need to be able to operate on the same frequency bands. Joint radar and communication systems have been proposed to alleviate the spectrum congestion. Such systems require careful design in order to maintain sufficient quality of service for the communication and the radar operation.

In this paper, we have proposed an improvement to the beamforming approach using the power level of the radar transmit beampattern to transit also communication signal with minimal interference with the radar operation. The proposed method allows better control over the beampattern in the direction of the communication receiver providing lower communication error level, especially if there is uncertainty in the direction of the communication receiver. Furthermore, we proposed a phase coding methods to transmit the communication symbols with minimal impact on the radar beampattern. The proposed approaches are significantly simpler than the methods described previously in the literature.

7. REFERENCES

[1] Hugh Griffiths, Lawrence Cohen, Simon Watts, Eric Mokole, Chris Baker, Mike Wicks, and Shannon Blunt, "Radar spectrum engineering and management: Technical and regulatory issues," *Proceedings of the IEEE*, vol. 103, no. 1, pp. 85–102, 2015.

[2] B. Paul, A. R. Chiriyath, and D. W. Bliss, "Survey of RF communications and sensing convergence research," *IEEE Access*, vol. 5, pp. 252–270, 2017.

[3] Fan Liu, Christos Masouros, Athina P. Petropulu, Hugh Griffiths, and Lajos Hanzo, "Joint radar and communication design: Applications, state-of-the-art, and the road ahead," *IEEE Transactions on Communications*, vol. 68, no. 6, pp. 3834–3862, 2020.

[4] Alex R. Chiriyath, Bryan Paul, and Daniel W. Bliss, "Radar-communications convergence: Coexistence, cooperation, and co-design," *IEEE Transactions on Cognitive Communications and Networking*, vol. 3, no. 1, pp. 1–12, 2017.

[5] Musa Furkan Keskin, Visa Koivunen, and Henk Wymeersch, "Limited feedforward waveform design for OFDM dual-functional radar-communications," *IEEE Transactions on Signal Processing*, vol. 69, pp. 2955–2970, 2021.

[6] A. Hassanien, M. G. Amin, E. Aboutanios, and B. Himed, "Dual-function radar communication systems: A solution to the spectrum congestion problem," *IEEE Signal Processing Magazine*, vol. 36, no. 5, pp. 115–126, 2019.

[7] Aboulnasr Hassanien, Moeness G. Amin, Yimin D. Zhang, and Fauzia Ahmad, "Dual-function radar-communications: Information embedding using sidelobe control and waveform diversity," *IEEE Transactions on Signal Processing*, vol. 64, no. 8, pp. 2168–2181, 2016.

[8] Aline de Oliveira Ferreira, Raimundo Sampaio-Neto, and José Mauro Fortes, "Robust sidelobe amplitude and phase modulation for dual-function radar-communications," *Transactions on Emerging Telecommunications Technologies*, vol. 29, no. 9, pp. e3314, 2018.

[9] Petre Stoica, Jian Li, and Yao Xie, "On probing signal design for MIMO radar," *IEEE Transactions on Signal Processing*, vol. 55, no. 8, pp. 4151–4161, Aug. 2007.

[10] D.R. Fuhrmann and G.S. Antonio, "Transmit beamforming for MIMO radar systems using partial signal correlation," in *Conference Record of the Thirty-Eighth Asilomar Conference on Signals, Systems and Computers*, Nov. 2004, vol. 1, pp. 295–299.

[11] Zhi-Quan Luo, Wing-Kin Ma, A.M.-C. So, Yinyu Ye, and Shuzhong Zhang, "Semidefinite relaxation of quadratic optimization problems," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 20–34, May 2010.

[12] Aboulnasr Hassanien, Moeness G. Amin, Yimin D. Zhang, and Fauzia Ahmad, "Phase-modulation based dual-function radar-communications," *IET Radar, Sonar & Navigation*, vol. 10, no. 8, pp. 1411–1421, 2016.

[13] Michael Grant and Stephen Boyd, "CVX: Matlab software for disciplined convex programming, version 2.1," http://cvxr.com/cvx, Mar. 2014.

[14] Michael Grant and Stephen Boyd, "Graph implementations for nonsmooth convex programs," in *Recent Advances in Learning and Control*, V. Blondel, S. Boyd, and H. Kimura, Eds., Lecture Notes in Control and Information Sciences, pp. 95–110. Springer-Verlag Limited, 2008.