MONOTONIC GENERALIZED NASH GAMES WITH APPLICATION TO THE MANAGEMENT OF ENERGY-AWARE ALOHA NETWORKS

Wenbo Wang and Amir Leshem

Faculty of Engineering, Bar-Ilan University, Ramat-Gan, Israel, 52900.

ABSTRACT

Generalized Nash games differ from strategic form games by allowing the strategy set available for each player to depend on the strategies selected by the other players. The strong dependence of the strategies of the players make these generalized games harder to analyze. While convex generalized games are well understood, the case where the constraint sets and rewards are non-convex is significantly more complicated. In this paper we analyze a family of monotonic generalized games (not necessarily convex). We provide uniqueness and existence theorems for these games as well as rapidly converging algorithm for obtaining a Nash equilibrium. We then use the proposed solution to optimize access probability and energy consumption in ALOHA networks, where users have fixed but heterogeneous QoS requirements.

***Index Terms*—** Generalized Nash equilibrium, energy-aware networks, distributed optimization, ALOHA.

1. INTRODUCTION

Generalized Nash games form a family of models capturing interactions among multiple players, where each player's choice of actions affects not only the outcome of the other players but also their sets of available actions or even the feasibility of finding an available action. As such, they are significantly more complicated to analyze than classical games. Significant amount of research on generalized convex games has been done in the context of Quasi-Variational Inequalities (QVI), see, e.g., [1] for excellent overview of the field. However, to apply the solution tools in the framework of QVI, the local objective of a play in the considered game has to satisfy certain theoretical properties, such as (strong) convexity in its utility function, or more generally, monotonicity in the game mapping (also known as monotone games in this case) w.r.t. to the local decision variables over closed convex constraint [2–4]. However, when generalized games have non-convex constraints or non-convex utilities, much less is known.

In this paper, we study a category of generalized non-cooperative game where convexity does not hold. We provide a novel existence and uniqueness theorem for a category of generalized Nash games with possibly non-convex constraints and cost functions and provide a rapidly converging algorithm which terminates either in a GNE o allows one of the players to inform the others that the problem is infeasible, which is very important for implementation.

Random access schemes have been widely used for data transmission of a large number of users sharing a common channel. Game theoretical tools are widely used in wireless networking to analyze sharing techniques. Works on networking games can be found e.g., in [5–7]. Random access games were studied in [8–17]. Game theoretic techniques were used in [8, 9, 11–13] to analyze single-channel ALOHA networks. Game theoretic analysis of the multi-channel ALOHA protocol have been studied, e.g. by [18], [15].

Wenbo Wang is supported by the Israel Science Foundation grant 1644/18 and ISF-NRF grant 2277/16

In [8, 9, 11, 13], distributed optimization algorithms of single-channel ALOHA networks using game theoretic tools are studied, where the utility of each user increases with the successful transmission probability. Here, we consider a similar model. Specifically, in [9,11] energy-efficient Nash equilibria under user-rate demands have been established. Another related work considered a non-cooperative power control game in CDMA networks with energy-efficiency perspectives [10], where the goal is to maximize the number of reliable bits transmitted per joule of energy consumed in a distributed fashion.

In this paper we discuss the problem of random access by many Internet-of-Things (IoT) devices each with heterogeneous but relatively low data rate requirements. For such devices energy consumption is most important, while data rates constraints are not very demanding. A protocol of choice for such devices is the ALOHA protocol which is the simplest to implement and requires very little processing. Moreover, distributed processing to optimize the access probability is necessary. Therefore, we focus on the case of ALOHA protocol with fixed rate requirements for the users and aim at minimizing the energy consumption of each device. To achieve that we introduce channel awareness into the framework of ALOHA (see [19] for a similar model). In contrast to the symmetric access strategies, our proposed algorithm enables the IoT devices to exploit the diversity of their own channels while maintaining a QoS constraint for each device. We formulate the problem as a generalized Nash game with non-convex constraints coupling the access probabilities of the individual devices. The utility of each device decreases with the consumed energy. This allows us to apply the proposed distributed access algorithm based on our theoretical findings. Finally, we show in the numerical simulations that our proposed algorithm is able to achieve energy efficiency when compared with the traditional slotted ALOHA mechanism and a group of other channel/rate-aware ALOHA-based schemes.

Throughout this paper, bold-faced letters (e.g., $\boldsymbol{\theta}$ and $\mathbf{U}$) and light-faced letters (e.g., θ) are used to denote vectors and scalar quantities, respectively. Calligraphic letters (e.g., $\mathcal{N}$) or upper-case bold-faced Greek letters (e.g., $\boldsymbol{\Theta}$) are used to denote sets or spaces. The rest of the paper is organized as follows: Section 2 provides a theoretical framework of distributed strategy searching for a family of monotonic generalized Nash games. Section 3 applies this framework of distributed strategy searching to the scenario of channel-aware slotted ALOHA for IoT networks. With numerical simulations, Section 4 demonstrates the convergence property of the strategy searching scheme, as indicated by our theoretical studies in Section 3. The efficiency of the proposed algorithm is demonstrated through performance comparison with standard ALOHA and a reference channel-aware ALOHA algorithm. Section 5 concludes the paper.

2. MONOTONIC GENERALIZED NASH GAMES

In this section we define a family of monotonic generalized Nash games and prove existence and convergence of a specific dynamics to a generalized Nash equilibrium.

Definition 1. *A game in strategic form is a 3-tuple $\mathcal{G} = (\mathcal{N}, \mathbf{\Theta}, \mathbf{U})$, where $\mathcal{N} = \{1, \ldots, N\}$ is a set of players. $\mathbf{\Theta} = \prod_{n=1}^{N} \mathbf{\Theta}_n$ is the set of strategy profiles and $\mathbf{U} = (u_1, ..., u_N) : \mathbf{\Theta} \longrightarrow \mathbb{R}^N$ is a vector of utility functions of the players.*

A generalized game is a 3-tuple $\mathcal{G} = (\mathcal{N}, \mathbf{\Theta}, \mathbf{U}))$ where the set of strategies has further constraints $\mathbf{\Theta} \subseteq \prod_{n=1}^{N} \mathbf{\Theta}_n$, i.e., not every strategy profile $\boldsymbol{\theta}$ is possible. We denote $\boldsymbol{\theta} = (\theta_n, \theta_{-n})$ to emphasize the n'th coordinate.

Define the locally feasible strategies for player n given a strategy profile θ_{-n} of the other players by:

$$\mathbf{\Theta}_n(\theta_{-n}) = \{\theta : (\theta, \theta_{-n}) \in \mathbf{\Theta}\}. \tag{1}$$

A generalized Nash Equilibrium (GNE) in a (generalized) non-cooperative game $\mathcal{G}$ is a strategy profile $\boldsymbol{\theta}^* = (\theta_n^*, \theta_{-n}^*)$ when $\forall n \in \mathcal{N}$, θ_n^* is the solution to the

$$\theta_n^* = \arg\max_{\theta} u_n(\theta, \theta_{-n}^*) \tag{2}$$

According to [20], to ensure the existence of a GNE, we typically need that the local feasible strategy sets $\Theta_n(\theta_{-n})$ are non-empty, closed and convex, and the utility functions $u_n(\theta_n, \theta_{-n})$ are quasi-convex w.r.t. θ_n on $\Theta_n(\theta_{-n}), \forall n \in \mathcal{N}$. However, this condition may not necessarily be satisfied for the category of problems (see the example in [21]) we are considering with the model in (16). As a result, the standard KKT condition-based methods [1, Chapter 10] for GNE solutions may not apply, and we need to find an alternative approach for the analysis and search of the GNE points.

In this paper we are interested in the following family of generalized games where the feasible sets are defined by monotonic and continuous functions:

Definition 2. *A generalized Nash game $\mathcal{G} = (\mathcal{N}, \mathbf{\Theta}, \mathbf{U})$ is monotonic if the following condition holds:*

- *Strategies are real numbers and*

$$\mathbf{\Theta} \subseteq \prod_{n=1}^{N} [\underline{\theta}_n, \bar{\theta}_n]. \tag{3}$$

- *For each player n, $u_n(\theta_n, \theta_{-n})$ is continuous and monotonically decreasing in θ_n.*
- *For each player n and θ_{-n}, the locally feasible set $\mathbf{\Theta}_n(\theta_{-n})$ is defined by*

$$\mathbf{\Theta}_n(\theta_{-n}) = \{\theta : q_n(\theta, \theta_{-n}) \geq \bar{q}_n\} \tag{4}$$

where q_n is a continuous function. $q_n(\theta, \theta_{-n})$ is monotonically increasing in θ and monotonically decreasing in each coordinate of θ_{-n}.

A vector $\boldsymbol{\theta}$ is feasible iff for all $n \in \mathcal{N}$

$$q_n(\theta_n, \theta_{-n}) \geq \bar{q}_n.$$

Note that since $\mathbf{\Theta}$ is bounded and $q_n(\theta_n, \theta_{-n})$ for each n are continuous $\mathbf{\Theta}$ is compact. Our main theorem asserts that if a monotonic generalized Nash game has a feasible point, then a GNE exists. The theorem is constructive and allows us to obtain the GNE iteratively. Furthermore, by monotonicity of u_n we will obtain that all u_n are maximized by the GNE.

Theorem 1 (Existence). *Assume that $\mathcal{G} = (\mathcal{N}, \mathbf{\Theta}, \mathbf{U})$ is a monotonic generalized Nash game. and $\boldsymbol{\theta}$ is a feasible strategy profile. Then, the game $\mathcal{G}$ has a GNE.*

Proof. The proof is composed of two parts. We first construct a convergent sequence of feasible strategy profiles starting from a feasible profile $\boldsymbol{\theta}$. Then, we show that the limit of the convergent sequence is a GNE.

Let $\theta_n(0) = \theta_n$ for all $n \in \mathcal{N}$. Assume that we defined a feasible point $\theta_n(t)$ for each n. If for all n $q_n(\theta_n(t), \theta_{-n}(t)) = \bar{q}_n$, then we achieved a GNE, since no player can reduce $\theta_n(t)$ and by monotonicity $\theta_n(t)$ must be decreased in order to increase the utility of player n. Otherwise, for each n define:

$$\theta_n(t+1) = \min\{\theta : \underline{\theta}_n \leq \theta, q(\theta, \theta_{-n}(t)) \geq \bar{q}_n\}. \tag{5}$$

As discussed above, if the process terminates in finite time we achieved a GNE. Otherwise, we obtained an infinite decreasing sequence $\boldsymbol{\theta}(t), t = 0, 1, \ldots$. By compactness of $\mathbf{\Theta}$

$$\boldsymbol{\theta}^\infty = \lim_{t \to \infty} \boldsymbol{\theta}(t) \in \mathbf{\Theta}. \tag{6}$$

By construction, $\boldsymbol{\theta}(t)$ is feasible for all t, i.e for all n and for all t:

$$q_n(\theta_n(t), \theta_{-n}(t)) \geq \bar{q}_n, \tag{7}$$

and therefore by continuity of q_n, $\boldsymbol{\theta}^\infty$ is also feasible. Assume that for some n

$$q_n(\theta_n^\infty, \theta_{-n}^\infty) > \bar{q}_n. \tag{8}$$

By monotonicity for all t

$$q_n(\theta_n^\infty, \theta_{-n}^\infty) > q_n(\theta_n^\infty, \theta_{-n}(t)) \tag{9}$$

By continuity of q_n there is a sufficiently large t_0 such that

$$q_n(\theta_n^\infty, \theta_{-n}(t_0)) > \bar{q}_n. \tag{10}$$

This is a contradiction since this implies by construction that $\theta_n^\infty > \theta_n(t+1)$. □

With the finding obtained in Theorem 1, we further show in Theorem 2 that the feasibility of any instance of a monotonic generalized Nash game can be identified following asynchronous best response. Furthermore, asynchronous best response is guaranteed to converge if the set of GNE in game is non-empty.

Theorem 2 (Convergence). *Assume that $\mathcal{G} = (\mathcal{N}, \mathbf{\Theta}, \mathbf{U})$ is a monotonic generalized Nash game. Then the asynchronous best response algorithm initialized with $\boldsymbol{\theta} = (\underline{\theta}_1, ..., \underline{\theta}_n)$ either converges to a GNE or it identifies that there is no feasible solution.*

Due to the page limit, the proof of this theorem will be given in the full version of this paper. We note that the proof for Theorem 2 uses a similar argument to the proof for Theorem 1, by constructing a sequence of strategies beginning with the lowest value $\underline{\theta}_n$ and then iteratively increasing it by solving a best response dynamics. The proof relies on monotonicity and continuity and does not require any property of differentiability or convexity. Finally, we note that if one of the players needs a strategy value higher than $\underline{\theta}_n$ for its local best response, it notifies all the other players that the problem is infeasible.

When the utility of each player is independent of the actions of the other players the following holds:

Corollary 1. *Assume that $\forall n \in \mathcal{N}$, $u_n(\theta_n, \theta_{-n})$ is independent of the adversaries strategies θ_{-n}. Then, if $\boldsymbol{\theta}$ is a GNE, it is socially optimal.*

Proof. Assume that $\boldsymbol{\theta}$ is a Nash equilibrium. Assume that $\boldsymbol{\theta} = (\theta_1, \ldots, \theta_N)$ is not the social optimum. and $\boldsymbol{\theta}' = (\theta_1', \ldots, \theta_N')$ is a feasible point with

$$\sum_{n=1}^{N} u_n(\theta_n) < \sum_{n=1}^{N} u_n(\theta_n'). \tag{11}$$

Therefore, there is some n for which

$$u_n(\theta_n) < u_n(\theta_n'). \tag{12}$$

By the decreasing monotonicity of $u_n(\theta)$ we know that $\theta_n' < \theta_n$. Therefore, by the monotonicity properties of $q_m(\theta_m, \theta_{-m})$ we easily observe that (θ_n', θ_{-n}) is also feasible and $u_n(\theta_n', \theta_{-n}) > u_n(\theta_n, \theta_{-n})$ contradicting the fact that θ is a Nash equilibrium.

□

Remark 1. *It is worth noting that the convergence property of conventional potential games (e.g., based on best replies), cannot be directly applied to the generalized NE problem here. It is known that even if game $\mathcal{G}$ (with a generalized payoff function) is a generalized ordinal potential game, simultaneous best response may not lead to a GNE from an arbitrary feasible point [22].*

3. DISTRIBUTED OPTIMIZATION OF ALOHA NETWORKS

Consider a network where N synchronized IoT devices attempt to transmit to the Access Point (AP) over a shared wireless channel using time-slotted ALOHA. We assume that the devices are backlogged and always have packets to transmit. The uplink channel is block-fading and each IoT device experiences a fixed channel gain within one time slot (e.g., during the transmission of one block). For the n-th IoT device, the received signal at the AP when no collision occurs is

$$y_n(t) = h_n(t)x_n(t) + \eta(t), \tag{13}$$

where $h_n(t)$ is the complex channel coefficient at time t for device n, $x_n(t)$ is the transmitted signal and $\eta(t)$ is the additive white Gaussian noise. We assume channel reciprocity in Time Division Duplexing (TDD), and each IoT device is able to perfectly estimate the channel at the beginning of each time slot, using the beacon signals broadcasted by the AP. We adopt the transmission strategy in [21] such that each IoT device only transmits when the channel condition is sufficiently good, namely, with a channel gain threshold $\|h(t)\|^2 \geq \alpha_n$. Then, for a generalized channel gain distribution $f(|h_n|^2)$, the probability for device n to transmit with threshold α_n at a given time slot is determined by

$$\theta_n(\alpha_n) = \int_{\alpha_n}^{\infty} f(|h_n|^2) d|h_n|^2, \tag{14}$$

which is a monotonically decreasing function of α_n. With (14), we have the probability of successful transmission for device n as follows:

$$q_n(\theta_n, \theta_{-n}) = \theta_n(\alpha_n) \prod_{i \neq n} (1 - \theta_i(\alpha_i)), \tag{15}$$

which monotonically decreases w.r.t. α_n and increases w.r.t. α_{-n}.

We consider that each user adopts a constant transmit power P_n at the AP. Then, the average power consumed by each device for packet transmission can be expressed as $P_n\theta_n(\alpha_n)$. Our goal of the decentralized access protocol design is to ensure that each device maintains a minimum expected transmit power while guaranteeing the successful transmission probability to be above a certain threshold $\overline{q}_n$. Then,

Algorithm 1 Asynchronous best response-based search of GNE for the channel access game ($n \in \mathcal{N}$)

Require: $\forall n \in \mathcal{N}$: Select $\theta_n = \underline{\theta}_n$, select $\chi > 0$

repeat
  for all $n = 1, \ldots, N$, asynchronously **do**
    $\theta_n' \leftarrow \theta_n$
    Given θ_{-n}, compute

$$\theta_n \leftarrow \arg\max_{\theta_n} -c_n(\theta_n, \theta_{-n}) \text{ s.t. } \theta_n \in \Theta_n(\theta_{-n}) \tag{17}$$

  end for
until $\forall n \in \mathcal{N}$: $\|\theta_n' - \theta_n\| \leq \chi$ **or** $\exists n \in \mathcal{N}$: θ_n is an infeasible solution to (17)

from a device-centric perspective, we can formulate the following local power minimization problem for device $n = 1, \ldots, N$:

$$\alpha_n^* = \arg\min_{\alpha_n} \ c_n(\alpha) = P_n\theta_n(\alpha_n) \tag{16}$$

$$\text{s.t.} \quad q_n(\alpha_1, \ldots, \alpha_N) = \theta_n(\alpha_n) \prod_{i \neq N} (1 - \theta_i(\alpha_i)) \geq \overline{q}_n, \tag{16a}$$

$$\underline{\alpha}_n \leq \alpha_n \leq \overline{\alpha}_n, \tag{16b}$$

From (14), we note that there exists a one-to-one mapping from α_n to θ_n. Then, we can conveniently transform (16) into an equivalent optimization problem w.r.t. $\boldsymbol{\theta}$, with $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_N]^{\mathrm{T}}$ being the vector of joint strategies of all the IoT devices. Then, (15) becomes a monotonically increasing function of θ_n and decreasing function w.r.t. θ_i ($i \neq n$). This indicates that the conditions in Definition 2 is satisfied by the local optimization problem (16). Therefore, we obtain an instantiation of the generalized monotonic Nash game from (16) as $\tilde{\mathcal{G}} = (\mathcal{N}, \boldsymbol{\Theta}, \mathbf{U} = \{u_n(\theta_n) = -c(\theta_n)\}_{n \in \mathcal{N}})$. Then, we can directly apply our theoretical findings in Section 2 and derive a decentralized NE strategy searching scheme for $\tilde{\mathcal{G}}$ as in Algorithm 1.

It is worth noting that the solution to (17) is based on the assumption that the channel statistics function $\theta_n(\alpha_n)$ is known in advance of the construction of the game (equivalently, Algorithm 1). However, in practical scenarios, the channel coefficient of each user may be sampled from an unknown distribution. Then, device n needs to perform the empirical estimation of $\theta_n(\alpha_n)$ based on the beacons of the AP to establish the one-to-one mapping from α_n to θ_n. Similarly, given the stationary adversary strategies of IoT device n, θ_{-n}, $q_n(\theta_n, \theta_{-n})$ can also be estimated through some certain strategy-payoff estimation process. The estimation can be performed by introducing an additional estimation process that is organized in mini-slots at the beginning of each time slot for channel-access decision making. Due to the page limit, we refer the readers to [23, Section V.A] for the details of the estimation techniques regarding the derivation of empirical channel statistics in (14). Meanwhile, since the payoff and constraint functions of the formulated threshold selection game $\tilde{\mathcal{G}}$ as defined in (16) satisfy the conditions as given by Definition 2 and Corollary 1, the NE of game $\tilde{\mathcal{G}}$, as obtained following Algorithm 1, conveniently provides the global optimal solution of access probabilities for the considered network as well.

4. SIMULATION RESULT

In this section we demonstrate in simulations the performance of the proposed method when applied to a network of IoT devices applying energy aware ALOHA with distributed optimization based on our generalized Nash game formulation. Assume that each IoT device expe-

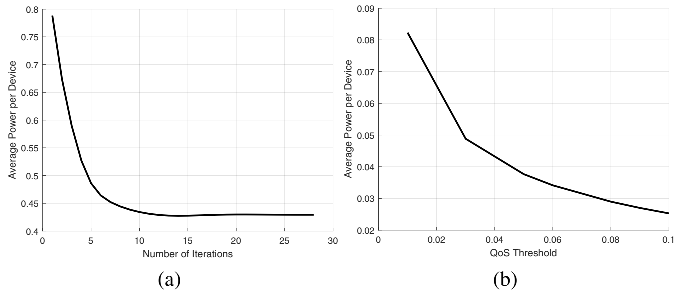

Fig. 1. Performance evaluation in a network of 10 IoT devices. (a) Evolution of the average power with Algorithm 1. (b) Evolution of the average power w.r.t. the QoS threshold $\overline{q}$ in (16a).

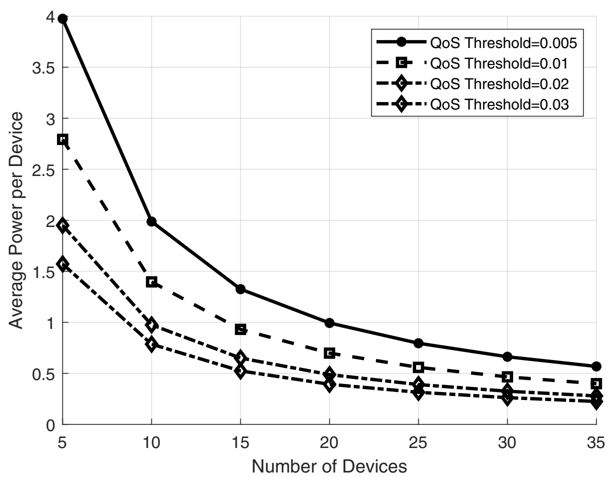

Fig. 2. Impact of the QoS threshold $\overline{q}$ on power consumption as the size of network increases.

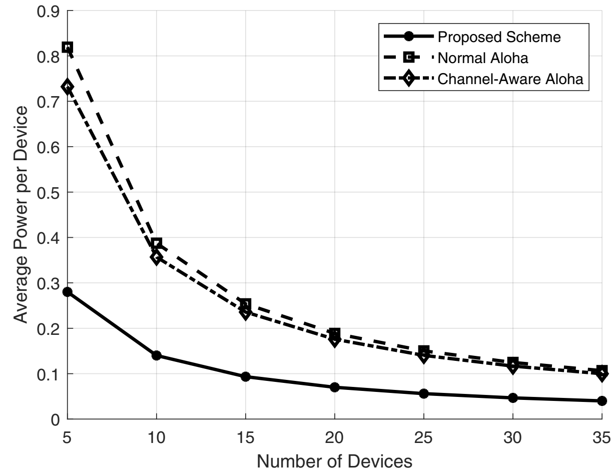

Fig. 3. Performance comparison with standard slotted ALOHA and Channel-Aware ALOHA [21].

riences independent Rayleigh fading over the shared channel. Then, using the energy-aware performance metric given in (16), we are interested in demonstrating (a) the efficiency in terms of strategy convergence with our proposed Algorithm 1, (b) the impact of system parameters (e.g., the network size and the QoS threshold) on the performance of the devices, and (c) the gain of channel awareness in our proposed algorithm when compared with the existing schemes in the literature.

In the numerical experiments corresponding to Figure 1, we demonstrate the convergence tendency of the proposed scheme with Figure 1(a), where the network size is fixed to 10 devices and the QoS threshold $\overline{q}$ is uniformly set to 0.1. As we can read from Figure 1(a), the proposed algorithm is able to converge quickly within a few (roughly 15) iterations for the considered network. Figure 1(b) demonstrates the impact of QoS threshold $\overline{q}$ on the power consumption of the 10-device network. The decreasing curve in Figure 1(b) indicates that as the QoS requirement becomes stricter, each IoT device tends to choose a higher channel coefficient threshold at the NE. This in return leads to a relatively smaller expected transmit power on average for each device.

In Figure 2, we demonstrate the impact of network size on the performance of the proposed algorithm. We can interpret from the figure that a lower QoS threshold or a larger network size corresponds to a higher power consumption. The former case matches our observation with Figure 1(b), and the latter case indicates that as the network gets more congested, the expected power of each device decreases since the probability of channel access decreases at the new NE.

Finally, we compare the performance of our proposed scheme with that of standard slotted ALOHA (i.e., with homogeneous strategies) and that of "Channel-Aware ALOHA" as proposed in [21]. The original objective function of each player in "Channel-Aware ALOHA" is adapted to align with (16) for performance comparison. As we can observe from Figure 3, "Channel-Aware ALOHA" is able to achieve a better performance than the standard slotted ALOHA, while our proposed scheme achieves the best performance out of the three algorithms. The reason is that "Channel-Aware ALOHA" searches for a symmetric strategy that dominates the homogeneous strategies of standard slotted ALOHA, while our proposed scheme in general converges to asymmetric strategies, which further improves the network performance.

5. CONCLUSION AND DISCUSSION

In this paper, we studied a special category of generalized Nash games, where each player's payoff function and constraint functions are monotonic. Instead of relying on convexity by quasi-variational inequality, we provided a framework of decentralized equilibrium searching based on asynchronous best response. Our theoretical results show that the proposed method is able to determine the GNE existence as well as to guarantee convergence to a GNE when exists. We also provided a rapidly converging algorithm. The results were applied to the problem of energy-aware slotted ALOHA, where we find the minimal energy required to meet given throughput constraints. Numerical simulation results show that by adopting the proposed scheme, a network is able to achieve higher energy efficiency when compared with the standard slotted ALOHA and other channel-aware ALOHA mechanisms.

It is worth noting that the proposed category of monotonic generalized Nash games is capable of addressing a plethora of practical allocation problems in wireless networks, beyond the channel-aware ALOHA game as concerned in this paper. Especially, when the constraint of a local user/link is constructed on the basis of the target level of a certain QoS function that is determined by the mutual interference among the links (e.g., effective channel capacity or bit-error rate), it is usually possible to obtain the property of monotonicity as required by Definition 2. Then, our proposed distributed equilibrium-searching algorithm can be conveniently applied. Our future research direction will include the application of the proposed framework of monotonic generalized Nash games to the scenarios such as power allocation in multi-input-multi-out communication networks.

6. REFERENCES

[1] F. Facchinei and J.-S. Pang, *Finite-dimensional variational inequalities and complementarity problems*. Springer Science & Business Media, 2007.

[2] C. Cenedese, G. Belgioioso, S. Grammatico, and M. Cao, "An asynchronous distributed and scalable generalized Nash equilibrium seeking algorithm for strongly monotone games," *European Journal of Control*, vol. 58, pp. 143–151, 2021.

[3] B. Franci and S. Grammatico, "Stochastic generalized Nash equilibrium seeking in merely monotone games," *IEEE Trans. Autom. Control*, 2021.

[4] P. Yi and L. Pavel, "An operator splitting approach for distributed generalized Nash equilibria computation," *Automatica*, vol. 102, pp. 111–121, 2019.

[5] E. Altman, T. Boulogne, R. El-Azouzi, T. Jiménez, and L. Wynter, "A survey on networking games in telecommunications," *Computers & Operations Research*, vol. 33, no. 2, pp. 286–311, 2006.

[6] I. Menache and A. Ozdaglar, "Network games: Theory, models, and dynamics," *Synthesis Lectures on Communication Networks*, vol. 4, no. 1, pp. 1–159, 2011.

[7] J. Park and M. van der Schaar, "The theory of intervention games for resource sharing in wireless communications," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 1, pp. 165–175, 2012.

[8] A. MacKenzie and S. Wicker, "Selfish users in ALOHA: a game-theoretic approach," *IEEE Vehicular Technology Conference*, vol. 3, pp. 1354–1357, 2001.

[9] Y. Jin and G. Kesidis, "Equilibria of a noncooperative game for heterogeneous users of an ALOHA network," *IEEE Commun. Lett.*, vol. 6, no. 7, pp. 282–284, 2002.

[10] F. Meshkati, M. Chiang, H. V. Poor, and S. C. Schwartz, "A game-theoretic approach to energy-efficient power control in multicarrier CDMA systems," *IEEE J. Sel. Areas Commun.*, vol. 24, no. 6, pp. 1115–1129, 2006.

[11] I. Menache and N. Shimkin, "Rate-based equilibria in collision channels with fading," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 7, pp. 1070–1077, 2008.

[12] H. Inaltekin and S. B. Wicker, "The analysis of Nash equilibria of the one-shot random-access game for wireless networks and the behavior of selfish nodes," *IEEE/ACM Trans. Netw.*, vol. 16, no. 5, pp. 1094–1107, 2008.

[13] U. Candogan, I. Menache, A. Ozdaglar, and P. Parrilo, "Competitive scheduling in wireless collision channels with correlated channel state," in *GameNets*, Istanbul, Turkey, May 2009, pp. 621–630.

[14] L. Chen, S. H. Low, and J. C. Doyle, "Random access game and medium access control design," *IEEE/ACM Trans. Netw.*, vol. 18, no. 4, pp. 1303–1316, 2010.

[15] K. Cohen and A. Leshem, "Distributed game-theoretic optimization and management of multichannel aloha networks," *IEEE/ACM Trans. Netw.*, vol. 24, no. 3, pp. 1718–1731, 2015.

[16] E. Altman, R. El Azouzi, and T. Jiménez, "Slotted Aloha as a game with partial information," *Computer networks*, vol. 45, no. 6, pp. 701–713, 2004.

[17] J. Choi, "Multichannel NOMA-Aloha game with fading," *IEEE Trans. Commun.*, vol. 66, no. 10, pp. 4997–5007, 2018.

[18] K. Cohen, A. Leshem, and E. Zehavi, "Game theoretic aspects of the multi-channel ALOHA protocol in cognitive radio networks," *IEEE J. Sel. Areas Commun.*, vol. 31, no. 11, pp. 2276 – 2288, 2013.

[19] I. Menache and N. Shimkin, "Rate-based equilibria in collision channels with fading," *IEEE J. Sel. Areas Commun.*, vol. 26, no. 7, pp. 1070–1077, 2008.

[20] F. Facchinei and C. Kanzow, "Generalized Nash equilibrium problems," *Annals of Operations Research*, vol. 175, no. 1, pp. 177–211, 2010.

[21] Y. Jin, G. Kesidis, and J. W. Jang, "A channel aware MAC protocol in an ALOHA network with selfish users," *IEEE J. Sel. Areas Commun.*, vol. 30, no. 1, pp. 128–137, 2012.

[22] F. Facchinei, V. Piccialli, and M. Sciandrone, "Decomposition algorithms for generalized potential games," *Computational Optimization and Applications*, vol. 50, no. 2, pp. 237–262, 2011.

[23] W. Wang, A. Leshem, D. Niyato, and Z. Han, "Decentralized learning for channel allocation in IoT networks over unlicensed bandwidth as a contextual multi-player multi-armed bandit game," *IEEE Trans. Wireless Commun.*, p. early access, 2021.