

ON THE ACQUISITION OF STATIONARY SIGNALS USING UNIFORM ADCS

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ABSTRACT

In this work, we consider the acquisition of stationary signals using uniform analog-to-digital converters (ADCs), i.e., employing uniform sampling and scalar uniform quantization. We jointly optimize the pre-sampling and reconstruction filters to minimize the time-averaged mean-squared error (TMSE) in recovering the continuous-time input signal for a fixed sampling rate and quantizer resolution and obtain closed-form expressions for the minimal achievable TMSE. We show that the TMSE-minimizing pre-sampling filter omits aliasing and discards weak frequency components to resolve the remaining ones with higher resolution when the rate budget is small. In our numerical study, we validate our results and show that sub-Nyquist sampling often minimizes the TMSE under tight rate budgets at the output of the ADC.

Index Terms— analog-to-digital conversion, estimation, filtering.

1. INTRODUCTION

Analog-to-digital conversion of continuous-time (CT) processes plays a key role in digital signal processing systems. This conversion involves two steps: First, the input is sampled, yielding a discrete-time (DT) representation of the CT signal, and then the samples are mapped onto a finite bit representation, i.e., the samples are quantized. Such acquisition is typically implemented using uniform analog-to-digital converters (ADCs), which sample at a fixed rate and convert each sample into a digital representation using a uniform partition of the real line. Traditionally, sampling and quantization have been studied independently, with classic results including the Shannon-Nyquist sampling theorem [1, 2] and the 6dB-per-bit rule-of-thumb for high-resolution quantization [3]. A comprehensive overview on works on sampling and quantization can be found in [4, 5] and [6], respectively.

While sampling and quantization are often studied separately, ADCs are typically implemented as part of an overall acquisition system, which also includes analog and digital filters. The choice of these filters can contribute to the ability to reconstruct a signal from its digital representation [7–10]. The mean squared error (MSE)-minimizing pre-filter and recovery filter when considering only sampling and only quantization have been studied in [7] and [8], respectively. The work [9] considered both sampling and quantization and numerically optimized the pre-sampling and reconstruction filters

with respect to the MSE distortion in recovering a CT wide-sense stationary (WSS) Gaussian process. Therein, the optimal pre-sampling filter was only approximated, as it was derived without taking quantization into account, and the resulting MSE in recovering the input was evaluated numerically. In [10], the authors studied a similar setting and analytically characterized the MSE-minimizing pre-sampling and recovery filters as well as the corresponding minimal achievable MSE. However, none of these works considered both sampling and quantization, while faithfully modeling the quantization distortion.

The fundamental distortion limits in recovering WSS Gaussian processes from digital representations acquired using filtering, sampling, and quantization have been studied in [11, 12]. To characterize the limits, these works allowed the quantization procedure to implement any form of lossy source coding, resulting in a setup denoted as *analog-to-digital compression* (ADX). Implementing such acquisition systems involves complex vector quantizers that jointly map an arbitrarily large number of samples onto a discrete representation. In addition, the authors of [11, 12] also considered a pulse-code modulation (PCM) setting, where the acquisition system uses scalar quantizers, which map each sample onto a discrete representation using the same mapping. Nonetheless, the effect of quantization in that setting is based on a model which only holds when using fine-resolution non-uniform quantization whose decision regions are tailored to the input distribution. Consequently, the resulting model does not reflect the operation of practical acquisition systems, especially when using low-resolution and uniform ADCs. In our previous work [13], we circumvented this problem by considering non-subtractive dithered quantization and aimed to recover a random parameter vector which is a linear function of an observed multivariate input process.

In contrast, we consider recovering univariate CT WSS signals in this work. Particularly, we focus on acquisition systems utilizing conventional uniform ADCs, and jointly design the pre-sampling filter and the reconstruction filter to minimize the time-averaged mean squared error (TMSE) in recovering the input when operating under a rate budget at the output of the ADC. To be able to optimize the overall system without imposing distortion models which hold for high-resolution ADCs, we adopt the approach used in [13–15] and model the uniform ADCs as implementing non-subtractive dithered quantization within their dynamic range [16]. The resulting model is known to be a faithful approximation of the distortion induced by conventional (including non-dithered) uniform quantizers at arbitrary resolutions for a broad family of input distributions [17]. Under this model, we analytically characterize the TMSE-minimizing linear pre-sampling and linear recovery filter as well as the minimal achievable MSE for a given sampling rate and quantizer resolution. Our solution for the pre-sampling filter is shown to be superior to the solution for the PCM system given in [11, 12]. Moreover, we show numerically that combining the derived filters with conventional, i.e., non-dithered quantizers, yields a reduced MSE, hence, demonstrating the practical value of our contributions.

Throughout this work, random quantities are denoted by sans-serif letters, e.g., \mathbf{x} , whereas x is a deterministic quantity. We use j , $*$, $\mathbb{E}\{\cdot\}$, and $\text{FT}\{\cdot\}$ to denote the imaginary unit, convolution, stochastic expectation, and Fourier transform (FT), respectively. We use $\mathbb{1}_{\mathcal{A}(x)}$ to denote the indicator function, which is 1 when the

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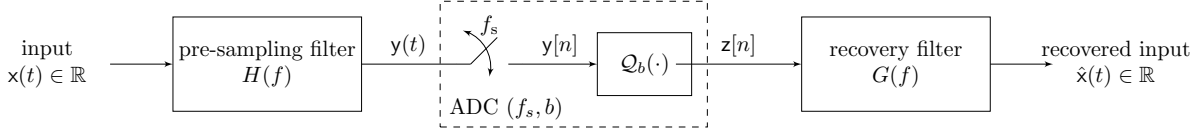


Fig. 1. Overview of the system model. Our goal is to recover the process $x(t)$ under a rate constraint $R = f_s \cdot b$.

condition $\mathcal{A}(x)$ holds and 0 otherwise, and $(x)^+ = \max(0, x)$. The sets of natural, integer, real, and complex numbers are written as \mathbb{N} , \mathbb{Z} , \mathbb{R} , and \mathbb{C} , respectively.

2. SYSTEM MODEL

2.1. Acquisition System

We consider the acquisition of a zero-mean WSS CT process $x(t) \in \mathbb{R}$, $t \in \mathbb{R}$, into a digital representation. We focus on bandlimited inputs, i.e., we assume that the power spectral density (PSD) of $x(t)$, denoted $S_x(f)$, is bandlimited with support $(-\frac{f_{\text{nyq}}}{2}, \frac{f_{\text{nyq}}}{2})$, i.e., $S_x(f) = 0$ for all $|f| \geq \frac{f_{\text{nyq}}}{2}$. The conversion system consists of a linear pre-sampling filter $H(f) \in \mathbb{C}$, an ADC implementing uniform sampling and scalar uniform quantization, and a linear recovery filter $G(f) \in \mathbb{C}$.

The resulting acquisition system, which is illustrated in Fig. 1, filters the input by the pre-sampling filter $H(f)$ and subsequently uniformly samples it with sampling rate f_s . The resulting samples are

$$y[n] = y(nT_s) = (x * h)(nT_s), \quad n \in \mathbb{Z}, \quad (1)$$

with $h(t) = \text{FT}^{-1}\{H(f)\}$ and $T_s = \frac{1}{f_s}$. After sampling, $y[n]$ is quantized by a uniform scalar mid-rise quantizer with an amplitude resolution of b bits, i.e., it can produce 2^b distinct output values. The (one-sided) dynamic range of the quantizer is denoted as $\gamma > 0$, and the mid-rise quantization function is

$$q_b(x') = \begin{cases} \Delta \left(\left\lfloor \frac{x'}{\Delta} \right\rfloor + \frac{1}{2} \right), & \text{for } |x'| < \gamma \\ \text{sign}(x') \left(\gamma - \frac{\Delta}{2} \right), & \text{otherwise,} \end{cases} \quad (2)$$

where $\Delta = \frac{2\gamma}{2^b}$ is the quantization step size, $\lfloor \cdot \rfloor$ denotes rounding to the next smaller integer, and $\text{sign}(\cdot)$ is the signum function. The overall bit rate is thus $R = f_s \cdot b$ bits per second.

Similar to [13, 14], we model the quantizers as implementing *non-subtractive dithered quantization* to obtain an analytically tractable system model. Such quantizers add a random *dither* signal to the input before quantization [16]. Hence, the quantizer outputs are given by

$$z[n] = Q_b(y[n]) = q_b(y[n] + w[n]) = y[n] + e[n]. \quad (3)$$

Here, $w[n]$ denotes the zero-mean dither random process, which is independent and identically distributed (i.i.d.) and mutually independent of the input process, while $e[n]$ is the quantization distortion. For non-overloaded ADCs, i.e., for inputs whose magnitude does not exceed γ , dithering can ensure that the first and second moments of $e[n]$ are independent of the input while minimizing the latter by choosing the probability density function of $w[n]$ to be a triangular distribution with a width of 2Δ [18, Sec. III.C]. Then, to obtain a negligible overload probability, we set the dynamic range γ to a multiple η of the standard deviation of the dithered input, i.e.,

$$\gamma^2 = \eta^2 \mathbb{E}\{(y[n] + w[n])^2\}. \quad (4)$$

By Chebychev's inequality, (4) guarantees that the overload probability is not larger than η^{-2} for any input distribution [19, eq. (5-88)].

The motivation for using the above model stems from the fact that it rigorously yields a tractable distortion model. In particular, for a vanishing overload probability, it follows from [16, Th. 2] that the autocorrelation function of $e[n]$ is given by $R_e[l] = \mathbb{E}\{e[n+l]e^T[n]\} = \frac{\Delta^2}{4}\delta[l]$, where $\delta[l]$ denotes the Kronecker delta function. While the resulting model of the quantization error rigorously holds for non-overloading non-subtractive dithered quantizers, it also approximately holds for conventional, i.e., non-dithered uniform quantizers applied to a broad range of inputs, and particularly sub-Gaussian signals [17]. This is also numerically verified in Section 4.

2.2. Problem Formulation

We consider the recovery of $x(t)$ from its digital representation $z[n]$. To this aim, we focus on *shift-invariant linear recovery* (cf. [20]), i.e., a linear recovery filter $G(f)$ is employed, which yields

$$\hat{x}(t) = \sum_{n \in \mathbb{Z}} g(t - nT_s) z[n], \quad (5)$$

where $g(t) = \text{FT}^{-1}\{G(f)\}$. Our goal is to find the pre-sampling filter $H(f)$ and the recovery filter $G(f)$, which minimize the reconstruction error for a fixed sampling rate f_s and quantizer resolution b . Because $\hat{x}(t)$ is *cyclostationary* with period T_s (cf. [21, Ch. 12]), we aim to minimize the TMSE [20], i.e.,

$$\min_{H(f), G(f)} \frac{1}{T_s} \int_0^{T_s} \mathbb{E}\{|\hat{x}(t) - x(t)|^2\} dt. \quad (6)$$

Note that while we model and optimize a practical acquisition system architecture in this work, the main focus of the works [11, 12] was to characterize the fundamental distortion limit of *any* acquisition system employing a sampling rate f_s under the constraint that the samples are encoded with a bit rate R . Hence, the ADX results from [11, 12] provide a lower bound on the minimum achievable TMSE in (6).

3. UNIFORM ADC BASED ACQUISITION SYSTEM

Here, we derive the acquisition system which minimizes the TMSE in recovering $x(t)$. We obtain the TMSE-minimizing recovery filter and pre-sampling filter in Subsection 3.1 and discuss our solution in Subsection 3.2. Due to page limitations, all proofs can be found in [22].

3.1. TMSE Minimizing Filters

We begin by obtaining the shift-invariant linear recovery filter $G(f)$, which minimizes the TMSE given in (6), for a given pre-sampling filter $H(f)$, a fixed sampling rate f_s , and a fixed quantizer resolution b . The result is summarized in the following proposition.

Proposition 1. *For a given pre-sampling filter $H(f)$, the TMSE minimizing linear recovery filter $G_o(f)$ is given by*

$$G_o(f) = \frac{S_x(f) H^*(f)}{S_y(e^{j2\pi f T_s}) + \kappa T_s \int_{-\frac{f_s}{2}}^{\frac{f_s}{2}} S_y(e^{j2\pi f' T_s}) df'} \quad (7)$$

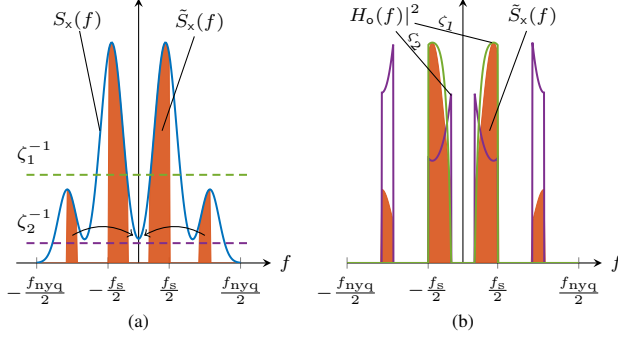


Fig. 2. Illustration of the operation of $|H_o(f)|^2$, where ζ_1 and ζ_2 correspond to a very low and a moderate ADC amplitude resolution.

where $S_y(e^{j2\pi f T_s}) = \frac{1}{T_s} \sum_{k \in \mathbb{Z}} |H(f - k f_s)|^2 S_x(f - k f_s)$ denotes the PSD of $y[n]$ and it holds $\kappa = \frac{\eta^2}{2^{2b}} (1 - \frac{2\eta^2}{3 \cdot 2^{2b}})^{-1}$. The resulting minimum achievable TMSE is given by

$$\text{TMSE}(H(f)) = \int_{\mathbb{R}} \left(S_x(f) - \frac{|H(f)|^2 S_x^2(f)}{T_s S_z(e^{j2\pi f T_s})} \right) df, \quad (8)$$

where $S_z(e^{j2\pi f T_s}) = S_y(e^{j2\pi f T_s}) + \kappa T_s \int_{-\frac{f_s}{2}}^{\frac{f_s}{2}} S_y(e^{j2\pi f' T_s}) df'$ denotes the PSD of $z[n]$.

Our next goal is to find the TMSE-minimizing pre-sampling filter, denoted as $H_o(f)$, by minimizing the TMSE expression given in (8) w.r.t. $H(f)$. The result is summarized in the following theorem.

Theorem 1. For a fixed sampling rate f_s , and quantizer resolution b , the TMSE minimizing pre-sampling filter $H_o(f)$ is characterized by

$$|H_o(f - k f_s)|^2 = \begin{cases} \left(\frac{(\sqrt{\zeta \tilde{S}_x(f)} - 1)^+}{2^{2b} \tilde{S}_x(f)} \right), & k = \tilde{k}(f), |f| < \frac{f_s}{2} \\ 0 & \text{otherwise,} \end{cases} \quad (9)$$

with $\tilde{k}(f) = \arg \max_{k \in \mathbb{Z}} S_x(f - k f_s)$, $\tilde{S}_x(f) = S_x(f - \tilde{k}(f) f_s)$, and ζ chosen such that $\kappa T_s \int_{-\frac{f_s}{2}}^{\frac{f_s}{2}} \left(\sqrt{\zeta \tilde{S}_x(f)} - 1 \right)^+ df = 1$. Furthermore, the resulting minimum achievable TMSE is

$$\text{TMSE}_o = \int_{\mathbb{R}} S_x(f) df - \int_{-\frac{f_s}{2}}^{\frac{f_s}{2}} \frac{(\sqrt{\zeta \tilde{S}_x(f)} - 1)^+ \tilde{S}_x(f)}{(\sqrt{\zeta \tilde{S}_x(f)} - 1)^+ + 1} df. \quad (10)$$

Theorem 1 characterizes the operation of the TMSE-minimizing pre-sampling filter $H_o(f)$. Note that the phase of $H_o(f)$ can be chosen arbitrarily, whereas the phase of $G_o(f)$ depends on $H(f)$. The optimal choice of the pre-sampling filter, i.e., $H_o(f)$, accounts for both aliasing induced by sub-Nyquist sampling as well as distortion due to low-resolution quantization: In particular, the filter preserves only the most dominant spectral components aliased to each frequency after uniform sampling with rate f_s (via the parameter $\tilde{k}(f)$). Furthermore, it nullifies the weak spectral modes which are likely to be indistinguishable after uniform quantization (via the parameter ζ).

The operation of the analog filter is illustrated in Fig. 2. First, Fig. 2.(a) depicts a multi-modal input PSD $S_x(f)$, the resulting $\tilde{S}_x(f)$, and two different water-filling thresholds ζ_1^{-1} and ζ_2^{-1} , which correspond to a very low and a moderate ADC amplitude resolution, respectively. Then, Fig. 2.(b) shows the resulting TMSE-minimizing pre-sampling filter $|H_o(f)|^2$ for ζ_1 and ζ_2 . For a very low ADC amplitude resolution, i.e., for ζ_1 , only the dominant spectral components

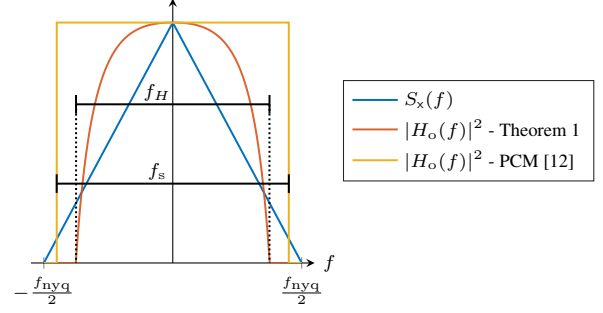


Fig. 3. Comparison of $|H_o(f)|^2$ from Theorem 1 to the filter of [12] for $R = 1$ bit per Nyquist interval.

of $\tilde{S}_x(f)$ are preserved, whereas in case of a moderate ADC amplitude resolution, i.e., for ζ_2 , all spectral components of $\tilde{S}_x(f)$ are preserved and $|H_o(f)|^2$ converges to a whitening filter.

3.2. Discussion

Together, Proposition 1 and Theorem 1 characterize the TMSE minimizing acquisition system, which utilizes a uniform scalar ADC. The analysis presented in this work is different from that provided in the PCM setting of [12, Sec. V], which is carried out *assuming* that the quantization distortion can be modeled as an additive zero-mean distortion, which is white, i.e., temporally uncorrelated, and uncorrelated with the quantizer input. In contrast, we can *guarantee* the same properties for any non-overloading quantizer input distribution by employing non-subtractive dithered quantization, which, however, complicates the derivation.

While our derived recovery filter $G_o(f)$ and the corresponding TMSE expression from Proposition 1 are similar to those obtained in [12, Prop. 5], i.e., both are Wiener filters and differences are due to different quantization distortion models, the proposed pre-sampling filters $H_o(f)$ also differ. In particular, for symmetric input PSDs $S_x(f)$ which are non-increasing for $f > 0$, i.e., for unimodal PSDs $S_x(f)$, the authors of [12] assume that (8) is minimized by choosing $H(f)$ as a low-pass filter with cut-off frequency $\frac{f_s}{2}$ [12, Sec. V.B]. This means that $H(f)$ corresponds to a conventional anti-aliasing filter. In contrast, Theorem 1 proves that the TMSE-minimizing pre-sampling filter, i.e., $H_o(f)$, may discard weak frequency components of $S_x(f)$, which cannot be resolved with the given quantizer resolution. For example, again considering unimodal PSDs $S_x(f)$, the frequency support of $H_o(f)$ may be given by $\mathbb{1}_{|f| < \frac{f_H}{2}}(f)$ with $f_H < f_s$, as illustrated in Fig. 3. This allows to resolve the remaining frequency components with higher accuracy, because the quantization distortion is proportional to the variance of the samples at the quantizer input, i.e., $\mathbb{E}\{e^2[n]\} \propto \mathbb{E}\{y^2[n]\}$. Consequently, the minimal TMSE is achieved by a trade-off between the distortion due to the pre-sampling filter and the quantization distortion. Note that a similar result has been obtained in [8], where the authors only consider quantization. In Section 4 we verify our claim and show that the pre-sampling filter proposed in Theorem 1 can outperform the one considered in [12].

While the pre-sampling filter proposed here is generally different from the one employed in [12], they are identical up to a scaling factor for a rectangular input PSD $S_x(f)$, because in this case, there are no weak frequency components to be discarded. For such rectangular input PSDs, one can specialize (10) to a closed-form expression. In the following, we compare this expression to the normalized TMSE of

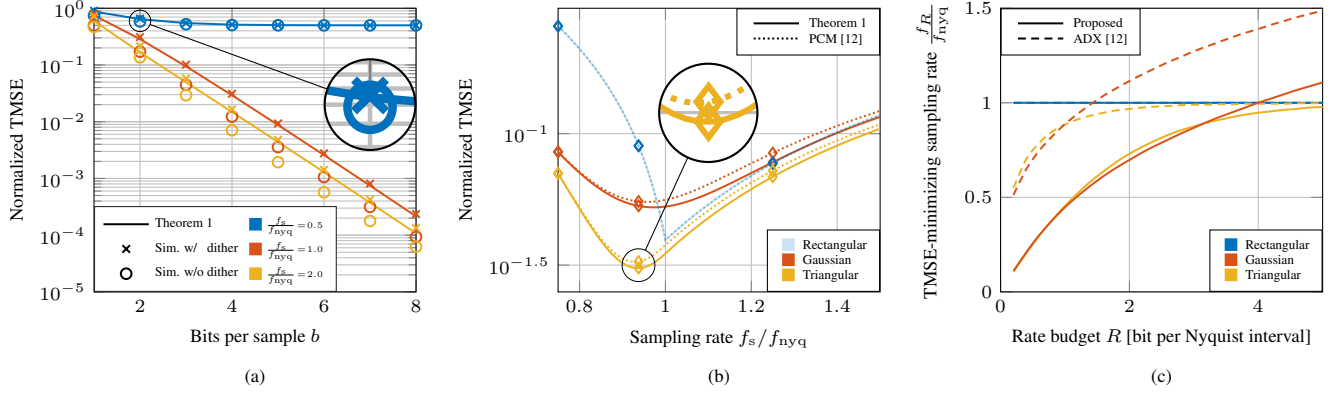


Fig. 4. In (a), we evaluate the TMSE over an increasing number of bits per sample b for different sampling rates f_s . We also compare the TMSE predicted by Theorem 1 to the simulated TMSE when employing non-subtractive dithered and conventional, i.e., non-dithered, quantizers. In (b), we compare the achievable TMSE when employing our proposed pre-sampling filter to the performance when employing the one proposed in [12], where we modify the result from [12] by using the quantization distortion resulting from our system model to enable a fair comparison. Diamonds illustrate the sampling rates corresponding to integer values of b , i.e., $b = R/f_s \in \mathbb{N}$. In (c), we evaluate the sampling rates f_R , which minimize the TMSE for a fixed rate budget $R = f_s \cdot b$. Note that we set the 3-dB bandwidth of the Gaussian PSD to $\frac{f_{nyq}}{2}$ in this work.

the solutions from [12], which yields the normalized TMSE

$$\frac{\text{TMSE}_o(f_s, b)}{\mathbb{E}\{|x(t)|^2\}} = 1 - \frac{\min(f_s, f_{nyq})}{f_{nyq}} \times \begin{cases} (1 + T_s \min(f_s, f_{nyq}) \bar{\kappa} 2^{-2b})^{-1}, & \text{Proposed} \\ (1 + T_s \min(f_s, f_{nyq}) c_q 2^{-2b})^{-1}, & \text{PCM [12]} \\ (1 - 2^{-2b}), & \text{ADX [12]} \end{cases} \quad (11)$$

with $\bar{\kappa} = \eta^2 (1 - \frac{2\eta^2}{3 \cdot 2^{2b}})^{-1}$. In (11), it holds $c_q = \frac{\sqrt{3}\pi}{2}$ for non-uniform scalar quantization of Gaussian inputs [6, p. 2329], as considered in [12]. Furthermore, the factor ' $\min(f_s, f_{nyq}) f_{nyq}^{-1}$ ', corresponds to the loss due to sub-Nyquist sampling, ' $\mathcal{X} \cdot 2^{-2b}$ ', $\mathcal{X} \in \{1, \bar{\kappa}, c_q\}$ stems from the loss due to quantization, and the factor ' $T_s \min(f_s, f_{nyq})$ ' shows a reduction of the quantization distortion when employing temporal oversampling. It can be shown that the TMSE for PCM in (11) is lower than that of Theorem 1, while both are outperformed by ADX, which corresponds to a lower bound on the minimum achievable distortion for any practical system. However, the PCM TMSE expression assumes non-uniform quantization and is only valid for Gaussian input distributions, while the proposed TMSE is derived for uniform quantization and holds for arbitrary input distributions when employing non-subtractive dithered quantization.

The analysis in this work is limited to single branch sampling. In [23, Sec. 2.5], it was shown that single branch sampling is sufficient for unimodal input PSDs. However, for multi-modal input PSDs, the performance can generally be improved by employing multi-branch sampling, which is beyond the scope of this work.

4. NUMERICAL RESULTS

In this section, we provide a numerical study focusing on Gaussian processes $x(t)$. For the considered equivalent quantizer model given in (3), it has been shown in [13, Sec. IV.B] that the overload probability needs to decrease with increasing b to ensure its validity. Hence, in the following, we increase η with b , i.e., we set $\eta(b) = 0.25b + 1.75$, as proposed in [13, Sec. IV.B].

First, we validate our theoretical results in Fig. 4(a), asserting that the TMSE derived in Theorem 1, where non-overloaded ADCs are assumed, is indeed achievable by the proposed system with and without

dithering. Here, we consider a rectangular input PSD $S_x(f)$. It can be seen that the TMSE predicted by (10) is a close match to the simulated TMSE, when employing dithered quantizers, as we assume in our system model. The mismatch is likely due to a small but non-zero overload probability. Furthermore, we observe that a lower TMSE can be achieved when using the same filters but employing conventional non-dithered quantizers. This asserts the validity of our derivations.

Next, in Fig. 4(b), we compare the resulting TMSE when employing the pre-sampling filter proposed in Theorem 1 to the one derived in [12, Sec. V.B], while using the quantization noise model derived in this work, e.g., we set $c_q = \bar{\kappa}$ in (11). Here, we fix the rate budget to $R = f_s \cdot b = 3.75$ bit per Nyquist interval. It can be seen that the proposed solution achieves a marginally lower TMSE for non-flat input PSDs. This shows the superiority of the proposed pre-sampling filter, which may discard weak frequency components. Furthermore, the evaluation shows that the TMSE is minimized for sampling rates $f_s < f_{nyq}$, for non-flat input PSDs, which is known from ADX [11, 12].

Finally, in Fig. 4(c), we compare the sampling rates, denoted as f_R , which minimize the TMSE for a fixed rate budget R in the proposed system and for the fundamental ADX limit [12, eq. (26)]. Note that the values of f_R are searched numerically. It can be seen that for low rate budgets R and non-flat input PSDs, the TMSE is minimized by employing sub-Nyquist sampling. Notably, f_R is much lower in the proposed system compared to the fundamental ADX limit. This demonstrates the effectiveness of sub-Nyquist sampling for practical systems operating under tight rate budgets and employing uniform ADCs.

5. CONCLUSION

In this work, we studied an acquisition system for WSS signals employing uniform sampling and uniform quantization. For a fixed sampling rate and quantizer resolution, we obtained closed-form expressions for the TMSE-minimizing pre-sampling and recovery filters, as well as the resulting minimum achievable TMSE. We showed that the proposed solution for the pre-sampling filter is superior to a previously proposed solution for the PCM setting from [12]. Furthermore, our numerical results demonstrated the validity of our model and, most notably, that the TMSE is often minimized by employing considerable sub-Nyquist sampling for non-flat input PSDs and low rate budgets.

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