COMPRESSIVE PHASE RETRIEVAL BASED ON SPARSE LATENT GENERATIVE PRIORS

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ABSTRACT

We address the problem of compressive phase retrieval (CPR) based on generative prior. The problem is ill-posed and requires structural assumptions. CPR techniques impose sparsity prior on the signal to perform reconstruction from compressive phaseless measurements. Recent developments in data-driven signal models in the form of generative priors have been shown to outperform sparsity priors with significantly fewer measurements. However, it is possible to improve upon the performance of generative prior based methods by introducing structure in the latent-space. We propose to introduce structure on the signal by enforcing sparsity in the latent-space via proximal method while training the generator. The optimization is called as proximal meta-learning (PML). Enforcing sparsity in the latent space naturally leads to a union-of-submanifolds model in the solution space. The overall framework of imposing sparsity along with PML is called as *sparsity-driven latent space sampling* (SDLSS). We demonstrate the efficacy of the proposed framework over the state-of-the-art deep phase retrieval (DPR) technique on MNIST and CelebA datasets. We evaluate the performance as a function of the number of measurements and sparsity factor using standard objective measures. The results show that SDLSS performs better at higher compression ratio and has faster recovery compared with DPR.

***Index Terms*—** Compressive phase retrieval, sparsity, generative models, proximal meta-learning.

1. INTRODUCTION

Compressive phase retrieval (CPR) is the problem of reconstructing a signal $\boldsymbol{x} \in \mathbb{R}^n$ from its compressed phaseless measurements $\boldsymbol{y} \in \mathbb{R}^m$. Mathematically, the measurement model is described as follows:

$$\boldsymbol{y} = |\mathbf{A}\boldsymbol{x}| \quad \text{s.t.} \quad y_i = |\langle \mathbf{a}_i, \boldsymbol{x}^* \rangle|, \quad i = 1, \dots, m, \tag{1}$$

where $\mathbf{A} \in \mathbb{R}^{m \times n}$ is the sensing matrix with Gaussian i.i.d entries such that $\mathbf{a}_{i,j} \sim \mathcal{N}(0, \frac{1}{m})$ and $|\cdot|$ acts element-wise. The reconstruction problem is ill-posed since $m \ll n$, with compression ratio $\frac{n}{m}$ and only magnitude measurements are available. To increase the fidelity of the recovered signal, structural assumptions or priors are imposed on the signal $\boldsymbol{x}$. Classical compressive phase retrieval algorithms assume sparsity ($\boldsymbol{x}$ or its representation in a suitable basis) [1] or non-negativity. In this paper we propose a generative prior with sparse latent input to model the true data distribution of the signal $\boldsymbol{x}$. The generative prior constraints the signal $\boldsymbol{x}$ to lie in the range space of the generator model.

1.1. Related Literature

We review prior methods used for standard phase retrieval (PR) and CPR. The ill-posedness in the phase retrieval problem is overcome by imposing structural constraints on the signal or its representation in the transform domain (Fourier or Wavelet). The constrained optimization problem could be convex or non-convex. Classical non-convex iterative error reduction algorithms recover phase from Fourier magnitude measurements with non-negativity constraint [2, 3]. Gerchberg-Saxton [3] and Fienup [2] algorithms involve back and forth iterative Fourier transformations between the object and its Fourier domain to estimate the phase. The aforementioned methods lack global convergence guarantees. Eldar and Mendelson [4] considered random measurements instead of Fourier and derived uniqueness and recovery guarantees. Recent non-convex approaches include alternating minimization algorithm [5] with geometric rate of convergence, Wirtinger Flow algorithm [6], where the algorithm is given a *spectral initialization*. Wang et al. formulated sparse PR as an amplitude based empirical loss function and proposed an iterative algorithm named sparse truncated amplitude flow (SPARTA) [1].

Convex approaches include compressive phase retrieval via lifting [7, 8], where the signal to be recovered is transformed into a rank-one matrix recovery problem by lifting the signal into the space of positive semi-definite matrices. Rank-one recovery problem is NP hard in general and is relaxed by replacing with trace-norm minimization. PhaseCut [9] and PhaseMax [10] are other convex relaxation techniques used to solve the phase retrieval problem. Recently, plug-and-play approaches namely SPAR [11], BM3D-PRGAMP [12] have been proposed to capture the complex signal structure in case of noisy and reduced observations, respectively. Phase retrieval problem has also been solved using deep learning approaches in application-specific settings such as Holography [13] and Ptychography [14].

We consider compressive phase retrieval using a generative prior (CPRGM) [15–17]. The prior is learned from the training data using a generative adversarial network (GAN) [18]. Recently, generative models have been used to solve the compressive sensing (CS) [19] and deep phase retrieval (DPR) [15] problems. Bora et al. replaced the traditional sparsity prior and enforced the constraint that the signal $\boldsymbol{x}$ lies in the range-space of the pretrained generator $\mathbf{G}_{\boldsymbol{\theta}} : \mathbb{R}^k \to \mathbb{R}^n$ such that $\boldsymbol{x} \approx \mathbf{G}_{\boldsymbol{\theta}}(\boldsymbol{z})$, where $\boldsymbol{z} \in \mathbb{R}^k$ and $\boldsymbol{\theta}$ denote the latent variable and the parameter of the generator, respectively.

Compressive phase retrieval using generative models has two limitations: First, the recovered signal $\boldsymbol{x}$ is constrained to lie in the range-space of the generator $\mathbf{G}_{\boldsymbol{\theta}}$. Second, the reconstruction is slow because the latent space is optimized through stochastic gradient-descent (SGD) and requires several restarts.

1.2. Motivation and Contribution

Most of the dimensions in high-dimensional data are redundant. Obtaining an effective latent code that captures the complex structure of the observed data is a challenge. Due to their efficiency in representation, sparse codes have been used in many learning and recognition systems. We extend the aforementioned capability of sparse coding technique to probabilistic generative models, allowing for efficient representations in a general setting. We adapt the Set-Restricted Eigenvalue Condition ($\mathcal{S}$-REC) loss proposed in [20] and use SDLSS framework to jointly learn the generative model and optimize the latent space for efficient representation [21]. We demonstrate the efficacy of SDLSS based on PML with application to compressive phase retrieval on standard datasets such as MNIST, and celebA and show that it outperforms the state-of-the-art DPR [15] technique.

2. UNION OF SUBMANIFOLDS

Motivated from the union of subspaces model introduced in [22], we review the union-of-submanifolds [20] model for the sparsity driven generative prior. Sparse latent space results in the range-space of the generator to be modelled as the union-of-submanifolds. We assume $\boldsymbol{x} \in \mathbb{R}^n$ to lie in the range-space of the generator network.

$$\boldsymbol{x} = \mathbf{G}_{\boldsymbol{\theta}}(\boldsymbol{z}) \quad \text{s.t} \quad \|\boldsymbol{z}\|_0 \leq s. \tag{1}$$

The generator $\mathbf{G}_{\boldsymbol{\theta}} : \mathbb{R}^k \to \mathbb{R}^n$ can be interpreted as a nonlinear dictionary that maps the sparse latent variable $\boldsymbol{z} \in \mathbb{R}^k$ to a submanifold $\mathcal{S} \in \mathbb{R}^n$. The sparsity s assumption on the input latent space divides the latent space into $\binom{k}{s}$ subspaces $\{\mathcal{W}_i\}$, such that the generator model $\mathbf{G}_{\boldsymbol{\theta}}$ transforms each subspace $\mathcal{W}_i$ to a corresponding submanifold $\{\mathcal{S}_i\}$. Thus, the range-space of the generator $\mathbf{G}_{\boldsymbol{\theta}} : \mathbb{R}^k \to \mathbb{R}^n$ comprises $\binom{k}{s}$ number of submanifolds $\{\mathcal{S}_i\}$ given as:

$$\mathcal{S}_{s,\mathbf{G}_{\boldsymbol{\theta}}} = \bigcup_i \mathcal{S}_i, \tag{2}$$

where $\mathcal{S}_i$ is the submanifold generated by $\mathbf{G}_{\boldsymbol{\theta}}(\boldsymbol{z})$, with $\|\boldsymbol{z}\|_0 \leq s$ and $\mathcal{S}_{s,\mathbf{G}_{\boldsymbol{\theta}}}$ is the range-space of generator $\mathbf{G}_{\boldsymbol{\theta}}$ with s-sparse latent variable $\boldsymbol{z}$.

We review the definition of Set Restricted Eigen Value Condition ($\mathcal{S}$-REC) [19] imposed on the sensing matrix $\mathbf{A} \in \mathbb{R}^{m\times n}$ so that the difference of any two vectors lying in the set $\mathcal{S}$ (range-space of the generator) is far away form the null-space of $\mathbf{A}$.

Definition 1. *(Bora et al. [19]) Let $\mathcal{S} \subseteq \mathbb{R}^n$, for some parameters $\gamma > 0$, and $\delta \geq 0$, a matrix $\mathbf{A} \in \mathbb{R}^{m\times n}$ is said to satisfy the $\mathcal{S}$-REC$(\mathcal{S}, \gamma, \delta)$ if $\forall \boldsymbol{x}_1, \boldsymbol{x}_2 \in \mathcal{S}$,*

$$\|\mathbf{A}(\boldsymbol{x}_1 - \boldsymbol{x}_2)\|_2 \geq \gamma\|\boldsymbol{x}_1 - \boldsymbol{x}_2\|_2 - \delta. \tag{3}$$

3. SPARSITY-DRIVEN LATENT SPACE SAMPLING FOR COMPRESSIVE PHASE RETRIEVAL

Most of the signals are not analysis-sparse. Instead, they are synthesis-sparse, which can be enforced by means of a suitable dictionary or a nonlinear model. Generative models with a fixed dimension for the input latent space enforce the reconstructed signal to lie in the range-space of the generator represented by a single manifold [23]. However, natural signals do not lie on a single manifold modelled by the range-space of the generator, but rather on union-of-submanifolds as discussed in [23].

The SDLSS framework allows the signal $\boldsymbol{x}$ to lie in the union-of-submanifolds $\mathcal{S}_{s,\mathbf{G}_{\boldsymbol{\theta}}}$ spanned by the generator $\mathbf{G}_{\boldsymbol{\theta}}$. Similar to Restricted Isometry Property (RIP) [24] imposed on the measurement matrix $\mathbf{A}$ in CS, a strong RIP condition is proposed in [25] for stable sparse recovery in CPR. For CPR with generative prior, we adapt the $\mathcal{S}$-REC loss [20] to place a condition on the ℓ_2-norm of a vector coming from the set $\mathcal{S}$ and its linearly transformed version. The optimization problem to learn the latent space jointly with the parameters of the generator takes the following form:

$$\begin{aligned} \min_{\boldsymbol{z},\boldsymbol{\theta}} \|\boldsymbol{z}\|_0, \quad \text{s.t.} \quad & \|\boldsymbol{y} - |\mathbf{A}\mathbf{G}_{\boldsymbol{\theta}}(\boldsymbol{z})|\|_2 \leq \epsilon, \text{and} \\ & \||\mathbf{A}(\boldsymbol{x}_1 - \boldsymbol{x}_2)|\|_2 \geq \gamma\|\boldsymbol{x}_1 - \boldsymbol{x}_2\|_2 - \delta, \end{aligned} \tag{4}$$

where $\delta \geq 0$ and $\gamma > 0$ are user-defined parameters, $\boldsymbol{z} \in \mathbb{R}^k$ is the latent space with sparsity at most s, and $\boldsymbol{\theta}$ denotes the generator parameter. Since $\boldsymbol{z}$ is assumed to be s-sparse, the ℓ_0 pseudo-norm is implemented by means of a hard-thresholding operator that retains the s largest magnitudes in $\boldsymbol{z}$ and sets the rest to zero.

Thus, the proposed methods introduce sparsity in the continuous latent space. The optimization considered in Eq. (4)

Algorithm 1 Sparsity Driven Latent Space Sampling (SDLSS) for CPR.

Input: data = $\{\boldsymbol{x}_i\}_{i=1}^N$, sensing matrix $\mathbf{A}$, generator $\mathbf{G}_{\boldsymbol{\theta}}$, learning rate α, number of latent optimization steps T, measurement error threshold ϵ and sparsity factor s.
repeat
  Initialize generator network parameter $\boldsymbol{\theta}$ respectively.
  for $i = 1$ **to** N **do**
    Measure the signal $\boldsymbol{y}_i = |\mathbf{A}\boldsymbol{x}_i|$
    Sample $\boldsymbol{z}_i \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$
    for $t = 1$ **to** T **do**
      $\hat{\boldsymbol{z}}_i = \mathcal{P}_s\left(\mathbf{z}_i - \beta \nabla_{\boldsymbol{z}} f(\boldsymbol{y}_i, \mathbf{z}_i)\right)$
    end for
  end for
  $\mathcal{L} = \mathcal{L}_{\mathbf{G}} + \mathcal{L}_{\mathbf{A}}$
  Update $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \alpha \frac{\partial \mathcal{L}}{\partial \boldsymbol{\theta}}$
until $\|\boldsymbol{y} - |\mathbf{A}\mathbf{G}_{\boldsymbol{\theta}}(\hat{\boldsymbol{z}})|\|_2 \leq \epsilon$

is expressed in an unconstrained form as:

$$\min_{\boldsymbol{z}, \boldsymbol{\theta}} \left(\mathcal{L}_{\mathbf{G}} + \mathcal{L}_{\mathbf{A}}\right), \tag{5}$$

where

$$\mathcal{L}_{\mathbf{G}} = \mathop{\mathbb{E}}_{\boldsymbol{z}} \left\{ \|\boldsymbol{y} - |\mathbf{A}(\mathbf{G}_{\boldsymbol{\theta}}(\boldsymbol{z}))|\|_2^2 + \|\boldsymbol{z}\|_0 \right\}, \quad \text{and}$$
$$\mathcal{L}_{\mathbf{A}} = \mathop{\mathbb{E}}_{\boldsymbol{x}_1, \boldsymbol{x}_2} \{ (\||\mathbf{A}(\boldsymbol{x}_1 - \boldsymbol{x}_2)|\|_2 + \delta - \gamma \|\boldsymbol{x}_1 - \boldsymbol{x}_2\|_2)^2 \}.$$

The generator model $\mathbf{G}_\theta$ is trained jointly along with latent space optimization using PML to minimize the expected measurement loss $\mathcal{L}_{\mathbf{G}}$ and the adapted S-REC loss $\mathcal{L}_{\mathbf{A}}$. The vectors $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$ in S-REC loss function $\mathcal{L}_{\mathbf{A}}$ are sampled from the true data distribution $p_{\text{data}}(\boldsymbol{x})$ and the generator $\mathbf{G}_{\boldsymbol{\theta}}(\boldsymbol{z})$, respectively, and the latent variable $\boldsymbol{z}$ is sampled from the standard normal distribution $\mathcal{N}(\mathbf{0}, \boldsymbol{I})$. The reconstructed signal is given as $\hat{\boldsymbol{x}} = \mathbf{G}_{\boldsymbol{\theta}}(\hat{\boldsymbol{z}})$, where $\hat{\boldsymbol{z}}$ is the optimized latent space variable.

We use the proximal meta-learning (PML) algorithm discussed in [20] to jointly optimize the latent variable $\boldsymbol{z}$ and the parameter $\boldsymbol{\theta}$ of the generative model. PML combines the idea of proximal gradient-descent together with meta-learning [26] to promote sparsity in the latent space while optimizing the cost in Eq. (5). The sparse coding of each data sample in a batch is considered as a task $\mathcal{T}_i$ and the corresponding latent variable $\boldsymbol{z}_i$ (such that $\boldsymbol{x}_i = \mathbf{G}_{\boldsymbol{\theta}}(\boldsymbol{z}_i)$ and $\boldsymbol{y}_i = |\mathbf{A}\boldsymbol{x}_i|$) as the parameter associated with task $\mathcal{T}_i$. The latent space is optimized to adapt to each task $\mathcal{T}_i$ with a few (up to five) proximal gradient-descent updates on the latent variable $\boldsymbol{z}_i$:

$$\begin{aligned}\hat{\boldsymbol{z}}_i &= \arg\min_{\boldsymbol{z}_i} \left(f(\boldsymbol{y}_i, \boldsymbol{z}_i) = \|\boldsymbol{y}_i - |\mathbf{A}\mathbf{G}_{\boldsymbol{\theta}}(\boldsymbol{z}_i)|\|_2\right), \\ &= \mathcal{P}_s\left(\boldsymbol{z}_i - \beta \nabla_{\boldsymbol{z}} f(\boldsymbol{y}_i, \boldsymbol{z}_i)\right),\end{aligned}$$

where β is the learning rate, and $\mathcal{P}_s$ is the hard-thresholding operator. The model parameter $\boldsymbol{\theta}$ is trained by optimizing the measurement loss $\mathcal{L}_{\mathbf{G}}$ and adapted S-REC loss $\mathcal{L}_{\mathbf{A}}$ across all the tasks $\mathcal{T}_i$. The parameter updates are performed via stochastic gradient-descent (SGD) [27]:

$$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} - \alpha \frac{\partial(\mathcal{L}_{\mathbf{G}} + \mathcal{L}_{\mathbf{A}})}{\partial \boldsymbol{\theta}}, \tag{6}$$

where α is the corresponding meta step-size parameter. The SDLSS approach for CPR is summarized in Algorithm 1.

4. EXPERIMENTAL RESULTS

The performance validation is conducted on the standard datasets such as MNIST and CelebA, and compared with state-of-the-art DPR [15]. In DPR, a pre-trained generator is used as a prior and the image is reconstructed by optimizing the latent space, which typically takes about 100 - 200 iterations of SGD and several random restarts. In contrast, the training process of the proposed SDLSS method involves learning the parameters of the generator along with optimizing the latent space. The latent space optimization takes only a few (about 3 to 5) SGD steps during training. Once the generator is trained, the test image reconstruction complexity is also about the same – 3 to 5 steps of SG optimization of latent space.

MNIST images are grayscale with dimensionality 28×28, whereas CelebA images are colour with dimensionality 64×64. The generator model is a two-layered feed-forward neural network for MNIST with 500 neurons in each hidden layer and leaky ReLU as the activation. In contrast, a standard DCGAN generator [28] is used for CelebA. The maximum dimension of the latent space is set equal to the dimension of the generator output ($28 \times 28 = 784$ for MNIST, and $64 \times 64 = 4096$ for CelebA). The sensing operator $\mathbf{A} \in \mathbb{R}^{m \times n}$ is a random Gaussian matrix with i.i.d. entries such that $\mathbf{a}_{i,j} \sim \mathcal{N}(0, \frac{1}{m})$. The ablation study is done on MNIST. The efficacy of the proposed SDLSS model is demonstrated in comparison with DPR [15] in terms of objective metrics: peak signal-to-noise ratio (PSNR = $-10\log(\|\boldsymbol{x} - \mathbf{G}_{\boldsymbol{\theta}}(\boldsymbol{z})\|_2^2)$ dB), with peak value considered to be unity and structural similarity index measure (SSIM). The metrics are averaged over all the test images of a batch. A batch size of 64 images is considered for all the datasets.

The effect of imposing sparsity on the latent space with respect to per-pixel reconstruction error (RE $= 10\log(\|\boldsymbol{x} - \mathbf{G}_{\boldsymbol{\theta}}(\boldsymbol{z})\|_2^2/n)$ dB), where n is the total number of pixels, is shown in Fig. 1. We observe that there exists an optimal sparsity factor for which the reconstruction error is minimum. The performance improvement of SDLSS over DPR in terms of PSNR and SSIM is demonstrated in Fig. 2. The PSNR values of the reconstructed images using the proposed SDLSS method are higher than DPR and show a consistent increase with the increase in the number of measurements. The SSIM values of the proposed method are lower compared to DPR at lower measurement range and show an improvement for

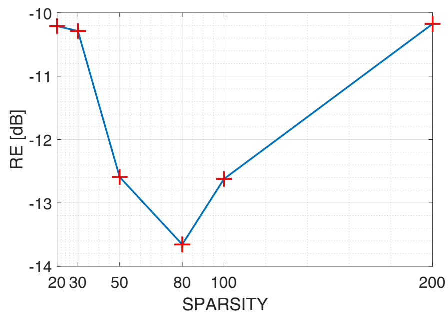

Fig. 1. Reconstruction error on MNIST test data as a function of sparsity s for a given measurement dimension $m = 20$ and latent dimension $k = 784$.

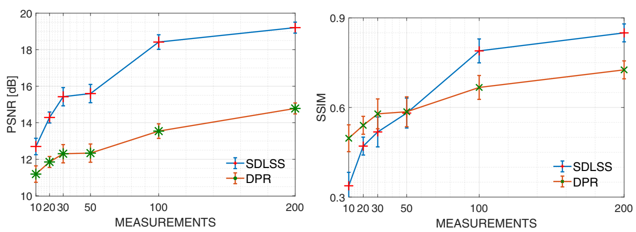

Fig. 2. Evaluation of SDLSS and DPR on MNIST test dataset as a function of the measurement dimension m for the latent dimension k = 100 and sparsity s = 80: (a) PSNR vs. m, and (b) SSIM vs. m.

Table 1. Performance comparison between DPR and the SDLSS (**ours**) approaches on CelebA dataset. The latent space dimension $k = 4096$.

METHOD	METRIC	$m = 500$	$m = 1000$
DPR	PSNR [DB]	19.40 ± 0.35	19.80 ± 0.35
	SSIM	0.7078 ± 0.020	0.7238 ± 0.017
SDLSS	PSNR [DB]	$\mathbf{20.04 \pm 0.37}$	$\mathbf{20.41 \pm 0.35}$
(s=100)	SSIM	$\mathbf{0.7511 \pm 0.019}$	$\mathbf{0.7622 \pm 0.017}$

higher set of measurements. The visual quality of the reconstructed images for MNIST is shown in Fig. 3. The images generated by SDLSS preserve the structure and have sharper features (highlighted using red boxes) compared to those generated by DPR. This experiment demonstrates that sparsity enhances the representation of images in the latent space. To further strengthen the claim, experiments are carried on CelebA dataset. From Fig. 4 we observe that the reconstructed images have sharper facial features in SDLSS than DPR. Table 1 shows the PSNR and SSIM values along with standard deviation calculated for different measurement dimensions. The values are averaged across all the test images in the batch.

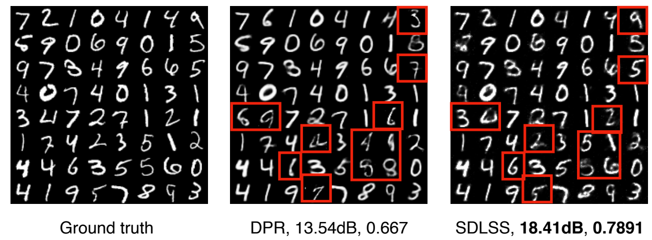

Fig. 3. MNIST: Reconstructed images with corresponding PSNR/SSIM values for $m = 100$. SDPR has $k = 100$, SDLSS has $k = 784$ and sparsity factor $s = 80$. The SDLSS gives images with sharper features (highlighted in red).

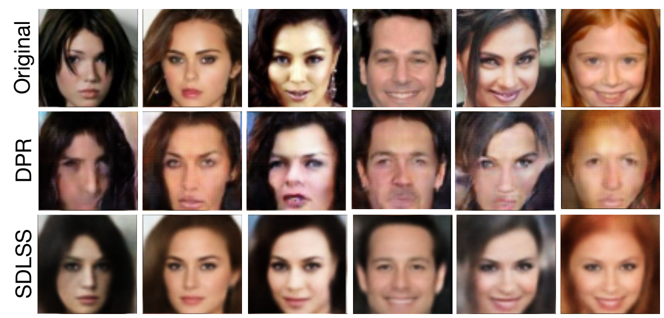

Fig. 4. Performance illustration on CelebA dataset. Row one: ground-truth image; Rows two and three: images reconstructed by DPR and SDLSS, respectively. $m = 500$ and $k = 4096$. The sparsity factor s in SDLSS is set to 100.

5. CONCLUSION

We considered the problem of compressive phase retrieval (CPR), where a signal is reconstructed from compressed phaseless measurements. A structural assumption of generative prior is imposed on the underlying signal $\boldsymbol{x}$ with sparsity enforced on the input latent space dimension — this corresponds to the union-of-submanifolds signal model, which represents the range-space of the generator. The $\mathcal{S} - REC$ loss proposed for generative prior based compressive sensing (CS) [20] is suitably modified for CPR. We learn the generator parameters jointly with latent space optimization to minimise the adapted $\mathcal{S} - REC$ loss and the measurement loss using PML. The proposed method shows better performance in comparison to state-of-the-art DPR [15] with faster reconstruction on standard datasets such as MNIST and CelebA.

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