DIRECT LOCALIZATION: AN ISING MODEL APPROACH

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ABSTRACT

Accurate indoor localization is a challenging problem in a multipath environment. In order to tackle this problem, several methods have been proposed. Direct localization is one of these methods that makes use of a two-dimensional search in a planar geometry. In this paper, we use a compressed sensing framework in the direct localization technique to estimate the location of a user in an indoor multipath environment. We form a penalized ℓ_0-norm structure for this problem, and then convert this structure to an Ising energy problem.The Ising energy problem is solved using Markov Chain Monte Carlo (MCMC). Our simulation results show that our approach improves the estimation accuracy compared to the existing methods in the literature.

Index Terms— Direct localization, Compressed sensing, Ising model, Digital Annealer

1. INTRODUCTION

With the emergence of location-based services, accurate knowledge of location information has become important. Unfortunately, Global Positioning System (GPS) cannot provide precise location information in indoors and dense environments. Recently, positioning systems use WiFi-based localization methods to increase the positioning accuracy in indoor environments [1, 2]. One of the categories of WiFi-based localization uses the angle of arriving wavefront, called the angle-of-arrival (AoA), to localize users [2–8].

In the AoA-based methods, the received signal at each base station is processed to estimate the AoA and next the location of the user is estimated by triangulation. Some techniques, use Maximum Likelihood (ML) algorithms to estimate AoA, [9]. Other techniques, such as MUSIC [10] and ESPIRIT [11], estimate AoA based on the eigen-structure of the input covariance matrix. Some other methods consider the signal sparsity and estimate the AoA using compressed sensing [7, 8, 12].

Although, AoA-based localization can have a good performance in free space, it will have bias in a multipath environment [13]. This is mainly due to the fact that in a multipath environment, we need to find line-of-sight (LoS) path which can be easily dampened or obstructed in dense environments [14]. In addition, AoA-based localization is a two-step procedure which makes the algorithm to be suboptimal.

To overcome these problems, the *direct localization* approach can be used [15]. In direct localization, the location of user is directly estimated from the received signal data in the base stations; it requires a search over the potential locations of the source. This method was first developed in [16] to estimate the location of users. In [15] and [17], direct localization approach was applied to LoS AoA estimation. Later, in [18] this approach was used to estimate the location of users in a multipath environment considering both LoS and non-line of sight (NLoS) AoA.

In this paper, we propose a direct localization method by co-processing the received signal at all the base stations. In order to co-process our data, we take advantage of the fact that LoS path for all the base stations originates from a common location in the area, while non-LoS (NLoS) paths have a random nature. In other words, when we grid the area, the strongest signal received at all the base stations comes from one of the grid points. Considering this fact, we employ a compressed sensing framework for our direct localization approach.

In particular, we reformulate our problem as an ℓ_0-norm minimization problem inspired by [8]. This ℓ_0-norm minimization problem is NP-hard. Thus, we convert it to an Ising energy problem which can be solved using Markov Chain Monte Carlo (MCMC) methods. In our approach, we keep the ℓ_0-norm objective function of compressed sensing and our approximation comes from the Ising energy formulation. Simulation results show that this approach outperforms the method of [18], which utilizes ℓ_1-norm relaxation.

The rest of the paper is organized as follows: Section 2 introduces our system model. In Section 3, we explain our methodology and formulations in detail. The performance of our method and its comparison to the existing literature are given in Section 4. Finally, Section 5 concludes the paper.

2. SYSTEM MODEL

Let us consider a two-dimensional multipath scenario, where P access points receive the signal emitted from a WiFi user. Each access point is equipped with a uniform linear array (ULA) of antennas with M elements. The user is located at point $[x_u, y_u]^T$ and the pth access point is located at $[x_a^p, y_a^p]^T$.

The wavefrom of the signal arriving at each access point

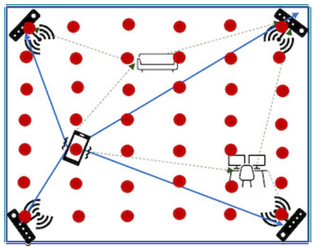


Fig. 1: A two dimensional environment and a uniform grid over it

is considered to be planar. In this case, the $M \times 1$ received signal vector at access point p, arriving from the angle θ_p, and with the time of flight (ToF) τ_p, from the broadside of the array, is

$$\mathbf{y}_p = \mathbf{a}(\theta_p, \tau_p)s_p + \mathbf{n}_p \tag{1}$$

where $s_p \in \mathbb{C}$ is a scalar that represents the complex signal amplitude in the multipath environment, $\mathbf{n}_p \in \mathbb{C}^{M\times 1}$ is the complex white Gaussian noise vector, and $\mathbf{a}(\theta_p, \tau_p) \in \mathbb{C}^{M\times 1}$ is the complex array manifold vector, which for a ULA is formulated as

$$\mathbf{a}(\theta_p, \tau_p) = \left[\Omega_{\tau_p}, \Omega_{\tau_p}\Gamma_{\theta_p}, \cdots, \Omega_{\tau_p}\Gamma_{\theta_p}^{(M-1)}\right]^T \tag{2}$$

where $\Gamma_{\theta_p} = e^{-j2\pi\frac{d}{\lambda}\sin(\theta_p)+j\frac{\pi d^2}{\lambda r_p}\cos^2(\theta_p)}$, r_p is the distance between the source and pth access point, $\Omega_{\tau_p} = e^{-j2\pi\Delta f\tau_p}$, d is the distance between two adjacent elements of the array and λ is the wavelength. Note that $\tau_p = \frac{r_p}{C}$, where C is the speed of light.

3. METHODOLOGY

3.1. Problem formulation

In this part, we explain our methodology for localization, which is based on Compressed Sensing. Consider a uniform grid in the environment as shown in Fig. 1. The set of grid points is denoted by $\mathcal{S}$:

$$\mathcal{S} = \{\hat{\mathbf{p}}_1, \hat{\mathbf{p}}_2, \cdots, \hat{\mathbf{p}}_G\} \tag{3}$$

where G is the total number of grid points and $\hat{\mathbf{p}}_j = [\hat{x}^j, \hat{y}^j]$.

Now, assume the user is located at point $\hat{\mathbf{p}}_j \in \mathcal{S}$. Access point p measures the received signal coming from the user located at this point at the angle $\theta_p^{(j)}$. This arriving angle is related to the position of the user by

$$\theta_p^{(j)} = \arctan\left(\frac{\hat{y}^j - y_p}{\hat{x}^j - x_p}\right) + \frac{\pi}{2} - \pi I(\hat{x}^j < x_p) \tag{4}$$

The $-\pi I(\hat{x}^j < x_p)$ is added to resolve the ambiguity in angles generated by $\arctan(.)$ (i.e. four quarters of the unit circle).

To use compressed sensing, we approximate our received CSI measurements, at access point p, with

$$\mathbf{y}_p = \Psi_p\mathbf{s}_p + \mathbf{n}_p \tag{5}$$

where $\mathbf{y}_p$ is the $M \times 1$ complex vector of received CSI, $\mathbf{s}_p$ is a $G \times 1$ complex vector of signal amplitudes for different grid points, $\mathbf{n}_p$ the $M \times 1$ vector of white Gaussian noise, Ψ_p is a $M \times G$ array manifold matrix and is given by

$$\Psi_p = [\mathbf{a}(\theta_p^{(1)}, \tau_p^{(1)}), \mathbf{a}(\theta_p^{(2)}, \tau_p^{(2)}), \cdots, \mathbf{a}(\theta_p^{(G)}\theta_p^{(G)})] \tag{6}$$

where $\mathbf{a}(.)$ is defined in (2). Because we assume that the user is located at one of those G grid points, in an environment with no reflections, only one of the elements of $\mathbf{s}_p$ is non-zero, and the non-zero component falls unto the same coordinate in the vector. In practice, however, the signal arriving at each access point is the combination of the LOS signal and several non-line of sight (NLOS) wavefronts reflected from walls and objects in the environment. Since the LOS signal for all access points is originated at the same location in space, the LOS component of $\mathbf{s}_p$ are at the same coordinate. This is not, however, true for NLOS signal components since each access points experiences reflections from different locations in space.

Our goal is to estimate the location of the user which should be selected among one of those grid points. Since the signal component is not important in location estimation, we approximate the observation vector with

$$\tilde{\mathbf{y}}_p = \Psi_p\mathbf{x}_p \tag{7}$$

where $\mathbf{x}_p \in \{0, 1\}^G$ is a sparse $G \times 1$ binary vector. All the elements of $\mathbf{x}_p$ should be zero except the element which corresponds to the grid-point the user is located on. As discussed above, for all the access points, $\mathbf{x}_p$ is the same as the user is located at one of the grid points. Therefore, we can omit the p index and use $\mathbf{x}$ vector instead.

Next, in order to estimate the location of user we form the following optimization problem:

$$\begin{aligned} \min_{\mathbf{x}\in\{0,1\}^G} \quad & ||\mathbf{x}||_0 \\ \text{s.t.} \quad & ||\mathbf{y}_p - \Psi_p\mathbf{x}||_2^2 < \epsilon, \;\; p = 1, 2, \cdots, P \end{aligned} \tag{8}$$

where ϵ is the maximum allowable mismatch between the observations and reconstructions. It is important to note that this binary approximation restricts the feasible set of our solution since $\mathbf{x}$ is a binary vector. Thus, the solution of this BP problem is an upper-bound to the exact solution.

3.2. Ising energy modelling

3.2.1. Definition of Ising model

Ising model is a model proposed to describe the system of interaction between spins of atoms in statistical physics. The goal of this model is to minimize the energy of this system given by

$$E(\mathbf{x};\mathbf{b},\mathbf{W}) = -\sum_{i=1}^{K} b_i x_i - \sum_{i=1}^{K}\sum_{j=1}^{K} W_{i,j} x_i x_j \tag{9}$$

where K is the number of binary variables, b_i shows the bias term and $W_{i,j}$ denotes the connection weight. In this model the variable x_i represents the spin of atoms and is a binary value, which is originally considered to be $\{-1,+1\}$ to show the state of the spin. However, the Ising energy model can easily be converted to use $\{0,1\}$ values. This conversion changes (9) to a *quadratic unconstrained binary optimization (QUBO)* problem, which is commonly used in Quantum computing [19].

3.2.2. *Ising energy reformulation*

Problem (8) is an NP-hard problem. To solve this problem, we utilize the Ising energy model. Thus, we first modify (8) to a regularized minimization problem:

$$\min_{\mathbf{x}\in\{0,1\}^G} \quad ||\mathbf{x}||_0 + \gamma \sum_{p=1}^{P} ||\mathbf{y}_p - \Psi_p \mathbf{x}||_2^2 \tag{10}$$

Next, we format (10) as an Ising energy model. In this case, we have G binary variables. We need to calculate the bias and the connection weights of the variables as well. Since $x_i \in \{0,1\}$, we have

$$||\mathbf{x}||_0 = \sum_{i=1}^{G} x_i. \tag{11}$$

Moreover, we can rewrite the second term in (10) as follows:

$$\begin{aligned}\sum_{p=1}^{P} ||\mathbf{y}_p - \Psi_p\mathbf{x}||_2^2 &= \sum_{p=1}^{P} (\mathbf{y}_p - \Psi_p\mathbf{x})^H(\mathbf{y}_p - \Psi_p\mathbf{x}) \\ &= \sum_{p=1}^{P} \mathbf{y}_p^H\mathbf{y}_p - y_p^H\Psi_p\mathbf{x} - \mathbf{x}^H\Psi_p^H y_p + \mathbf{x}^H\Psi_p^H\Psi_p\mathbf{x} \\ &= \sum_{p=1}^{P} ||\mathbf{y}_p||_2^2 - 2\Re(y_p^H\Psi_p\mathbf{x}) + \mathbf{x}^H\Psi_p^H\Psi_p\mathbf{x}\end{aligned} \tag{12}$$

Now, let us rewrite (10) as an Ising energy model using (11) and (12). We should also consider that when $x_i \in \{0,1\}$, $x_i^2 = x_i$:

$$\begin{aligned}\min_{\mathbf{x}\in\{0,1\}^G} &\sum_{i=1}^{G} (1 + \gamma \sum_{p=1}^{P} (-\sum_{m=1}^{M} 2\Re(y_{p,m}^H \Psi_{p_{m,i}} + \Psi_{p_{m,i}}^2)) x_i \\ &+ \sum_{i=1}^{G} \sum_{j=1, j\neq i}^{G} \sum_{p=1}^{P} \sum_{m=1}^{M} \gamma \Psi_{p_{m,i}}^H \Psi_{p_{m,j}} x_i x_j\end{aligned} \tag{13}$$

So, based on (14), the bias term of the Ising formulation is:

$$\begin{aligned} b_i = -(1 + \gamma \sum_{p=1}^{P} (-\sum_{m=1}^{M} 2\Re(y_{p,m}^H \Psi_{p_{m,i}} + \Psi_{p_{m,i}}^2)), \\ i = 1,2,...,G \end{aligned} \tag{14}$$

and the connection weight is:

$$\begin{aligned} W_{i,j} = \sum_{p=1}^{P}\sum_{m=1}^{M} \gamma \Psi_{p_{m,i}}^H \Psi_{p_{m,j}}, \\ i = j = 1,2,...,G, i \neq j \end{aligned} \tag{15}$$

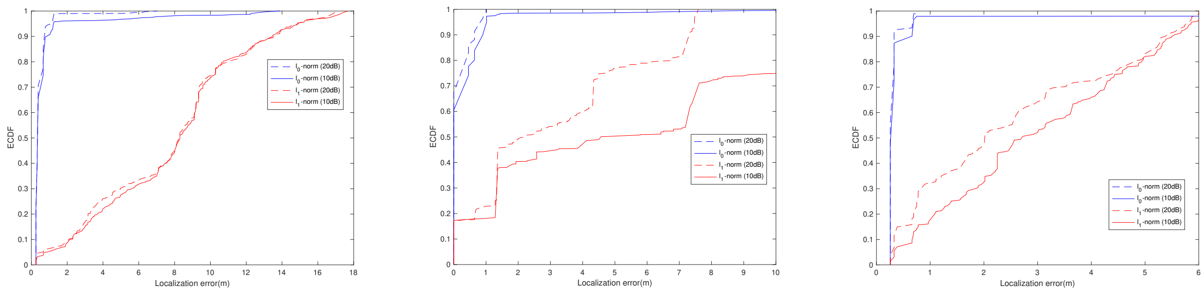
Although (14) is still NP-hard, it is a binary program and there exist efficient solvers for binary quadratic programs. In this paper, we use a Markov chain Monte-Carlo (MCMC) solver called the *Digital Annealer* (DA) [20]. DA is a computer architecture utilized to solve binary combinatorial problems that have an Ising energy formulation as their objective function. According to [20], the time-to-solution of DA for the fully connected spin-glass-problems has been about two times faster than the existing annealling and parallel tempering methods.

4. NUMERICAL RESULTS

In this section, we evaluate the performance of our proposed method. We utilize the WIM2 simulator [21] to generate the received signal in our scenarios. WIM2 simulator uses the WINNER II channel model, which is a geometry-based stochastic channel model (GSCM) that is parameterized for many scenarios including the ones with harsh multipath inference such as indoors and dense areas. In our simulations, this simulator provides 16 multipath clusters including LOS path.

In all of the scenarios we investigate here, we consider a $20m \times 20m$ area. A source is located randomly in this area. The carrier frequency is 5.25 GHz. Each access point is equipped with a uniform linear array (ULA) with 20 elements. We consider the following scenarios for this study:

- Triangular scenario: 3 access points are located at $[0m, 20m]$, $[20m, 15m]$, $[12m, 0m]$ and with orientation $[0, 0, \frac{2\pi}{3}]$, $[0, 0, \frac{5\pi}{4}]$, $[0, 0, \frac{5\pi}{3}]$ with respect to the global coordinate system (GCS);
- Rectangular scenario: 4 access points stations are located at $[0m, 20m]$, $[20m, 20m]$, $[20m, 0m]$, $[0m, 0m]$ and with the orientations $[0, 0, \frac{3\pi}{4}]$, $[0, 0, \frac{5\pi}{4}]$, $[0, 0, \frac{7\pi}{4}]$, $[0, 0, \frac{\pi}{4}]$ with respect GCS;
- Circular scenario: 6 access points form a circular pattern and are located at $[2m, 15m]$, $[10m, 20m]$, $[18m, 15m]$, $[18m, 5m]$, $[10m, 0m]$, and $[2m, 5m]$, with the orientations $[0, 0, \frac{2\pi}{3}]$, $[0, 0, \pi]$, $[0, 0, \frac{4\pi}{3}]$, $[0, 0, \frac{5\pi}{3}]$, $[0, 0, 0]$, $[0, 0, \frac{\pi}{3}]$ with respect to GCS.



(a) (b) (c)

Fig. 2: Comparison of the ECDF of the two methods for the (a) triangular scenario (b) rectangular scenario(c) circular scenario

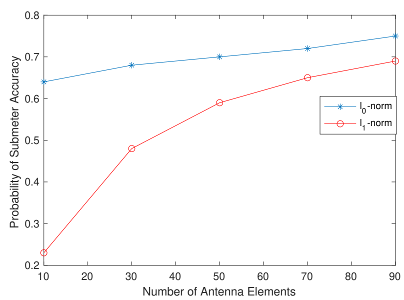


Fig. 3: Probability of sub-meter accuracy vs. number of antennas

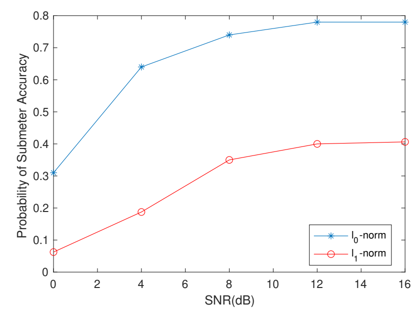


Fig. 4: Probability of sub-meter accuracy vs. SNR

In Fig. 2, we compare the empirical cumulative distribution function (ECDF) of the localization error of our method with [18] for different SNRs, (i.e. 10dB and 20dB) for all three scenarios (triangular, rectangular, and circular) with different number of access points. In [18], which is also based on compressed sending, the authors approximate ℓ_0-norm with an ℓ_1-norm relaxation to make it convex, and solve it using the second order cone programming (SOCP). However, we transform the ℓ_0-norm problem to an Ising energy problem by restricting the solution to be a binary vector. With this Ising transformation, the problem can be solved using DA. All of these experiments were conducted on the second generation of Digital Annealer environment. In the absence of DA, one can use off-the-shelf optimization tools (e.g. Gurobi, CPlex, etc) to solve the binary programming problem. Simulation results in Fig. 2 show that the binary approximation in our method performs significantly better than the ℓ_1-norm approximation in [18].

In Fig. 3, we compare the probability of sub-meter accuracy with respect to the number of antenna elements of our method and [18] for the rectangular scenario for SNR = 20dB. When the number of antenna elements of each access point increases, the angular resolution improves. Thus, in both methods the probability of sub-meter accuracy enhances while the number of antenna elements grows. However, as noticed, the proposed Ising model-based localization outperforms the ℓ_1-norm approximation.

In addition, in Fig. 4, the probability of sub-meter accuracy with respect to increase in SNR is compared for our proposed method and [18]. For both methods, the probability of sub-meter accuracy improves with the increase in SNR due to a decrease in the power of the noise. However, our proposed method outperforms [18] significantly in different SNRs.

5. CONCLUSION

This paper investigates the direct localization method in a multipath environment with an Ising energy model approach. It presents a method to co-process the received signal of all the access points to increase the accuracy of localization. The problem is formulated by the compressed sensing structure and is transformed to an Ising energy minimization problem. This NP-hard problem can be solved with Markov chain Monte-Carlo methods. Numerical results show that this method outperforms the existing methods in the literature.

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