SCALABLE DATA ASSOCIATION AND MULTI-TARGET TRACKING UNDER A POISSON MIXTURE MEASUREMENT PROCESS

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ABSTRACT

Measurement rates for both targets and clutter have been assumed to be known *a priori* in most existing tracking systems, whereas practically the rates may be unknown to users or time-varying. This paper therefore fills this gap by developing a Poisson mixture process tracker (PMPT) to capture the temporal characteristics of the rate parameters under a Poisson mixture measurement process. Specifically, the Generalized inverse Gaussian (GIG) distribution is proposed as a prior for Poisson rates, and two novel priors, the time independent GIG prior and the GIG Markov chain prior, are designed. In addition, a scalable inference framework is introduced to enable efficient data association and parallel updating of target states under a sequential Markov chain Monte Carlo (MCMC) scheme with linear complexity in the number of measurements and targets. Results show that our proposed method can provide a robust solution in highly dynamic detection probability environments.

Index Terms— data association, Generalized inverse Gaussian, scalability, sequential MCMC, multi-target tracking

1. INTRODUCTION

Data association is a challenging problem and considered intractable in large-scale multi-target tracking where traditional multi-target trackers, such as the joint probabilistic data association (JPDA) filter and the multiple hypothesis tracker (MHT), fail due to the requirement of exhaustive enumeration of all possible association hypotheses. To alleviate the computational demands of data association, various strategies, including the graphical models [1, 2], the sampling-based methods [3, 4], random finite set (RFS) [5, 6, 7], and the probabilistic MHT (PMHT) [8, 9], have been intensively studied. A non-homogeneous Poisson process (NHPP) measurement model has been proposed in [10, 11] to circumvent the computationally demanding data association step. A detailed discussion of PMHT and its relationship to the NHPP model can be found in [10, 12, 13, 14]. The method in [10] and the probability hypothesis density (PHD) filter both utilise a Poisson model, but the Bayesian recursion in [10] directly propagates the posterior density of the target state, while the PHD filter propagates the first-order moment of an RFS.

Most previous studies that adopted NHPP measurement models assume the Poisson rate either to be a known constant as in [10, 15] or as a function of the extended target state [16]. In practical use, the Poisson rates may be unknown to users and/or time-varying. Hence, the ability to estimate the underlying rates can be critical for robust tracking performance. Target measurement rate estimation has been considered in [17] for the extended target PHD filter with exponential forgetting for the prediction step. A similar prediction step has been adopted in [14] to estimate the NHPP model's target measurement rates using a batch expectation–maximization (EM) method. However, both of these algorithms assume a known constant clutter rate. Methods that solely estimate unknown clutter rates for the PHD filter can be found in [18].

This paper addresses both the data association and measurement rate estimation in multi-target tracking. Firstly, as an extension to the methods in [10], we develop a scalable data association and multi-target tracking method with linear complexity in the number of measurements and targets. The state estimation is readily parallelised under an online Gibbs sequential MCMC (SMCMC) scheme (see [19, 20, 21] and references therein), which has great potential in large-scale multitarget tracking. Compared to the methods in [15] that adopt a JPDA approximation scheme, our method theoretically converges to an optimal Bayesian filter with a sufficiently large MCMC sample size. Moreover, our method may have an advantage over RFS-based methods for extended target tracking as it does not require any extra measurement clustering and partitioning steps due to the relaxation of the one or zero measurement per target restriction. Secondly, this paper develops a multi-target Poisson mixture process tracker (PMPT), which considers the estimation of Poisson rates for both targets and clutter. Specifically, we propose a Generalized inverse Gaussian (GIG) prior model, expressed either as a time independent GIG prior or as a GIG Markov chain. This readily enables the proposed parallel tracker to capture the temporal characteristics of the rate parameters. Compared to the method in [14] that uses a heuristic Gamma random walk model and batch EM-based estimation, our GIG prior is estimated sequentially and is significantly more general: the three-parameter GIG is more flexible and known to be a better fit for highly dispersed detection data [22].

2. SCALABLE MULTI-TARGET TRACKER UNDER KNOWN POISSON RATES

Assuming that there are K objects moving independently, the overall state vector is defined as $X_n = [X_{1,n}, ..., X_{K,n}]^T$. Conditional on *known* Poisson rates $\Lambda_n = [\Lambda_{0,n}, ..., \Lambda_{K,n}]^T$, the measurement process is a NHPP from a superposition of measurements from the K targets and clutter ($i = 0$) with intensity $\lambda(Z_n|X_n) = \sum_{i=0}^{K} \lambda_i(Z_n|X_{i,n})$ [10]. More specifically, for each target at time step n, the intensity measure $m_{i,n}$ (i.e. the number of measurements arising from the i-th target) follows a Poisson distribution with rate $\Lambda_{i,n}$:

$$p(m_{i,n}|\Lambda_{i,n}) = \frac{\exp(-\Lambda_{i,n})(\Lambda_{i,n})^{m_{i,n}}}{m_{i,n}!} \tag{1}$$

The $\{m_{i,n}\}_{i=0}^{K}$ measurements of this NHPP are i.i.d. samples from a probability density function $p(Z_n|X_{i,n}) = \prod_{j=1}^{m_{i,n}} p(Z_{j,n}|X_{i,n})$, and each individual intensity is given by

$$\lambda_i(Z_{j,n}|X_{i,n}) = \Lambda_{i,n} p(Z_{j,n}|X_{i,n}) \tag{2}$$

2.1. Scalable Inference Scheme

The M point measurements received at time step n are denoted $Z_n = [Z_{1,n}, ..., Z_{M,n}]$. We define the data association as a random variable $\theta_n = [\theta_{1,n}, ..., \theta_{M,n}]$, with each component $\theta_{j,n} \in \{0, 1, ..., K\}$ ($\theta_{j,n} = 0$ indicating clutter) and $j \in \{1, \ldots, M\}$. In order to avoid expensive summations in the SMCMC method, we target the joint posterior distribution $p(X_{n-1:n}, \theta_n | Z_{0:n}, \Lambda_n)$ directly in the SMCMC at each time step n. Assume that the stationary distribution $p(X_{n-1}, \theta_{n-1} | Z_{0:n-1})$ at time step $n-1$ has been approximated empirically by a set of N_p unweighted samples from the converged chain $\{X_{n-1}^{(p)}, \theta_{n-1}^{(p)}\}_{p=1}^{N_p}$, the posterior distribution $p(X_{n-1:n}, \theta_n | \Lambda_n, Z_{0:n})$ can be approximated as:

$$p(X_{n-1:n}, \theta_n | Z_{0:n}, \Lambda_n) \propto p(Z_n | X_n, \theta_n) p(\theta_n | \Lambda_n, X_n) \times \sum_{i=1}^{N_p} p(X_n | X_{n-1}^{(i)}) \delta_{X_{n-1}^{(i)}}(X_{n-1}) \quad (3)$$

In order to sample from this updated distribution, an online Gibbs sampling SMCMC scheme is adopted here. In the sequel we present the full conditionals of θ_n, X_{n-1} and X_n. First, the conditional $p(\theta_n | X_{n-1:n}, Z_{0:n}, \Lambda_n)$ can be written as [15]:

$$p(\theta_n | X_{n-1:n}, Z_{0:n}, \Lambda_n) \propto \prod_{j=1}^{M} \Lambda_{\theta_{j,n}} p(Z_{j,n} | X_{\theta_{j,n},n}) \quad (4)$$

from which we can see that sampling of θ_n may be performed by independent parallel draws from the terms for $j = 1, ..., M$

$$p(\theta_{j,n} | Z_n, X_{n-1:n}, \Lambda_n) \propto \sum_{i=0}^{K} \Lambda_{i,n} p(Z_{j,n} | X_{i,n}) \delta_i(\theta_{j,n}) \quad (5)$$

where $p(Z_{j,n} | X_{i,n})$ is defined later, see (17).

Since the objects are moving independently, we can again sample $X_{i,n-1}$ and $X_{i,n}$ for every object $i = 1, ...K$ in parallel. The conditional $p(X_{i,n-1} | Z_{0:n}, X_{i,n}, \theta_n)$ for each object i is:

$$p(X_{i,n-1} | Z_{0:n}, X_{i,n}, \theta_n) \propto \sum_{p=1}^{N_p} p(X_{i,n} | X_{i,n-1}^{(p)}) \delta_{X_{i,n-1}^{(p)}}(X_{i,n-1}) \quad (6)$$

and conditional $p(X_{i,n} | Z_{0:n}, X_{i,n-1}, \theta_n)$ is given by:

$$p(X_{i,n} | Z_{0:n}, X_{i,0:n-1}, \theta_n) \propto p(\mathcal{Z}_{i,n} | X_{i,n}) p(X_{i,n} | X_{i,n-1}) \quad (7)$$

where the measurement set $\mathcal{Z}_{i,n}$ ($|\mathcal{Z}_{i,n}| = m_{i,n}$) includes the $m_{i,n}$ measurements $Z_{j,n}$ that satisfy $\theta_{j,n} = i$.

Finally, the online Gibbs sampling SMCMC scheme is summarised at time step n, iterated to convergence for $m = 0, 1, ..., N_{iter}$:

Step 1: Sample $\theta_{1,n}^{(m)}, ..., \theta_{M,n}^{(m)}$ from $p(\theta_{j,n} | Z_n, X_{n-1:n}^{(m-1)}, \Lambda_n)$ according to (5) in parallel;

Step 2: For every object $i = 1, ..., K$, sample $X_{i,n-1}^{(m)}$, $X_{i,n}^{(m)}$ from $p(X_{i,n-1} | Z_{0:n}, X_{i,n}^{(m-1)}, \theta_n^{(m)})$ and $p(X_{i,n} | Z_{0:n}, X_{i,n-1}^{(m)}, \theta_n^{(m)})$ in parallel.

3. SCALABLE MULTI-TARGET TRACKER UNDER UNKNOWN POISSON RATES

This section introduces the PMPT method which is a practical extension of the NHPP model in Section 2 where the true values of Poisson rates Λ_n for both targets and clutter are unknown and/or time-varying. We adopt a natural assumption that the Poisson rates of each object and the clutter are mutually independent, so that $p(\Lambda_{0:n}) = \prod_{i=0}^{K} p(\Lambda_{i,0:n})$. By treating $\Lambda_{i,n}$ as a random variable with a prior, or mixing density, $g(\Lambda_{i,n})$, the number of measurements from target i follows a Poisson mixture distribution defined as:

$$p(m_{i,n}) = \int_0^{\infty} p(m_{i,n} | \Lambda_{i,n}) g(\Lambda_{i,n}) d\Lambda_{i,n} \quad (8)$$

A common choice for the mixing density $g(\Lambda_{i,n})$ is Gamma distribution, which implies a Negative Binomial (NB) distributed measurements. We now incorporate a more flexible prior distribution, the GIG prior, for the rate parameters, which is conjugate to Poisson distribution and includes the Gamma distribution as a special case. The GIG distribution is a three-parameter distribution with density function:

$$\mathcal{GIG}(\Lambda; a, b, p) = \frac{(a/b)^{p/2}}{2\mathcal{K}_p(\sqrt{ab})} \Lambda^{p-1} \exp(-\frac{a}{2}\Lambda - \frac{b}{2\Lambda}) \quad (9)$$

where $\mathcal{K}_v(z)$ denotes the modified Bessel function of the second kind for order v and argument z. Note that the Gamma distribution $\mathcal{GIG}(\Lambda; \sqrt{2a}, 0, p)$, the inverse Gaussian $\mathcal{GIG}(\Lambda; a, b, -\frac{1}{2})$, and the inverse gamma $\mathcal{GIG}(\Lambda; 0, \sqrt{2a}, -p)$ may be obtained as special cases of the GIG (see [22]). Other mixing densities for the Poisson mixture process may be found in [23].

In the following, two priors for the Poisson rate, a time independent GIG prior and a GIG Markov chain prior, are proposed, and inference methods are described.

3.1. Poisson Rate with a Time Independent GIG Prior

Assume first that the Poisson rates are time independent, that is, $p(\Lambda_n | \Lambda_{0:n-1}) = \prod_{i=0}^{K} p(\Lambda_{i,n})$. In this case, each $\Lambda_{i,n}$ follows a time independent GIG prior $\mathcal{GIG}(\Lambda_{i,n}; a_i, b_i, p_i)$. By using (1) and (8), the number of measurements turns out to be Sichel distributed; compared to the Poisson distribution, it may be a better fit for overdispersed data [24].

Here we use an online Gibbs sampling scheme that iteratively samples Λ_n, θ_n, and $X_{n-1:n}$ with the objective distribution being $p(\Lambda_n, \theta_n, X_{n-1:n} | Z_{0:n})$. It can be shown that the full conditional of Λ_n is also a GIG distribution:

$$p(\Lambda_n | \theta_n, X_{n-1:n}, Z_{0:n}) \propto p(Z_n | \theta_n, X_n) p(\theta_n | \Lambda_n, X_n) p(\Lambda_n) = \prod_{i=0}^{K} \mathcal{GIG}(\Lambda_{i,n}; a_i + 2, b_i, m_{i,n} + p_i) \quad (10)$$

Conditional on Λ_n, $p(\theta_{j,n} | Z_n, X_{n-1:n}, \Lambda_n)$ for each measurement, $p(X_{i,n-1} | Z_{0:n}, X_{i,n}, \theta_n)$ and $p(X_{i,n} | Z_{0:n}, X_{i,n-1}, \theta_n)$ for each object i are equal to (5), (6) and (7), respectively and again sampling from (10), (5)-(7) in the Gibbs steps can all be processed in parallel.

A Rao-Blackwellisation (RB) scheme can be designed to improve the accuracy of the algorithm. By marginalising out the Poisson rate Λ_n, we only need to infer the joint probability density $p(\theta_n, X_{n-1:n} | Z_{0:n})$. Conditionals are computed for use in the Gibbs sampling steps. The conditional $p(\theta_{j,n} | \theta_{-j,n}, Z_{0:n}, X_{n-1:n})$ can be deduced as:

$$p(\theta_{j,n} | \theta_{-j,n}, X_{n-1:n}, Z_{0:n}) \propto \sum_{i=0}^{K} p(Z_{j,n} | X_{i,n}) \prod_{q=0}^{K} (\alpha_q \beta_q / 2)^{m_{q,n}} \times \mathcal{K}_{m_{q,n}+\gamma_q}(\alpha_q) \delta_i(\theta_{j,n}) \quad (11)$$

where shape parameters $\alpha_q = \sqrt{b_q(a_q+2)}$, $\beta_q = 2/(a_q+2)$ and $\gamma_q = p_q$. The conditional $p(X_{i,n-1}|Z_{0:n}, X_{i,n}, \theta_n)$ and conditional $p(X_{i,n}|Z_{0:n}, X_{i,n-1}, \theta_n)$ for each object i are equal to the equation (6) and (7), respectively. We note however that coupling effects mean that parallelisation will not be possible in the same way as for the non-RB versions.

3.2. Poisson Rate with a GIG Markov Chain Prior

For cases where the Poisson rate changes more gradually over time, we introduce a novel GIG first-order Markov chain as the prior for the Poisson rate, $p(\Lambda_n|\Lambda_{0:n-1}) = \prod_{i=0}^{K} p(\Lambda_{i,n}|\Lambda_{i,n-1})$; this is constructed such that the expectation of $\Lambda_{i,n}$ w.r.t. $p(\Lambda_{i,n}|\Lambda_{i,n-1})$ equals the previous time step's Poisson intensity $\Lambda_{i,n-1}$. Under this constraint, the transition density for $\Lambda_{i,n}$ is designed as:

$$p(\Lambda_{i,n}|\Lambda_{i,n-1}) = \mathcal{GIG}(\Lambda_{i,n}; \frac{r_c r_B}{\Lambda_{i,n-1}}, \frac{r_c \Lambda_{i,n-1}}{r_B}, p_i) \qquad (12)$$

where $r_B = \frac{\mathcal{K}_{p_i+1}(r_c)}{\mathcal{K}_{p_i}(r_c)}, r_c > 0$. When $p_i > 0.5$, $\frac{r_c+1}{r_c} < r_B < \frac{2p_i(r_c+1)}{r_c}$, and the double inequality is reversed for $0 < p_i < 0.5$, please see [25] for details.

Thus, our objective distribution is $p(\theta_n, X_{n-1:n}, \Lambda_{n-1:n}|Z_{0:n})$. Once again, a SMCMC with Gibbs sampling steps is adopted and conditionals for $\Lambda_{n-1}, \Lambda_n, X_{n-1}, X_n, \theta_n$ can be deduced. Specifically, conditional of Λ_{n-1} can be simplified to:

$$p(\Lambda_{n-1}|\theta_n, \Lambda_n, X_{0:n}, Z_{0:n}) \propto \sum_{i=1}^{N_p} p(\Lambda_n|\Lambda_{n-1}^{(i)})\delta_{\Lambda_{n-1}^{(i)}}(\Lambda_{n-1}) \qquad (13)$$

and the conditional of Λ_n is a GIG distribution in a form similar to (10) with parameters being:

$$\begin{aligned} & p(\Lambda_n|\Lambda_{n-1}, \theta_n, X_{n-1:n}, Z_{0:n}) \\ =& \prod_{i=0}^{K} \mathcal{GIG}(\Lambda_{i,n}; \frac{r_c r_B}{\Lambda_{i,n-1}} + 2, \frac{r_c \Lambda_{i,n-1}}{r_B}, m_{i,n} + p_i) \end{aligned} \qquad (14)$$

The remaining conditionals are of the same forms as before in (5)-(7), and therefore not repeated here.

4. RESULTS

4.1. System Model

For the d-th dimension in a spatial reference system, the dynamical model of kinematic state $X_i^d(t)$ for each object i is linear and Gaussian, modelled by a stochastic differential equation (SDE) [26]:

$$dX_i^d(t) = A_i X_i^d(t)dt + C_i dB_i^d(t) \qquad (15)$$

where $B_i^d(t)$ is a Brownian motion with covariance Q_w. The following continuous-time state-space model can be obtained by integrating (15) from t to $t+\tau$:

$$X_i^d(t+\tau) = F_i X_i^d(t) + \boldsymbol{w}_{i,t}^d, \quad \boldsymbol{w}_{i,t}^d \sim \mathcal{N}(0, P_{i,\tau}) \qquad (16)$$

with the transition matrix F_i and the dynamical noise covariance $P_{i,\tau}$ given by

$$F_i = e^{\tau A_i}, \quad P_{i,\tau} = e^{\tau A_i} P_0 e^{\tau A_i^T} + \int_0^\tau e^{tA_i} C_i Q_w C_i^T e^{tA_i^T}\, dt$$

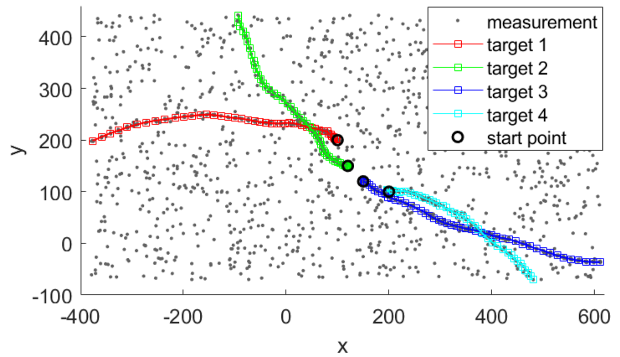

Fig. 1. Measurements, estimated tracks and ground-truth tracks of four targets; squares are mean estimates, and lines are ground truths

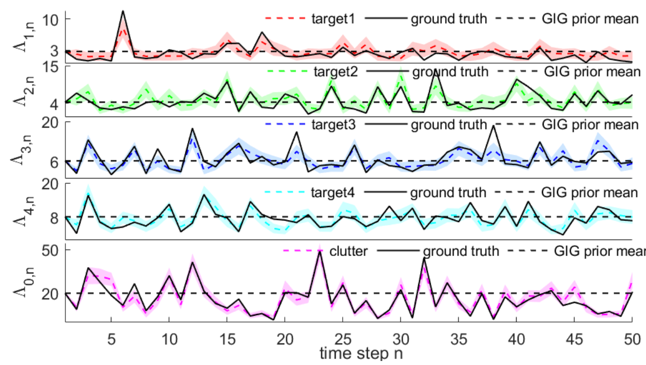

Fig. 2. Poisson rate of four objects and clutter over time; the shaded areas represent estimated Poisson rates $\pm 1\sigma$ (σ-standard deviation).

In simulations we adopt a nearly constant velocity (NCV) model with each $X_i^d(t) = [x_i^d(t), \dot{x}_i^d(t)]^T$, $A_i = [0\,1; 0\,0]$ and $C_i = [0; 1]$. Other dynamic models (e.g., [27, 28, 29]) can also be incorporated in our methods depending on the application scenarios.

In this paper the target shape is modeled via a Gaussian distribution. Each extended target originated measurement follows a linear and Gaussian model while the clutter measurement is uniformly distributed in the observation area of volume V, that is:

$$p(Z_{j,n}|X_{i,n}^d) = \begin{cases} \mathcal{N}(H_n X_{i,n}^d, Q_v), & i \neq 0; \quad \text{(object)} \\ \frac{1}{V}, & i = 0; \quad \text{(clutter)} \end{cases} \qquad (17)$$

where H_n is the observation mapping matrix, Q_v is the extended target's covariance, and the measurement error is assumed negligible compared to target extension.

4.2. Simulation

In the first simulation, four extended objects move independently in 2-D over 50 time steps. A known constant $Q_v = [1\ 0; 0\ 1]$ is assumed for the target's extent. The time interval is $\tau = 1$s. Trajectories are generated according to the models in Section 4.1. The simulated Poisson rates with a time independent GIG prior are shown in Fig 2. The GIG prior parameters are set to $a_{1:4} = 1$, $a_0 = 0.1$, $b_{1:4} = 1$, $b_0 = 10$, $\{p_i = i\}_{i=1}^4$ and $p_0 = 0.5$. For target rates, the GIG priors' means are around 2, 4, 6, 8; the clutter rate is overdispersed and a long tail property of GIG prior can be observed. Provided the time-varying Poisson rates, the measurement set is displayed in Fig. 1. The tracking performance and Poisson rate estima-

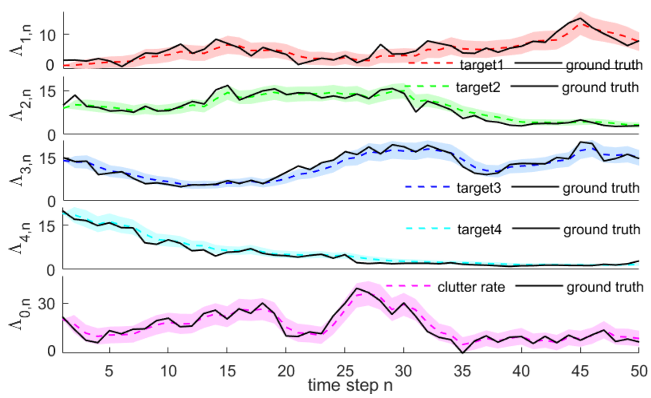

Fig. 3. Poisson rate of four objects and clutter over time;the shaded areas represent estimated Poisson rates $\pm 1\sigma$.

tion results from the online Gibbs sampler are shown in Fig. 1 and 2. A total of 150 samples is used with a burn-in time of 50 iterations per time step n. We can see that the measurement rates for both targets and clutter are well estimated and tracking performance remains good in a rapidly-changing detection environment with dense background clutter.

In the second simulation, trajectories of four extended targets are generated with the same setting as in the first simulation, except that the corresponding Poisson rates now follow a GIG Markov chain prior with parameters $r_c = 1$, $p_{0:4} = 30$. The true Poisson rates are displayed in Fig. 3. It can be seen that the Poisson rates change more gradually in comparison to the previous experiment. Each target related measurement rate is traversed at different scales so that the estimation performance can be assessed in different detection probability environments. The tracking performance is similar to Fig. 1 and thus we omit the estimation result due to limited space. From Fig. 3 we can see that our GIG Markov chain prior can well capture the time dependence of Poisson rates, and measurement rates are well estimated in both small rate and large rate scenarios.

4.3. Video Data

We also evaluate the proposed methods on the MOT Challenge benchmark datasets [30] in which the data were filmed using a static camera. We reduce the data to 2D by setting z coordinates to 0. The frame rate is 7 and a section of 60 time steps with 6 pedestrians is selected as the ground truth. Based on that, we additionally assume the measurement rates for both people and clutter follow the patterns shown in Fig. 4. The received measurements are displayed in Fig. 5. Here we only present the estimation result from using a GIG Markov chain prior as it is more suitable for the time dependent measurement rates shown in Fig. 4. The tracking performance and Poisson rate estimation results by using 200 samples with 50 burn-in time are shown in Fig. 4-5, and parameters for GIG prior are $r_c = 1$, $p_{0:6} = 5$. It can be observed that the Poisson rates for both clutter and targets are well estimated, and our method can react to the step changes of the measurement rates very responsively. With an accurate estimation of underlying Poisson rates, the tracking result is also satisfactory.

5. CONCLUSION

In this paper we develop a parallelisable Bayesian inference framework for data association and multi-target tracking where unknown and possibly time-varying Poisson rates (for targets and clutter) can

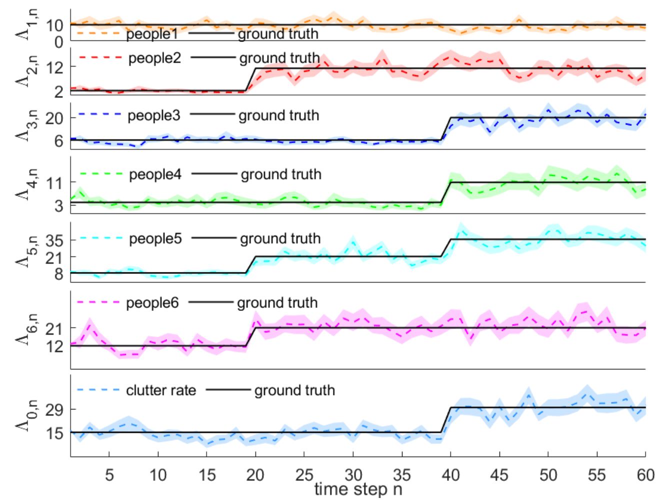

Fig. 4. Poisson rates of six people and clutter over time; shaded areas represent estimated Poisson rates $\pm 1\sigma$.

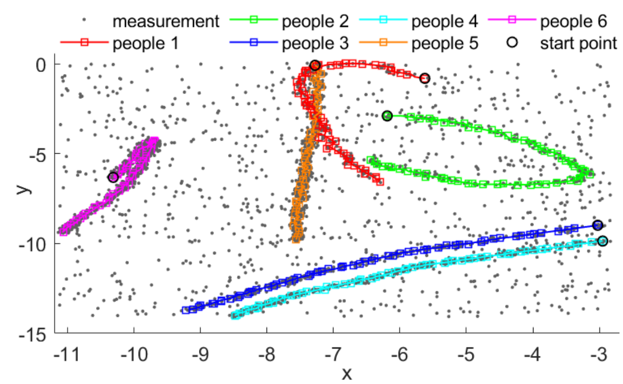

Fig. 5. Measurements, estimated tracks and ground truth tracks of six persons; squares are mean estimates, and lines are ground truths

be estimated in conjunction with the target kinematics and association variables. This relaxes the common (but restrictive) assumption of having to assign fixed Poisson rates to the trackers and hence sets us apart from conventional approaches. We have explored a powerful family of probability distributions, the Generalised inverse Gaussian (GIG) family, which offers a good fit for overdispersed data (compared to the Poisson). We demonstrate the effectiveness of the proposed methods using both simulated and real data. Future work will consider the cases with a varying number of targets. We will also extend our work to multiple group tracking with dynamical group interactions and/or intents.

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