JOINT BEAM SELECTION AND PRECODING BASED ON DIFFERENTIAL EVOLUTION FOR MILLIMETER-WAVE MASSIVE MIMO SYSTEMS

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ABSTRACT

Power consumption caused by radio frequency (RF) chains in millimeter-wave (mmWave) massive multiple-input multiple-output (MIMO) systems can be solved by beam selection. However, the spectral efficiency of traditional beam selection algorithms is unsatisfactory due to the reduction in the number of RF chains and the multiuser interference. This work proposes a differential evolution (DE)-based beam selection algorithm and an improved QR precoder to reduce power consumption and increase the performance of the systems. The proposed algorithm selects the optimal beams with DE-based beam selection for each user, which reduces the power consumption and the interference among each beam. In addition, to greatly decrease the multiuser interference, we propose an improved QR precoder by equalizing diagonals and using Tomlison-Harashima (TH) theory which can greatly reduce the computational complexity and improve the performance. The simulation results show that the proposed scheme outperforms some existing algorithms in the aspects of energy efficiency and spectral efficiency.

Index Terms— mmWave, beamspace, massive MIMO, beam selection, differential evolution

1. INTRODUCTION

Millimeter-wave (mmWave) massive multiple-input multiple-output (MIMO) can not only offer sufficiently large spectrum bandwidth and allow more antenna elements to be packaged in a given antenna aperture [1], but also substantially improve energy efficiency and spectral efficiency [2], [3]. Therefore, this technology is expected to meet the demand for 5G and beyond wireless communication systems [4]. Due to a large number of antenna elements, the number of required radio frequency (RF) chains becomes very large in mmWave massive MIMO systems. The RF chains can bring high hardware overhead and consume substantial energy, which are not affordable in realistic application. To surmount this intractable problem, beamspace MIMO (B-MIMO) is employed as a promising scheme to greatly decrease the number of RF chains. Because the mmWave channels are sparse in the beamspace [5], few beams are able to be appropriately chosen by beam selection, ultimately reducing the number of needed RF chains, hardware overhead and energy consumption [6].

Maximizing magnitude (MM)-based beam selection allows different users to share the same beam resources and causes the number of simultaneously active RF chains to vary with the channel and user topologies [7]. To reduce the interference of users that use the same beam, a maximizing signal-to-interference plus noise-ratio (SINR)-based (MS) incremental algorithm was proposed in [8]. Although the MS algorithm can improve the system performance, the massive complexity results in high time delay. In [9], interference-aware (IA) beam selection was proposed to maximize the sum rate for noninterference-user. Since one beam serves only one user for the IA scheme, the number of beams substantially increases with the number of users [10]. A beam selection algorithm using out-of-band spatial information was proposed in [11], which reduces the overhead of establishing a mmWave link. Moreover, the beamspace precoder determines the optimization of the beam selection algorithm, while the MM, MS and IA beam selection algorithms cannot significantly eliminate multiuser interference by employing a zero-forcing (ZF) precoder [12]. To further solve the multiuser interference problem, a joint beam selection and precoding scheme was proposed in [13]. This scheme uses QR decomposition-based beam selection to reduce the channel dimension, and a precoder is used to further eliminate the multiuser interference of the reduced-dimensional system. Although the QR-based precoder can obtain a higher achievable sum rate and power efficiency than the ZF precoder, users suffer from different effective channel gains and unfair user services that increase the bit error rate (BER). Therefore, it is vital to design a more efficient and fair joint solution for beam selection and precoding for mmWave massive MIMO systems.

In this paper, we propose a differential evolution (DE)-based beam selection and an improved QR precoder for mmWave massive MIMO systems. First, we propose to use DE to select a subset of beams for each user with finite numbers of iterations and obtain a near-optimal reduced-

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dimensional beamspace channel matrix. The proposed beam selection can reduce multiuser interference and improve the achievable sum rate with low complexity. Moreover, to optimize the fitness function of the proposed beam selection algorithm and improve the BER performance, we further propose an improved QR precoder that enables users to obtain the same effective channel gain by using equalizing diagonals and the Tomlison-Harashima (TH) precoding algorithm. Simulation results illustrate that the proposed scheme performs better than some recently proposed schemes in terms of achievable sum rate, energy efficiency and BER performance.

2. SYSTEM MODEL

We consider a downlink of a mmWave massive MIMO system, where the base station (BS) is equipped with N antennas and $N_{RF} = K$ RF chains and simultaneously serves K single-antenna users. The spacing among each element is $d = \lambda/2$ meters, where λ is the carrier wavelength [4], [9]. By using discrete Fourier transform (DFT), the conventional spatial channel can be converted to the beamspace. Then, the signal model of B-MIMO can be represented by

$$\mathbf{y} = \mathbf{H}_b^H \mathbf{P}_b \mathbf{s} + \mathbf{n}, \tag{1}$$

where $\mathbf{H}_b = \mathbf{UH} = [\mathbf{h}_{b,1}, \mathbf{h}_{b,2}, ..., \mathbf{h}_{b,K}]$ represents the beamspace channel matrix and $\mathbf{h}_{b,k}$ is the beamspace channel matrix of the kth user. $\mathbf{P}_b = \mathbf{UP}$ denotes the beamspace precoding matrix, where $\mathbf{P} \in \mathbb{C}^{N\times K}$ is the spatial precoding matrix satisfying $\mathrm{tr}\left(\mathbf{P}\mathbf{P}^{\mathrm{H}}\right) \leq \rho$, and ρ is the total transmit power. The spatial domain channel matrix of the system is $\mathbf{H} = [\mathbf{h}_1, \mathbf{h}_2, ..., \mathbf{h}_K]$, where the Saleh-Valenzuela channel model is employed and $\mathbf{h}_k = \sqrt{N/L+1}\sum_{l=0}^{L} \beta_k^{(l)} \mathbf{a}\left(\theta_k^l\right)$ [14]. Considering the critically sampled of N-element ULA antennas, the array steering vector can be represented as $\mathbf{a}(\theta) = \frac{1}{\sqrt{N}}[\exp(-j2\pi\theta i)]_{i\in\mathcal{I}(N)}$, where $\theta = 0.5\sin\phi$ is the spatial frequency corresponding to the physical angle of arrival ϕ, $\phi \in [-\pi/2\ , \pi/2\ ]$. The spatial DFT matrix $\mathbf{U}$ is expressed as $\mathbf{U} = \left[\mathbf{a}\left(\overline{\theta}_1\right), \mathbf{a}\left(\overline{\theta}_2\right), ..., \mathbf{a}\left(\overline{\theta}_N\right)\right]^H$, where $\overline{\theta}_n = \frac{1}{N}\left(n - \frac{N+1}{2}\right)$ for $n = 1, 2, ..., N$. $\mathbf{s} \in \mathbb{C}^{K\times 1}$ is the original signal vector satisfies $E\left(\mathbf{s}\mathbf{s}^H\right) = \mathbf{I}_K$, and $\mathbf{n} \sim \mathcal{CN}\left(0, \sigma^2\mathbf{I}_K\right)$ denotes the additive white Gaussian noise vector. N rows of $\mathbf{H}_b$ correspond to N orthogonal beams with space directions $\overline{\theta}_1, \overline{\theta}_2, ..., \overline{\theta}_N$, respectively. The reduced-dimensional B-MIMO system after beam selection can be given by $\widetilde{\mathbf{y}} = \widetilde{\mathbf{H}}_b^H \widetilde{\mathbf{P}}_b \mathbf{s} + \mathbf{n}$, where $\widetilde{\mathbf{H}}_b \in \mathbb{C}^{K\times K}$ is the reduced-dimensional beamspace channel matrix composed of K selected beams, and $\widetilde{\mathbf{P}}_b \in \mathbb{C}^{K\times K}$ is the reduced-dimensional digital precoding matrix.

3. THE PROPOSED JOINT BEAM SELECTION AND PRECODING SCHEME

The beamspace channel $\mathbf{H}_b$ is sparse, so that the number of dominant elements of $\mathbf{h}_{b,k} = \mathbf{U}\mathbf{h}_k$ is much less than N [15]. Therefore, we first select K beams out of N for the K users without suffering a considerable loss in the achievable sum-rate R_{sum}, which is defined as

$$R_{sum} = \sum\nolimits_{k=1}^{K} R_k = \sum\nolimits_{k=1}^{K} \log_2(1 + \mathit{SINR}_k), \tag{2}$$

where R_k is the data rate and the SINR for the kth user is

$$\mathit{SINR}_k = \frac{\rho \frac{|\alpha|^2}{K} \left|\mathbf{h}_k^H \mathbf{f}_k\right|^2}{\rho \frac{|\alpha|^2}{K} \sum_{m\neq k} \left|\mathbf{h}_k^H \mathbf{f}_m\right|^2 + \sigma^2}. \tag{3}$$

Specifically, we treat K users as K independent variables of a certain function, and the indices of the available beams are the range of values of the independent variables. Then, the optimization problem can be presented as

$$\begin{aligned} (Ib_1, Ib_2, ..., Ib_K)^{opt} &= \underset{Ib_1,...,Ib_K}{\arg\max}\ R_{sum} \\ &= \underset{Ib_1,...,Ib_K}{\arg\min}\ f\left(Ib_1, Ib_2, ..., Ib_K\right), \end{aligned} \tag{4}$$

where $Ib_k \in \mathcal{N}$ is the index value of the selected beam of the kth user, and the index set of available beams is denoted by $\mathcal{N} = \{1, ..., N\}$. $f(\cdot)$ is defined as a fitness function, which is utilized to evaluate the suitability of the individual (a possible beam subset). Due to the non-convex constraint $Ib_k \in \mathcal{N}$, the optimization problem (4) is non-convex. Inspired by DE [16], we propose a DE-based beam selection algorithm to obtain an approximately optimal reduced-dimensional beamspace channel matrix $\widetilde{\mathbf{H}}_b$.

The utility function is denoted by g_{nk}, which indicates the importance of the beam n for the kth user

$$\mathrm{g}_{nk} = \frac{|\mathbf{h}_{nk}|^2}{1 + I_{nk}}, \qquad k \in \mathcal{K},\ n \in \mathcal{N}, \tag{5}$$

where $\mathbf{h}_{nk}$ is the beamspace channel matrix of the nth beam for the kth user, $I_{nk} = \sum_{i\in\mathcal{K}\setminus k} |h_{ni}|^2$, and the set of users can be expressed as $\mathcal{K} = \{1, 2, ..., K\}$. The beam n_1 is more suitable for kth user than beam n_2, if $\mathrm{g}_{n_1k} > \mathrm{g}_{n_2k}$. To prevent the corresponding strongest beam from being occupied by other users, resulting in severe multiuser interference, we modify the mapping between Ib_k and indices of available beams as

$$Ib_k = \mathcal{B}_k\left(u_k\right),\ u_k \in \{1, ..., Card(\mathcal{B}_k)\}, \tag{6}$$

where $\mathcal{B}_k = \left\{i \in N : \mathrm{g}_{ik} \geq \gamma_k \max_i \mathrm{g}_{ik}\right\}$ is a set of the indices of few dominant beams for the kth user, $\gamma_k \in (0, 1)$ is the threshold that determines the number of feasible solutions, and u_k is the optimal individual for the kth user. Assume that the population size and the number of iterations are P_{size} and $T_{\max}$. Then, at the tth iteration, the ith individual is

$$U_i^t = \left(u_{i,1}^t, u_{i,2}^t, ..., u_{i,K}^t\right),\ i \in \{1, 2, ..., P_{size}\}, \tag{7}$$

where $u_{i,k}^t \in \{1, 2, ..., Card\left(\mathcal{B}_k\right)\}$ is a gene.

To enhance population diversity, we define the mutation mechanism as

$$V_i^{t+1} = U_{r1}^t + F \cdot \left(U_{r2}^t - U_{r3}^t\right), \tag{8}$$

where $V_i^{t+1} = \left(v_{i,1}^{t+1}, v_{i,2}^{t+1}, ..., v_{i,K}^{t+1}\right)$ is the ith transitional individual generated by the mutation operator; $r1, r2$ and $r3$ are random numbers chosen form the set $\{1, 2, ..., P_{size}\}$; $F \in (0, 1)$ is referred to contraction factor. To prevent transitional individuals from exceeding the search space, we define a boundary qualification criterion

$$v_{i,k}^{t+1} = \begin{cases} 1, & v_{i,k}^{t+1} < 1 \\ Card\left(\mathcal{B}_k\right), & v_{i,k}^{t+1} > Card\left(\mathcal{B}_k\right) \end{cases}. \tag{9}$$

Then, let current individual U_i^t and transitional individual V_i^{t+1} perform the crossover operation that further enhance the diversity of the population to generate alternative individual $W_i^{t+1} = \left(w_{i,1}^{t+1}, w_{i,2}^{t+1}, ..., w_{i,K}^{t+1}\right)$, in which

$$w_{i,k}^{t+1} = \begin{cases} v_{i,k}^{t+1}, & rand \leq CR \\ u_{i,k}^{t}, & others \end{cases}, \tag{10}$$

where $CR \in (0,1)$ is the cross-probability factor, which determines the probability that the transitional individual gene replaces the current individual gene; $rand$ is a random number chosen from the standard uniform distribution on interval $(0,1)$. Finally, a selection operation is defined to obtain the progeny population. The selection operation uses a greedy strategy, and the alternative individual W_i^{t+1} competes with the current individual U_i^t according to the fitness function $f(\cdot)$ value:

$$U_i^{t+1} = \begin{cases} W_i^{t+1}, & f\left(\mathcal{B}_1\left(w_{i,1}^{t+1}\right), ..., \mathcal{B}_k\left(w_{i,k}^{t+1}\right)\right) \\ & < f\left(\mathcal{B}_1\left(u_{i,1}^{t+1}\right), ..., \mathcal{B}_k\left(u_{i,k}^{t+1}\right)\right) \\ U_i^t, & others \end{cases}. \tag{11}$$

The proposed DE-based beam selection which is summarized in Algorithm 1 eliminates unfavorable mutations and gradually accumulates favorable mutations during the iteration, so that the genetic frequency of the population changes directionally that guarantees the individual to evolve in a certain direction (e.g., minimizing $f(\cdot)$). Thus, it can obtain the approximate global optimal solution by mutation, crossover and selection, and improve the achievable sum rate performance.

Although the beam selection reduces multiuser interference, precoding is widely used to further improve the system performance. When the channel $\widetilde{\mathbf{H}}_b$ is ill, the linear precoders degrade, and when $\widetilde{\mathbf{H}}_b$ is low rank, it cannot be used. To overcome the shortcomings of linear precoding, we further propose a nonlinear low-complexity improved QR precoding.

First, by using QR decomposition, we can obtain $\widetilde{\mathbf{H}}_b = \widetilde{\mathbf{Q}}\widetilde{\mathbf{R}}$, where $\widetilde{\mathbf{Q}} \in \mathbb{C}^{K \times K}$ is the unitary matrix and $\widetilde{\mathbf{R}} \in \mathbb{C}^{K \times K}$ is the upper triangular matrix. Then, we construct a matrix $\mathbf{\Lambda} \in \mathbb{C}^{K \times K}$ which is a diagonal matrix composed of the reciprocal of the diagonal elements of $\widetilde{\mathbf{R}}^H$. When $\widetilde{\mathbf{P}}_b = \alpha\widetilde{\mathbf{Q}}\mathbf{\Lambda}$, we can simplify system formula:

$$\widetilde{\mathbf{y}} = \alpha\widetilde{\mathbf{R}}^H\mathbf{\Lambda}\mathbf{s} + \mathbf{n} = \alpha\mathbf{L}\mathbf{s} + \mathbf{n}, \tag{12}$$

where $\mathbf{L} \in \mathbb{C}^{K \times K}$ is a lower triangular matrix consisting of diagonal elements equal to 1, α is a scale factor that makes $\mathrm{tr}\left(\widetilde{\mathbf{P}}_b^H\widetilde{\mathbf{P}}_b\right) \leq \rho$, which is defined as

$$\alpha = \sqrt{\rho/\mathrm{tr}\left(\mathbf{\Lambda}^H\mathbf{\Lambda}\right)} = \sqrt{\rho/\sum\nolimits_{k=1}^{K}\left|{}^{1}/{}_{r_{kk}}\right|^2}. \tag{13}$$

Additionally, we use TH precoding [17] at the transmitter and perform the same modulo operation for each user. Eq. (12) can be further simplified as $\widetilde{\mathbf{y}} = \alpha\mathbf{s} + \mathbf{n}$. Moreover, to reduce the computational complexity of DE-based beam selection, we define the fitness function as

$$f(\cdot) = \sum\nolimits_{k=1}^{K}\left|\frac{1}{r_{kk}}\right|^2. \tag{14}$$

If k exceeds rank of channel matrix $\widetilde{\mathbf{H}}_b$, let $r_{kk} = 10^{-3}$ to ensure that the individual corresponding to the full-rank channel matrix has stronger survivability than the individual corresponding to the low-rank channel matrix. The complexity of the above algorithm mainly depends on the calculation of g_{nk}, finding dominant beams for users and the calculation of $f(\cdot)$. The total complexity is $T_{max}P_{size}O\left(2K^3+2K\right)+O(4NK)$.

Algorithm 1 DE-based Beam Selection

Require: $\mathbf{H}_b$, γ_k, F, CR, $T_{\max}$
Ensure: $\widetilde{\mathbf{H}}_b$
  Phase I: Calculate g_{nk}, using Eq. (5)
  Phase II: Find $\mathcal{B}_k$
  Phase III: Evolving
  Initialize primary population U^0
  for $t = 0 \to T_{\max}$ **do**
    for $i = 1 \to P_{size}$ **do**
      Perform mutation to generate V_i^{t+1} by Eq. (8)
      for $k = 1 \to K$ **do**
        Perform boundary detection on $v_{i,k}^{t+1}$ by Eq. (9)
        Perform crossover to generate $w_{i,k}^{t+1}$ by Eq. (10)
      end for
      Calculate the fitness function $f(\cdot)$ value
      Perform selection operation to obtain U_i^{t+1} by Eq. (11)
    end for
  end for
  Find the individual U_j^{t+1} with the smallest fitness function value from U^{t+1}
  Calculate $Ib_1, Ib_2, ..., Ib_K$ corresponding to U_j^{t+1} by Eq. (6)
  $\widetilde{\mathbf{H}}_b = \mathbf{H}_b(m,:)$, $m \in \{Ib_1, Ib_2, ..., Ib_K\}$

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4. SIMULATION RESULTS

The proposed algorithm is compared with recently proposed QR-based, IA, MS and MM-2 schemes. One line-of-sight (LoS) component with $\beta_k^{(0)} \sim \mathcal{CN}(0,1)$ and two NLoS components with $\beta_k^{(l)} \sim \mathcal{CN}(0, 0.01)$, $l = 1,2$ are presented. The spatial frequencies $\theta_k^{(l)} (l = 0,1,2)$ are uniformly distributed in $[-1/2, 1/2]$ and independent of each other. The results of [18] showed that $T_{\max} = 1000$, $P_{size} = 10K$ and $\gamma_k = 0.04$ were the optimal choices, but for different functions, the values of F and CR were different.

We first study the optimal individual fitness value against the number of iterations. According to [19], we take three individual fitness values, and the values of F and CR are both within the range of $[0,1]$. It is clear from Fig. 1 that the fitness value of the optimal individual decreases as the number of iterations increases. This means that the population evolves toward minimizing the fitness function $f(\cdot)$ (i.e., maximizing the achievable sum rate R_{sum}). In addition, the proposed algorithm has a larger evolution rate at the contraction factor

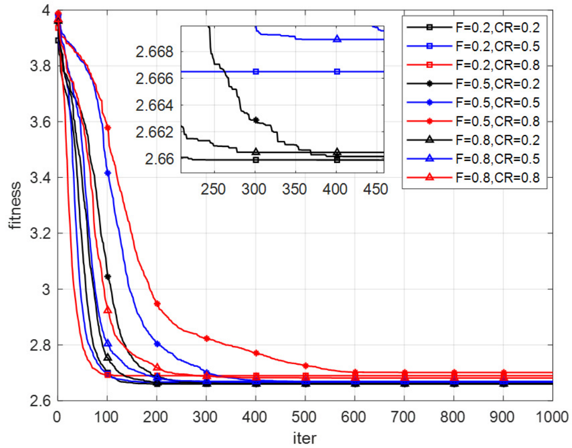

Fig. 1. Fitness comparison against the number of iterations.

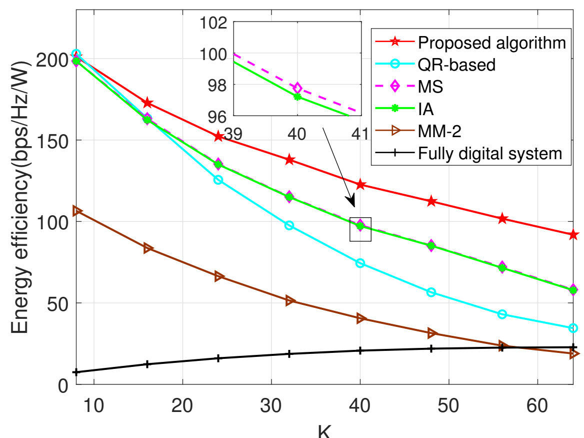

Fig. 2. Energy efficiency comparison against K for $N\!=\!256$ and SNR=20 dB.

$F = 0.2$ and achieves a smaller fitness value at the probability factor $CR = 0.2$. Therefore, in the following experiments, we choose $F = 0.2, CR = 0.2$.

Secondly, we study the energy efficiency against the number of users under the configuration N=256 and SNR=20 dB. The energy efficiency is defined as [13]

$$\zeta = \frac{R_{sum}}{\rho + N_{RF} P_{RF}}\,, \tag{15}$$

where P_{RF} denotes the energy consumed per RF chain. In this paper, the practical values $\rho = 32$ mW and $P_{RF} = 34.4$ mW are adopted [7], [9], [13]. In Fig. 2, we observe that the proposed scheme improves the energy efficiency significantly compared with other algorithms. With the increase of K, the energy efficiency gap between the proposed algorithm and other algorithms increases. Furthermore, the energy efficiency of the fully digital system is very low due to the large number of corresponding RF chains and the high energy consumption of each RF chain.

The achievable sum rate and BER against SNR are showed in Fig. 3. It is evident from Fig. 3(a) that although the achievable sum rate of all schemes improves as the SNR increases, the proposed scheme is close to that of the fully digital scheme

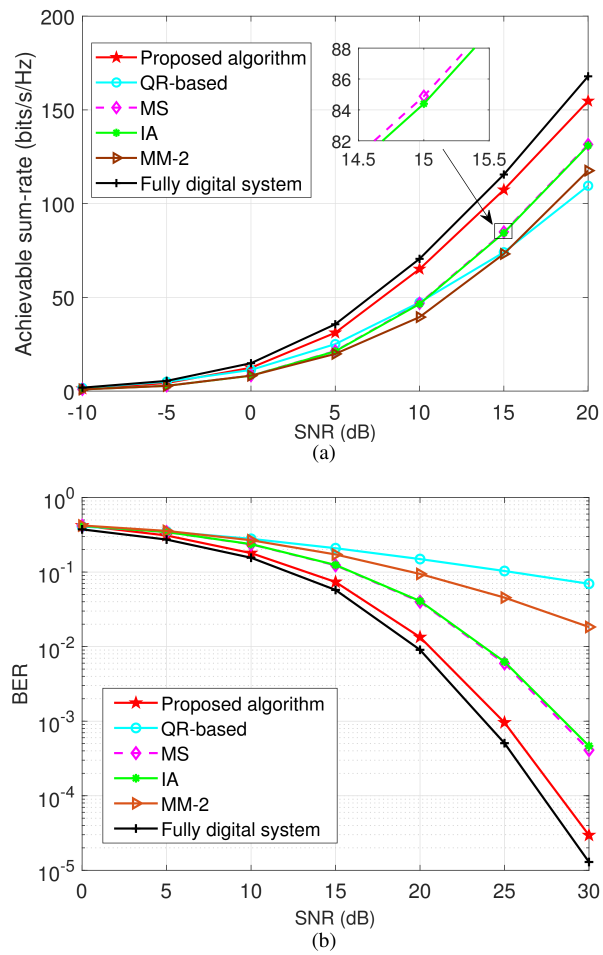

Fig. 3. Performance comparison against SNR for $N\!=\!256$ and $K\!=\!32$. (a) Achievable sum rate; (b) BER.

and significantly higher than that of other schemes. From Fig. 3(b), we observe that the BER performance of the proposed scheme surpasses that of MS, IA, MM-2 and QR-based schemes. The proposed algorithm requires an SNR of 24.9 dB to achieve a BER of 10^{-3}, while the MS and IA beam selection require SNR values of 28.3 dB and 28.5 dB, respectively, and the MM-2 and QR-based beam selection methods cannot even achieve a BER of 10^{-3}.

5. CONCLUSION

We propose a joint DE-based beam selection and improved QR precoding algorithm for millimeter-wave massive MIMO systems. We regard beam selection as an optimization problem, and leverage the DE algorithm to seek out the optimal beam for each user, which improves the sum rate performance. Meanwhile, aiming at the poor BER problem due to different channel gains, we further propose an improved QR precoder by equalizing diagonals and using TH theory. Through the simulation, it can be proven that the proposed method is better than some other methods in the aspects of achievable sum rate, energy efficiency and BER.

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