Notes on Nuclear and Particle Physics

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Part I
Nuclear Physics

1 Nucleon Diffusion

Assume an infinite square well potential, now with positive energy inside. Assume also $l = 0$ (valid for low energies). Given a impact parameter b we see

$$L \propto mvb \Rightarrow \frac{mvb}{\hbar} \text{ integer}$$

given the quantization of angular momentum. So, for $mvb << \hbar$, $l = 0$ is a reasonable assumption. Solving Schrodinger equations, for the radial part, we get

$$u\left(r\right) = \begin{cases} A \sin kr & r < R \quad k = \sqrt{\frac{2m(E+V_0)}{\hbar^2}} \\ \rho \sin\left(kr + \delta\right) & r > R \quad k = \sqrt{\frac{2mE}{\hbar^2}} \end{cases}.$$

Setting boundary conditions we get

$$k \cot\left(kR + \delta\right) = k_1 \cot k_1 R \tag{1}$$

and given E, V and R we may solve for δ.
Now consider an incident plane wave

$$\psi(z,t)=\mathrm{B}e^{i(kz-\omega t)} = e^{-i\omega t}\psi_{\text{inc}}\left(z\right)$$

$$\psi_{\text{inc}} = Be^{ikz} = B\sum_{l=0}^{\infty} i^l \left(2l+1\right) j_l\left(kr\right) P_l\left(\cos\theta\right).$$

For low energies, the $l = 0$ term dominates so

$$\psi_{\text{inc}} = B\frac{\sin kr}{kr} = \frac{B}{2ik}\left(\frac{e^{ikr}}{r} - \frac{e^{-ikr}}{r}\right)$$

where the positive and negative exponential correspond to outgoing and ongoing waves, respectively. Therefore, if we have elastic diffusion we get, in general (for a potential that goes to zero at infinity), the interaction changes the wave function only by a phase factor.

$$\psi(r)=\frac{B}{2ik}\left(\frac{e^{i(kr+\beta)}}{r}-\frac{e^{-ikr}}{r}\right)$$

is the solution for $r > R$, see

$$\psi(r)=\frac{C}{r}\sin\left(kr+\delta\right)=\frac{C}{2i}e^{-i\delta}\left(\frac{e^{i(kr+2\delta)}}{r}-\frac{e^{-ikr}}{r}\right)$$

identifying $\beta = 2\delta$ and $B = Cke^{-i\delta}$.

The diffused wave function is

$$\psi_{\text{dif}}=\psi-\psi_{\text{inc}}=\frac{B}{2ik}\frac{e^{ikr}}{r}\left(e^{2i\delta}-1\right).$$

To a positive δ parameter corresponds an attractive force, and repulsive force for negative δ. Calculating σ, we consider

$$j_{\text{inc}}=\frac{\hbar}{2mi}\left(\psi^*_{\text{inc}}\frac{\partial}{\partial z}\psi_{\text{inc}}-\frac{\partial}{\partial z}\psi^*_{\text{inc}}\psi_{\text{inc}}\right)=\frac{\hbar k}{m}\left|B\right|^2$$

$$j_{\text{dif}}=\frac{\hbar}{2mi}\left(\psi^*_{\text{dif}}\frac{\partial}{\partial z}\psi_{\text{dif}}-\frac{\partial}{\partial z}\psi^*_{\text{dif}}\psi_{\text{dif}}\right)=\frac{\hbar}{mkr^2}\left|B\right|^2\sin^2\delta$$

so

$$d\sigma=\frac{j_{\text{dif}}r^2d\Omega}{j_{\text{inc}}}\Rightarrow\frac{d\sigma}{d\Omega}=\frac{\sin^2\delta}{k^2}$$

and

$$\sigma_{\text{total}}=4\pi\frac{\sin^2\delta}{k^2}.$$

From equation 1 we get

$$\sin^2\delta=\frac{\left(\cos kR+\frac{\alpha}{k}\sin kR\right)^2}{1+\left(\frac{\alpha}{k}\right)^2}$$

where $\alpha = -k_1 \cot k_1 R$. Notice that $\sigma = 0$ for

$$\frac{k_1}{k}=\cot kR\tan k_1R\Rightarrow k_1\cot k_1R=k\cot kR.$$

For deuterium, $V_0 = 3.5$ MeV. Suppose $E \leq 10$ MeV, then $k_1 \approx 0.92\ \text{fm}^{-1}$ and $k \approx 0.016\ \text{fm}^{-1}$. Given that $R \approx 2$ fm we get, for low energies

$$\sigma=\frac{4\pi}{\alpha^2}\left(1+kR\right)^2=4.6\,\text{b}$$

to which we must add the parameters of the triplet $s = 1$ to match experimental data

$$\sigma^{\text{np}}=\frac{3}{4}\sigma^{\text{np}}_t+\frac{1}{4}\sigma^{\text{np}}_s.$$

For diffusion nn and pp, as the particles are identical, the corresponding wave function must be anti-symmetric (fermions) such that for $l = 0$ we have only diffusionof the singlet $s = 0$. Subtracting the effect of Coulomb interaction in pp, one gets

$$\sigma_s^{\mathrm{nn}} = \sigma_s^{\mathrm{pp}}$$

which implies the nuclear force is symmetric for intections nn and pp. Comparing with σ_s^{np}, we see that it is only slightly larger than σ_s^{pp}, so the nuclear force is approximately independent of the charge of the particles (for low energies).