Nuclear and Particle Physics - Solutions

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1 Nuclear Properties

1. The De Broglie wavelenght is
$$\lambda = \frac{h}{p}$$
so ones needs only determine an expression for the momentum. Let $E << mc^2$, then the electron is not relativistic, therefore
$$E = \frac{p^2}{2m} \Leftrightarrow p = \sqrt{2mE} \Rightarrow \lambda = \frac{h}{\sqrt{2mE}}$$
Let $E >> mc^2$, then the electro is ultra-relativistic, so
$$E^2 = m^2c^4 + p^2c^2 \Rightarrow E \approx pc \Rightarrow \lambda = \frac{hc}{E}$$

2. Problem 2
   (a) The cross section is given by the relation, for the Coulomb potential
$$\sigma\left(\theta\right) = \left(\frac{\mathcal{V}m}{2\pi\hbar^2}|V_{fi}|\right)^2 = \left(\frac{b}{E}\right)^2 \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$
   (b)
$$\sigma\left(\theta > \theta_0\right) = \left(\frac{b}{E}\right)^2 2\pi \int_{\theta_0}^{\pi} \frac{\sin\theta}{\sin^4\left(\frac{\theta}{2}\right)} d\theta = \left(\frac{b}{E}\right)^2 8\pi \int_{\theta_0}^{\pi} \frac{\cos\left(\frac{\theta}{2}\right)}{\sin^3\left(\frac{\theta}{2}\right)} d\left(\frac{\theta}{2}\right)$$
$$= \left(\frac{b}{E}\right)^2 8\pi \int_{\theta_0}^{\pi} \frac{d\left(\sin\left(\frac{\theta}{2}\right)\right)}{\sin^3\left(\frac{\theta}{2}\right)} = \left(\frac{b}{E}\right)^2 4\pi \left[\frac{1}{\sin^2\left(\frac{\theta}{2}\right)}\right]_{\pi}^{\theta_0}$$

3. Taking the average

$$< r^2 >= \int_{\mathcal{V}} \rho r^2 dV = 4\pi \int_0^R \rho_0 r^4 dr = \frac{4}{5}\pi\rho_0 R^5$$

For the nuclear form factor

$$F(\vec{q}) = \frac{1}{Ze}\int_{\mathcal{V}} \rho(\vec{r}) e^{i\langle\vec{q},\vec{r}\rangle} d^3r$$

noticing that $Ze = \frac{3}{4}\pi R^3$, one gets

$$F(q) = \frac{8\rho_0}{3R^3}\int_0^R drr^2 \int_0^\pi d\theta e^{iqr\cos\theta}\sin\theta = \frac{8\rho_0}{3R^3 q} i \int_0^R dr\, r\left[e^{iqr} - e^{-iqr}\right] = \frac{8\rho_0}{3R^3 q} i\left[I_+(R) - I_+(0) - I_-(R) + I_-(0)\right]$$

where the integral is solved by parts

$$I_\pm = \int xe^{\pm i\alpha x}dx = \frac{\pm}{i\alpha}\left(xe^{\pm i\alpha x} - \int e^{\pm i\alpha x}dx\right) = \frac{\pm}{i\alpha}\left(xe^{\pm i\alpha x} \mp \frac{1}{i\alpha}e^{\pm i\alpha x}\right)$$

and the result is

$$F(q) = \frac{8\rho_0}{3R^3q^2}\left[R\left(e^{iqR} - e^{-iqR}\right) - \frac{1}{iq}\left(e^{iqR} + e^{-iqR}\right)\right] = \frac{16\rho_0}{3R^3q^3}\left[qR\sin(qR) + \cos(qR)\right]$$

4. Derived in class.

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6. Problem 6

   (a) The density is given by

   $$\rho = \frac{M}{V} = \frac{3Am_p}{4\pi R^3} = \frac{3m_p}{4\pi r_0^3}$$

   (b) Solving for r_0 we get

   $$\frac{R}{r_0} = A^{\frac{1}{3}} \Rightarrow \frac{A}{V} = \frac{3}{4\pi r_0^3}$$

   (c) Substitute parameters.

7. The electric field is given by

$$E = \frac{Ze}{4\pi\epsilon_0}\frac{1}{r^2}$$

so one needs only substitute parameters.

8. We have the reactions

$$^{40}_{19}K \longrightarrow ^{40}_{20}Ca$$

$$n \longrightarrow p + e^- + \overline{\nu}_e$$

and

$$^{40}_{21}Sc \longrightarrow ^{40}_{20}Ca$$

$$p \longrightarrow n + e^+ + \nu_e$$

Adding the equations we see that

$$M\left(^{40}_{19}K\right) - M\left(^{40}_{20}Ca\right) = 1.310\,\text{MeV}$$

$$M\left(^{40}_{21}Sc\right) - M\left(^{40}_{20}Ca\right) = 12930 + 2m_e$$

and therefore

$$M\left(^{40}_{21}Sc\right)\text{-}\ M\left(^{40}_{19}K\right) = 2\left(M_H - m_n\right) + \Delta E_e = 13.952 - 1.310$$

9. From the Weizacker formula

$$E = a_V A - a_S A^{\frac{2}{3}} - a_C \frac{Z\left(Z-1\right)}{A^{\frac{1}{3}}} - a_A \frac{\left(A-2Z\right)^2}{A} \pm \delta\left(A, Z\right).$$

   (a) One needs only substitute $A = 51$ and $Z = 23$ to get the bonding energy. For energy per per nucleon devide by A.
   (b) The energy per proton is: $B(A,Z)$-B$(A-1,Z-1)$.
   (c) The energy per neutron is: $B(A,Z)$-B$(A-1,Z)$.

10. We aim to calculate de energy difference

$$\Delta E = B(A,Z)\text{-}2B\left(\frac{A}{2},\frac{Z}{2}\right) < 0.$$