

Nuclear and Particle Physics - Solutions

March 2, 2020

1 Nuclear Properties

1. The De Broglie wavelength is

$$\lambda = \frac{h}{p}$$

so one needs only determine an expression for the momentum. Let $E \ll mc^2$, then the electron is not relativistic, therefore

$$E = \frac{p^2}{2m} \Leftrightarrow p = \sqrt{2mE} \Rightarrow \lambda = \frac{h}{\sqrt{2mE}}$$

Let $E \gg mc^2$, then the electron is ultra-relativistic, so

$$E^2 = m^2c^4 + p^2c^2 \Rightarrow E \approx pc \Rightarrow \lambda = \frac{hc}{E}$$

2. Problem 2

- (a) The cross section is given by the relation, for the Coulomb potential

$$\sigma(\theta) = \left(\frac{\mathcal{V}m}{2\pi\hbar^2} |V_{fi}| \right)^2 = \left(\frac{b}{E} \right)^2 \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

- (b)

$$\begin{aligned} \sigma(\theta > \theta_0) &= \left(\frac{b}{E} \right)^2 2\pi \int_{\theta_0}^{\pi} \frac{\sin\theta}{\sin^4\left(\frac{\theta}{2}\right)} d\theta = \left(\frac{b}{E} \right)^2 8\pi \int_{\theta_0}^{\pi} \frac{\cos\left(\frac{\theta}{2}\right)}{\sin^3\left(\frac{\theta}{2}\right)} d\left(\frac{\theta}{2}\right) \\ &= \left(\frac{b}{E} \right)^2 8\pi \int_{\theta_0}^{\pi} \frac{d\left(\sin\left(\frac{\theta}{2}\right)\right)}{\sin^3\left(\frac{\theta}{2}\right)} = \left(\frac{b}{E} \right)^2 4\pi \left[\frac{1}{\sin^2\left(\frac{\theta}{2}\right)} \right]_{\pi}^{\theta_0} \end{aligned}$$

3. Taking the average

$$\langle r^2 \rangle = \int_V \rho r^2 dV = 4\pi \int_0^R \rho_0 r^4 dr = \frac{4}{5} \pi \rho_0 R^5$$

For the nuclear form factor

$$F(\vec{q}) = \frac{1}{Ze} \int_V \rho(\vec{r}) e^{i\langle \vec{q}, \vec{r} \rangle} d^3r$$

noticing that $Ze = \frac{3}{4}\pi R^3$, one gets

$$F(q) = \frac{8\rho_0}{3R^3} \int_0^R dr r^2 \int_0^\pi d\theta e^{iqr \cos \theta} \sin \theta = \frac{8\rho_0}{3R^3 q} i \int_0^R dr r [e^{iqr} - e^{-iqr}] = \frac{8\rho_0}{3R^3 q} i [I_+(R) - I_+(0) - I_-(R) + I_-(0)]$$

where the integral is solved by parts

$$I_\pm = \int x e^{\pm i\alpha x} dx = \frac{\pm}{i\alpha} \left(x e^{\pm i\alpha x} - \int e^{\pm i\alpha x} dx \right) = \frac{\pm}{i\alpha} \left(x e^{\pm i\alpha x} \mp \frac{1}{i\alpha} e^{\pm i\alpha x} \right)$$

and the result is

$$F(q) = \frac{8\rho_0}{3R^3 q^2} \left[R (e^{iqR} - e^{-iqR}) - \frac{1}{iq} (e^{iqR} + e^{-iqR}) \right] = \frac{16\rho_0}{3R^3 q^3} [qR \sin(qR) + \cos(qR)]$$

4. Derived in class.

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6. Problem 6

(a) The density is given by

$$\rho = \frac{M}{V} = \frac{3Am_p}{4\pi R^3} = \frac{3m_p}{4\pi r_0^3}$$

(b) Solving for r_0 we get

$$\frac{R}{r_0} = A^{\frac{1}{3}} \Rightarrow \frac{A}{V} = \frac{3}{4\pi r_0^3}$$

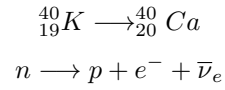
(c) Substitute parameters.

7. The electric field is given by

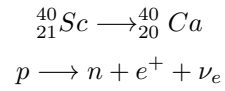
$$E = \frac{Ze}{4\pi\epsilon_0} \frac{1}{r^2}$$

so one needs only substitute parameters.

8. We have the reactions



and



Adding the equations we see that

$$\begin{aligned} M({}^{40}_{19}\text{K}) - M({}^{40}_{20}\text{Ca}) &= 1.310 \text{ MeV} \\ M({}^{40}_{21}\text{Sc}) - M({}^{40}_{20}\text{Ca}) &= 12930 + 2m_e \end{aligned}$$

and therefore

$$M({}^{40}_{21}\text{Sc}) - M({}^{40}_{19}\text{K}) = 2(M_H - m_n) + \Delta E_e = 13.952 - 1.310$$

9. From the Weizacker formula

$$E = a_V A - a_S A^{\frac{2}{3}} - a_C \frac{Z(Z-1)}{A^{\frac{1}{3}}} - a_A \frac{(A-2Z)^2}{A} \pm \delta(A, Z).$$

- (a) One needs only substitute $A = 51$ and $Z = 23$ to get the bonding energy. For energy per nucleon divide by A .
- (b) The energy per proton is: $B(A, Z) - B(A-1, Z-1)$.
- (c) The energy per neutron is: $B(A, Z) - B(A-1, Z)$.

10. We aim to calculate the energy difference

$$\Delta E = B(A, Z) - 2B\left(\frac{A}{2}, \frac{Z}{2}\right) < 0.$$