fish model

WHAM Model Components — Annotated

This document summarizes and annotates the **four key WHAM model components**:
1. **Numbers at Age (NAA)**
2. **Natural Mortality (M)**
3. **Selectivity (Sel)**
4. **Environmental Covariates (Ecov)**

1. Numbers at Age (NAA)

Equation (state update, log-scale):

$$\log N_{a,y} = \begin{cases} \log f(SSB_{y-1}) + \varepsilon_{1,y}, & a = 1 \\ \log N_{a-1,y-1} - Z_{a-1,y-1} + \varepsilon_{a,y}, & 1 < a < A \\ \log\left(N_{A-1,y-1}e^{-Z_{A-1,y-1}} + N_{A,y-1}e^{-Z_{A,y-1}}\right) + \varepsilon_{A,y}, & a = A \end{cases}$$

Model types:

- **NAA-1:** only recruitment deviations, IID normal.
  → Simplest; only new recruits are noisy.
- **NAA-2:** recruitment deviations AR(1) in time.
  → Recruitment good/bad years persist.
- **NAA-3:** full state-space, IID deviations at all ages.
  → Each age/year gets independent noise.
- **NAA-4:** 2D AR(1) across age × year.
  → Smooth correlated deviations, most realistic.

2. Natural Mortality (M)

General form:

$$\log M_{a,y} = \mu_{M_a} + \delta_{a,y}$$

- μ_{M_a}: mean mortality (can be age-specific, grouped, or Lorenzen curve).
- $\delta_{a,y}$: deviation (random effect).

Model types:

- **M-1:** fixed M, no random effects.
- **M-2:** IID deviations, $\delta_{a,y} \sim N(0, \sigma_M^2)$.
- **M-3:** 2D AR(1) across age × year.
  → Captures smooth variation by age and year.

3. Selectivity (Sel)

Assume **separability**:

$$F_{a,y} = F_y \cdot s_a$$

- F_y: fully selected fishing mortality in year y.
- s_a: selectivity at age a ($0 < s_a < 1$).

Parametric forms: logistic, double-logistic, decreasing-logistic.
Non-parametric option: estimate each s_a individually.

Random effect deviations on selectivity parameters ($\zeta_{p,y}$):

- **Sel-1:** fixed selectivity (no random effects).
- **Sel-2:** IID deviations (noise, uncorrelated).
- **Sel-3:** 2D AR(1) deviations (parameter × year correlation).

4. Environmental Covariates (Ecov)

Latent process X_y modeled via state-space; linked to population with lag ψ:

- $\psi = 1$: X_{y-1} affects recruitment in year y.
- $\psi = 0$: X_y affects natural mortality in year y.

Process models:

- **Random Walk:**

  $$X_{y+1} | X_y \sim N(X_y, \sigma_X^2)$$

  → Non-stationary, uncertainty grows.
- **AR(1):**

  $$X_y \sim N(\mu_X(1 - \phi_X) + \phi_X X_{y-1}, \sigma_X^2), \ |\phi_X| < 1$$

  → Stationary, mean-reverting; preferred for projections.

Observation model:

$$x_y | X_y \sim N(X_y, \sigma_x^2)$$

Cheat Sheet

- **NAA:** noise in abundance (from only recruits $\rightarrow$ all ages, IID vs correlated).
- **M:** natural mortality fixed $\rightarrow$ IID noise $\rightarrow$ correlated by age & year.
- **Sel:** fixed $\rightarrow$ IID random effects $\rightarrow$ correlated deviations.
- **Ecov:** external drivers, modeled as Random Walk or AR(1), linked by lag ψ.