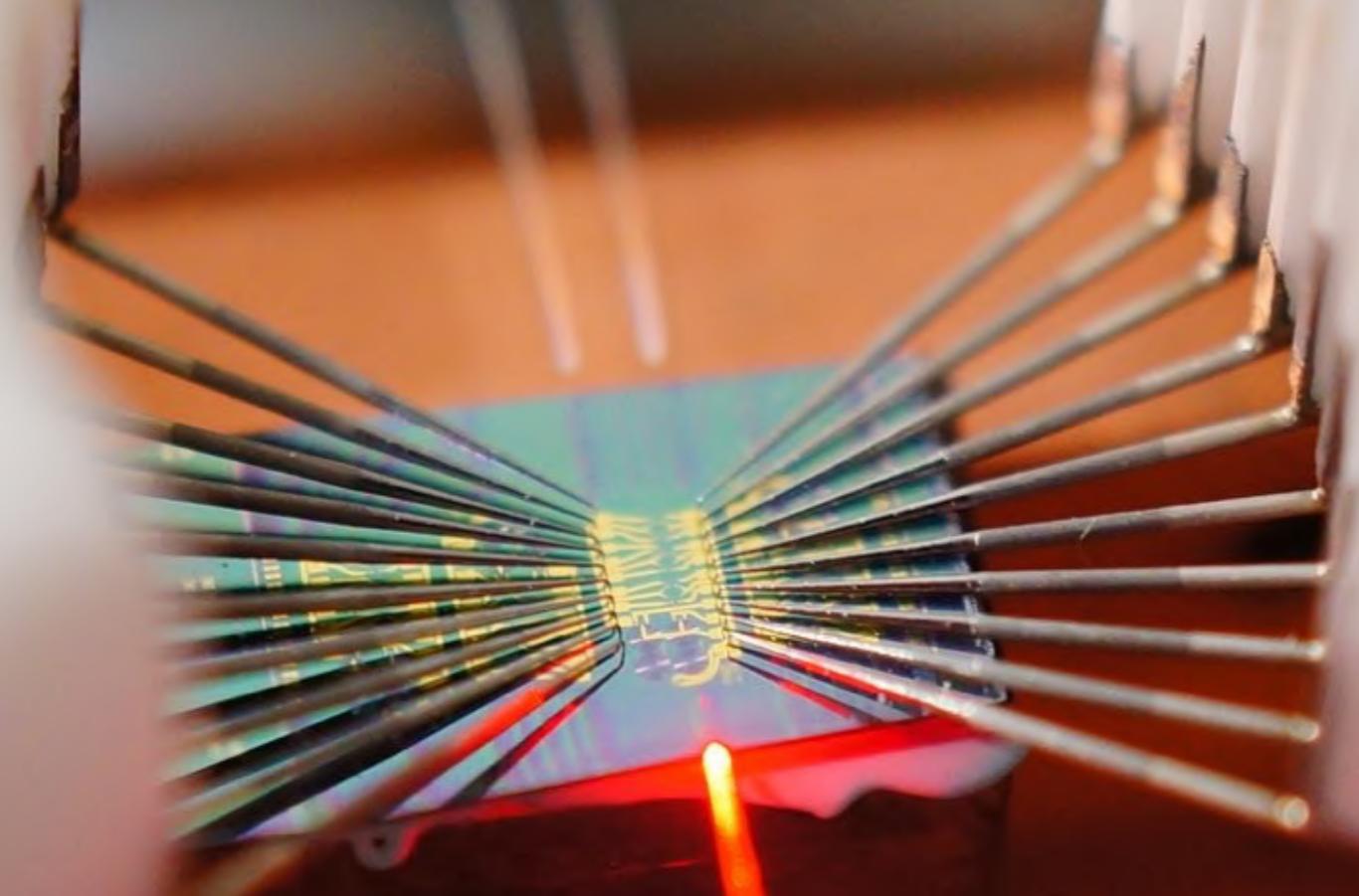


Witness assisted variational eigensolver



R. Santagati

Jianwei Wang

Andrea Gentile

Stefano Paesani

Jarrod McClean

Me



Google



Nathan Wiebe

Microsoft

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TORONTO

David Tew

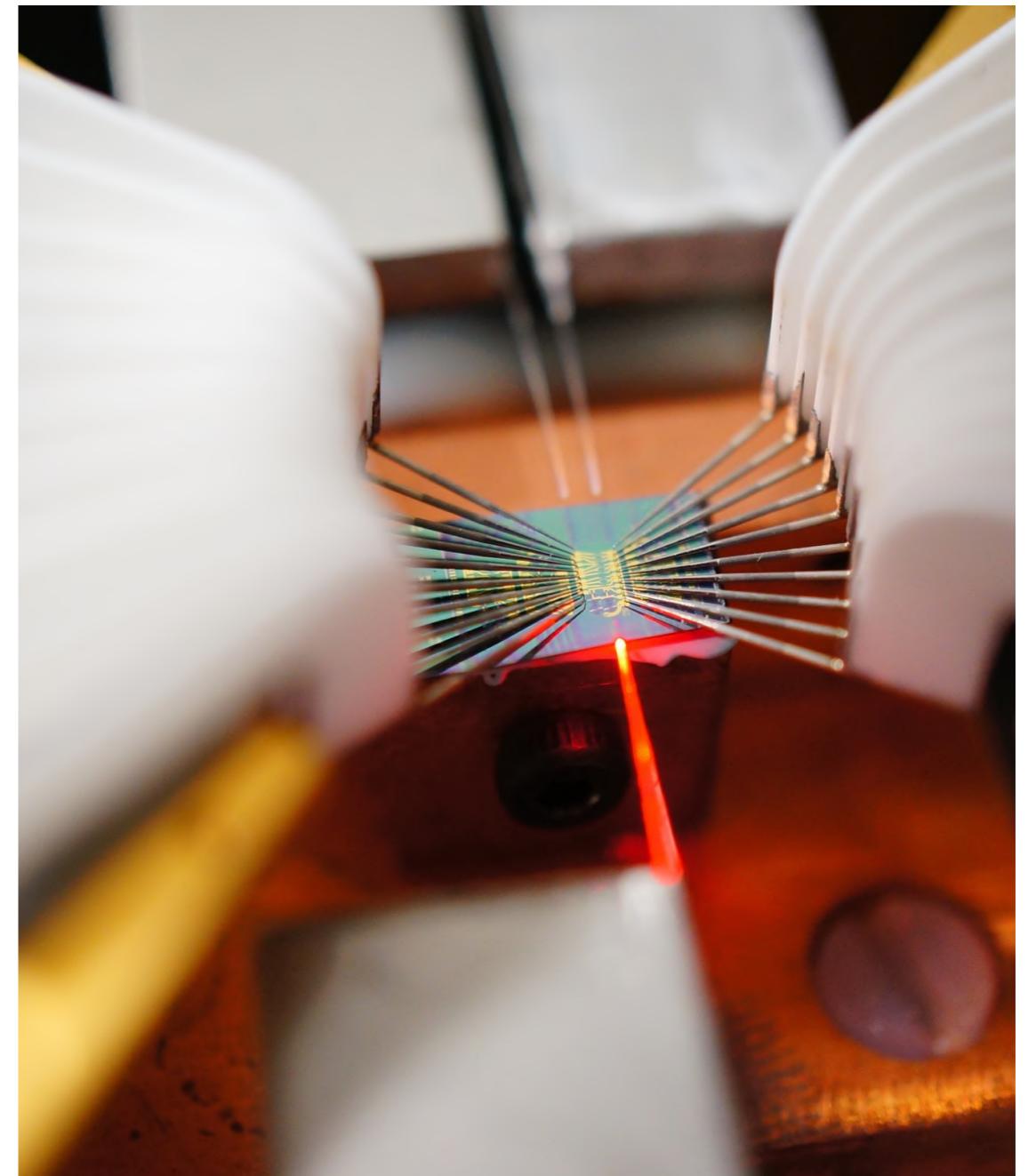


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Content

- Variational quantum eigensolvers
- Experimental implementations
- WAVES
- Experimental and simulation results
- (Optional) A bit of photonic quantum hardware



Variational quantum eigensolver

Given \hat{H} we want to find $|\psi\rangle$ s.t. $\hat{H}|\psi\rangle = \lambda|\psi\rangle$.

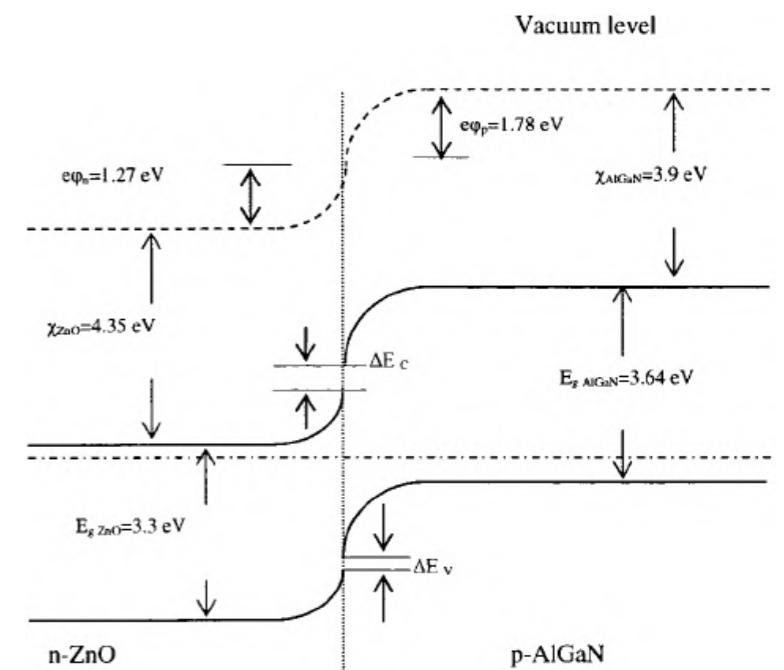
High precision
estimation of λ

Agriculture -> 100-200 qubits -> Design catalysts for efficient conversion of Ni to fertilizer

Carbon Capture -> 100-200 qubits ->
Catalysts to extract carbon dioxide from air
with less energy

Battery design -> 10^3 qubits

Drug discovery -> Molecular docking

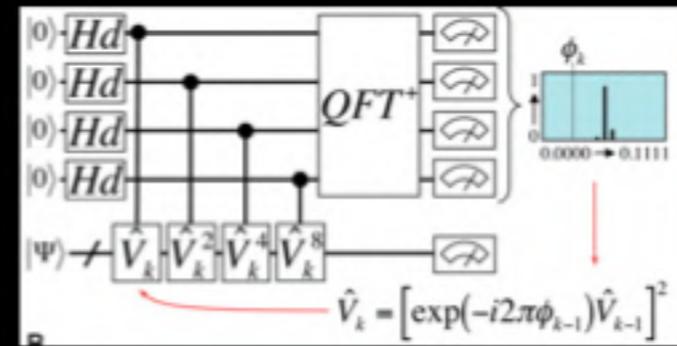


Feynman, R. P. Int. J. Theor. Phys. 21, 467 (1982).
Lloyd, S. Science 273, 1073–1078 (1996).
Aspuru-Guzik, A. et. al. Science 309, 1704 (2005).

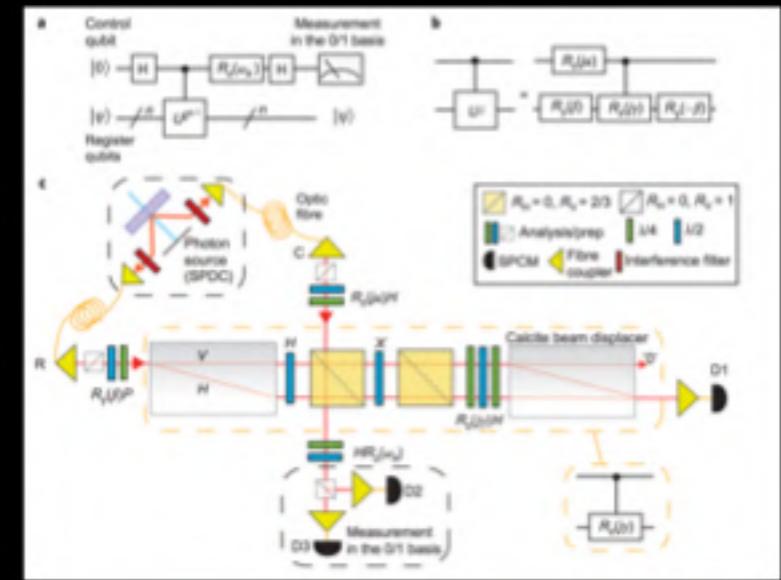
Canonical methods

Adiabatic state preparation

↓
PEA



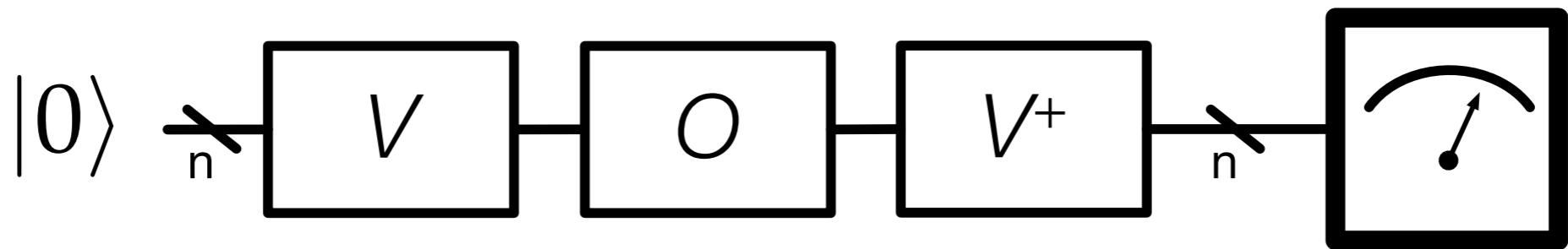
A. Aspuru-Guzik, Love,A. D. Dutoi Science (2005)



Lanyon et al. Nat Chem. (2010)

Measuring expectation values

$$\langle O \rangle_{|\Psi\rangle} = \frac{\langle \Psi | O | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$



Scalable if

$$O = \sum_{\alpha} h_{\alpha} O_{\alpha}$$

With a poly number of $O \downarrow \alpha$

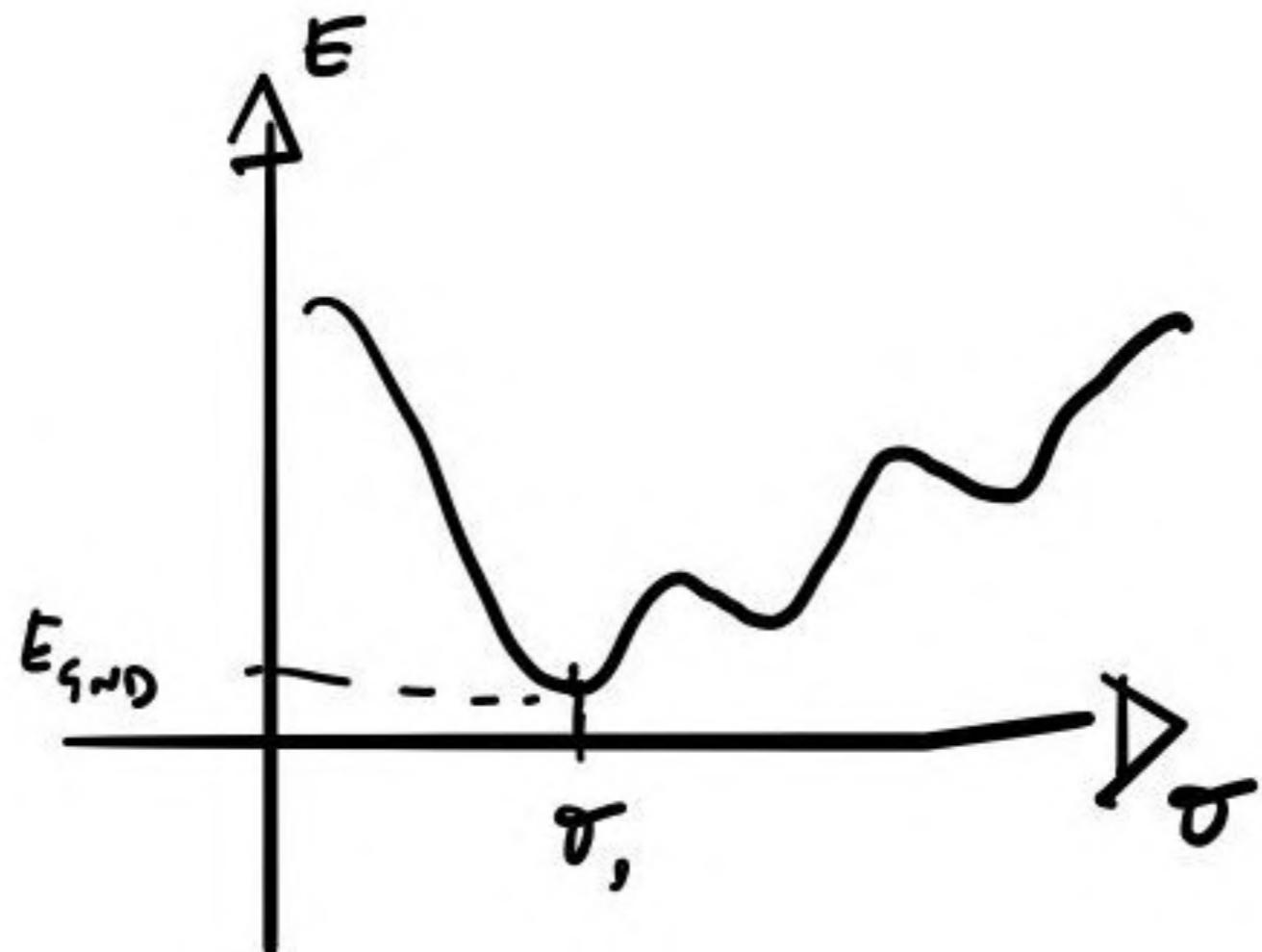
Variational Principle

$$\langle \mathcal{H} \rangle(\vec{\theta}) = \langle \psi(\vec{\theta}) | \mathcal{H} | \psi(\vec{\theta}) \rangle \geq E_{gnd}.$$

In VQE, we want to find the set of parameters that minimize the expectation value of the energy.

If our state preparation routine is general enough and the minimization protocol good enough we should find our ground state

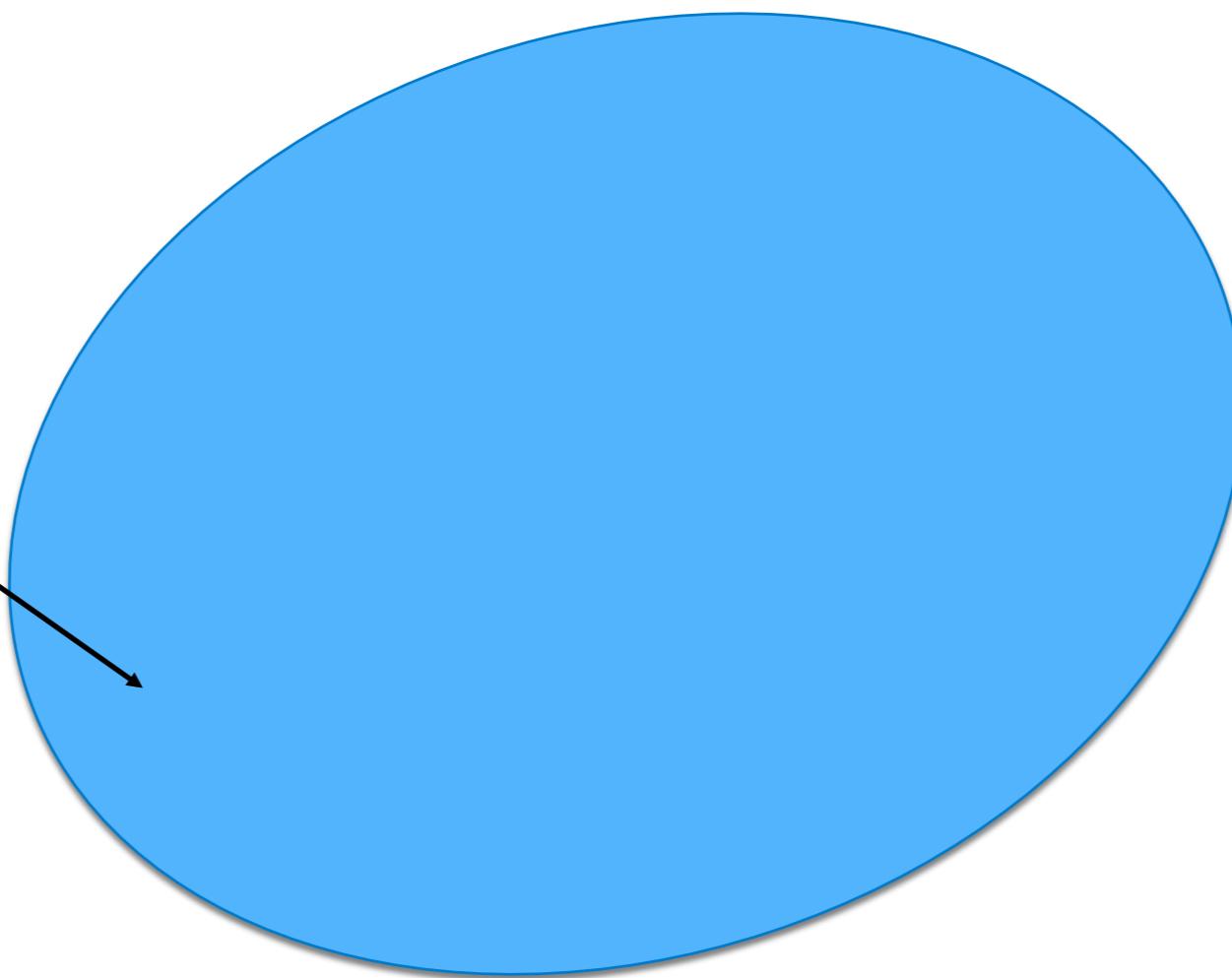
VQE



Variational Principle

$$\langle \mathcal{H} \rangle(\vec{\theta}) = \langle \psi(\vec{\theta}) | \mathcal{H} | \psi(\vec{\theta}) \rangle \geq E_{gnd}.$$

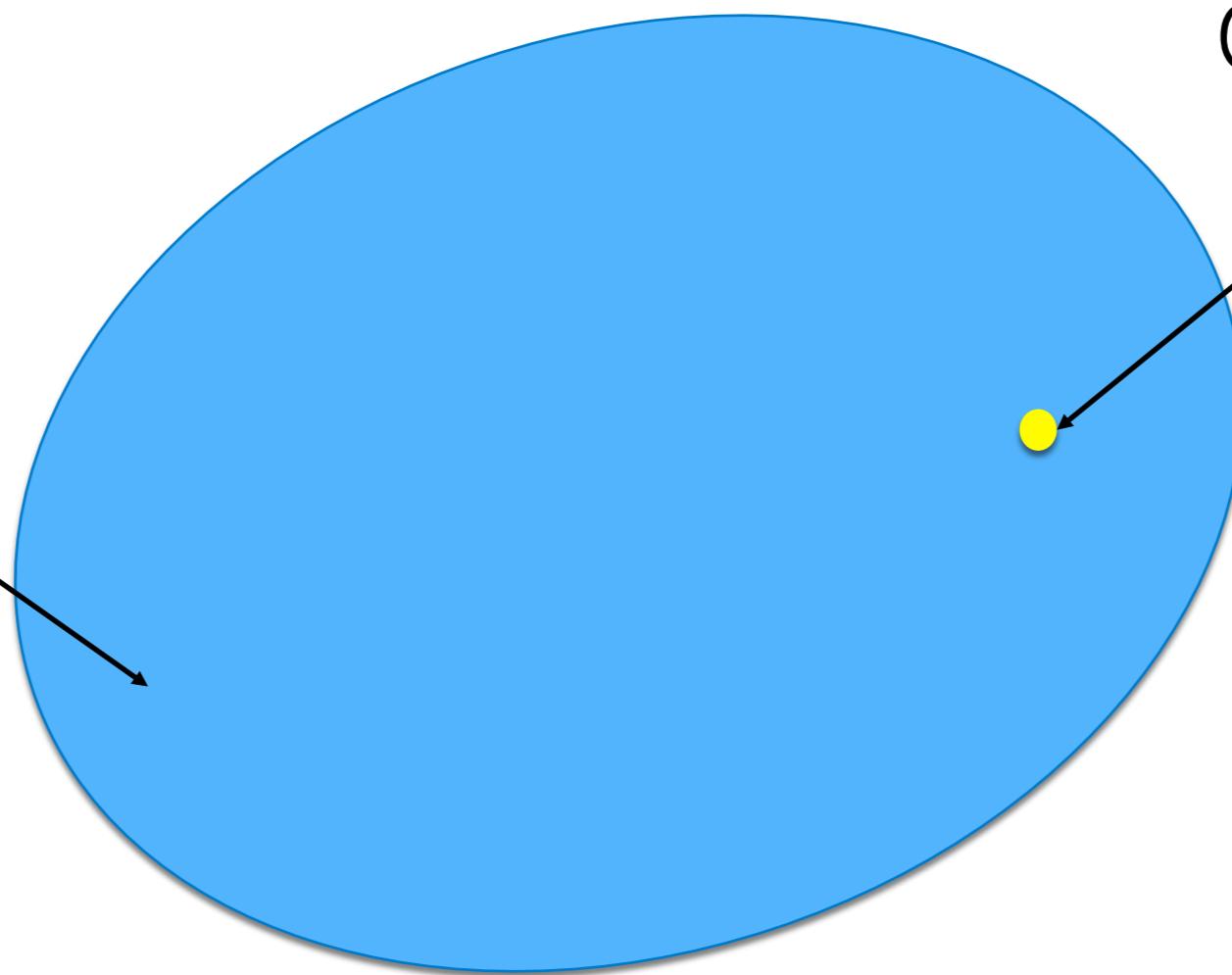
Hilbert space
of n qubits
(Exponential)



Variational Principle

$$\langle \mathcal{H} \rangle(\vec{\theta}) = \langle \psi(\vec{\theta}) | \mathcal{H} | \psi(\vec{\theta}) \rangle \geq E_{gnd}.$$

Hilbert space
of n qubits
(Exponential)

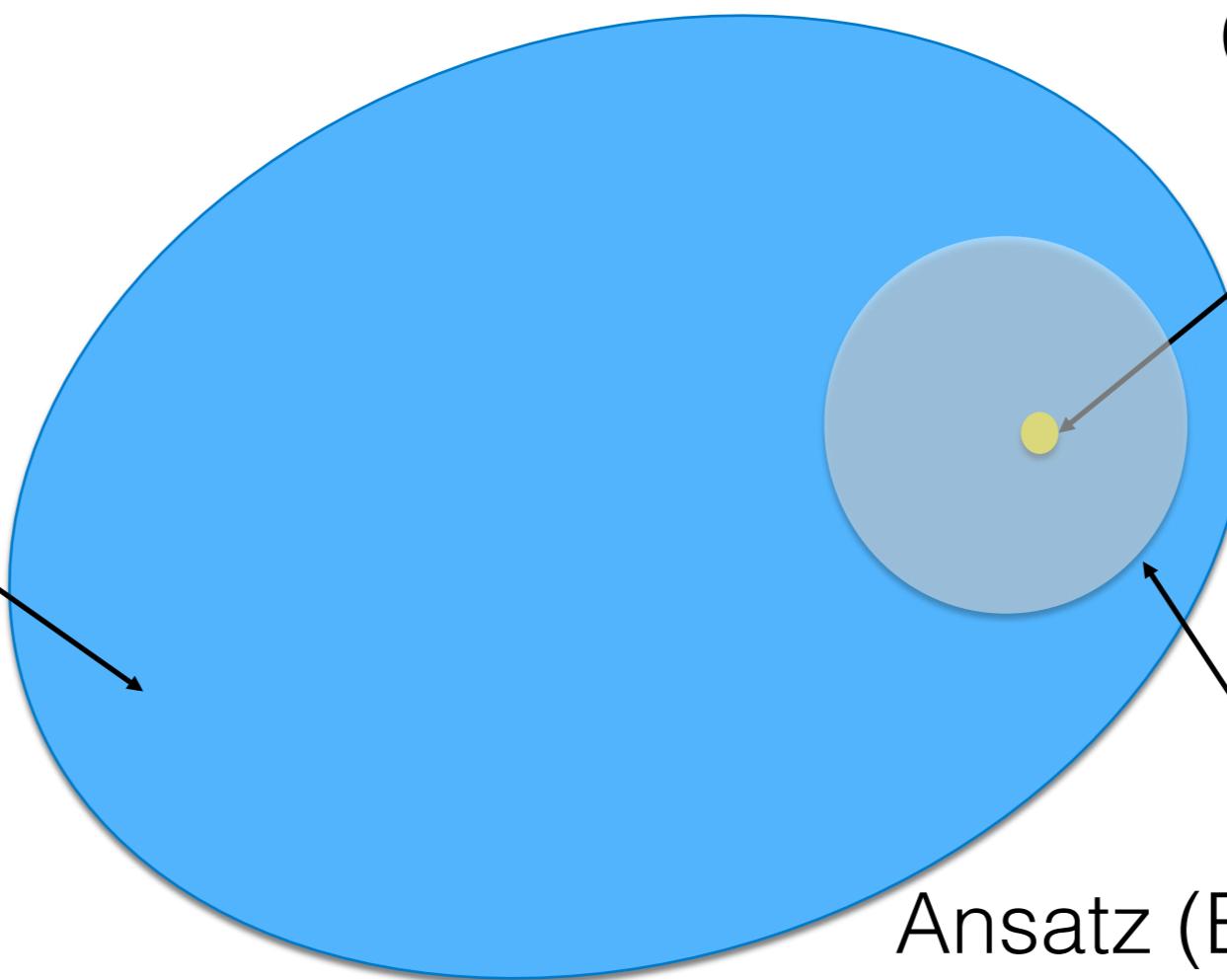


Ground state

Variational Principle

$$\langle \mathcal{H} \rangle(\vec{\theta}) = \langle \psi(\vec{\theta}) | \mathcal{H} | \psi(\vec{\theta}) \rangle \geq E_{gnd}.$$

Hilbert space
of n qubits
(Exponential)



Ground state

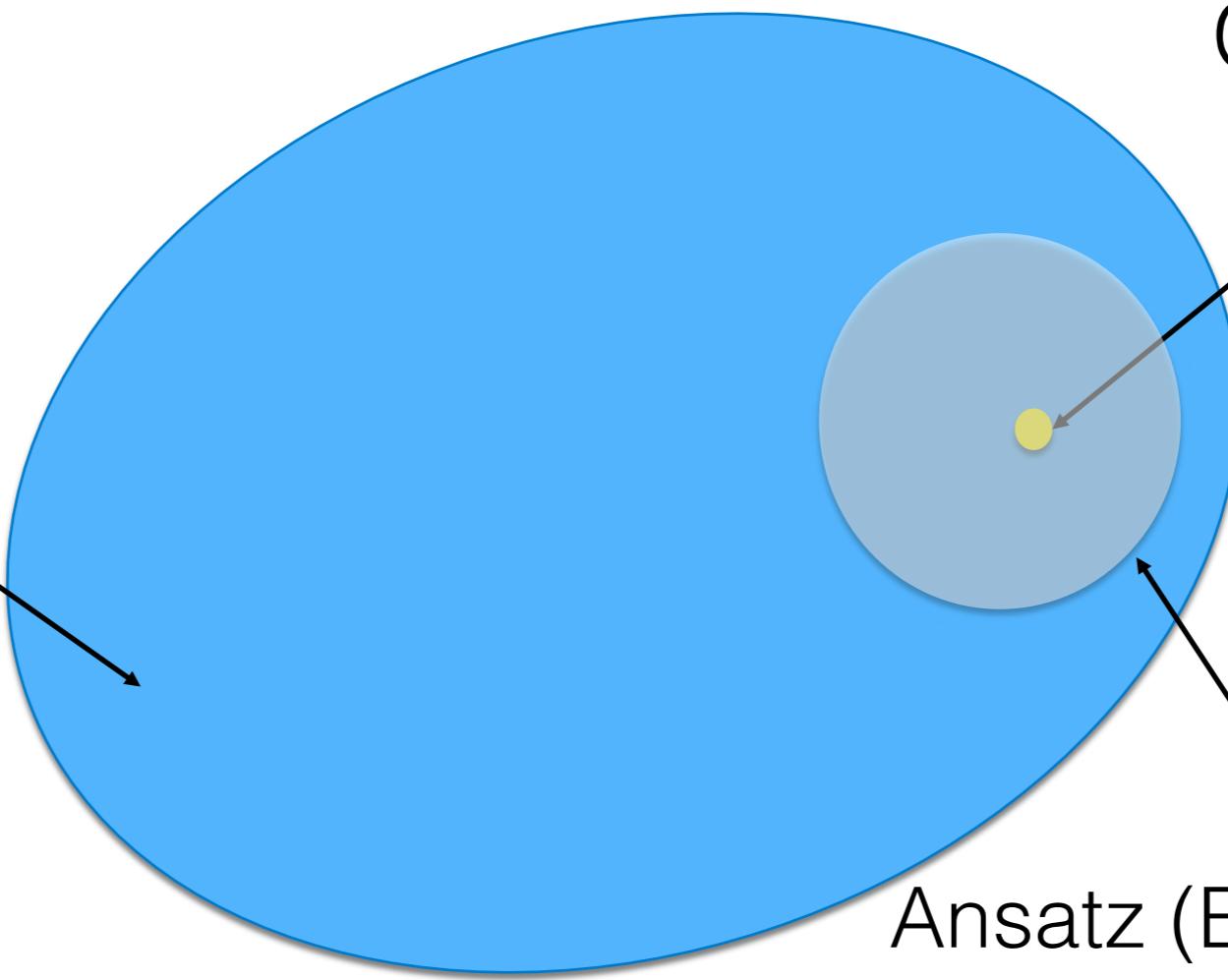
Ansatz (Educated guess)
- Space of quantum states
accessible with our
state preparation strategy

Variational Principle

$$\langle \mathcal{H} \rangle(\vec{\theta}) = \langle \psi(\vec{\theta}) | \mathcal{H} | \psi(\vec{\theta}) \rangle \geq E_{gnd}.$$

Hilbert space
of n qubits
(Exponential)

Needs to be Poly(n)
#gates



Ground state

Ansatz (Educated guess)
- Space of quantum states
accessible with our
state preparation strategy

Ansatz

Unitary coupled cluster

$$| \Psi \rangle = e^{T - T^\dagger} | \Phi \rangle_{\text{ref}} \quad (T - T^\dagger) \text{ Anti-hermitian}$$

$$T = T_1 + T_2 + \dots$$

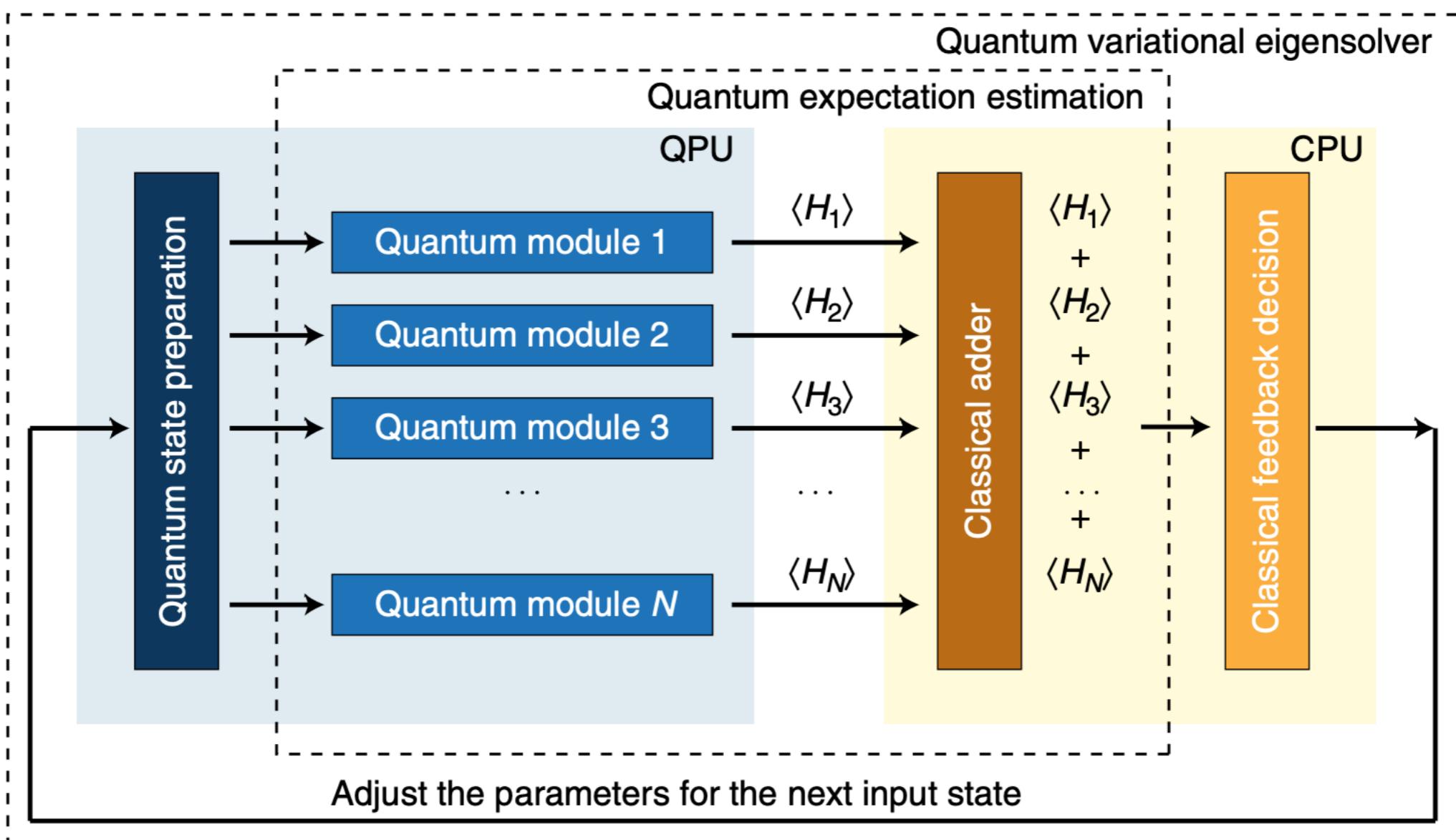
$$T_1 = \sum_{pr} t_p^r \hat{a}_p^\dagger \hat{a}_r \quad \text{Jordan-Wigner}$$

$$T_2 = \sum_{pqrs} t_{pq}^{rs} \hat{a}_p^\dagger \hat{a}_q^\dagger \hat{a}_r \hat{a}_s \quad \begin{aligned} \hat{a}_j &\rightarrow I^{\otimes j-1} \otimes \sigma_+ \otimes \sigma_z^{\otimes N-j} \\ \hat{a}_j^\dagger &\rightarrow I^{\otimes j-1} \otimes \sigma_- \otimes \sigma_z^{\otimes N-j} \end{aligned}$$

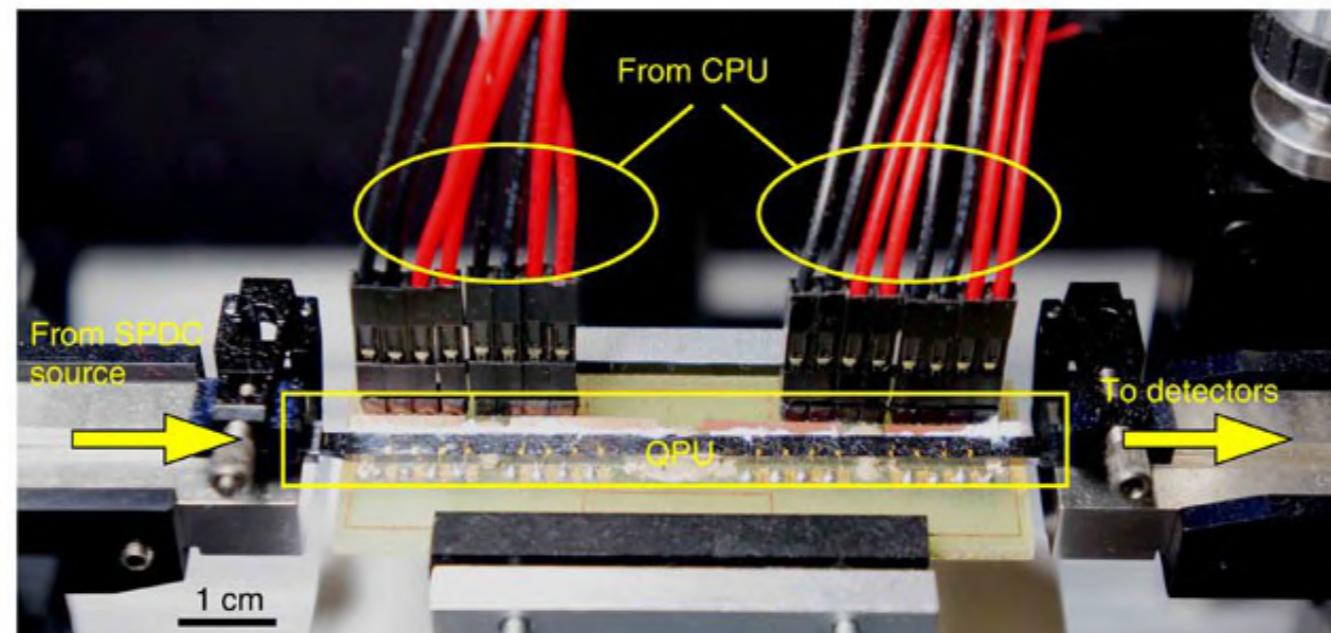
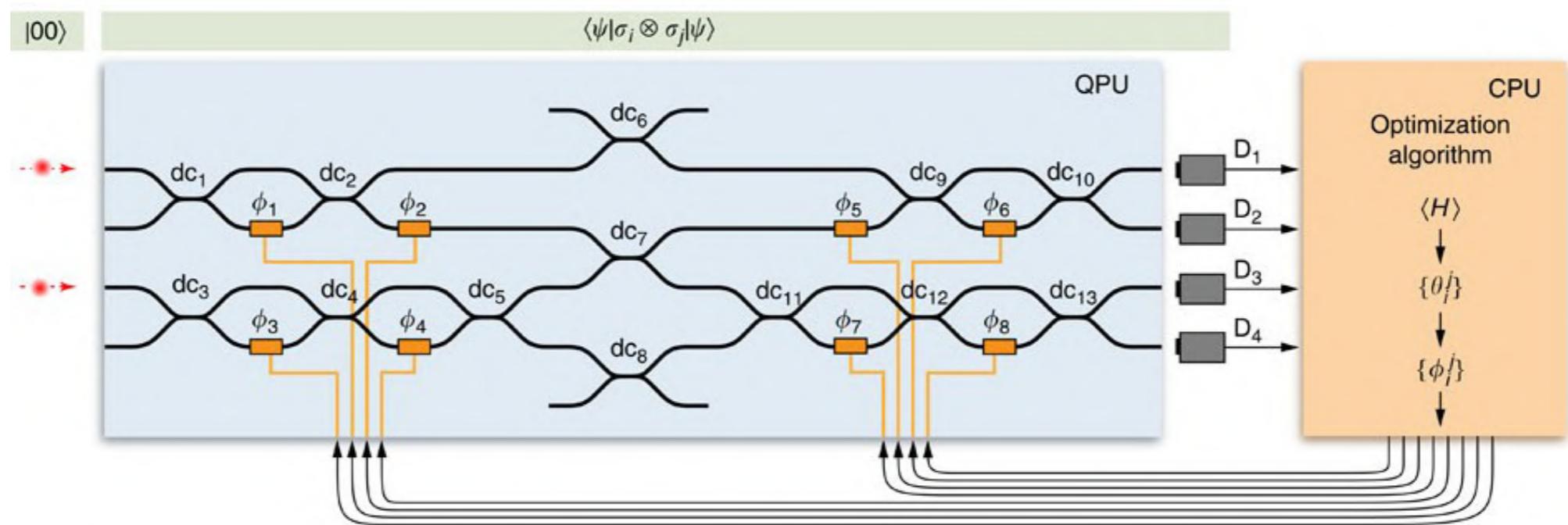
VQE

$$\langle \mathcal{H} \rangle(\vec{\theta}) = \langle \psi(\vec{\theta}) | \mathcal{H} | \psi(\vec{\theta}) \rangle \geq E_{gnd}.$$

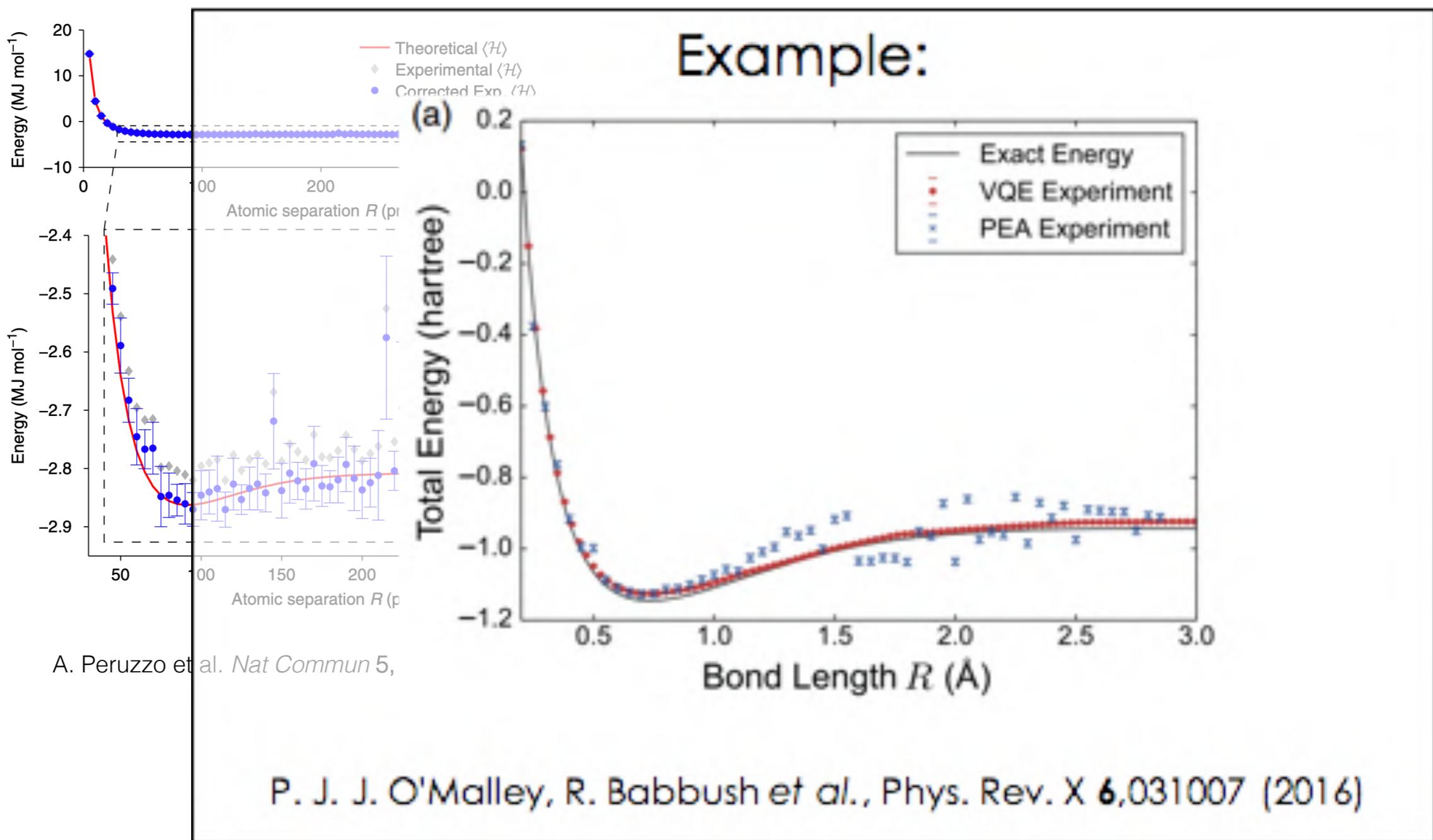
$$\mathcal{H} = \sum_{i\alpha} h_\alpha^i \sigma_\alpha^i + \sum_{ij\alpha\beta} h_{\alpha\beta}^{ij} \sigma_\alpha^i \sigma_\beta^j + \dots$$



First experimental dem.



VQE experiment



VQE summary

- VQE is great for noise resilience
- It is expected to be applied to pre-threshold devices
- As it is, it can target only the ground state but excited states are equally fundamental
- So far main solution was to use an expensive folding spectra method

WAVES - Witness Assisted Variational Eigenstate Solver

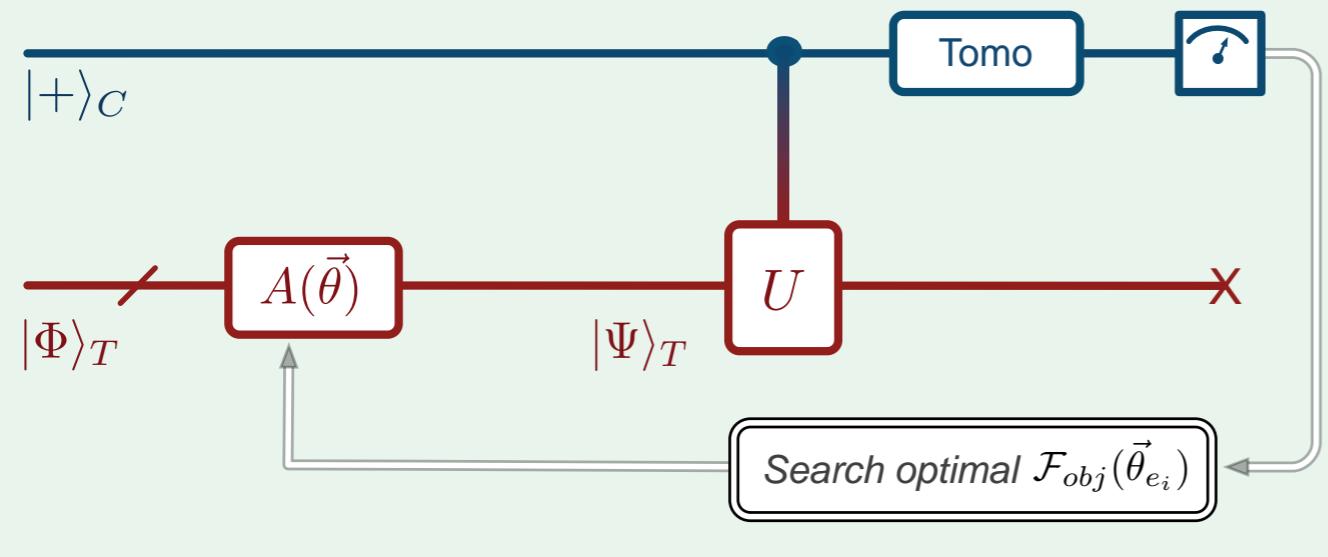
Ground-state search
using \mathcal{F}_{obj} and $\hat{E}_{p_0} = \hat{I}$

Excited-states search
using \mathcal{S} and guessed \hat{E}_{p_i}

Eigenvalues estimation
using IPEA

$$U = e^{-iHt}$$

Ground-state search:



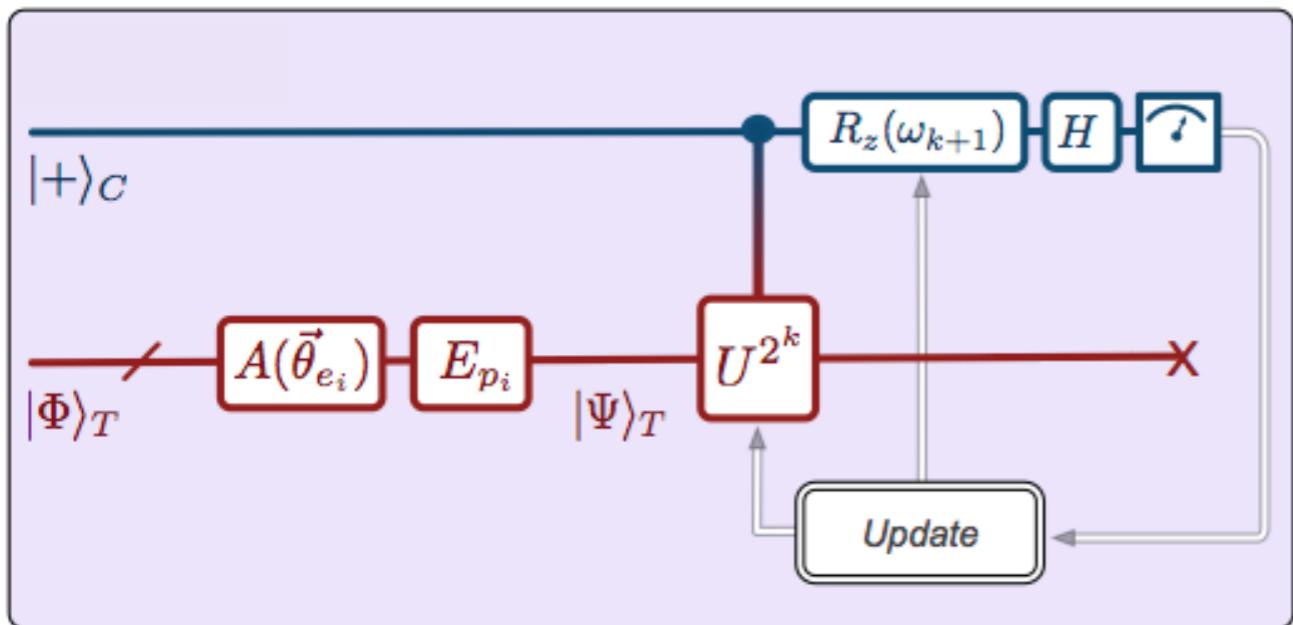
$$\hat{U} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$$

$$|+\rangle_C \rightarrow |0\rangle + e^{i\phi}|1\rangle$$

From the phase we can estimate the eigenvalue (i.e. the energy)

WAVES - Witness Assisted Variational Eigenstate Solver

If P is 1 $|\Psi_T\rangle$ is eigenstate
If P is <1 $|\Psi_T\rangle$ is NOT eigenstate



The purity can be used as eigenstate witness.

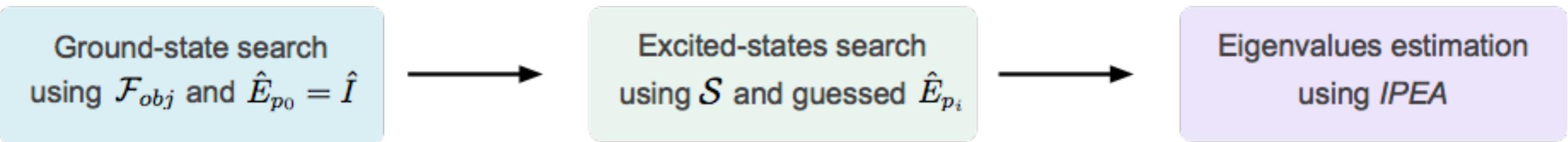
Helmholtz free energy

$$\mathcal{F}'_{obj}(\mathcal{S}, \mathcal{E}) = \mathcal{E} + T\mathcal{S}(\rho)$$

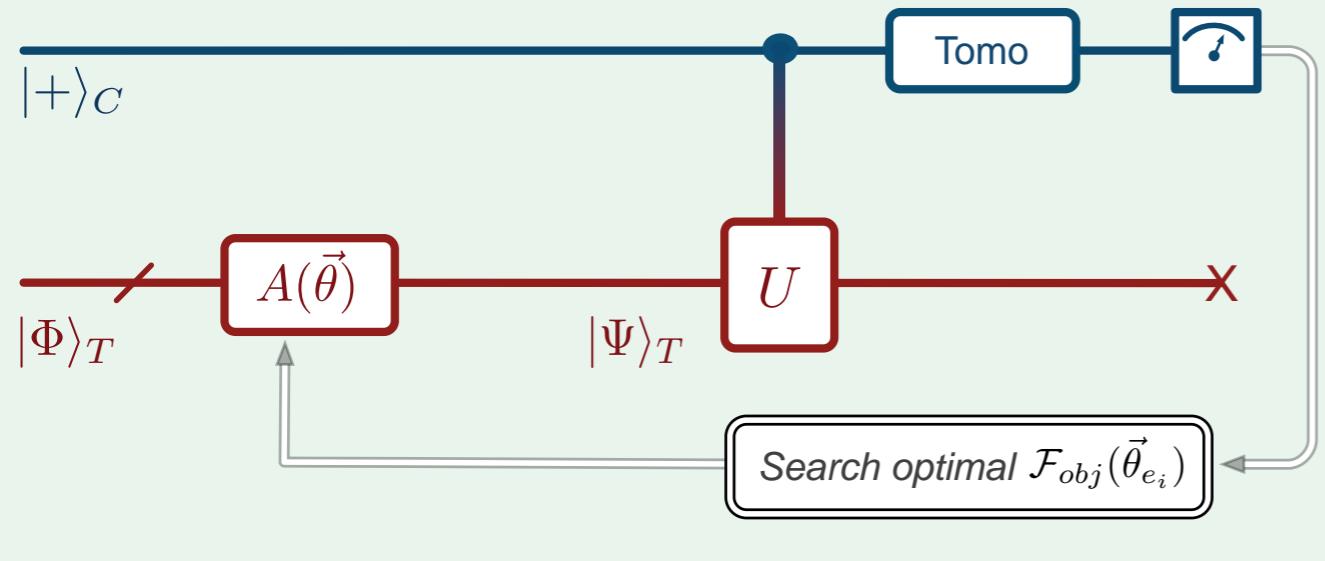
$$\mathcal{F}_{obj}(\mathcal{P}, \mathcal{E}) = \mathcal{E} - T\mathcal{P}$$

$$S = -Tr(\rho \ln \rho) \quad S = 1 - Tr(\rho^2)$$

WAVES - Witness Assisted Variational Eigenstate Solver



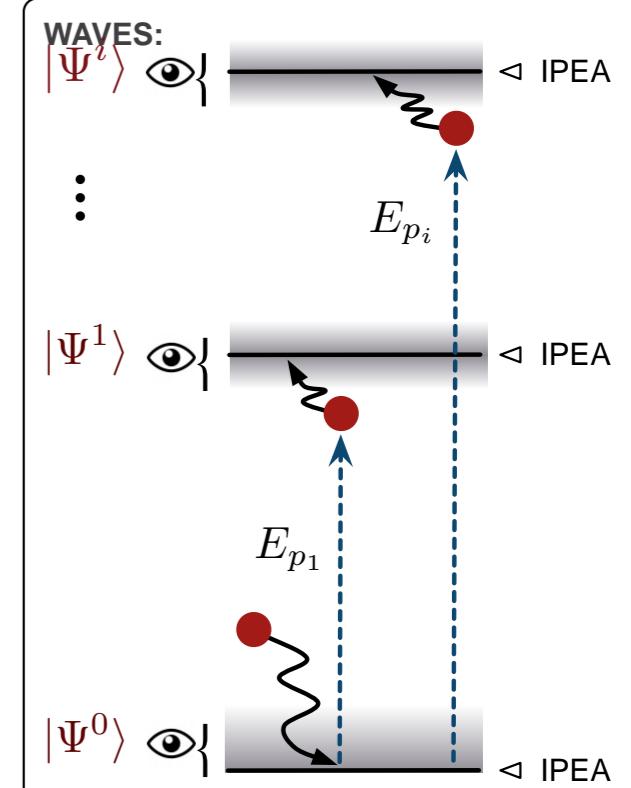
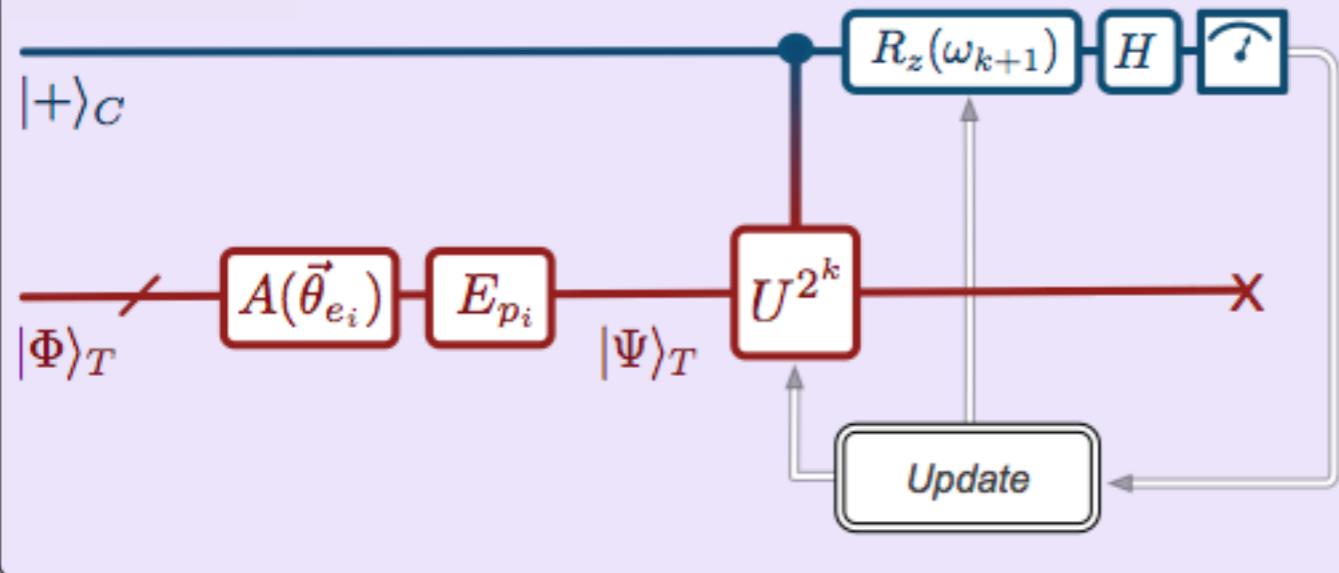
Ground-state search:

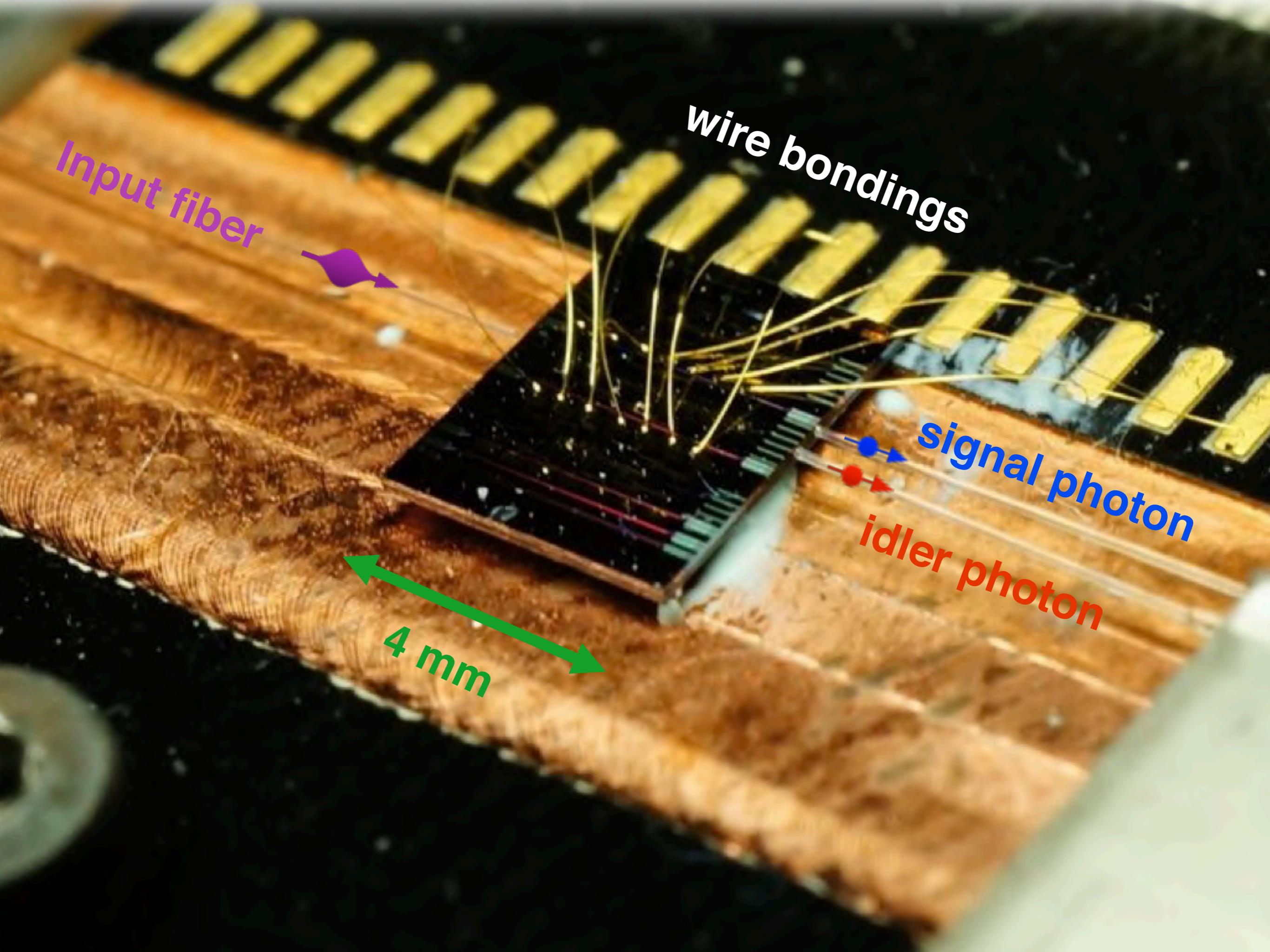


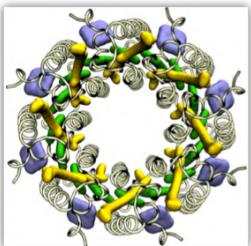
$$\mathcal{F}_{obj} = \mathcal{E} + TS$$

$$T \rightarrow \infty$$

Targeting excited states



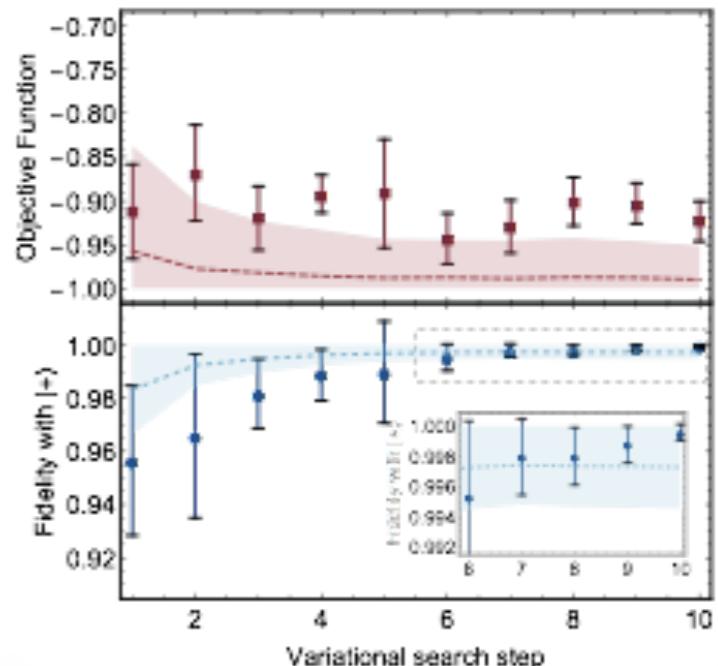
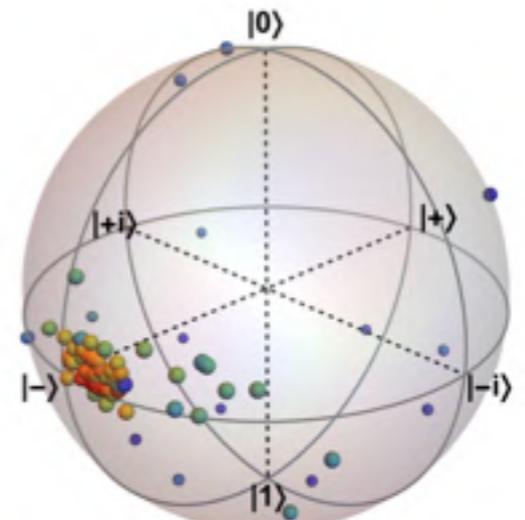
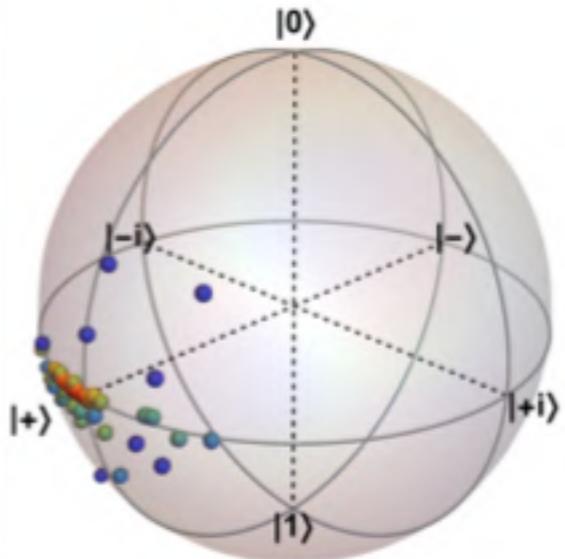
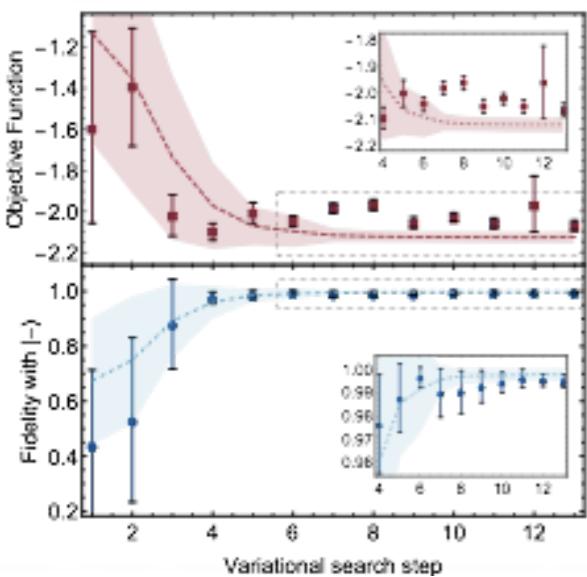




Experiment

Ground-state search
using \mathcal{F}_{obj} and $\hat{E}_{p_0} = \hat{I}$

Excited-states search
using \mathcal{S} and guessed \hat{E}_{p_i}



F=99.5%

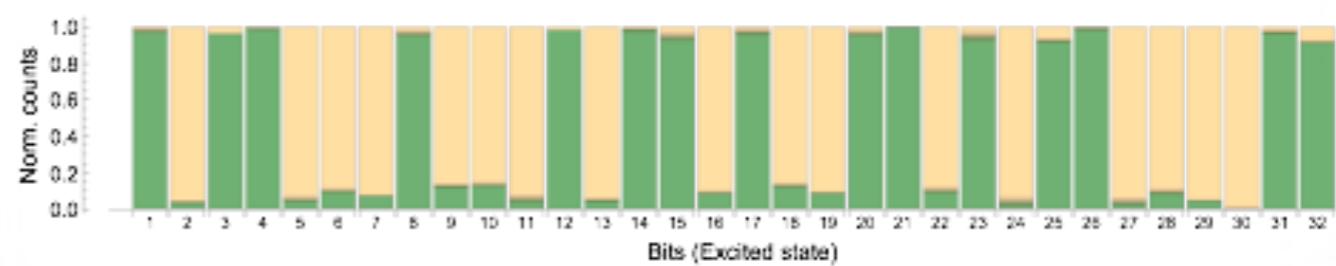
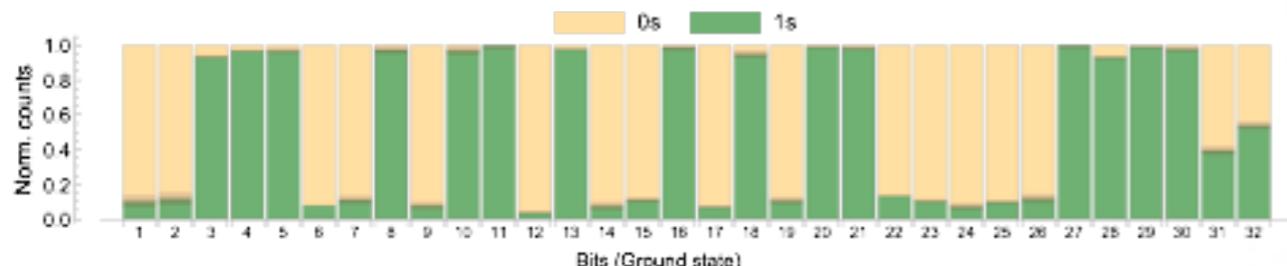
Santagati et. al (2016) in preparation

$$\mathcal{F}'_{obj}(\mathcal{S}, \mathcal{E}) = \mathcal{E} + T\mathcal{S}(\rho)$$

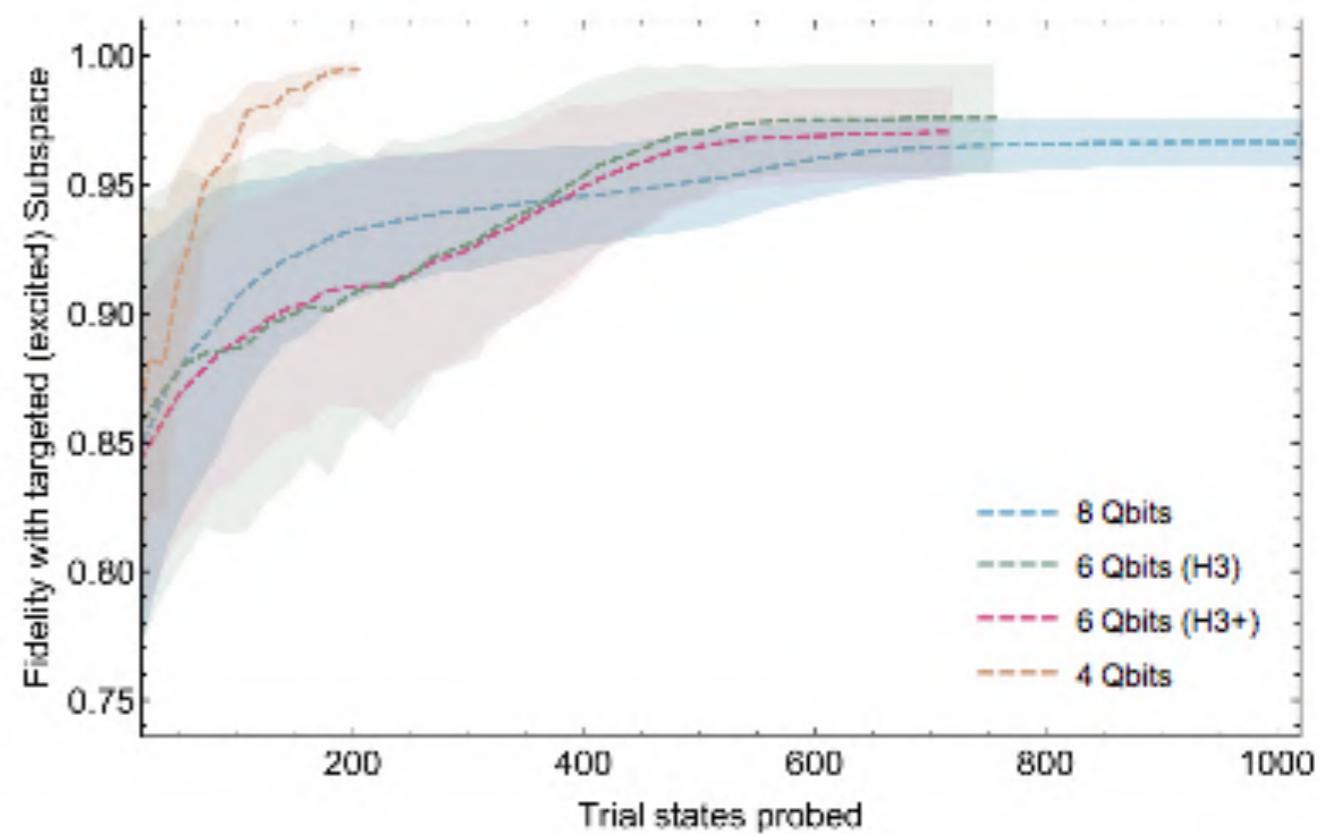
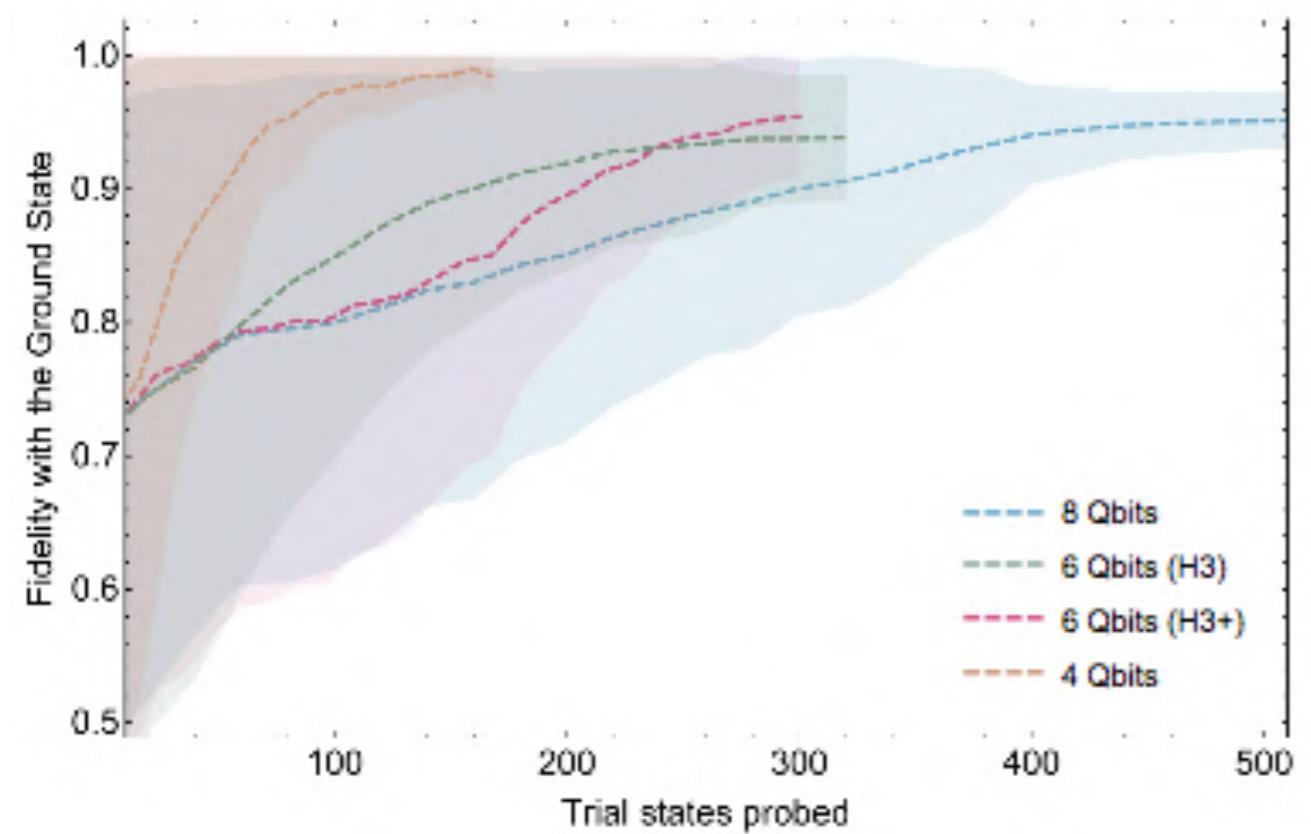
F=99.9%

IPEA

32 bits



WAVES: Simulations



Simulating H2, H3, H3+ and H4
with
4,6 and 8 qubits

Restrictions on the ansatz state preparation seems to limit the initial fidelity

Ansatz: UUCC

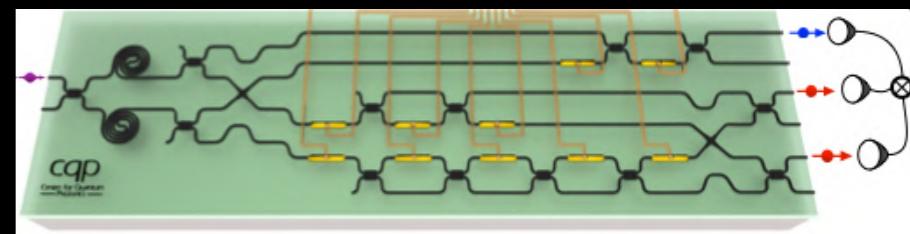
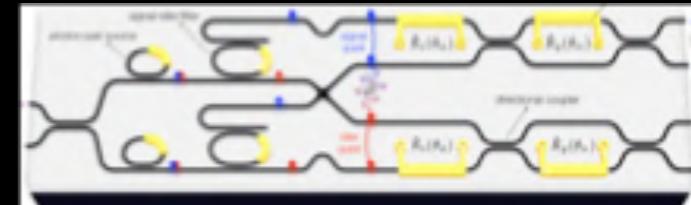
$$U(\vec{t}) = \exp \left[\sum_{ij} t_{ij} (a_i^\dagger a_j - a_j^\dagger a_i) + \sum_{ijkl} t_{ijkl} (a_i^\dagger a_j a_k^\dagger a_l - a_l^\dagger a_k a_j^\dagger a_i) \right]$$

Excitation operators

$$E_{ij} = \exp \left[\frac{\pi}{2} (a_i^\dagger a_j - a_j^\dagger a_i) \right]$$

CONCLUSION

- VQE
- Witness Assisted Variational Eigensolver
- Implementation in silicon quantum photonics

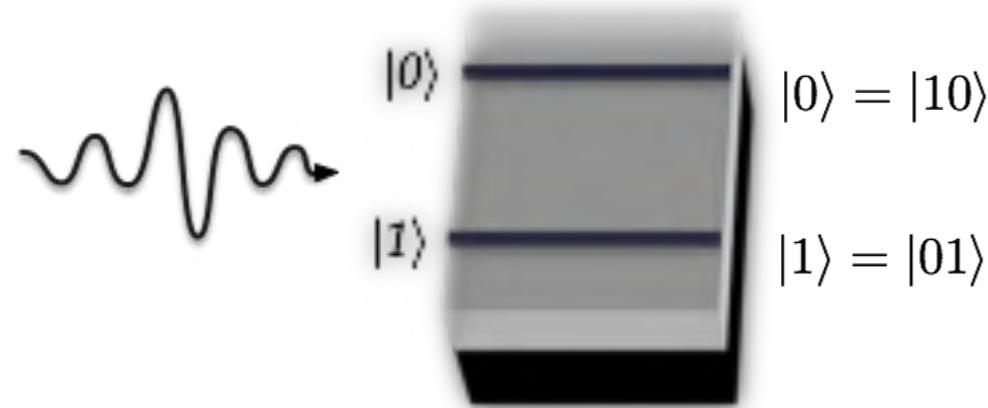


CCP

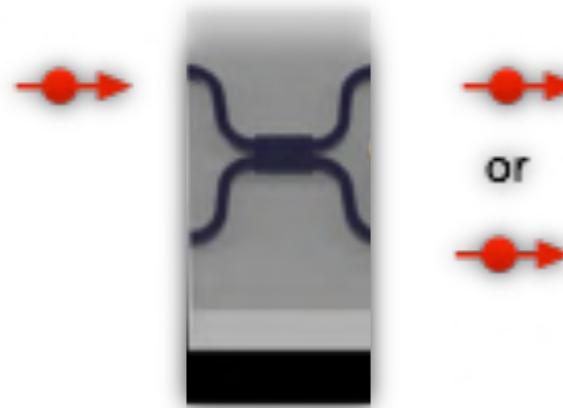


Q.I. with integrated photonics

path-encoded qubit



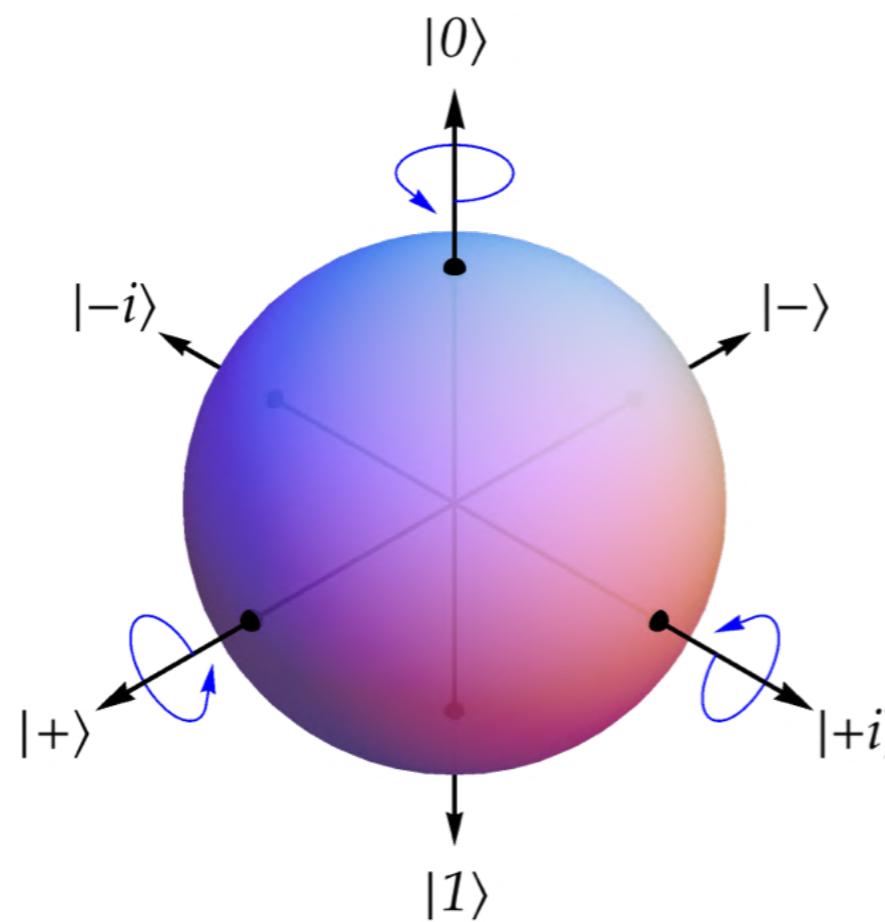
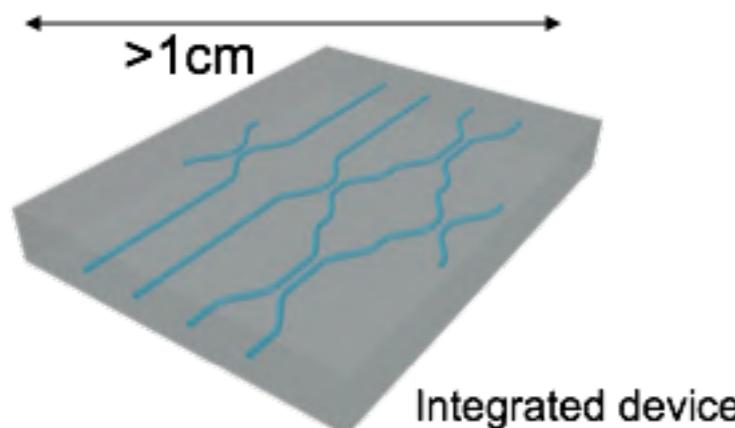
Directional Coupler



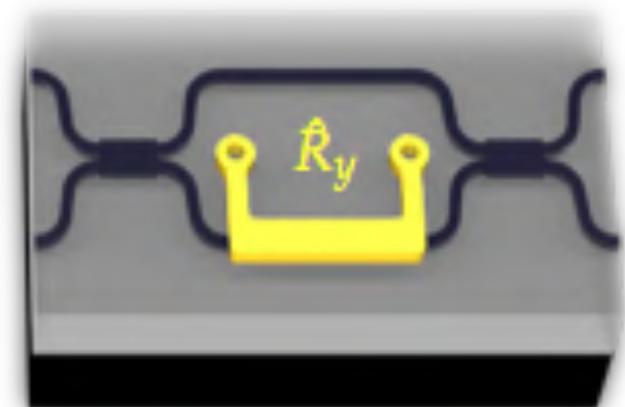
Termal phase shifter



- Path encode qubits
- very long coherence time
- single and 2 qubits gate

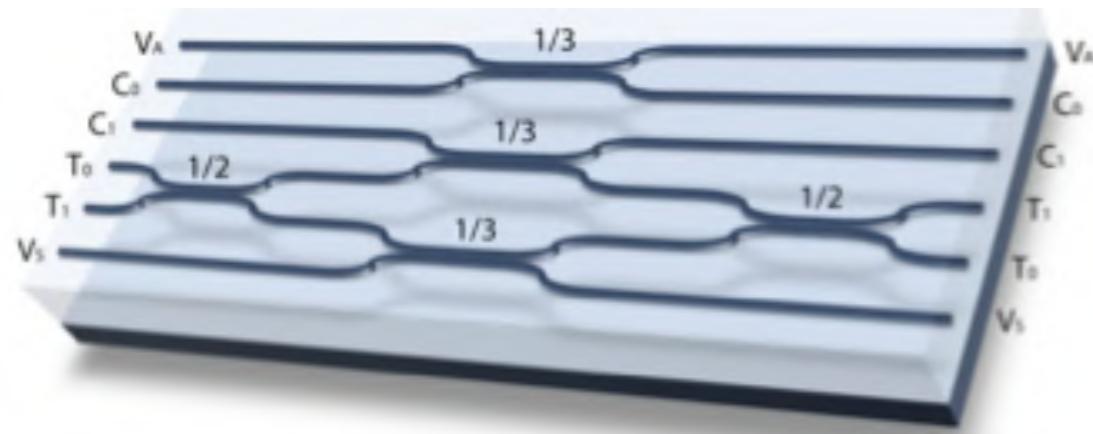


Interferometers



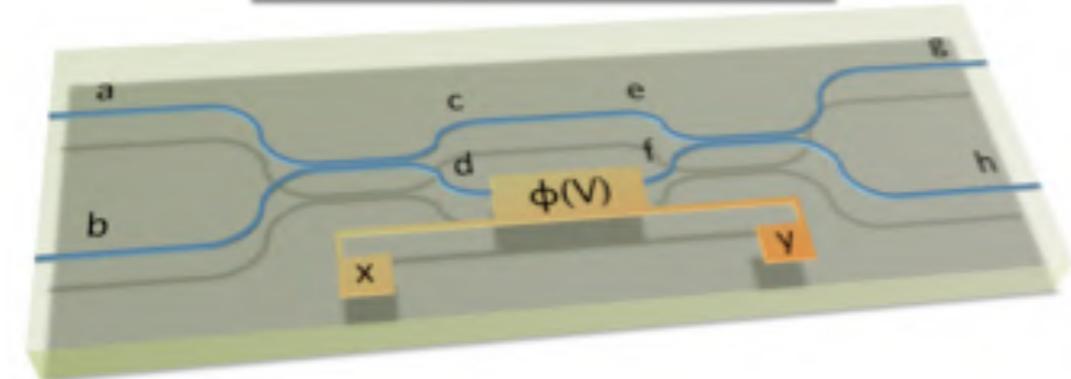
Silica on silicon integrated quantum photonic circuits

CNOT Gate



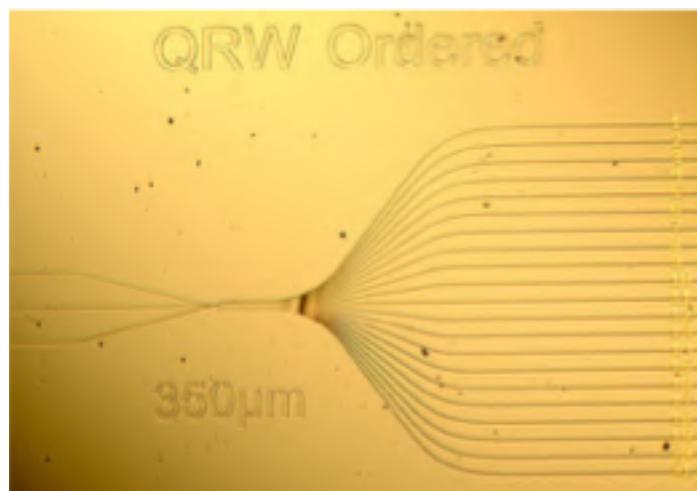
Politi, Cryan, Rarity, Yu, and O'Brien
Science 320, 5876 (2008)

Single qubit Gate



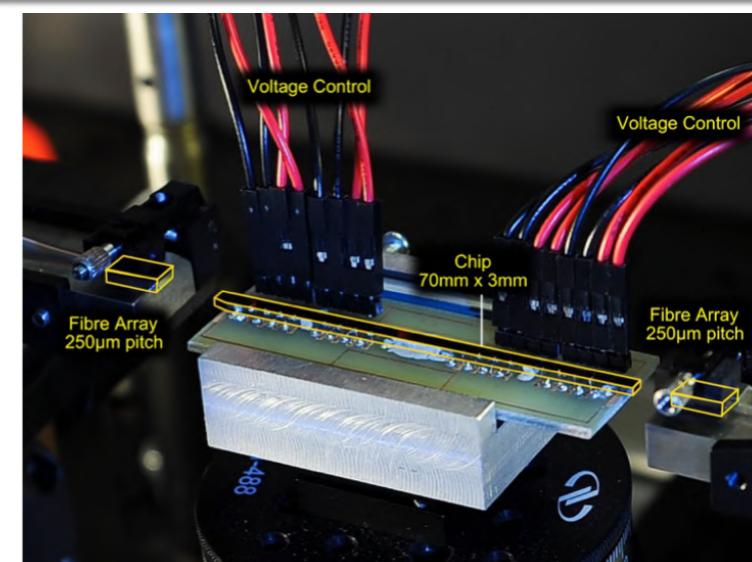
Politi, Cryan, Rarity, Yu, and O'Brien
Science 320, 5876 (2008)

Quantum Walks



Peruzzo, Lobino, Matthews, Matsuda, et al.
Science 329 1500 (2010)

Generate arbitrary states

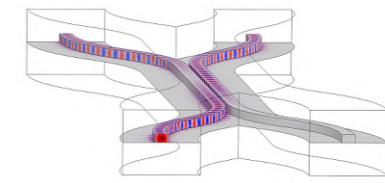
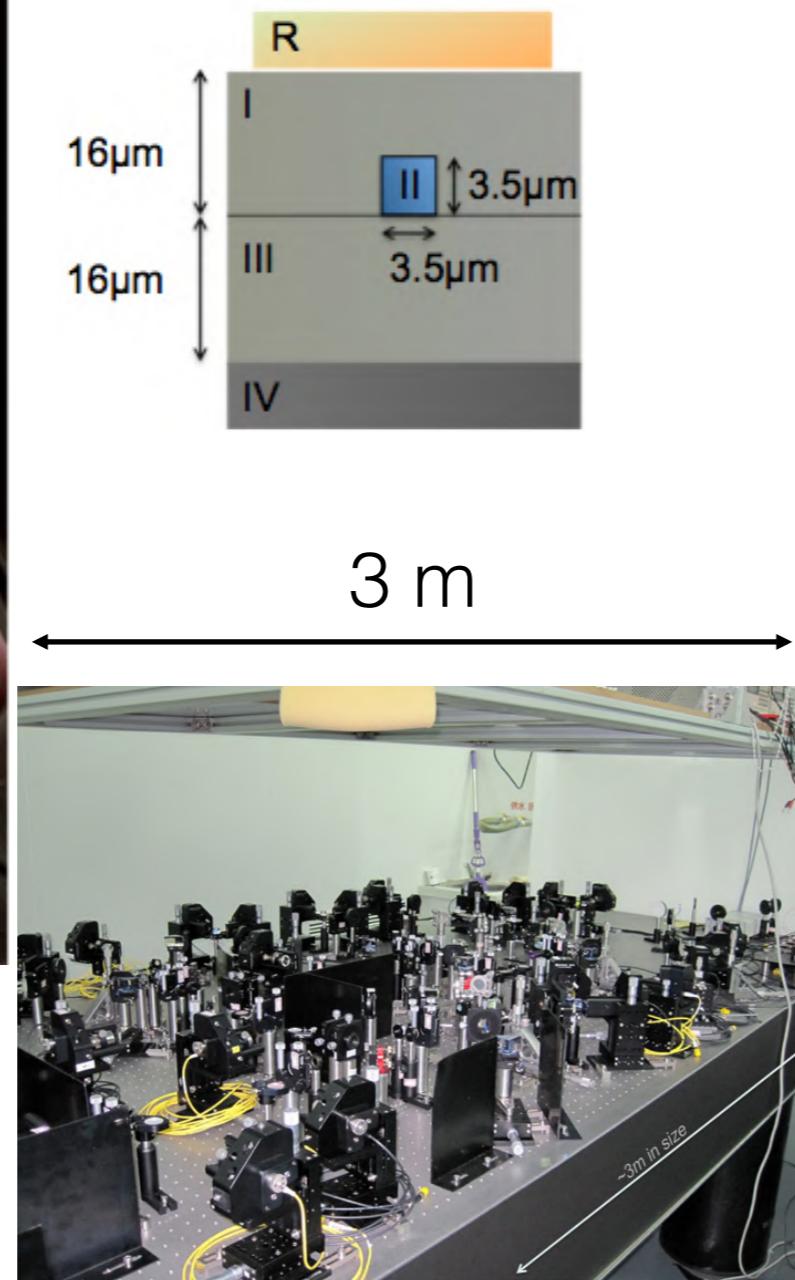


Shadbolt et al.
Nature Photonics (2011)

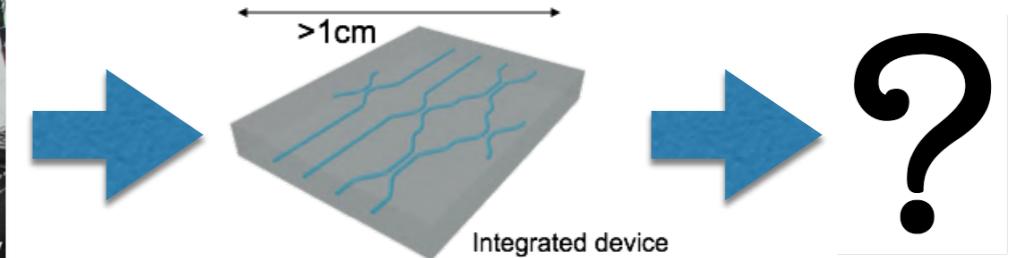
Silica on silicon integrated quantum photonic circuits



Politi, et al. *Science*,
320, 5876 (2008).



10^4 components ...

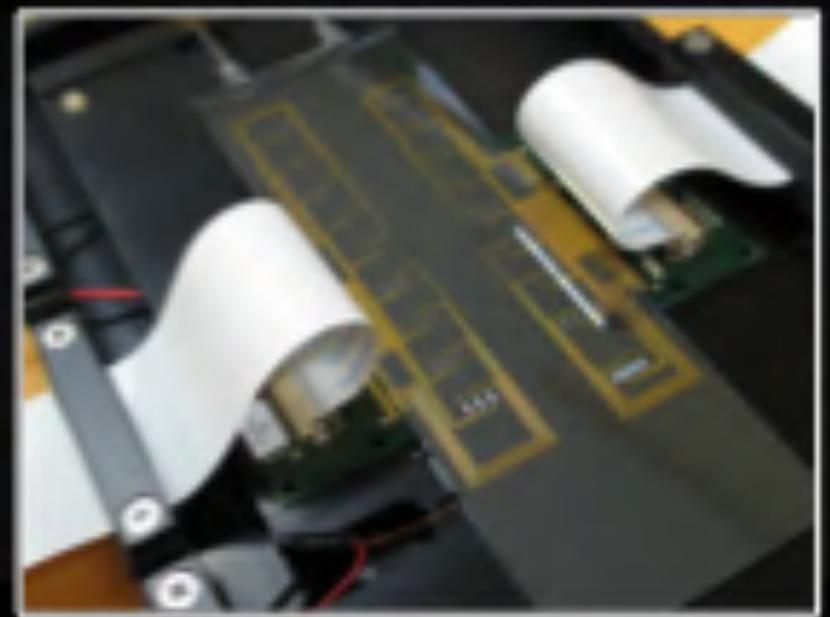
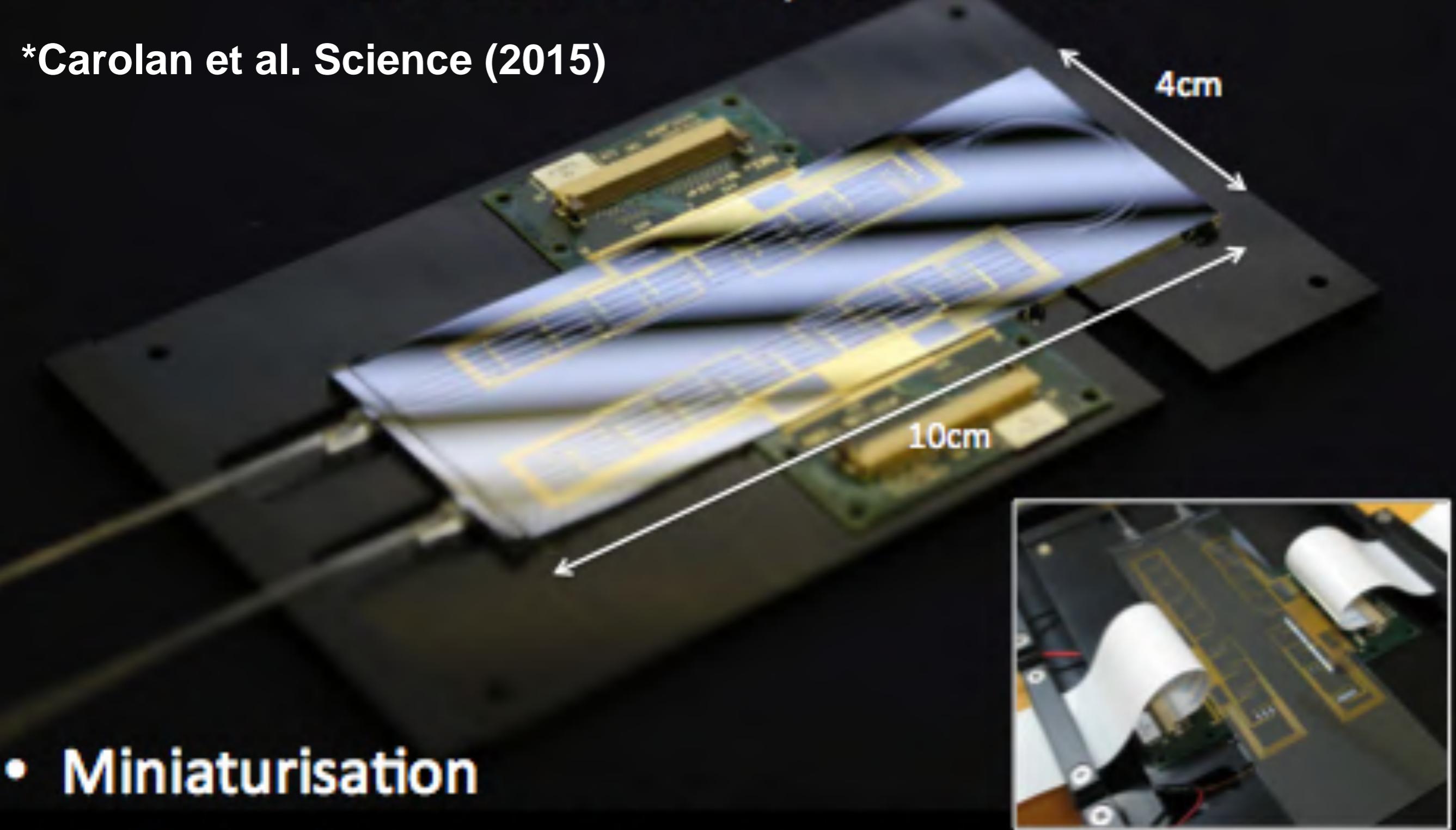


- **Size / Compactness**
 - **Stability**
 - **Complexity**
- **Route to scalability?**

- Silica waveguide on a silicon wafer
- Photo-lithographically processed with sub μm accuracy
- Low loss coupling to fibre-optics

Universal Linear Optics Processor

*Carolan et al. Science (2015)

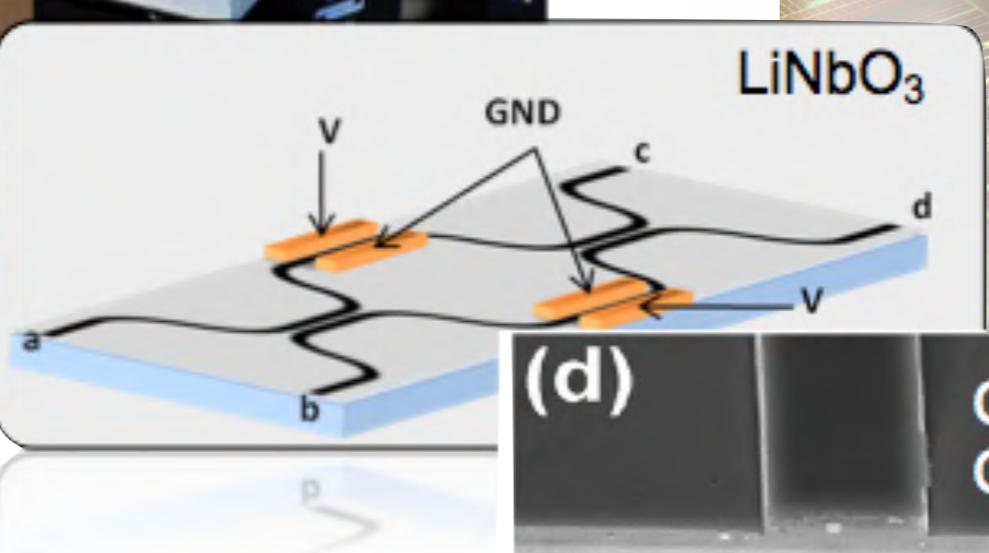
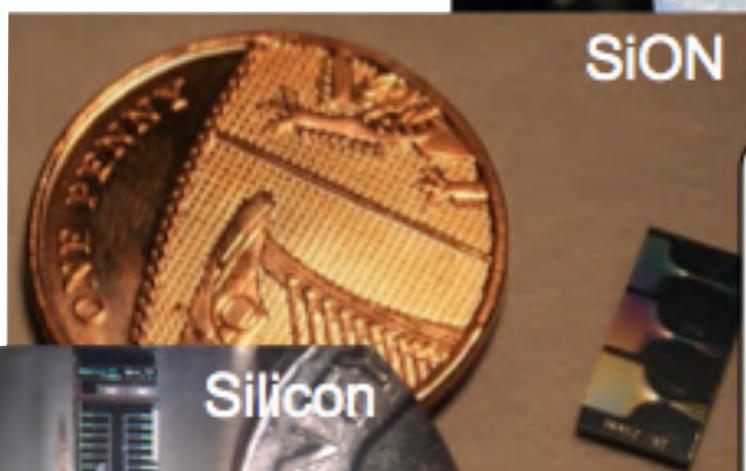
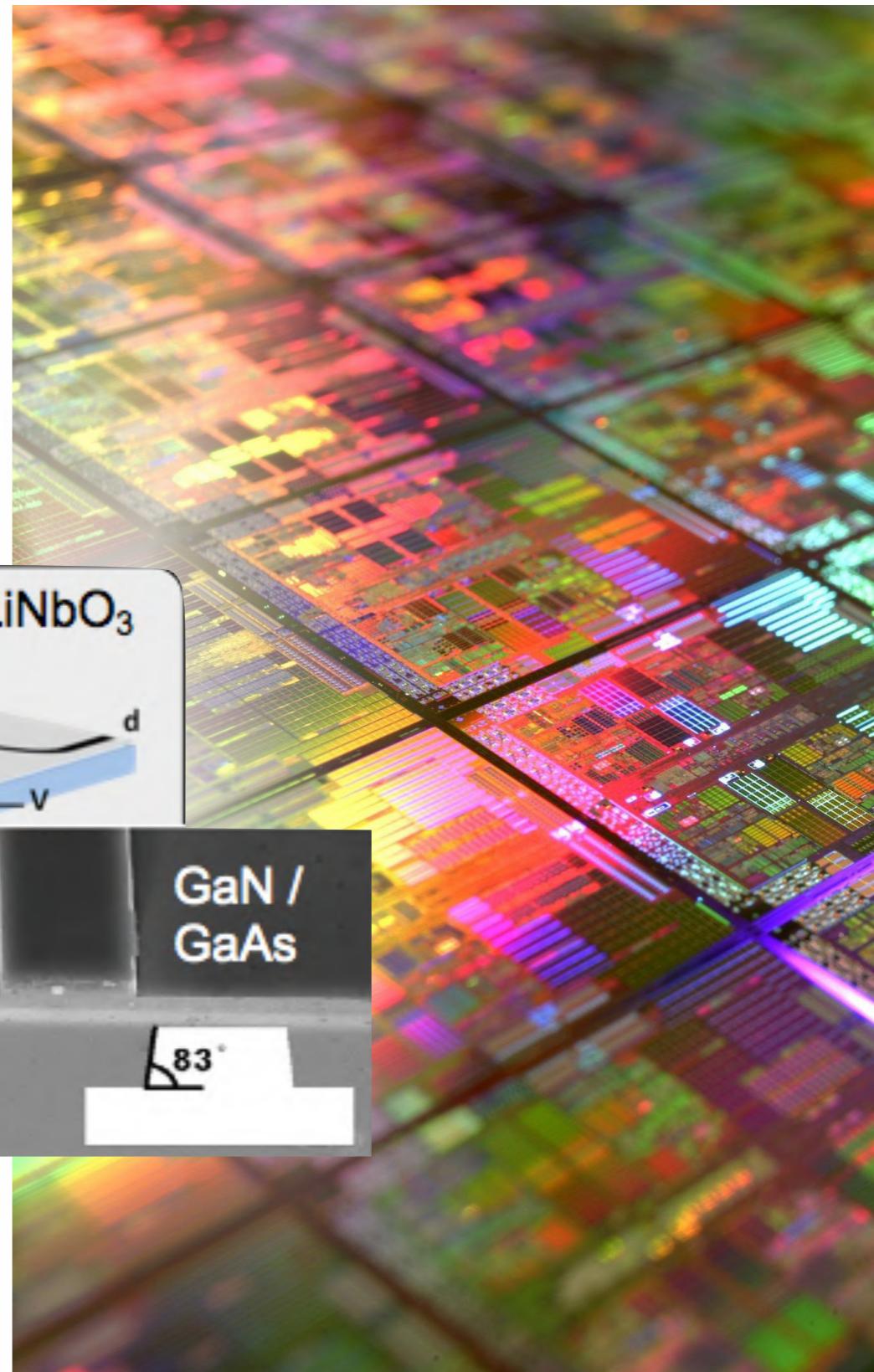
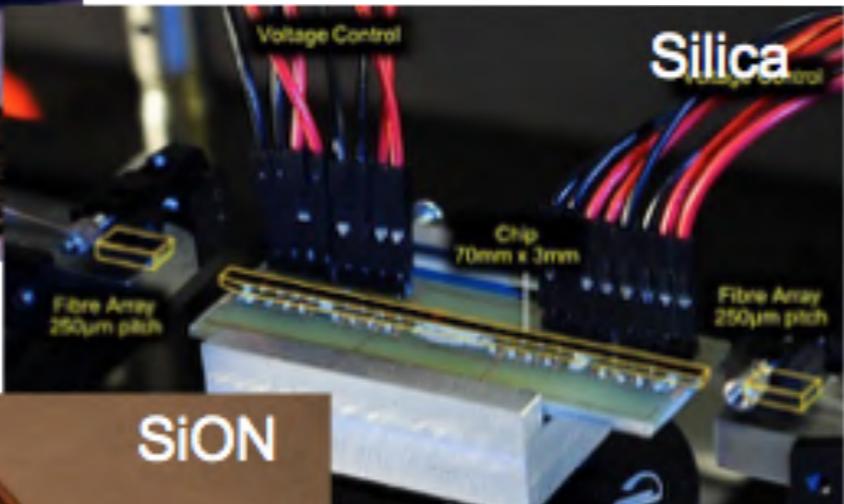


- **Miniaturisation**
- **Additional functionality** (sources, detectors)
- **Full integration**

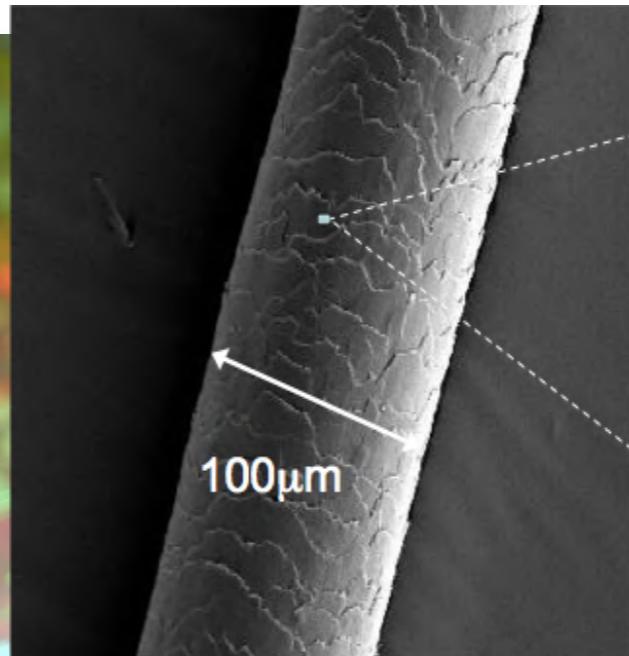
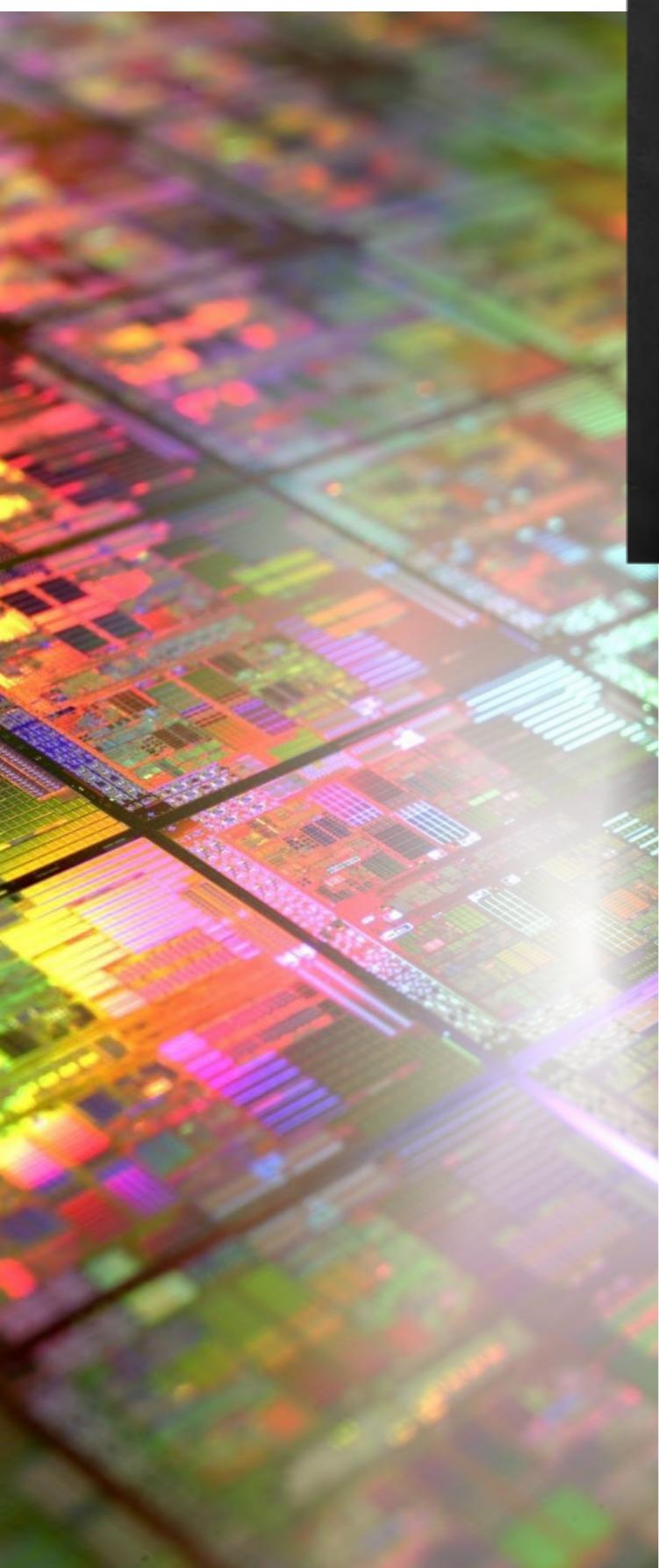


University of
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Materials choices

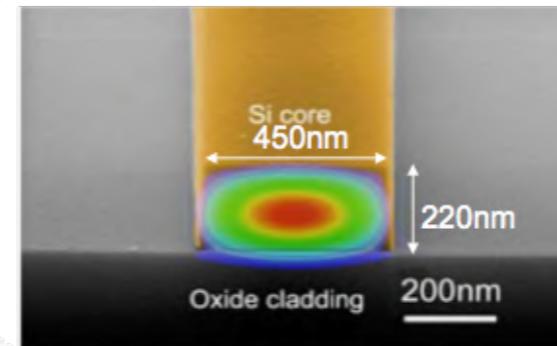


Silicon Quantum photonics



Si waveguide is 200 times smaller

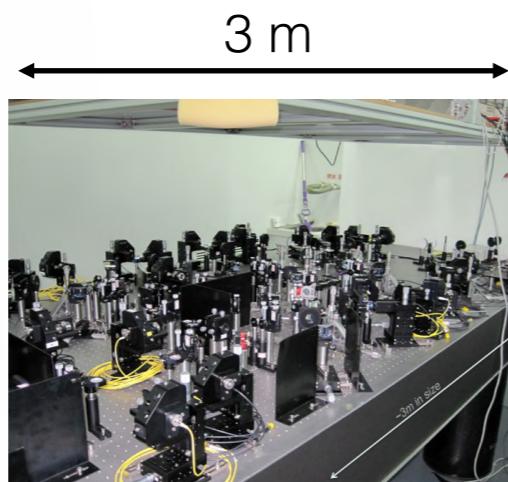
Silicon waveguide



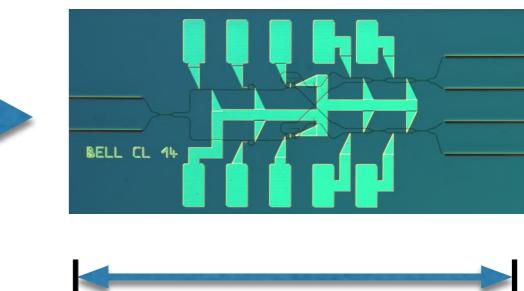
Ultra-high confinement of light

silicon

- Ultra-compact waveguides
 - Small bend radius ($<1\mu\text{m}$)
 - High component density
- Mature semiconductor fabrication processing
- High non-linearity efficient sources
- High confinement efficient detectors
- Filters have been demonstrated
- Integration with electronics



> 1 cm



1.4 mm

Silicon photonics components

spot-size converter

grating coupler



multimode coupler

directional coupler

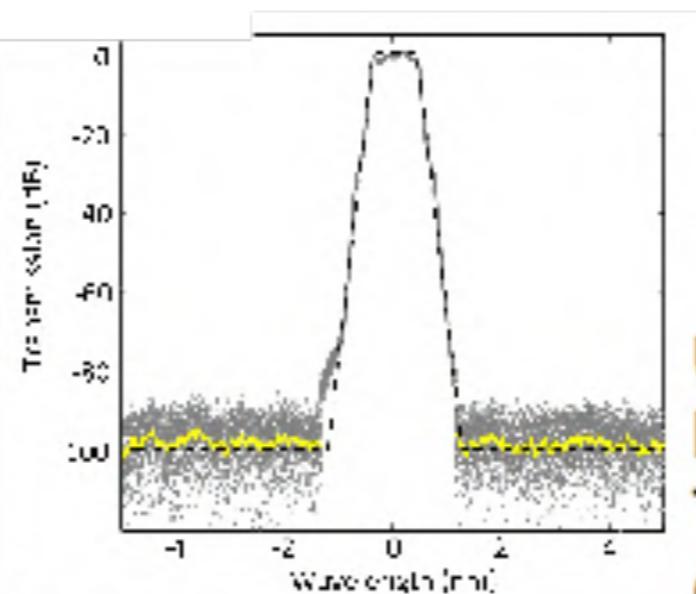
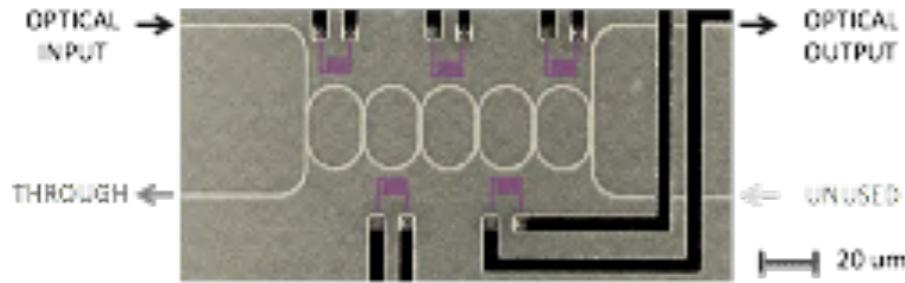
phase shifter

resonator

spiral

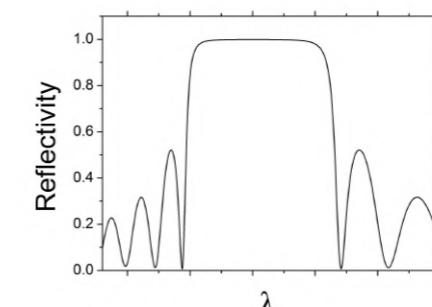
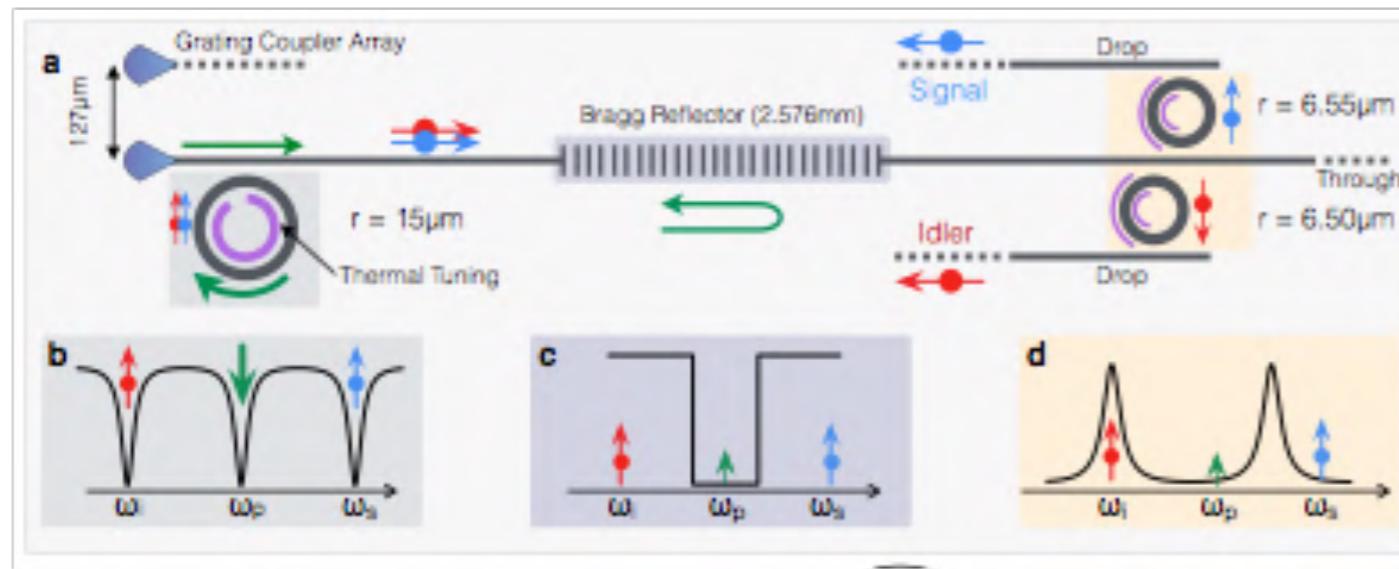
Integrated filters in SOI

>100 dB extinction ratio



UCSD: Ong, Kumar,
Mookherjea, IEEE Photon.
Technol. Lett. **25**, 1543
(2013).

Integration with photon source

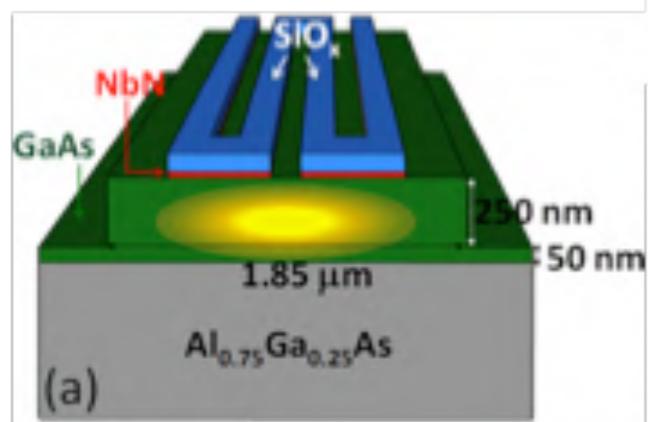


95 dB
CAR 50Hz

MIT: Harris, D. Grassani,
Simbula, Pant, Galli, Baehr-
Jones, Hochberg, Englund,
Bajoni, Galland, (2014).

Detectors

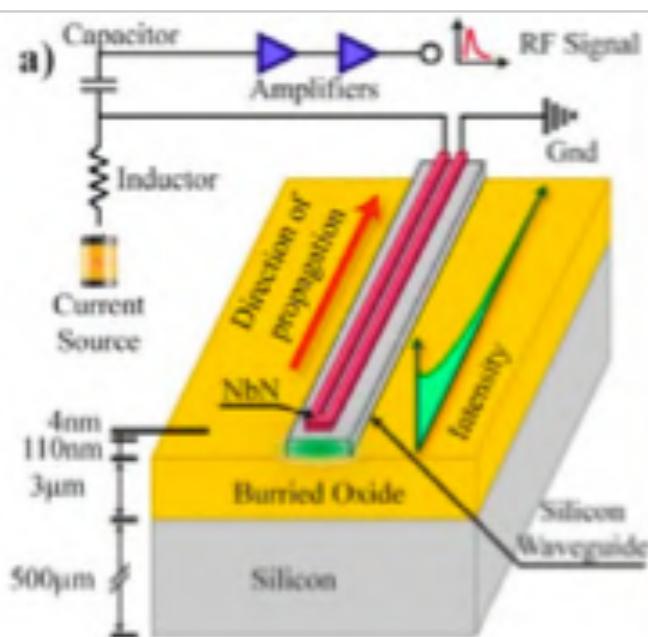
GaAs waveguide superconducting detector



$\eta=20\%$

Sprengers, Gaggero, Sahin, Jahanmirinejad, Frucci, Mattioli, Leoni, Beetz, Lermer, Kamp, Höfling, Sanjines, Fiore, Appl. Phys. Lett. **99**, 181110 (2011)

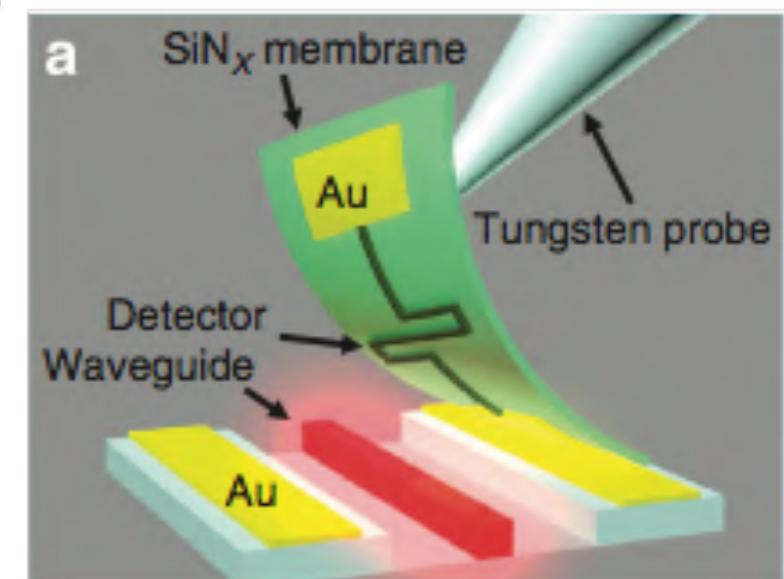
Silicon waveguide superconducting detector



$\eta=91\%$

Pernice, Schuck, Minaeva, Gol'tsman, Sergienko, Tang, Nat Comms **3**, 1325 (2012)

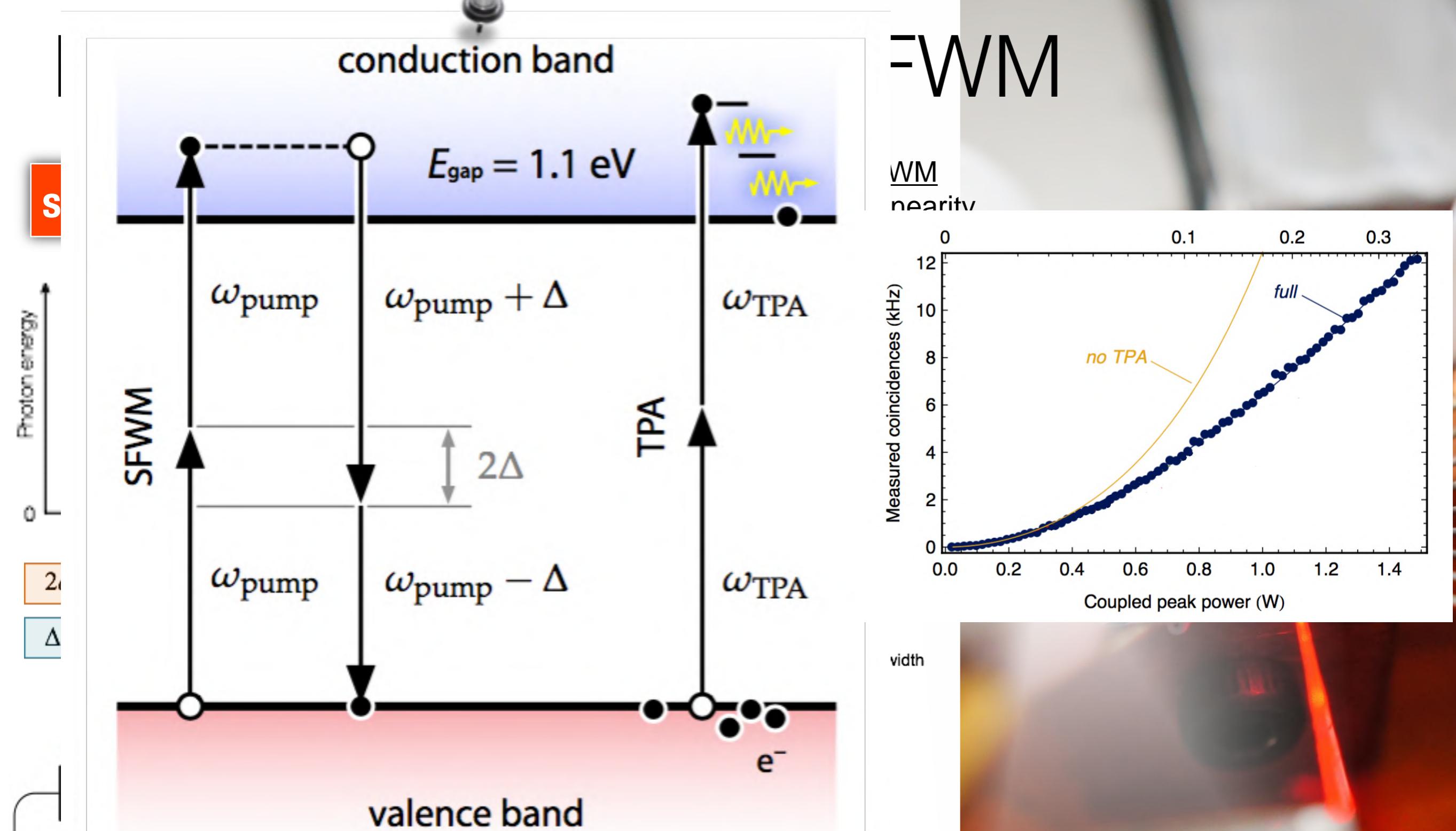
Flip-chip integration



Najafi, Mower, Harris, Bellei, Dane, Lee, Hu, Kharel, Marsili, Assefa, Berggren, Englund, Nat Comms **6**, 5873 (2015).



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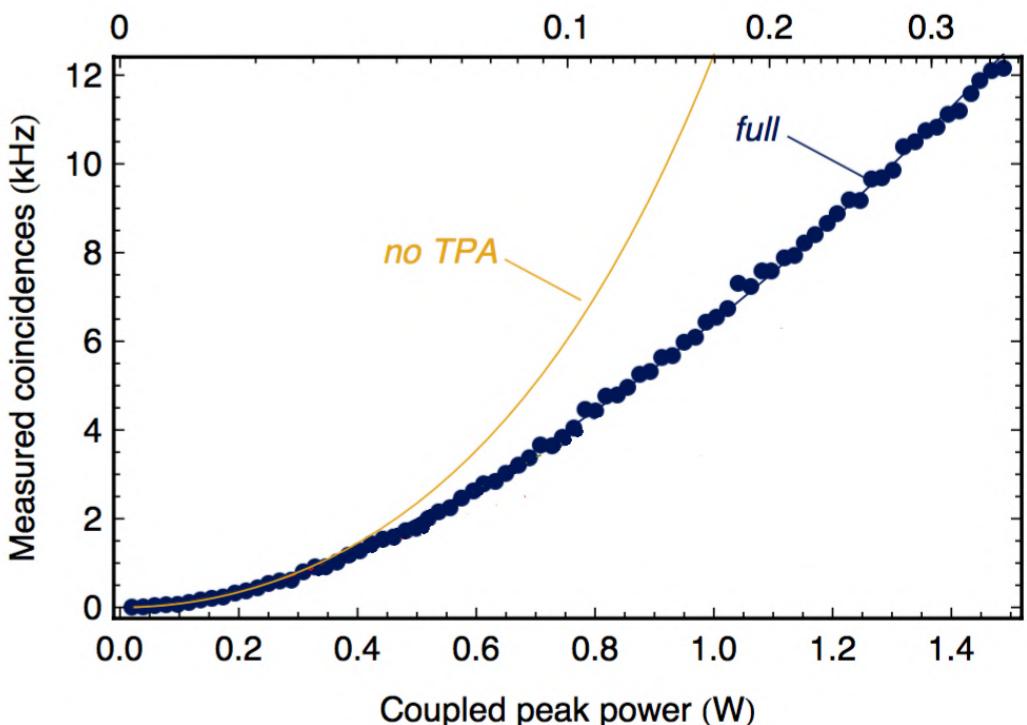


Priem, Dumon, Bogaerts et al.
2005

Silverstone et al. 2016

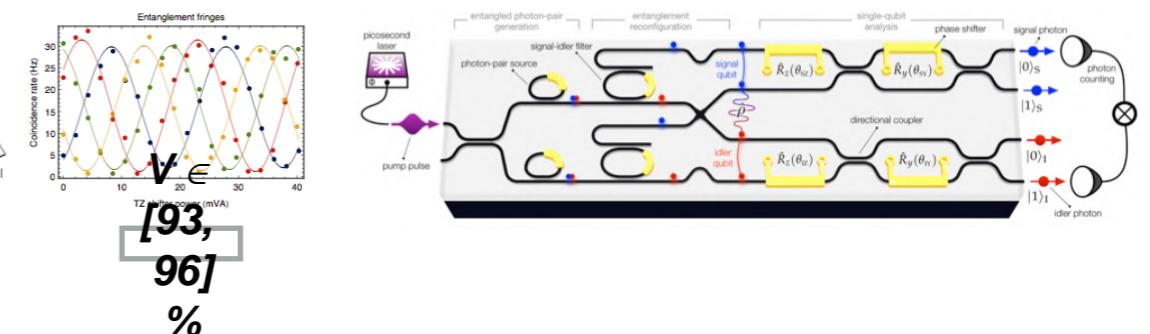
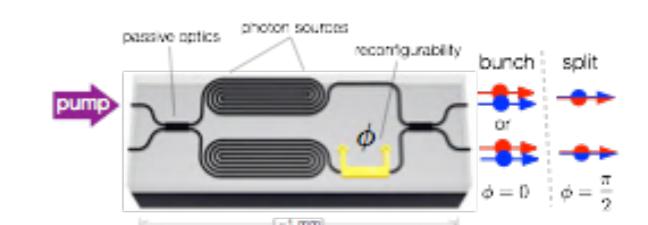
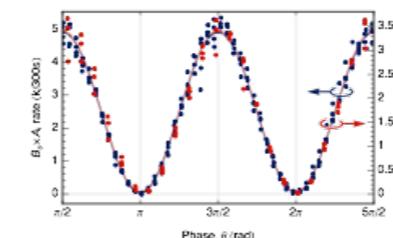
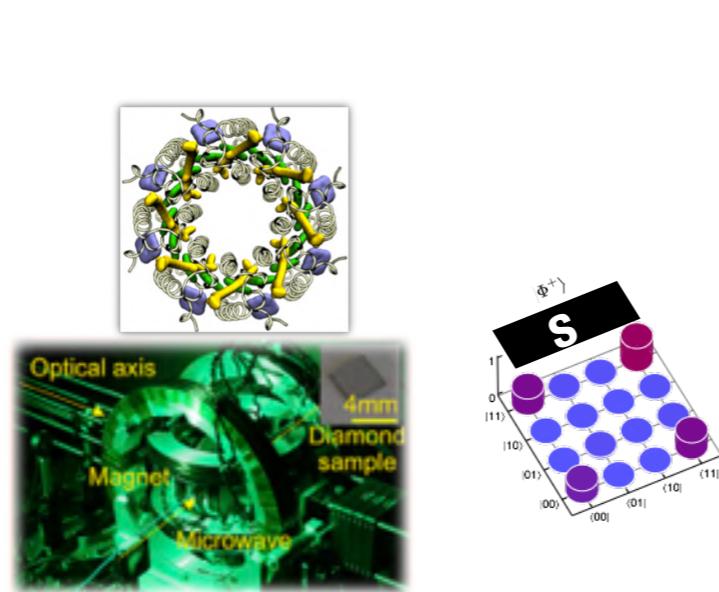
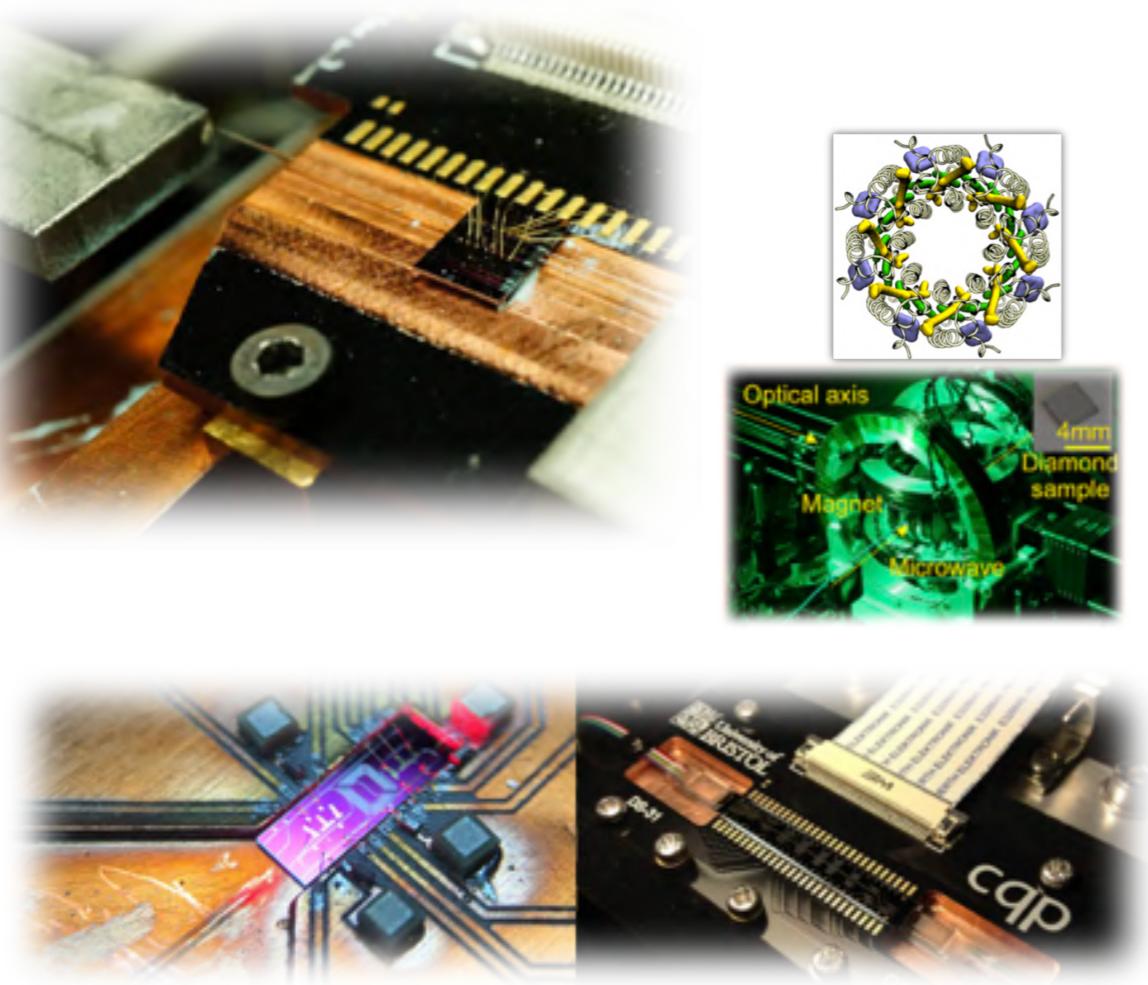
-WM

WM
nearity



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Quantum information processing and communication in Si QP



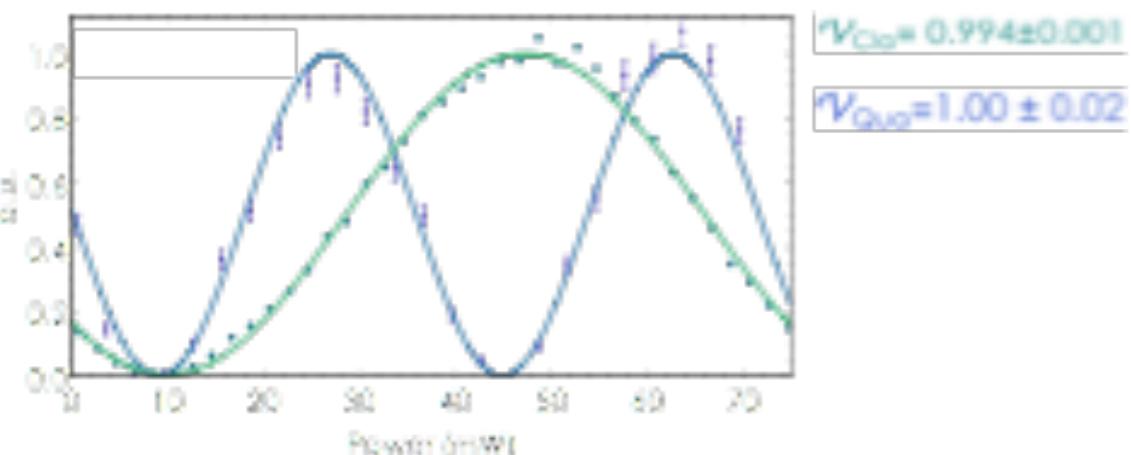
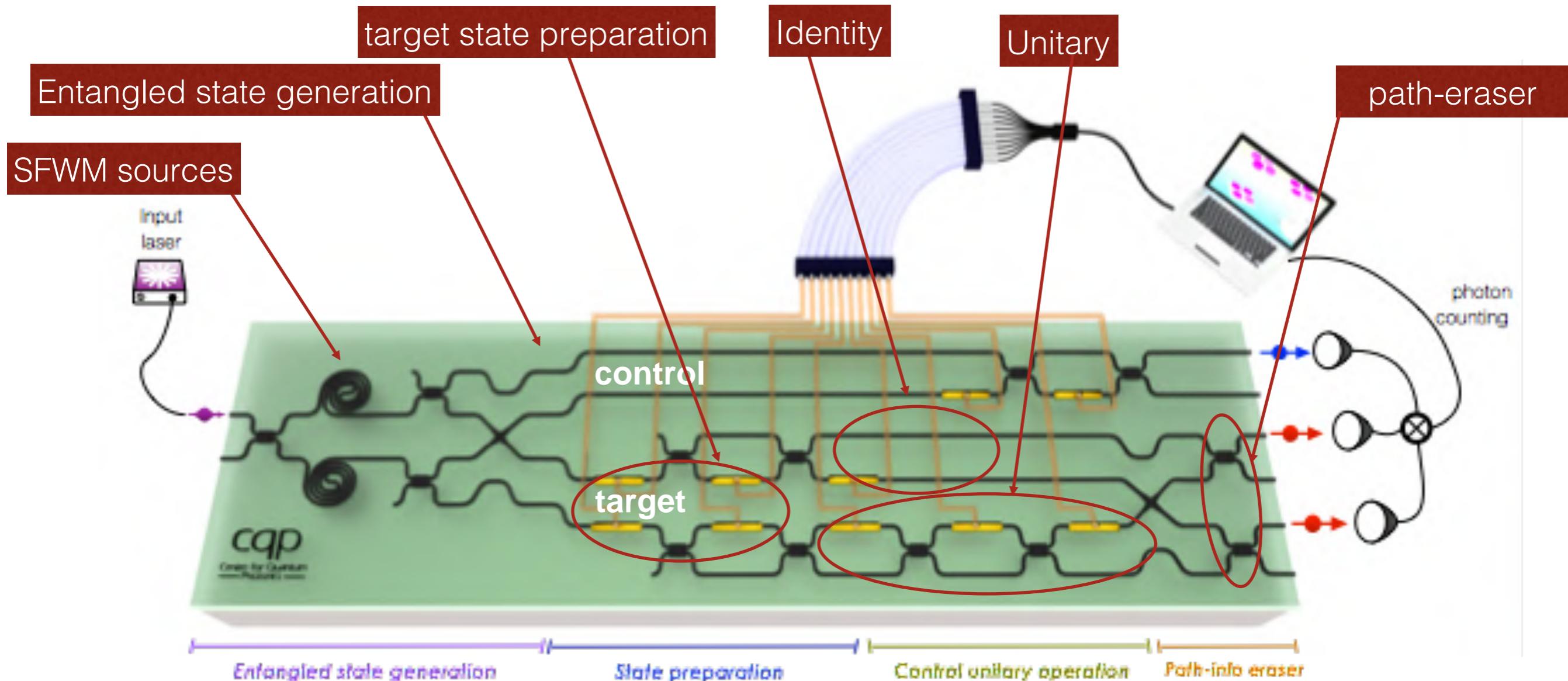
- Quantum simulation
- Bayesian Phase Estimation
- Hamiltonian Learning

Santagati et al. (2016)
Paesani et al. (2016)
Wang et al. (2016)



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CU in Integrated Optics



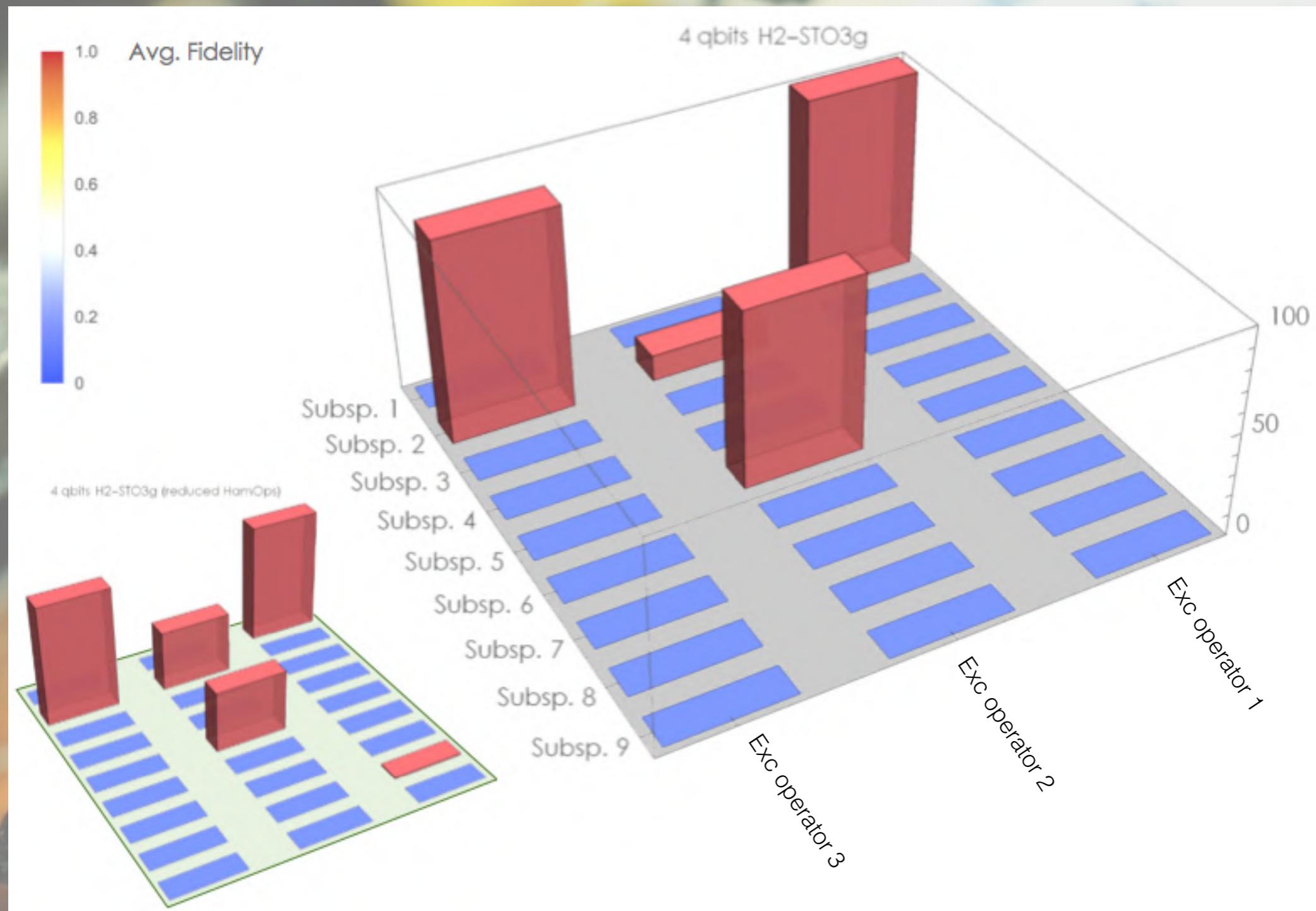
$$|0\rangle_C \otimes I |\psi\rangle_T + |1\rangle_C \otimes U |\psi\rangle_T$$

Zou et al. (2013)



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Simulations



Typical Silicon quantum photonics experiment

