Neither Contextuality nor Nonlocality Admits Catalysts

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- but there is a free transformation $d \otimes e \to d \otimes f$

If the resource theory is catalysis-free, this never happens. Writing $e \rightsquigarrow f$ for the existence of a free transformation, this is equivalent to saying that $d \otimes e \rightsquigarrow d \otimes f$ implies $e \rightsquigarrow f$ for any d, e, f.

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Overview

- As the resource theory of contextuality we use that of

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- The resource theory of non-locality: the n-partite version of the above

- Proof idea: if you can catalyze once you can catalyze arbitrarily many times. For big enough n this implies that one needs only a compatible (and hence non-contextual) part of d.[1]

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Formalising empirical data

A **measurement scenario** $S = \langle X_S, \Sigma_S, O_S \rangle$:

- X_S – a finite set of measurements
- Σ_S – a simplicial complex on X_S
  faces are called the **measurement contexts**
- $O_S = (O_x)_{x \in X_S}$ – for each $x \in X_S$ a non-empty outcome set O_x. Joint outcomes over $U \subseteq X_S$ denoted by $\mathcal{E}_S(U)$.

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a_0	b_1	$1/2$	0	0	$1/2$
a_1	b_0	$1/2$	0	0	$1/2$
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$X = \{a_0, a_1, b_0, b_1\}, \; O_x = \{0, 1\}$

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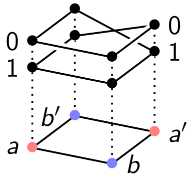
An **empirical model** $e = \{e_\sigma\}_{\sigma \in \Sigma}$ on S:

- each $e_\sigma \in \mathrm{Prob}\,(\mathcal{E}_S(\sigma))$ is a probability distribution over joint outcomes for σ.
- *generalised no-signalling* holds: for any $\sigma, \tau \in \Sigma_S$, if $\tau \subseteq \sigma$,

$$e_\sigma|_\tau = e_\tau$$



Contextuality

An empirical model $e = \{e_\sigma\}_{\sigma \in \Sigma}$ on a measurement scenario (X, Σ, O) is **non-contextual** if there is a distribution d on $\prod_{x \in X} O_x$ such that, for all $\sigma \in \Sigma$:

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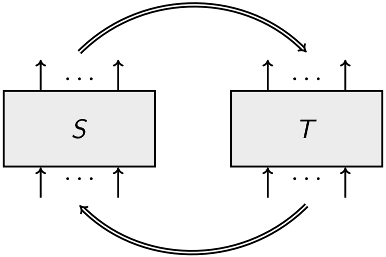
The import of Bell's and Kochen–Spekker's theorems is that there are behaviours arising from quantum mechanics that are contextual.

Maps between scenarios



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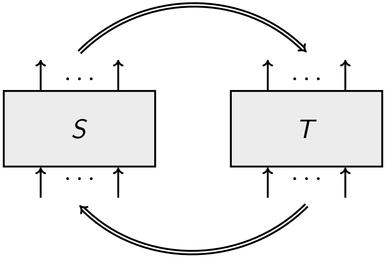

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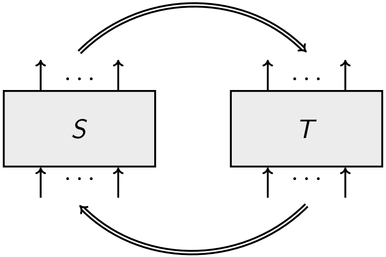

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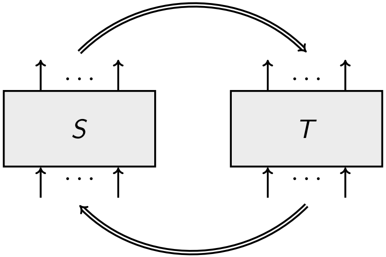

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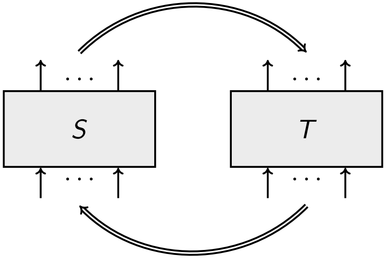

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Simpliciality of π means that contexts in Σ_T are mapped to contexts in Σ_S.

Simulations

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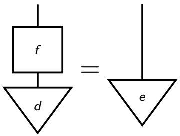

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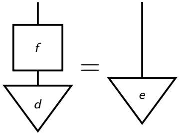

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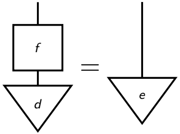

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But what if we want to

(i) let a measurement of T to depend on a measurement protocol of S?

(ii) use classical randomness?

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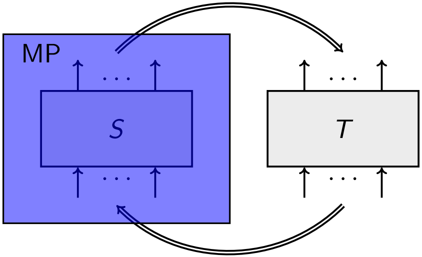
- measurement protocols are compatible if they can be combined consistently

Adaptive procedure



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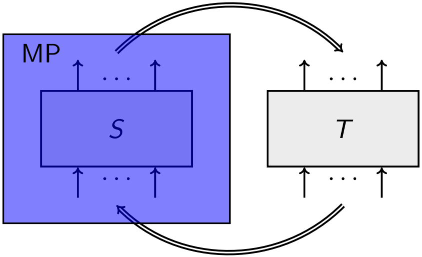

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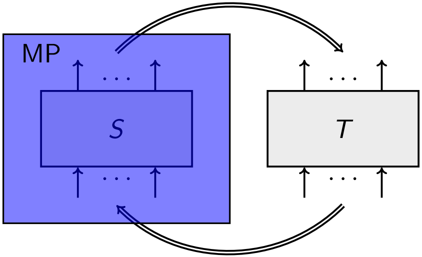

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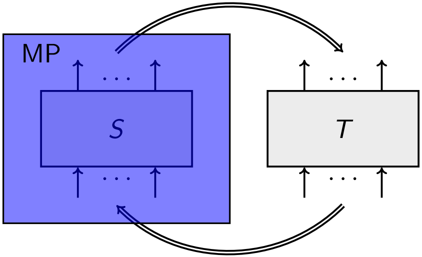

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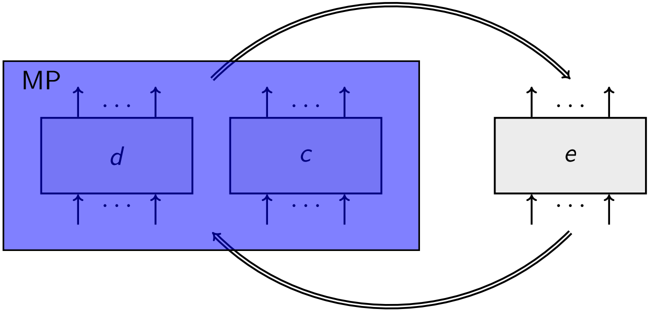
- map outcomes of $\mathrm{MP}(S)$ to outputs of T

Adaptive procedure with classical randomness



Requirement: c is noncontextual.

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We denote the existence of a simulation of e by d as $d \rightsquigarrow e$, read "d simulates e".

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The convertibility relation $\rightsquigarrow$ results in a resource theory of contextuality with nice properties:

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- Contextual fraction is a monotone
- Contextuality is equivalent to insimulability from a trivial model. Variants for logical and strong contextuality.

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Theorem (No-catalysis)

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Second step—if you can catalyze once, you can do so many times: we can get deterministic simulations $\mathrm{MP}(d \otimes (g^{\otimes n})) \to d \otimes (f^{\otimes n})$ for any n so that the i-th copy of f uses d and the i-th copy of g, but otherwise the copies of f are simulated similarly.

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$\square$

Non-locality

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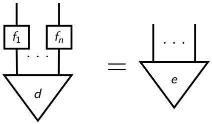

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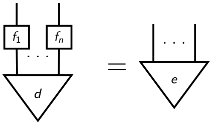

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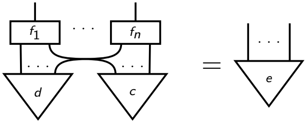

where c is local. This captures the LOSR-paradigm.

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Thus we can't use a PR box as a catalyst, even if we can freely use quantum correlations.

Questions...

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MK, "Neither Contextuality nor Nonlocality Admits Catalysts" (2021), Phys. Rev. Lett. 127, 160402
arXiv:2102.07637