

- A parameter is a numerical fact about a population
- A census with 100% response rate and no response-bias is the ideal method of exactly determining population parameter
- In most cases it is impossible to determine the parameter exactly, can be estimated using a sample (part of the population)
- Factors for a good estimate (for generalising)
 - 1 Sampling frame must contain population of interest
 - 2 Probability sampling (probability of being chosen is non-zero and known)
 - 3 Must be large enough
 - 4 High response rate
- Factors 1 and 2 result in selection bias if not enforced
- estimate = parameter + bias + random error
- Bias is influenced by 1, 2 and 4: aim for minimal selection bias and non response
- Random error is influenced by 3. Larger sample -> smaller error
- Selection bias if 1 or 2 not enforced. All non probability sampling (e.g. convenience/ volunteer sampling) results in selection bias

PROBABILITY SAMPLING

Simple random sampling (use of chance)

- Draw units from the population at random without replacement
- Chance of selection at every stage changes, but chance of ending up in final sample is the same
- + Tends to be good representation of population
- - Subject to non response

Systematic sample

- Selecting units from a list by applying a selection interval (that is randomly derived)
- + simpler selection process, can be treated like a simple random sample if numbers are assigned randomly
- May not be representative if sampling list is non-random

Stratified sampling

- Population divided into subgroups (strata), and a random sample is taken from each strata
- Good to use for estimations within subgroups in addition to estimating within population parameter
- Estimate of parameter is done by taking weighted average of subgroup estimates
- Able to get representation from every strata
- Need information about sample frame and stratum

- Cluster sampling
 - Population is broken down into clusters, then randomly sample a fixed number of clusters
 - All observations from selected clusters are included
 - + less tedious, less time consuming
 - - high variability due to dissimilar clusters or small number of clusters

NON-PROBABILITY SAMPLING

- Selection is done by human discretion rather than probability
- Includes convenience sampling and volunteer sampling

Convenience sampling

- Researcher uses subjects that are most easily available to participate in the study
- - Prone to selection bias (some parts of demographic left out)
- - Vulnerable to non-response bias

Volunteer sampling

- Researcher seeks volunteers to participate in the study
- - Non-response bias, where people who do not volunteer are left out
- - Selection bias when some members are not included

VARIABLES

Independent vs dependent variable

- Independent: subject to manipulation in a study
- Dependent: variable hypothesised to change depending on how independent variable is manipulated

Categorical variables

- Take category of label values, each observation can only be placed in one label and labels are mutually exclusive
- **Ordinal**: variables are categories come with natural order and numbers are used to represent it (e.g. happiness)
- **Nominal**: have no intrinsic ordering (e.g. eye colour)

Numerical variables

- Takes numerical values for which arithmetic operation makes sense
- **Discrete**: possible values of the variable form a set of numbers with gaps (e.g. MCs for modules)
- **Continuous**: can take on all possible numerical values (e.g. time)

Summary statistics

- Measures of central tendencies: mean, median, mode
- Measures of dispersion: Standard deviation, interquartile range
- Standard deviation and IQR does not relate to spread pattern
- First quartile is the 25th percentile of data values
- Standard deviation = square root of variance ≠ spread pattern

$$s^2 = \frac{\sum (x_i - \bar{x})^2}{n - 1}$$

Mean

- Mean is defined as the sum of all data points divided by the number of data points
- **Adding a constant value to all data points changes that mean by that constant value**
- **Multiplying a value c to all data points results in the mean being multiplied by that constant c**

Median

- Median of a numerical value in a data set is the middle value of the variable after arranging all the values in ascending / descending order
- Does not indicate total value, frequency or distribution
- **Adding a constant value to all data points changed the median by that constant value**
- **Multiplying a value c to all data points results in the median being multiplied by constant c**

Mode

- The mode is that value that appears most frequently in the data set
- When describing distribution of points of a discrete variable, mode is the "peak" of the distribution graph

IQR ≠ spread pattern

- Interquartile range is the difference between the third quartile and first quartile
- IQR is always non-negative
- **Adding a constant to all data points does not change the IQR**
- **Multiplying a value c to all data points results in the IQR being multiplied by |c|**

- Experimental studies
- Primary goal is to prove a cause and effect relationship between 2 variables
 - To establish cause and effect, the independent variable should be the only variable that results in a change in the dependent variable
 - Researchers can assign participants to control and experimental groups (random assignment is best)
 - Random assignment: random draw without replacement
 - Groups can have different sizes as long as sizes are quite large

- Blinding
- Blinding is done to guard against human bias.
 - In a single-blinded experiment, either participants or evaluators do not know whether they are in (or evaluating) the treatment or control groups
 - In a double-blinded experiment, both participants and evaluators do not know whether they are in (or evaluating) the treatment or control groups
 - Researchers still know which group is subjected to which treatments

- Observational studies
- In an observational study, participants self assign to each of the respective groups (may be unethical to assign)
 - Observational studies can only establish association between variables
 - Association does not imply causation
 - **A confounder is a third variable associated with both the independent and dependent variables**
 - Confounder must be a different variable than the dependent and independent variables
 - The more confounders that the study can control and still show association results in stronger evidence for a genuine relationship

- Comparison
- Comparison method is used to see the effect of treatments on outcomes
 - Comparison of outcomes between a treatment and control group
 - Control and treatment groups should be similar
 - Subjects should be put in control and treatment groups randomly
 - A large number of participants should be used. Law of large numbers ensures all other variables are almost equally present in both groups

- RATES**
- **Marginal** rate is the probability of an event occurring; not conditioned on another event occurring
 - **Joint** rate is the probability of 2 events occurring together (i.e. intersection of the probabilities of 2 events, $P(A \text{ and } B)$; denominator is total number
 - **Conditional** rate is the probability of event A occurring given that event B occurs, $P(A|B)$; denominator is B (given condition)

- ASSOCIATION**
- A and B are associated with each other if rate $(A|B) \neq \text{rate}(A | \text{not } B)$
 - A and B are positively associated if rate $(A|B) > \text{rate}(A | \text{not } B)$
 - A and B are negatively associated if rate $(A|B) < \text{rate}(A | \text{not } B)$
 - A and B are interchangeable due to symmetry of rates, i.e. rate $(A|B) > \text{rate}(A | \text{not } B)$ iff rate $(B|A) > \text{rate}(B | \text{not } A)$
 - Symmetry rule:
 - $\text{rate}(A|B) > \text{rate}(A|NB) \leftrightarrow \text{rate}(B|A) > \text{rate}(B|NA)$
 - $\text{rate}(A|B) < \text{rate}(A|NB) \leftrightarrow \text{rate}(B|A) < \text{rate}(B|NA)$
 - $\text{rate}(A|B) = \text{rate}(A|NB) \leftrightarrow \text{rate}(B|A) = \text{rate}(B|NA)$

- BASIC RULE OF RATES**
- rate(A) is always between rate $(A|B)$ and rate $(A|NB)$
 - Given rate $(A|B) = x$ and rate $(A|C) = y$, with B and C disjoint, $\min\{x,y\} \leq \text{rate}(A | B \cup C) \leq \max\{x,y\}$
 - The closer rate (B) gets to 100%, the closer rate (A) gets to rate $(A|B)$
 - Rate (A) is exactly in between rate $(A|B)$ and rate $(A| \text{not } B)$ if rate (B) = 50%

- SIMPSON'S PARADOX**
- Relationship between rates in subgroups is reversed / disappears when subgroups are combined
 - **Sure sign of confounder**
 - To determine is a variable is a confounder, data must be collected on it
 - Allocate factors proportionately to remove association between variable and treatment type
 - Randomized assignment gives equal proportion most of the time, but is not always possible as people cannot be forced
 - To control confounder, slicing is used
 - Slicing: subgroup analysis is used (conclude based on subgroup numbers instead of misleading overall data)

	Hair type				
	Straight		Curly		
Colour	Male	Female	Male	Female	Total
Red	7	9	8	5	29
Brown	35	20	12	16	83
Blonde	51	55	38	27	171
Black	22	25	19	24	90
Total	115	109	77	72	373

The marginal rate, rate(Curly), is (a); while the joint rate, rate(non-Black and Female) is (b) %.

To **calculate the marginal rate**, rate(Curly), we take the column totals of all Curly-haired persons (both Male and Female) divided by the grand total of everyone in the data set, $\frac{(77 + 72)}{373} \approx 39.95\%$

To **calculate the joint rate**, rate(non-Black and Female), we take the count of “Females with non-black hair” divided by once again the grand total of everyone in the data set, i.e. $\frac{(9 + 20 + 55 + 5 + 16 + 27)}{373} \approx 35.39\%$

	Female	Male	Total
Gamer	48	96	144
Non-Gamer	72	64	136
Total	120	160	280

To **calculate the conditional rate**,
 $\text{rate}(\text{Female} | \text{Gamer}) = \frac{48}{144} = 0.33$,
 $\text{rate}(\text{Female} | \text{Non-Gamer}) = \frac{72}{136} = 0.53$.
 Since $\text{rate}(\text{Female} | \text{Gamer}) < \text{rate}(\text{Female} | \text{Non-Gamer})$, there is negative association between being female and being a gamer.