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2016
MCM/ICM
Summary Sheet

(Your team's summary should be included as the first page of your electronic submission.)
Type a summary of your results on this page. Do not include the name of your school, advisor, or team members on this page.

Water is so significant for living things that without water there will be no life on the earth, while many regions are facing the trouble that the poor ability to provide clean water to meet the needs of its population. Our target is to build a measuring system to determine weather is a region lack of water or not, meanwhile predict the situation of water resources of a region in 15 years (2015-2029). Besides, we are required to come up with an intervention plan aiming to mitigate water scarcity.

To tackle the issues mentioned above, we build 6 models and conclude our tasks.

In task 1, combining the **Analytic Hierarchy Process (AHP)** and **Fuzzy Synthetic Evaluation (FSE)**, we use **Multi-goal Fuzzy Synthetic Evaluation** to obtain an index which can measure the ability to provide clean water of a region. First, according to related data we choose 14 indicators that affect the index. Then, we get the weight matrix by AHP and find 5 most important indicators. Next, with the help of FSE we get the judgment matrix and work out the index. Finally, we test the model and put forward a standard to measure a region's water shortage degree.

In task 2, we pick China to analyze on the basis of model built in task 1. On the ground of data from World Bank, we calculate the index of China in 2013. What's more, we give further analysis about social and environmental drivers combining actual situation in China.

In task 3, to predict the situation of water resources of China in 15 years, we establish 5 models for 5 indicators separately using **ARIMA model**, **Logistic model** and **linear fitting**. Via the models we have built , we calculate the value of 5 indicators in 15 years, then we figure out the index of China in 15 years to predict the water situation. In the end, we analyze the effect of the citizens.

In task 4, we deal regulation coefficients according to the conditions. There are two thresholds to set regulation coefficients, n is in a limitation. When total water use is larger than the amount of fresh water can be cycled, using strong intervention, or taking soft intervention.

In task 5, we set a series of regulation coefficients to simulate China's intervention plan. From the output, the Carrying Capacity of Water Resources of China is becoming better and better. It proved our intervention plan is effective.

Additionally, we analyze the strengths and weaknesses of our models.

Are we heading towards a thirsty planet?

Contents

1 Introduction

Water is very important for living things. Without water there can be no life on earth. All animals and plants need water. Man also needs water.

It is generally believed that there is a good supply of fresh water. But to our disappointment, the fact is just the opposite. According to the United Nations, 1.6 billion people (one quarter of the world's population) experience water scarcity. A number of regions are to seek in the ability to provide clean water to meet the needs of its population.

Take China as an example, China is confronting about water shortage issues for a long time because of its numerous population (more than 20 percent of world's population) and lack of freshwater (less than 7 percent).

To address the problem above and provide a fifteen-year (2015-2029) water strategy for China, we conclude sub-problems to tackle in our paper.

● Build a model to measure the ability of a region to provide clean water to meet the needs of its population.

● Build a model to show what the water situation will be in 15 years in a region.

● Propose intervention plans on the basis of all the drivers of water scarcity.

● Build a model to project water availability into the future and the susceptibility to water scarcity of our chosen region.

2 Solution of Task 1

2.1 Introduction

In this task, we need to judge can a region provide clean water to meet the needs of its population or not, which requires us to find out a method to develop a standard to measure it quantitatively. We build two models to tackle this problem. **Model 1**: using AHP to determine the evaluation indicators which affect the region's ability of provide clean water. **Model 2**: According to the evaluation indicators determined in Model 1 figure out an index to describe the region's ability of provide clean water quantitatively.

2.2 Model 1: Analytic Hierarchy Process(AHP) Model

2.2.1 Selecting the Preliminary Indicators

In this task, we decide to use **Carrying Capacity of Water Resources** to describe the local ability to provide clean water. According to the analysis, we have found the following relationship: **Firstly**, water supports the local social and economic development, then the local social and economic development will influence the water back with positive and negative factors. On one hand local social and economic development push the speed of technology development, on the other hand, local social and economic development generate a lot of polluted water into the water resource, it reduces the quality of water resource. So the local social and economic development is one of the important factors to measure Carrying Capacity of Water Resources. **Secondly**, the local condition of water resources is another important factor. People have more water, the more water can be used. **Thirdly**, the water supply capacity limits people using water. **Fourthly**, water demand reflects people's water usage, the more water people need, the more stress on the Carrying Capacity of Water Resources.

In summary, **local social and economic development**, **the local condition of water resources**, **the water supply capacity** and **water demands** are the 4 main factors. We list 4 main factors and their sub factors in Table 1.

Table 1: The main factors and their sub factors affecting the water carrying capacity

TYPE	DETAIL
the local condition of water resources B_1	C_1:Per capita water resources C_2:Precipitation C_3:The total amount of groundwater
the local social and economic developmentB_2	C_4: Per capita GDP C_5:Population density C_6:The proportion of population has access to clean water C_7:Health facilities
water supply capacityB_3	C_8:Reservoir storage C_9:Per capita water supply C_{10}:Wastewater reuse rate C11:Renewable internal freshwater resources
water demandB_4	C_{12}:Per capita industrial water C_{13}:Per capita agricultural water C_{14}:Per capita domestic water

By going through Water Resources Bulletin over the years supplied by Chinese Ministry of Water Resources, we found that the amount of annual supply and storage capacity are equal. After consideration, we believe that China is in a stage of rapid development, in short supply, so the provided water resources are all utilized with no remaining. Therefore, we only consider the demand of water (ie the supplied amount of water is equal to the demand of water, the amount of industrial water supply per capita is equal to the per capita industrial water demand, the amount of agricultural water supply per capita is equal to the per capita agriculture water demand, etc.). Through group discussions, we believe that per capita water resources is more representative than the total water resources, so we only take per capita water resources into consideration limited by space and time. Treatment method is similar as for total GDP and per capita GDP, total industrial water and per capita industrial water, total agricultural water and per capita agricultural water, domestic water and per capita domestic water.

The 14 evaluation indicators listed above involve most of the factors that have an effect on the Carrying Capacity of Water Resources, that is to say we can take the factors as parameters.

2.2.2 Correlation Analysis and Determining

Then we use **Analytic Hierarchy Process (AHP)** to sort the evaluations. Filter by AHP and calculate the evaluation index weight distribution can greatly reduce the subjective factor, so that complex problems of each factor is divided into orderly layers are linked, so principled.

Table 2.1: The comparison matrix from A to B

A	B_1	B_2	B_3	B_4
B_1	1	2	8	2
B_2	1/2	1	1/3	1/2
B_3	1/8	3	1	1/7
B_4	1/2	2	7	1

Table 2.2: The comparison matrix from B_1 to C

B_1	C_1	C_2	C_3
C_1	1	5	6
C_2	1/5	1	2
C_3	1/6	1/2	1

Table 2.3: The comparison matrix from B_2 to C

B_2	C_4	C_5	C_6	C_7
C_4	1	4	5	5
C_5	1/4	1	2	2
C_6	1/5	1/2	1	1
C_7	1/5	1/2	1	1

Table 2.4: The comparison matrix from B_3 to C

B_3	C_8	C_9	C_{10}	C_{11}
C_8	1	1/3	2	2
C_9	3	1	2	2
C_{10}	1/2	1/2	1	1
C_{11}	1/2	1/2	1	1

Table 2.5: The comparison matrix from B_4 to C

B_4	C_{12}	C_{13}	C_{14}
C_{12}	1	1	1
C_{13}	1	1	1
C_{14}	1	1	1

Table 3: The result of levels of total sort and consistency of test

C	B_1	B_2	B_3	B_4	The sort result of C	Consistency
C_1	0.7258				0.3324	
C_2	0.1721				0.0788	0.0279
C_3	0.1020				0.0467	
C_4		0.5993			0.1064	
C_5		0.1892			0.0336	0.0104
C_6		0.1058			0.0188	
C_7		0.1058			0.0188	
C_8			0.2471		0.0230	
C_9			0.4359		0.0130	0.0579
C_{10}			0.1585		0.0083	
C_{11}			0.1585		0.0083	
C_{12}				0.3333	0.1039	
C_{13}				0.3333	0.1039	0.0000
C_{14}				0.3333	0.1039	

After the total level of sorting, we check the consistency of judgment throughout the hierarchical model. And all levels of consistency test result CR is less than 0.1, indicating that all levels of the sort has satisfactory consistency. And the A to B matrix in CR is 0.0212<0.10, indicating a satisfactory overall level sorting consistency.

From the all evaluations, we can clearly find **C_1 Per capita water resources**, **C_4 Per capita GDP**, **C_{12} Per capita industrial water**, **C_{13} Per capita agricultural water**, **C_{14} Per capita domestic water**, are the most important factors. We choose them in our judging model, take normalization processing to calculate the weight of each index weight factor:

$$a_i = C_i \Big/ \sum_{i=1}^{5} C_i$$

And we can get the weight matrix A = (0.443, 0.142, 0.138, 0.138, 0.138).

2.2.3 Strengths and Weaknesses

Strengths:

- The model is on the basis of a **systematic analysis method**, which makes the factors pretty clear and definite.
- The model can find out important factors **conveniently and practically**.

Weaknesses:

- The model contains some subjective components which may lead to inaccuracy in the following study.

2.3 Model 2: Multi-goal Fuzzy Synthetic Evaluation

2.3.1 Introduction

Since there are not specific or clear concepts of our evaluations, we use **Fuzzy Synthetic Evaluation (FSE) Model** to make an overall impression of Carrying Capacity of Water Resources, which is mainly influenced by 5 evaluations (Model 1). In addition, we determine an index to measure the ability of a region to provide clean water to meet the needs of its population.

2.3.2 Rating and Classifying Comprehensive Evaluation

According to analysis above, we define our **evaluation set U** is made up of an *n* (n=5) sub-object, namely,

$$U = \{u_1, u_2, u_3, u_4, u_5\},$$

in which $u_1 \rightarrow C_1$, $u_2 \rightarrow C_4$, $u_3 \rightarrow C_{12}$, $u_4 \rightarrow C_{13}$, $u_5 \rightarrow C_{14}$. And its **comment set V** is made up of an *m* sub-object. Considering different effect of evaluations on Carrying Capacity of Water Resources, the influence level is classified into 3 grades, that is

$$V = \{V_1, V_2, V_3\}.$$

We can rate each classification of every evaluation on the basis of different situation (Table 4). **Grade V_3** represents the situation of water resources is poor, that is to say the carrying capacity of water resources, with little potential for further development and utilization, is close to saturation. **Grade V_1** represents the situation of water resources is good, but the water using degree and development planning are in an early stage. That is to say, in this period, the carrying capacity of water resources is large and water resources have a certain guarantee to meet the development of economic. **Grade V_2** intermediates between Grade V_1 and Grade V_3. In this situation, water resources has a certain scale and potential for further development and utilization, but corresponding steps must be taken.

To reflect the carrying capacity of water resources of each grade better and quantitatively descript the influence level, we score V between 0-1. Here, we simply suppose a_1=0.95, a_2=0.5, a_3=0.05. The higher score of a_i, the more potential to explore and utilize water resources.

Table 4: Comprehensive evaluation of the value classification

evaluation set U	unit	comment set V

			V1	V2	V3
C_1 Per capita water resources	u_1	m^3 /year	>5000	5000-500	<500
C_4 Per capita GDP	u_2	current US$	>40000	40000-10000	<10000
C_{12} Per capita industrial water	u_3	m^3 /year	<50	50-250	>250
C_{13} Per capita agricultural water	u_4	m^3 /year	<100	100-250	>250
C_{14} Per capita domestic water	u_5	m^3 /year	<50	50-200	>200
	scores		a_1=0.095	a_2=0.5	a_3=0.05

2.3.3 Determining the Subordinate Function

Element μ_{nm} judgment matrix R is the subordinate function of u_n and V_m (n=5,m=3). Subordinate functions of $\mathbf{u_1}$, $\mathbf{u_2}$ are shown as below.

$$\mu_{V1}(u_n)\begin{cases}\frac{1}{2}\left(1+\frac{u_n-k_1}{u_n-k_2}\right) & u_n \geq k_1\\ \frac{1}{2}\left(1-\frac{u_n-k_1}{k_2-k_1}\right) & k_1 > u_n \geq k_2\\ 0 & u_n \leq k_2\end{cases}$$

$$\mu_{V2}(u_n)\begin{cases}\frac{1}{2}\left(1-\frac{u_n-k_1}{u_n-k_2}\right) & u_n \geq k_1\\ \frac{1}{2}\left(1+\frac{k_1-u_n}{k_1-k_2}\right) & k_1 > u_n \geq k_2\\ \frac{1}{2}\left(1+\frac{u_n-k_3}{k_2-k_3}\right) & k_2 > u_n \geq k_3\\ \frac{1}{2}\left(1-\frac{k_3-u_n}{k_2-u_n}\right) & k_3 > u_n\end{cases}$$

$$\mu_{V3}(u_n)\begin{cases}0 & u_n \geq k_2\\ \frac{1}{2}\left(1-\frac{u_n-k_3}{k_2-k_3}\right) & k_3 \leq u_n < k_2\\ \frac{1}{2}\left(1+\frac{k_3-u_n}{k_2-u_n}\right) & u_n < k_3\end{cases}$$

Subordinate functions of $\mathbf{u_3}$, $\mathbf{u_4}$, $\mathbf{u_5}$ are shown as below.

$$\mu_{V1}(u_n)\begin{cases}\frac{1}{2}\left(1+\frac{u_n-k_1}{u_n-k_2}\right) & u_n < k_1\\ \frac{1}{2}\left(1-\frac{u_n-k_3}{k_2-k_3}\right) & k_1 < u_n \leq k_2\\ 0 & u_n > k_2\end{cases}$$

$$\mu_{V2}(u_n)\begin{cases}\frac{1}{2}\left(1-\frac{u_n-k_1}{u_n-k_2}\right) & u_n<k_1\\ \frac{1}{2}\left(1+\frac{k_1-u_n}{k_1-k_2}\right) & k_1\le u_n<k_2\\ \frac{1}{2}\left(1+\frac{u_n-k_3}{k_2-k_3}\right) & k_2\le u_n<k_3\\ \frac{1}{2}\left(1-\frac{k_3-u_n}{k_2-u_n}\right) & k_3\le u_n\end{cases}$$

$$\mu_{V3}(u_n)\begin{cases}0 & u_n\le k_2\\ \frac{1}{2}\left(1-\frac{u_n-k_3}{k_2-k_3}\right) & k_2\le u_n<k_3\\ \frac{1}{2}\left(1+\frac{k_3-u_n}{k_2-u_n}\right) & u_n\ge k_3\end{cases}$$

Judging from the function above, we can work out membership r_m, and $r_1=\mu_{v1}(u_n)$, $r_2=\mu_{v2}(u_n)$, $r_3=\mu_{v3}(u_n)$. Then, we can figure out **judgment matrix R**:

$$R=|r_1,r_2,r_3|=\begin{vmatrix}r_{11} & \cdots & r_{1m}\\ \vdots & \ddots & \vdots\\ r_{n1} & \cdots & r_{nm}\end{vmatrix}$$

The value of comprehensive evaluation about carrying capacity of water resources is **B=A·R**. Then we can work out the **index α**:

$$\alpha=(a_1,a_2,a_3)\cdot B^T$$

The higher index, indicating the larger carrying capacity of water resources, and various social economic development of all sectors has a certain safeguards.

2.3.4 Testing

In order to test reliability of the model, we pick 8 nations, which are in different water resources situation (according to the UN water scarcity map), and work out the index separately (Table 5).

Table 5: Comprehensive evaluation of the value classification

	judgment matrix R	B	α
CHN	$R_{CHN}=\begin{vmatrix}0 & 0.76492 & 0.23508\\ 0 & 0.41648 & 0.58352\\ 0.23194 & 0.76806 & 0\\ 0.02774 & 0.97226 & 0\\ 0.46493 & 0.53507 & 0\end{vmatrix}$	$B_{CHN}=(0.10000\,,0.71200\,,0.18700)$	0.46035
RUS	$R_{RUS}=\begin{vmatrix}0.96304 & 0.03696 & 0\\ 0 & 0.64958 & 0.35042\\ 0 & 0.39698 & 0.60302\\ 0.51927 & 0.48073 & 0\\ 0.21088 & 0.78912 & 0\end{vmatrix}$	$B_{RUS}=(0.52739\,,0.33864\,,0.13298)$	0.67698
FRA	$R_{FRA}=\begin{vmatrix}0.00843 & 0.99157 & 0\\ 0.57453 & 0.42547 & 0\\ 0 & 0.27400 & 0.72600\\ 0.58416 & 0.41584 & 0\\ 0.24946 & 0.75054 & 0\end{vmatrix}$	$B_{FRA}=(0.20036\,,0.69845\,,0.10019)$	0.54458
QAT	$R_{QAT}=\begin{vmatrix}0 & 0.33632 & 0.66368\\ 0.89448 & 0.10552 & 0\\ 0.65798 & 0.34202 & 0\\ 0.43828 & 0.56172 & 0\\ 0.28128 & 0.71872 & 0\end{vmatrix}$	$B_{QAT}=(0.31712\,,0.38787\,,0.29401)$	0.50990

	judgment matrix R	B	α
USA	$R_{USA}=\begin{vmatrix}0.83062 & 0.16938 & 0\\ 0.73195 & 0.26805 & 0\\ 0 & 0.09141 & 0.90859\\ 0 & 0.34274 & 0.67526\\ 0 & 0.45855 & 0.54145\end{vmatrix}$	$B_{USA}=(0.47190, 0.23381, 0.29329)$	0.57987
ZAF	$R_{ZAF}=\begin{vmatrix}0 & 0.46356 & 0.53644\\ 0 & 0.41413 & 0.58587\\ 0.63175 & 0.36825 & 0\\ 0.38146 & 0.61854 & 0\\ 0.34375 & 0.65625 & 0\end{vmatrix}$	$B_{ZAF}=(0.18726, 0.49090, 0.32084)$	0.43939
IND	$R_{IND}=\begin{vmatrix}0 & 0.53253 & 0.46747\\ 0 & 0.31854 & 0.68146\\ 0.63427 & 0.36573 & 0\\ 0 & 0.42065 & 0.57935\\ 0.53836 & 0.46164 & 0\end{vmatrix}$	$B_{IND}=(0.16182, 0.45337, 0.38381)$	0.39961
ISL	$R_{ISL}=\begin{vmatrix}0.99808 & 0.00192 & 0\\ 0.66657 & 0.33343 & 0\\ 0.53165 & 0.46835 & 0\\ 0.20954 & 0.79046 & 0\\ 0 & 0.29957 & 0.70043\end{vmatrix}$	$B_{ISL}=(0.63909, 0.26325, 0.09666)$	0.70043

From the Table 5, the index α of CHN, QAT, ZAF, IND is relatively low, and the index α of RUS, FRA, USA, ISL is relatively high. So we can rate α=0.54 to measure the region's ability to provide clean water. Namely, if a region's index α>0.54, its water resources can meet the needs of population. To the contrary, if a region's index α<0.54, it lacks of enough water resources to meet the needs of population.

The index α we get is basically in accordance with the facts (the UN water scarcity map), which proves that our model is effective and convincing.

2.3.5 Strengths and Weaknesses

Strengths:

- Quantization: The model gives accurately quantitative evaluation by precise number processing.
- Information-rich: The model gives rich information by vector matrix operation. It is a good solution to the ambiguity and uncertainty.

Weaknesses:

- The practicability of this model is limited in some degree by computational complexity.
- The indicator weight vector is determined subjectively, leading to the index cannot be very accurate. So this model may be not useful to problems which requesting high accuracy.

3 Solution of Task 2

3.1 Introduction

In this task, we need to pick a region where water is either heavily or moderately overloaded and describe the reasons why and how the scarcity happens. Meanwhile, social and environmental drivers must be taken into consideration. All the analysis is based on the 2 models we have built.

3.2 Analytic Hierarchy

3.2.1 Why and How Water is Scarce in the Region

According to the UN water scarcity map, we pick China to measure the ability of providing clean water to meet the needs of its population. As we work out in Task 1,α_{CHN}=0.46035,which represents the ability to provide clean water is at a relatively low level. Related data in 2013 are as follows (Table 6).

Table 6 Evaluation indicators data of China in 2013

Per capita total water resources	Per capita GDP	Per capita industrial water	Per capita agricultural water	Per capita domestic water
2059.69	6991.853	103.611	288.904	55.2609

**All the numbers are from World Bank Data. The units are neglected in the table.*

As we can see, Per capita total water resources in China is at V_2 level, Per capita GDP in China is at V_3 level, Per capita industrial water in China is at V_2 level, Per capita agricultural water in China is at V_2 level, Per capita domestic water in China is at V_2 level.

China's Per capita total water resources is far below the standard. Its physical scarcity is serious. And China's Per capita GDP is low, that means China's related science and technology is not mature (such as desalination, sewage treatment). So it's economic scarcity is not optimistic. Besides, the amount of per capita industrial water, per capita agricultural water and per capita domestic water is not low, which further aggravates the scarcity of water.

3.2.2 Further Analysis

Although we have identified the main reasons for China's water shortage, they are still impacted by the specific environment and economic situation in China.

Environmental drivers→Physical scarcity

● Due to the large span broad geographic territory, China almost cover all terrain, mountains, plateaus and hills make up about two-thirds of the total land area.

● Yangtze River is the largest river in China, the other major rivers are the Yellow River, Pearl River, Heilongjiang River, Huaihe River. The number of rivers and lakes in China is large, but per capita water resources less.

● China's ecological environment is deteriorating, mainly as follows: soil erosion, air pollution, waste pollution and so on. In the north, soil erosion and desertification are serious, and forest resources dropped rapidly, which significantly increased water pollution.

Social divers→Economic

●GDP

● Science and Technology
● New or potential backup water source (such as desalination)

4 Solution of Task 3

4.1 Introduction

To solve this problem, we need to predict the future direction of the evaluations. To predict C_1 (C_1: Per capita total water resources), we draw a **fitting curve** to get the value of the total water resources, and we need to predict the number of total people in the future by **Logistic Model**. To predict C_4 (C_4: GDP per capita), we use **ARIMA Model** to get the future GDP value. To predict C_{12}(C_{12}:Per capita industrial water),C_{13}(C_{13}:Per capita agricultural water),C_{14}(C_{14}:Per capita domestic water), we build Industrial Water Change Model, Agricultural Water Change Model, Domestic Water Change Model respectively.

4.2 Model Establishment

4.2.1 Model 1: Population Model

Table 7 Notation

Notation	Explanation
t	time(per year)
$x(t)$	population at t
k	maximum population that China allowed
r	natural population growth rate

Assumption

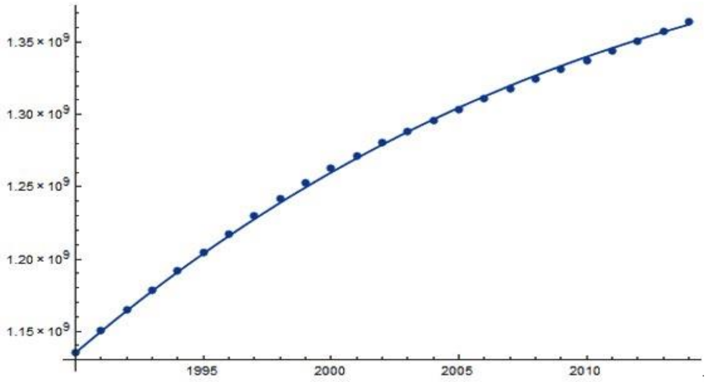
● The start time is 1990.
● $x_0 = 1.13519 \times 10^9$
● According to the research, we get $k = 1.45 \times 10^9$

$$x(t)=\frac{k}{1+\left(\frac{k}{x_0}-1\right)e^{-(t-t_0)r}}$$

Via calculation, we get

$$r = 0.0608960632868,$$
$$RNew = 0.9986141325652581$$

Picture 1



It fits data very well. We get the fitting graph Picture 1.

Then Model 1 can be shown as

$$X(t)=\frac{1.45*10^9}{1+0.27732484e^{0.0608960632868(1990-t)}}$$

Note: RNew is a recently arisen statistical parameterused to determine the non-linear regression fit, $Q=\sum(y-y')^2$ and $\sum y^2$, and $RNew=\sqrt{1-(Q/\sum y^2)}$.

RNew equals to R^2 on the interpretation of the fit equation.

4.2.2 Model 2: ARIMA(1,1,0) Model

GDP (Gross Domestic Product) is a key indicator of national accounting and measuring overall economic health of a country or region. It is the market value of all final goods and services in a country during the given period. There are many factors influencing GDP, it's difficult to build a model of GDP. We can use ARIMA(1,1,0) Model to predict the value of GDP in 15 years(ARIMA Model can achieve short-term forecast).

Assumption

- GDP_0=9490602600148.49, GDP_1=10354831729340.4

$\ln GDP_t = 0.0436 + 1.534\ln GDP_{t-1} - 0.534\ln GDP_{t-2} + v_t$

We can trust our model by residuals ADF test (Residuals are stationary random sequence). To make the calculation more convenient, we use the average value of v_t. v_t =0.000361444. The model as follow:

$$\ln GDP_t = 0.0436+1.534\ln GDP_{t-1} - 0.534\ln GDP_{t-2}+0.000361444$$

And we can predict the future GPA in 15 years. The results are as follows:

Table 8 The prediction of GDP in China in 15 years

2015	2016	2017	2018	2019
1.13356587064550 ×10^{13}	1.24316037430954 ×10^{13}	1.36465321599876 ×10^{13}	1.49878356685231 ×10^{13}	1.64654577988242 ×10^{13}
2020	**2021**	**2022**	**2023**	**2024**
1.80913865299124 ×10^{13}	1.98794159036679 ×10^{13}	2.18450677549105 ×10^{13}	2.40056123731226 ×10^{13}	2.63801534561543 ×10^{13}
2025	**2026**	**2027**	**2028**	**2029**
2.89897578871356 ×10^{13}	3.18576196873473 ×10^{13}	3.50092526994679 ×10^{13}	3.84727096712154 ×10^{13}	4.22788273182687 ×10^{13}

4.2.3 Model 3: Agricultural Water Change Model

Agricultural water is mainly used for farmland irrigation, which is associated with cultivated area and water consumption unit of arable land.

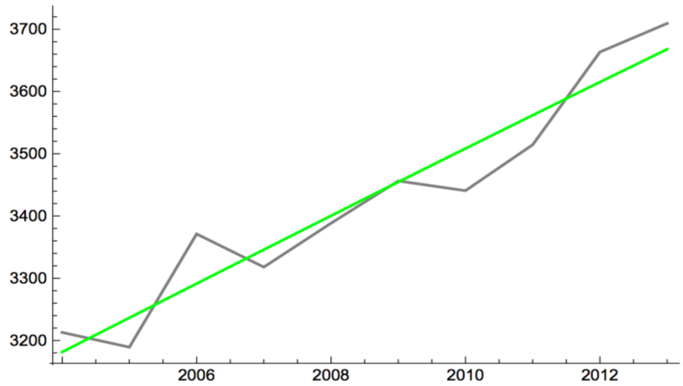
Table 9 Notation

Notation	Explanation
A	cultivated area
$\mathbf{C_A}$	Agricultural water
$\frac{C_A}{A}$	Water consumption per unit of arable land

It is easy to know:

$$C_A = A \times \frac{C_A}{P}$$

Using linear fitting to fit water consumption per unit of arable land, the result is as follow:

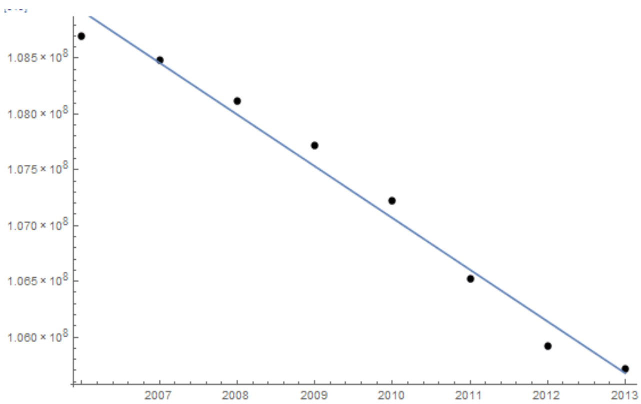


The actual value is the grey line, and the fitting line is the green one. RNew=0.986508. It fits data well. We can finally get:

$$\frac{C_A}{A} = \frac{6000}{1+0.0919157e^{-0.0368502(-2003+t)}}$$

Picture 2

Then we use logistic fitting to fit water consumption per unit of arable land, the result is as follow (Unit: ha):



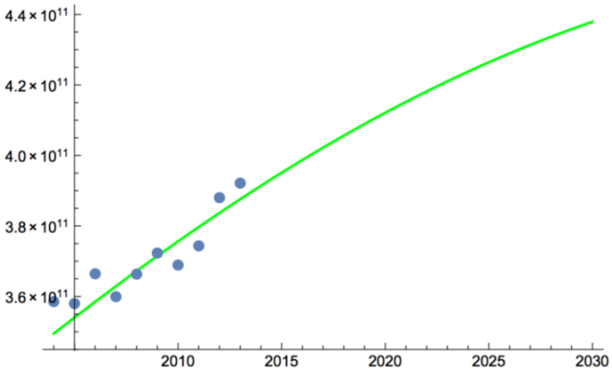
The points are the actual values is, and the fitting line is the blue one. RNew=0.998598. It fits data well. And we finally get

$$A = 1.03964 \times 10^9 - 463964t$$

Then we can figure out: $C_A = \frac{6000\times(1.03964\times10^9+463964t)}{1+0.919157e^{-0.0368502(-2003-t)}}$

Picture 3

The result is as follow:



The points are the actual values, and the fitting line is the green one. RNew=0.985606 . It fits data well.

Picture 4

4.2.4 Model 4: Domestic Water Change Model

Domestic water consumption is closely related to population, we use per capita water consumption and population to describe domestic water consumption.

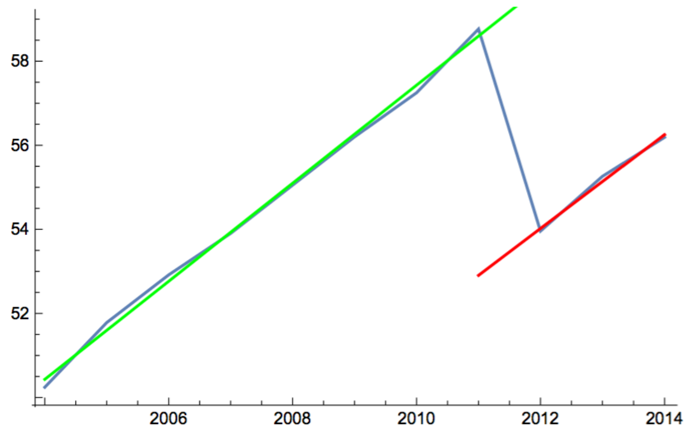
Table 10 Notation

Notation	Explanation
P	Population
C_M	Domestic water
$\frac{C_M}{P}$	Domestic water consumption per capita

It is easy to know:

$$C_M = P \times \frac{C_M}{P}$$

Draw the actual points of domestic water consumption per capita and connect them, and we can find the slope of green line(slope is 1.11543) equals to the slope of green line(slope is1.16654). Because China began recording data from 1998, so we can't get a total view. We think the increasing speed is a value.



RNew=0.998405. It fits data well. So we can work out:

$$\frac{C_M}{P} = -2190.23 + 1.11543t$$

(from 2012 to a short-term time).

$$C_M = x(t) \times \frac{C_M}{P}$$

Picture 5

4.2.5 Model 5: Industrial Water Change Model

Industrial Water is related to development of industrial scale and water consumption per unit of industrial scale. We use industrial GDP to describe the development of industrial scale.

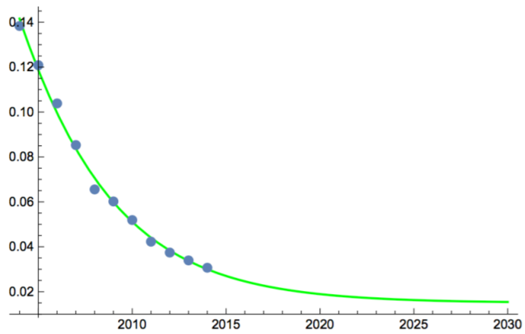
Table 11 Notation

Notation	Explanation
IGDP	Industrial GDP
$\mathbf{C_I}$	Industrial Water
$\frac{C_I}{IGDP}$	water consumption per unit Industrial GDP (cubic meters)
$\frac{d\,IDGP}{d\,t}$	Industrial GDP water consumption (cubic meters)

It is easy to know:

$$C_I = \frac{C_I}{IGDP} \times \frac{C_I}{I}$$

Then, we fit by logistic model, and the result is as follow:



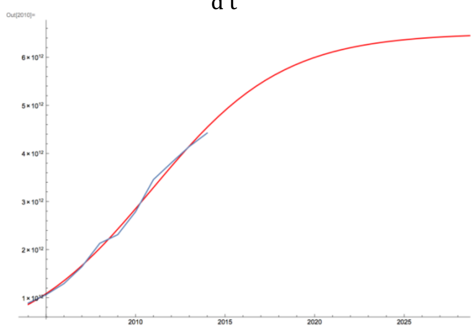
The points are the actual values , and the fitting line is the green one.

RNew=0.968531. It fits data well.

We can get: $\frac{C_I}{IGDP} = 0.015 + \frac{1}{1+5.50515e^{0.225495(-2003+t)}}$

Picture 6

Then, we fit $\frac{d\,IGDP}{d\,t}$, the result is as follow:

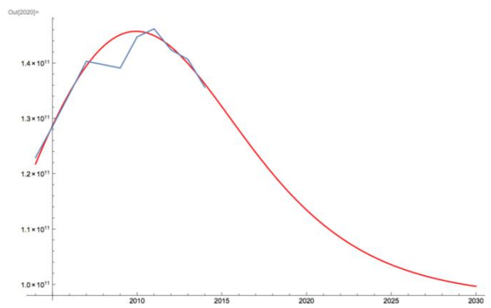


The actual value is the grey line, and the fitting line is the red one. RNew=0.969601. It fits data well.

We can get: $\frac{d\,IGDP}{d\,t} = \frac{6.5\times10^{12}\left(0.015+\frac{1}{1+5.50515e^{0.225495(-2003+t)}}\right)}{1+8.61601\,e^{-0.272377(-2003+t)}}$

Picture 7

Finally, we fit C_I, the result is as follow:



The actual value is the grey line, and the fitting line is the red one.

RNew=0.984121. It fits data well.

And we can get:

$$C_I = \frac{6.5 \times 10^{12}}{1 + 8.61601e^{-0.272377(-2003+t)}}$$

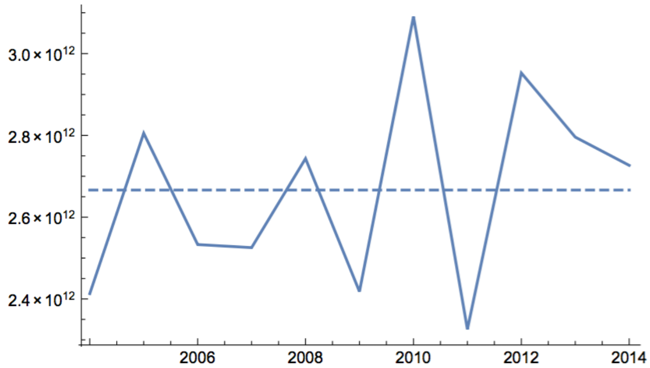
Picture 8

4.2.6 Others

About predicting the total water resources, the change is not discontinuous. So we get the average value as the value of total water resources.

Table 12 Notation

Notation	Explanation
R	Total water resources

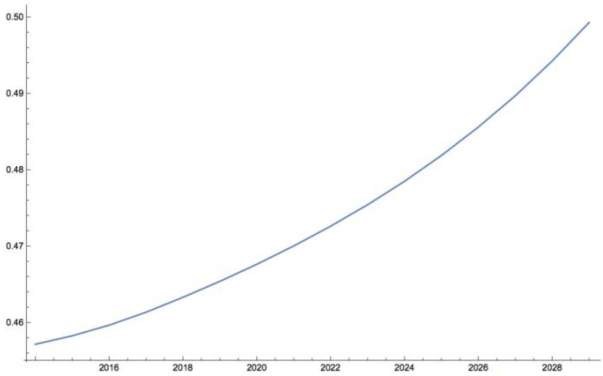


Via calculation, $R = 2.66634 \times 10^{12}$

Picture 9

4.2.7 Water Situations in 15 Years

Through the above models, we can predicate each value of each period of C_1, C_4, C_{12}, C_{13}, C_{14}. Also we can calculate Carrying Capacity of Water Resources by Task 1. And the result is as follows (The line is not via fitting):



Picture 9

We can see China's Carrying Capacity of Water Resources shows upward trend, and the Growth rate is increasing.

From environmental drivers' effect, per capita total water resources in China is in v1 level, the water environment provides enough water resources to China, so China can develop industrial. It helps China increasing GDP very quickly. On one hand, GDP will make China rich, China will have more ability to grow technology, it can increase the conversion rate of sewage treatment and desalination. On the other hand, it can make people become rich, they will have more chances to get better education, that makes them know the importance and scarcity of water, then Per capita domestic water will decrease. Those factors will make the water condition become better in the future. This constitutes a mutually positive effect. So people life in China will become better and better, they will become rich and knowledgeable.

4.3 Strengths and Weaknesses

Strengths

- The model is simple and flexible.

Weaknesses

- Limitation: In order to get better prediction results, the model requires a relatively large number of historical data (≥50), which limits its practicability.

5 Solution of Task 4

5.1 Introduction

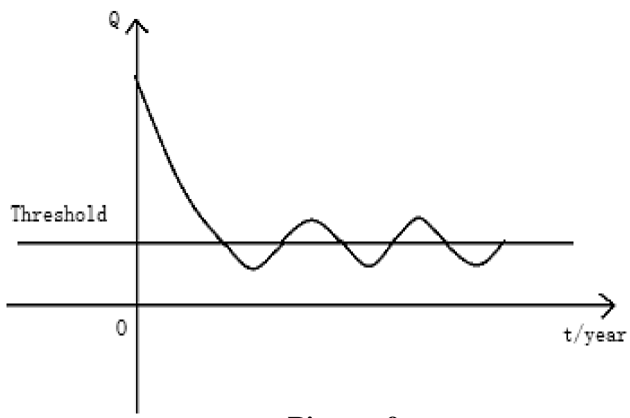
We design an intervention planbased on criteria. When water consumption is lower the amount of fresh water can be cycled, the local environment will not be damaged. By searching from Internet, we getthe amount of fresh water can be cycledis 30.4% of the total water resources. 1.1 times of the amount of fresh water can be recycled is our plan's threshold.

5.2 Notation

Table 12 Notations

Notation	Explanation
A_t	Per capita total water use in t
P_t	Population in t
Q_t	Total water use in t
W	The amount of fresh water can be cycled

As we can see, it is simple to find out: $Q_t = A_t \times P_t$



Picture 9

According to **Picture 9**, we can find:

When $Q_t < W$, we can increase the growth speed of GDP and Industrial GDP.

When $W<Q_t<kW$ ($1<k<1.7$), we can do a soft intervention to make it return to normal. After intervention, Q_t may be lower than W.

When $Q_t>kW$, we must do a strong intervention. But we can't decrease GDP (We can't let the economy condition back, so that previous efforts made in vain). So we make a strong measure to the growth rate of GDP and population.

After the trend is stable, we will examine the value of threshold. If threshold< 0, the intervention plan is false, and the region can't be less susceptible to water scarcity. If threshold > 0, total water use in the region will be stable. And we can calculate water carrying capacity of the region by Task 1. If the result is good, it means the region get of the dry condition, or the region is still lack of water resources.

5.3 Details of Intervention Plan

Table 13 Details of intervention plan

Intervention indicators	What to do	Why
Industrial water	Decrease the growth speed of industrial IGDP	We can't decrease it let the economy condition back, but we should limit the industrial scale.
	Decrease water consumption per unit Industrial GDP	We can improve water utilization plant to reduce it.
Agricultural water	Reduce arable land	It means we should improve crop yields per unit area and limit population.
	Improve the utilization of water	Make water distribution more reasonable, develop desalination technology.
Domestic water	Improve the utilization of domestic water	Promote the awareness of water conservation, improve the price of water.
GDP	Decrease the growth speed of industrial GDP	GDP has forward factors, such as developing technology and sewage treatment can help us improve the utilization of water. But we should also limit it because it will more water consumption.
Population	Limit population growth	Today's population is too large, it gives heavy stress to the earth. We can implement family planning policy, better at early trends of population growth(to avoid aging).

To surrounding areas, this intervention plan will make GDP increase gradually slowed down (To give the state a certain buffer time) and make economy maintain at a certain level finally, there may be some fluctuations. The state will have enough time to fit the change (In fact, the level of intervention is adjustable), and it won't make economic recession.

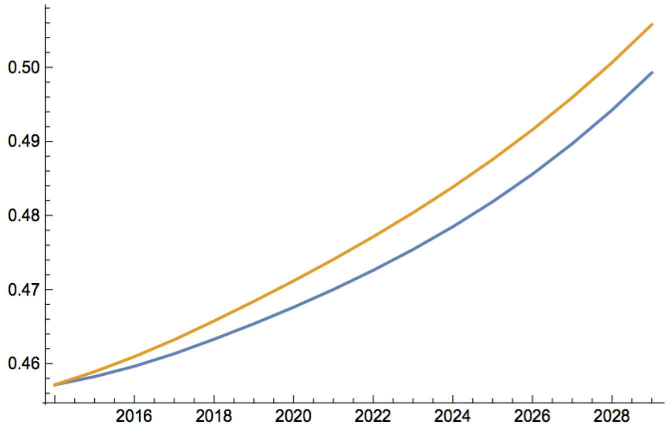
To the entire water ecosystem, the intervention plan will decrease the growth speed of water consumption, give enough time to industrial scale change firstly. Then slow the water resources reduction. Finally, water requirement will equal to W, the nation doesn't need more water from water resources, then total water resources will increase at a slow speed(Nature create water at a slow speed). The Carrying Capacity of Water Resources of the nation will improve progressively.

In a larger context, if every country can use this intervention plan, the world's total water resources will develop towards a positive trend, and it won't lead to economic recession. This is the advantage. And the disadvantage is, if only one country or some countries use this intervention plan, their developing speed will behind other country. If the country is not advanced to a certain level, it will fall behind comparing to other countries, that maybe lead to war. And because water is flowing, other country's polluted water flow into the nation which use this intervention plan, that makes the achievement of the country become vain. But other countries have the ability to become bigger. In other view, this intervention plan limit country's development.

6 Solution of Task 5

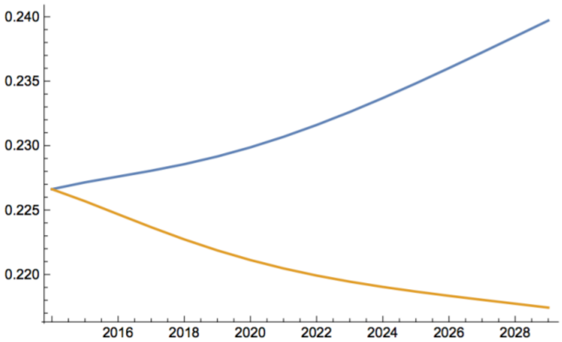
We set related index for over intervention plan. We decrease the growth speed of industrial IGDP and GDP, limit population, reduce arable land, improve the utilization of water about industrial, agriculture and domestic water. When $Q_t < W$, we do nothing due to the complexity.

The intervened result is as follows:

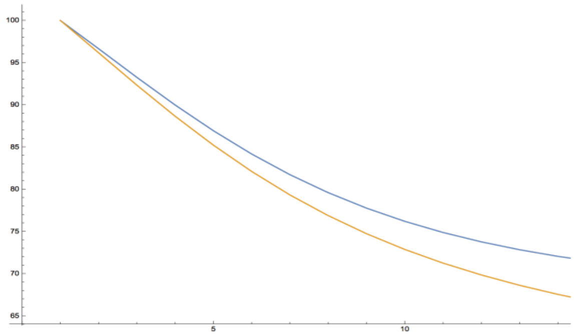


Picture 10 The trend of Carrying Capacity of Water Resources

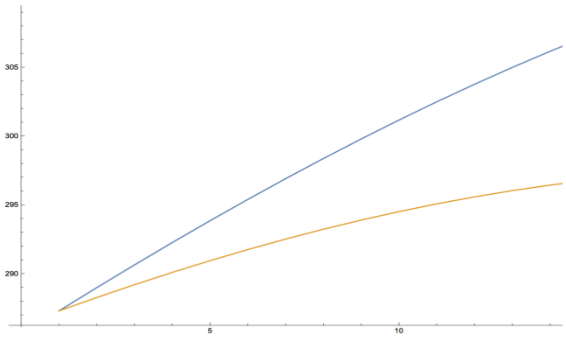
From the picture, we can see the Carrying Capacity of Water Resources has increasingly, it means our intervention plan is effective. So the influences of lacking water to China will become less. Each of evaluations is as follows:



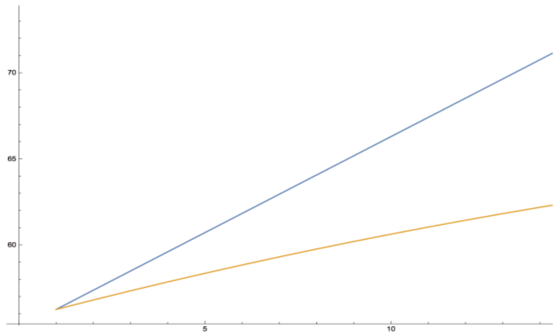
Picture 11 The total water volume/W



Picture 12 Per capita industrial water



Picture 13 Per capita agricultural water



Picture 14 Per capita domestic water

Water will be the critical issue in the future. One of the reasons is that water is nonrenewable resources. It suits Hubbert Bell Curve. And water is used in all kinds of fields.

For China, it may be not easily happened. But for some country, when the threshold <0 after intervention plan, it means the condition of lacking water is difficult to deal. For those countries, when Q_t > total water resources, it means dry.

7 Conclusions

- Confirm the weight of each evaluation is very important. A tiny change bring a big different.
- To make your model more accurate, you need more data and more factors.
- To do different intervention according to the condition will make the result more suit people's goal.

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