Fast calculation of the exact radiological path for a three-dimensional CT array[a)]

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Ready availability has prompted the use of computed tomography (CT) data in various applications in radiation therapy. For example, some radiation treatment planning systems now utilize CT data in heterogeneous dose calculation algorithms. In radiotherapy imaging applications, CT data are projected onto specified planes, thus producing "radiographs," which are compared with simulator radiographs to assist in proper patient positioning and delineation of target volumes. All these applications share the common geometric problem of evaluating the radiological path through the CT array. Due to the complexity of the three-dimensional geometry and the enormous amount of CT data, the exact evaluation of the radiological path has proven to be a time consuming and difficult problem. This paper identifies the inefficient aspect of the traditional exact evaluation of the radiological path as that of treating the CT data as individual voxels. Rather than individual voxels, a new exact algorithm is presented that considers the CT data as consisting of the intersection volumes of three orthogonal sets of equally spaced, parallel planes. For a three-dimensional CT array of N^3 voxels, the new exact algorithm scales with $3N$, the number of planes, rather than N^3, the number of voxels. Coded in FORTRAN-77 on a VAX 11/780 with a floating point option, the algorithm requires approximately 5 ms to calculate an average radiological path in a 100^3 voxel array.

Key words: radiological path, inhomogeneity correction, CT

INTRODUCTION

In radiation therapy applications, computer tomography (CT) data are utilized in various dose calculation and imaging algorithms. For example, some radiation treatment planning systems now utilize two-dimensional CT data for pixel-based heterogeneous dose calculations. Other systems forward project three-dimensional CT data onto specified planes, thus forming "radiographs," which are compared with simulator radiographs to assist in proper patient positioning and delineation of target volumes. All such applications, whether in inhomogeneity calculations or imaging applications, essentially reduce to the same geometric problem: that of calculating the radiological path for a specified ray through the CT array.

Although very simple in principle, elaborate computer algorithms and a significant amount of computer time is required to evaluate the exact radiological path. The amount of detail involved was recently emphasized by Harauz and Ottensmeyer,[1] who stated that even for the two-dimensional case, their algorithm for calculating the exact radiological path grew more and more unwieldy and time consuming, while remaining unreliable. For three dimensions, they concluded that determining the exact radiological path is not viable. This paper describes an *exact*, efficient, and reliable algorithm for calculating the radiological path through a three-dimensional CT array.

Denoting a particular voxel density as $\rho(i,j,k)$ and the length contained by that voxel as $l(i,j,k)$, the radiological path may be written as

$$d = \sum_i \sum_j \sum_k l(i,j,k)\rho(i,j,k). \tag{1}$$

Direct evaluation of Eq. (1) entails an algorithm which scales with the number of terms in the sums, that is, the number of voxels in the CT array. The following describes an algorithm that scales with the sum of linear dimensions of the CT array.

METHOD

Rather than independent elements, the voxels are considered as the intersection volumes of orthogonal sets of equally spaced, parallel planes. Without loss of generality, Fig. 1 illustrates the two-dimensional case, where pixels are considered as the intersection areas of orthogonal sets of equally spaced, parallel lines. The intersections of the ray with the lines are calculated, rather than intersections of the ray with

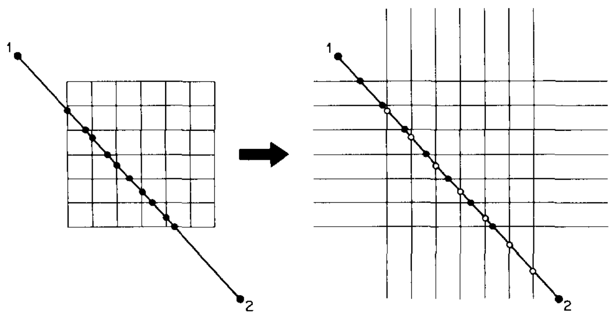

FIG. 1. The pixels of the CT array (left) may be considered as the intersection areas of orthogonal sets of equally spaced, parallel lines (right). The intersections of the ray with the pixels are a subset of the intersections of the ray with the lines. The intersections of the ray with the lines are given by two equally spaced sets: one set for the horizontal lines (filled circles) and one set for the vertical lines (open circles). The generalization to a three-dimensional CT array is straightforward.

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