CS 111 homework5

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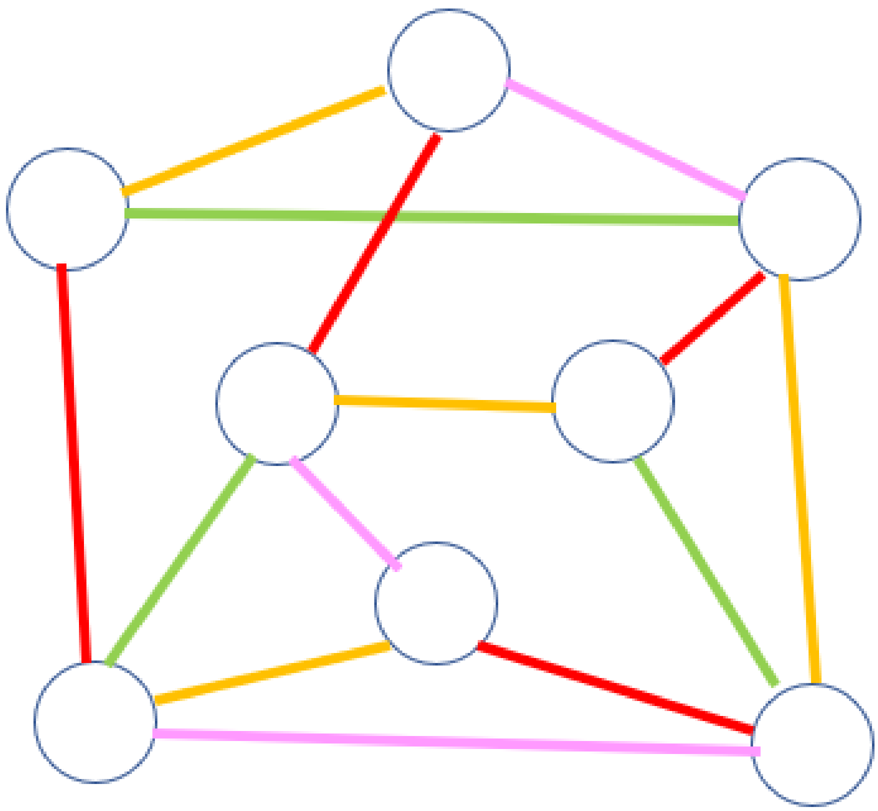
1 Problem 1

a).



(41).png

b). Using induction to prove that for a graph with total m edges and maximum degree of D, if D $\geq$ 1 then G can be edge-colored with at most 2D -1 colors.
p(m) ::= the theorem to prove
Base case: m = 1, then D = 1, the graph can be edge-colored with 1 color.

Prove P(1) is correct.

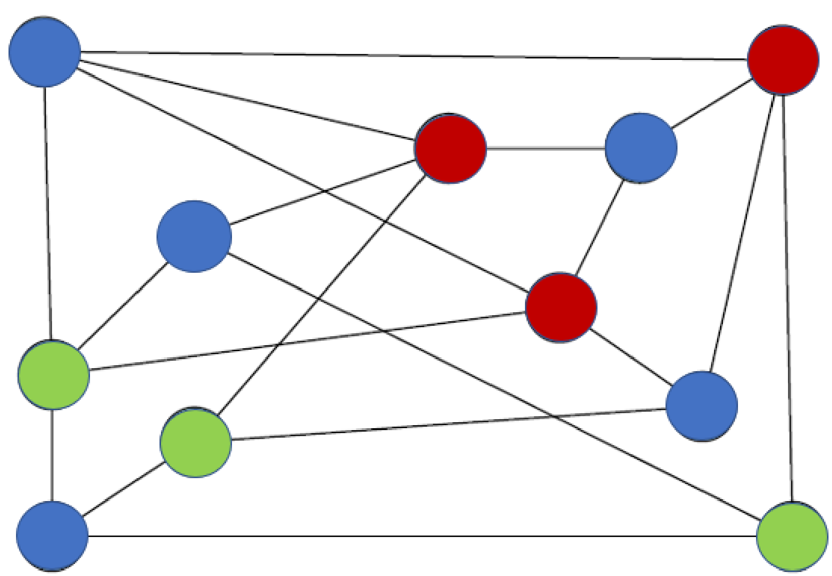
Assumption: p(m) is correct. Need to prove P(m+1) is correct
Induction on the number of edges:
let G be a graph with m+1 edges and with maximum degree D. Now we remove one edge e, the maximum degree is less or equal to D and the graph only has m edges now. Then we can apply the assumption: this graph can be edge-colored with 2D-1 colors. Now we add the edge e back, where the maximum degree stay with D. We know that there are 2D-1 colors available, and 2D-1 $>$ D because 2D-1-D = D-1 , D $\geq$ 1. Therefore, there are enough colors to color m+1 edges

2 Problem 2

a). Graph G does not has a perfect matching. To determine whether a bipartite graph has perfect matching, we use Hall's theorem.
For the set X={1,3,4}, it has a neighbor set N = {5,9}
$\Rightarrow \mid X \mid = 3 > \mid N \mid = 2$ According to Hall's theorem, the graph G does not has a perfect matching.
b). Graph H has a perfect matching: they are {0,7} {1,5} {2,6} {3,9} {4,8}

3 Problem 3

a).



(43).png
The minimum number of color can be used to color the graph is 3.

b). Because each vertex has a degree at least 3. Therefore, if we only use two colors. There mush be some adjacent vertex have the same color. Therefore, there is not way to reduce the number of color.