CS111 homework 1

Ruiwen He

October 15th 2018

1 Problem I

(a). $f(n) = (3n+1) * \sum_{i=1}^{i=3n+1} i^2 + 2$
Explanation: Outer loop runs 3n+1 times. Inner loop needs to do the summation because the value of i is changing.
(b). f(n)=(3n+1) * $\sum_{i=1}^{i=3n+1} i^2 + 2$

$$= (3n+1) * (\sum_{i=1}^{i=3n+1} i^2 + \sum_{i=1}^{i=3n+1} 2) = (3n+1) * (n(n+1)(2n+1)/6 + (3n+1) * 2)$$

$$= n^4 + \frac{11}{6}n^3 + n^2 + \frac{1}{6}n + 18n^2 + 12n + 2$$

(c). therefore f(n) = O(n^4)
because it is a polynomial of degree 4.
f(n) = $\Omega(n^4)$
because f(n) $\geq$ n^4 when n $\geq$ 0 $therefore \quad f(n) = \theta(n^4)$

2 Problem II

Proof: prove by induction that $3.3^n \leq B_n \leq 2(3.4^n)$ for all n ≥ 0
$BaseCase:$ $B_0 = 2 \leq 2 * 3.4^0 = 2 \qquad B_1 = 5 \leq 2 * 3.4^1 = 6.8$
Assume $B_n \leq 2 * (3.4^n)$ for all n $\geq$ 0
$Induction:$ $B_{n+1} = 3B_n + B_{n-1} \leq 3 * 2 * (3.4)^n + 2 * 3.4^{n-1}$
$= \frac{22.4}{3.4^2} * 3.4^{n+1} < 2 * 3.4^{n+1}$
therefore B_n = O(3.4^n)
Base Case: $B_0 = 2 \geq 1 \qquad B_1 = 5 \geq 3.3$
Assume $B_n \geq 3.3^n$ for all n $\geq$ 0
$Induction:$ $B_{n+1} = 3B_n + B_{n-1} \leq 3 * (3.3)^n + 3.3^{n-1}$
$= \frac{10.9}{3.3^2} * 3.3^{n+1} > 3.3^{n+1}$
therefore $B_n = \Omega(3.3^n)$

3 Problem III

(a). $f(n) = \frac{1}{2}n^5 + 7n^4 - n^3 + 13n$
=O(n^5) because $f(n)$ is a polynomial of degree 5
=Ω(n^5) because $f(n) \geq \frac{1}{100}n^5$ when $n \geq 1$
therefore: $f(n) = \theta(n^5)$

(b). $f(n) = 3 + 2n^{-2} + \frac{1}{(log^2 n)}$
because $n^2 > log^2 n$ for $n \geq 1 \Longrightarrow \frac{1}{n^2} < \frac{1}{log^2 n}$ for $n \geq 1$
therefore $n^{-2} = O(\frac{1}{log^2 n})$ *and* $3 = O(\frac{1}{log^2 n}) \Longrightarrow f(n) = O(\frac{1}{log^2 n})$
because $f(n) > \frac{1}{log^2 n}$ for all n ≥ 2 f(n) = $\Omega(\frac{1}{log^2 n})$
therefore $f(n) = \theta(\frac{1}{log^2 n})$

(c).$f(n) = n(7n^3 logn + 9nlog^5 n) + 15n^3$
$= 7n^4 logn + 9n^2 log^5 n + 15n^3$
$15n^3 = O(n^4 logn)$ because $n^3 < n^4 logn$ for $n > 1$
$9n^2 log^5 n = 9n^2 * log^4 n * logn = 9n^2 lognO(n) = O(n^4 logn)$
therefore $f(n) = O(n^4 logn)$
$f(n) > n^4 logn$ for $n \geq 1$ $\Longrightarrow$ f(n) = $\Omega(n^4 logn)$
therefore $f(n) = \theta(n^4 logn)$

(d).$n^2 2^n + 13n^4 + n^3 logn$
$n^4 = O(2^n)$ and $n^3 logn = O(2^n) * O(n^2) = O(2^n n^2)$ $\Longrightarrow$ f(n) = O($n^2 2^n$)
$f(n) \geq n^2 2^n$ for all $n \geq 1 \Longrightarrow$ f(n) = $\Omega(n^2 2^n)$
therefore $f(n) = \theta(n^2 2^n)$

(e) $n^5 3^n + n4^n$
$n^5 3^n = n * n^4 * 3^n = n * O((\frac{4}{3})^n * 3^n) = O(n4^n)$ therefore $f(n) = O(n4^n)$
$f(n) \geq n4^n$ for all $n \geq 1 \Longrightarrow$ f(n) = $\Omega(n4^n)$
therefore $f(n) = \theta(n4^n)$