homework4

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1 Problem 1

i).$T_n = 5T_{n/3} + 4$ Explanation: call PrintAs([n/3]) 5 times and 4 times print("A") in the program
According to Master Theorem: $a = 5, b = 3, c = 4, d = 0 \Rightarrow$a>b^d
therefore, $T_n = \theta(n^{\log_b a}) = \theta(n^{\log_3 5})$

ii).$T_n = 6T_{n/2} + 10n^3$ Explanation: call PrintBs([n/2]) 6 times and $10n^3$ times print("B") in the program
According to Master Theorem: $a = 6, b = 2, c = 10, d = 3 \Rightarrow$a<b^d
therefore, $T_n = \theta(n^d) = \theta(n^3)$

iii).$T_n = 4T_{n/2} + 20n^2$ Explanation: call PrintCs([n/4]) 4 times and $20n^2$ times print("C") in the program
According to Master Theorem: $a = 4, b = 2, c = 20, d = 2 \Rightarrow$a=b^d
therefore, $T_n = \theta(n^d \log(n)) = \theta(n^2 log(n))$

b). According to the question:

$$T_n = \begin{cases} 1 & n = 1 \\ 4T_{n/2} + 3 & n \geq 2 \end{cases}$$

therefore: $T_n = \theta(n) when n = 1$
when $n \geq 2$: because $T_n = 4T_{n/2} + 3 \Rightarrow a = 4, b = 2, c = 3, d = 0$
T$_n = \theta(n^{log_2 4})$

2 Problem 2

a).According to the question:
$1 \leq D \leq 7$
$2 \leq C \leq 11$
$4 \leq R \leq 25$
$0 \leq T \leq 6$
to let all variables have lower bound with 0:

$0 \leq D_1 \leq 6 \quad D_1 = D - 1$
$0 \leq C_1 \leq 9 \quad C_1 = C - 2$
$0 \leq D_1 \leq 21 \quad R_1 = R - 4$
$D + C + R + T = 25 \Rightarrow \mathrm{D}_1 + C_1 + R_1 + T = 18$
$S(D_1 \leq 6 \cap C_1 \leq 9 \cap R_1 \leq 21 \cap T_1 \leq 6) = S_{total} - S(D_1 \geq 7 \cup C_1 \geq 10 \cup R_1 \geq 22 \cup T_1 \geq 7)$
According to the inclusion and exclusion principle:

Inclusion-Exclusion Principle

$$\begin{aligned}|S_1 \cup S_2 \cup S_3 \cup S_4| = {} & |S_1| + |S_2| + |S_3| + |S_4| \\ & -|S_1 \cap S_2| - |S_1 \cap S_3| - |S_1 \cap S_4| - |S_2 \cap S_3| - |S_2 \cap S_4| - |S_3 \cap S_4| \\ & +|S_1 \cap S_2 \cap S_3| + |S_1 \cap S_2 \cap S_4| + |S_1 \cap S_3 \cap S_4| + |S_2 \cap S_3 \cap S_4| \\ & -|S_1 \cap S_2 \cap S_3 \cap S_4|\end{aligned}$$

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we can get:
$S(D_1 \leq 6 \cap C_1 \leq 9 \cap R_1 \leq 21 \cap T_1 \leq 6) = \binom{21}{3} - [\binom{14}{3} + \binom{11}{3} + \binom{0}{3} + \binom{14}{3} - \binom{4}{3} - \binom{0}{3} - \binom{7}{3} - \binom{0}{3} - \binom{4}{3} - \binom{0}{3} + \binom{0}{3} + \binom{0}{3} + \binom{0}{3} + \binom{0}{3} - \binom{0}{3}] = \frac{21!}{3!18!} - \frac{14!}{3!11!} - \frac{11!}{3!8!} - \frac{14!}{3!11!} + \frac{4!}{3!1!} + \frac{7!}{3!4!} + \frac{7!}{3!4!} = 480$

b). $\mid P \cup Q \cup R \mid = \mid P \mid + \mid Q \mid + \mid R \mid - \mid P \cap Q \mid - \mid P \cap R \mid - \mid R \cap Q \mid + \mid P \cap Q \cap R \mid$
$121 = \mid P \mid + 2 \mid P \mid + 4 \mid P \mid - 11 - 7 - 10 + \mid P \cap Q \cap R \mid$
$\Rightarrow \mid P \mid = \frac{149 - |P \cap Q \cap R|}{7}$
Due to the fact that $\mid P \mid$ and $\mid P \cap Q \cap R \mid$ should be both integer and $1 \leq \mid P \cap Q \cap R \mid \leq 11$
the only combination is $\mid P \mid = 21, \mid P \cap Q \cap R \mid = 2$ and $\mid P \mid = 20, \mid P \cap Q \cap R \mid = 9$