$$S_1 = \begin{bmatrix} y \\ \dot{y} \\ \psi \\ \dot{\psi} \end{bmatrix} \qquad S_2 = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

$$\dot{S}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-4C_a}{m\dot{x}} & 0 & -\dot{x} + \frac{2C_a(l_r - l_f)}{m\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2(l_r - l_f)C_a}{I_z\dot{x}} & 0 & -\frac{2(l_f^2 + l_r^2)C_a}{I_z\dot{x}} \end{bmatrix} S_1 + \begin{bmatrix} 0 & 0 \\ \frac{2C_a}{m} & 0 \\ 0 & 0 \\ \frac{2l_fC_a}{I_z} & 0 \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix}$$

$$\dot{S}_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m} \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix} + \begin{bmatrix} 0 \\ \dot{\psi}\dot{y} - fg \end{bmatrix}.$$

Since we are only considering lateral dynamics, we can write the system as:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-4C_a}{m\dot{x}} & 0 & -\dot{x} + \frac{2C_a(l_r - l_f)}{m\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2(l_r - l_f)C_a}{I_z\dot{x}} & 0 & -\frac{2(l_f^2 + l_r^2)C_a}{I_z\dot{x}} \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ \frac{2C_a}{m} & 0 \\ 0 & 0 \\ \frac{2l_fC_a}{I_z} & 0 \end{bmatrix}, \quad C = I(4,4).$$

Set $e_1 = y - y_{des}$, $e_2 = \psi - \psi_{des}$:

$$\dot{\psi}_{des} = \frac{V_x}{R} \qquad \frac{V_x^2}{R} = V_x \dot{\psi}_{des}$$

$$\ddot{e}_1 = \ddot{y} + V_x(\dot{\psi} - \dot{\psi}_{des}) \qquad e_2 = \psi - \psi_{des} \qquad \dot{e}_1 = \dot{y} + V_x(\psi - \psi_{des})$$

Set $E_1 = \begin{bmatrix} e_1 \\ \dot{e}_1 \\ e_2 \\ \dot{e}_2 \end{bmatrix}$

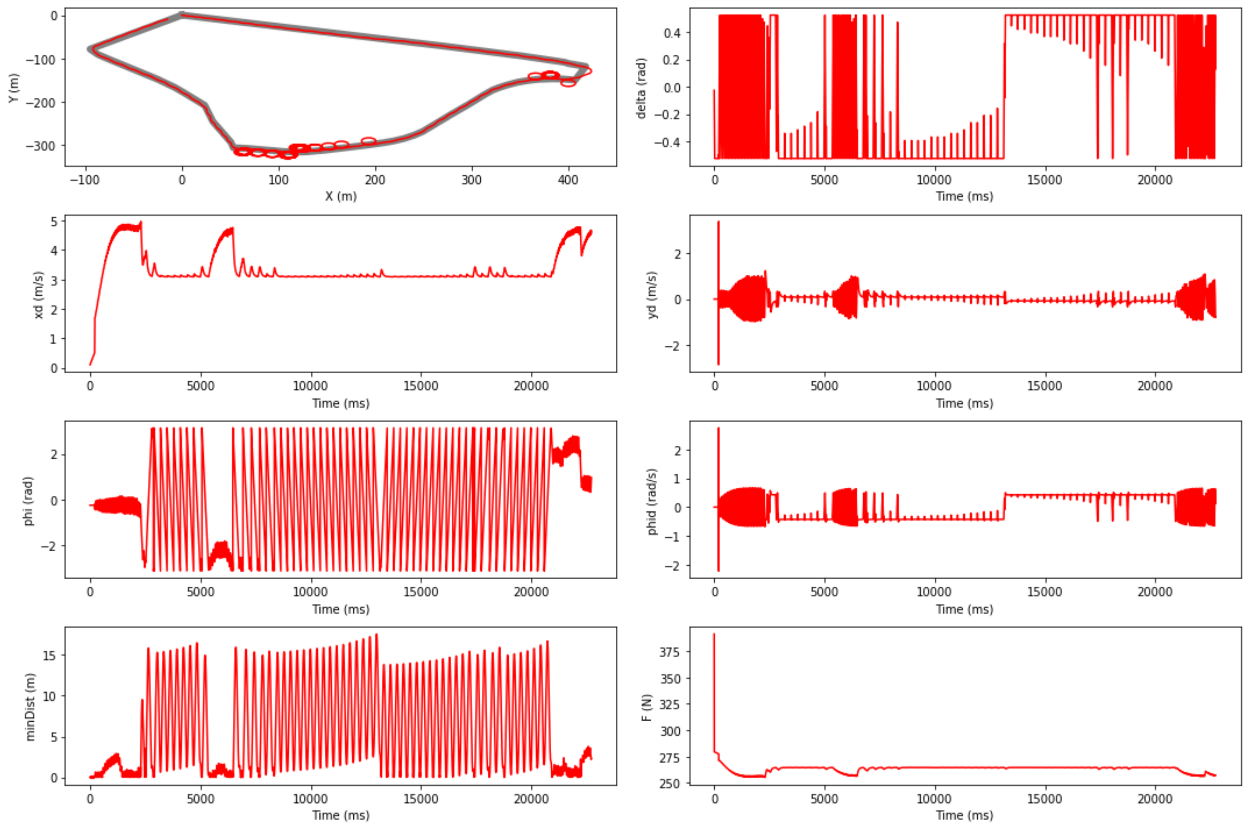
$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{2C_{af} + 2C_{ar}}{m\dot{x}} & \frac{2C_{af} + 2C_{ar}}{m} & \frac{-2C_{af}l_f + 2C_{ar}l_r}{m\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{2C_{af}l_f - 2C_{ar}l_r}{I_z\dot{x}} & \frac{2C_{af}l_f - 2C_{ar}l_r}{I_z} & -\frac{2C_{af}l_f^2 + 2C_{ar}l_r^2}{I_z\dot{x}} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ \frac{2C_{af}}{m} \\ 0 \\ \frac{2C_{af}l_f}{I_z} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{4C_a}{m\dot{x}} & \frac{4C_a}{m} & \frac{2C_a(l_r - l_f)}{m\dot{x}} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{2C_a(l_r - l_f)}{I_z\dot{x}} & \frac{2C_a(l_f - l_r)}{I_z} & -\frac{2(l_f^2 + l_r^2)C_a}{I_z\dot{x}} \end{bmatrix}$$

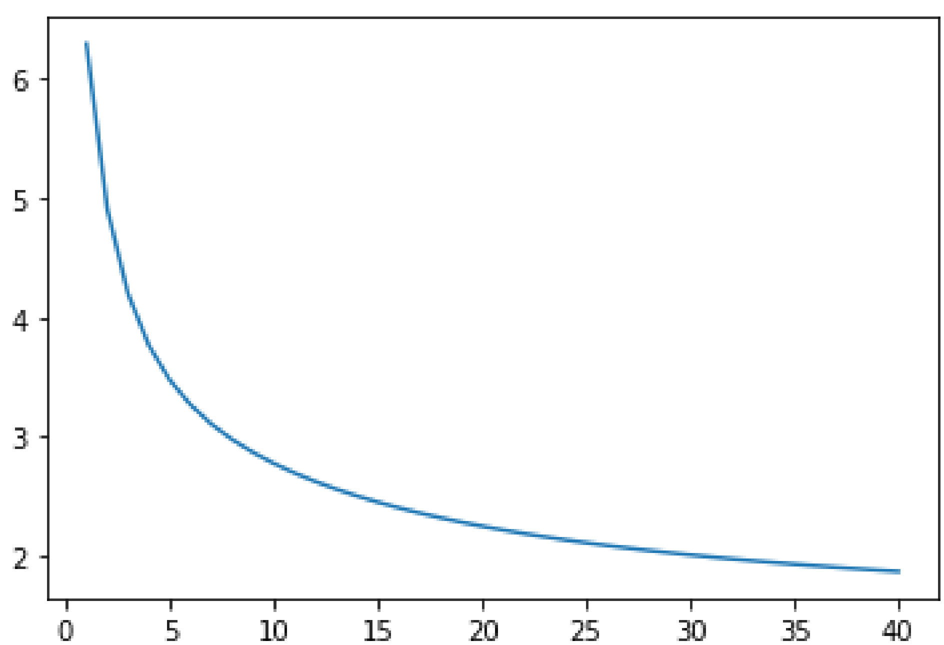
$$\dot{x}=2: \quad A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -15 & 30 & 4.5 \\ 0 & 0 & 0 & 1 \\ 0 & 2.6914 & -5.3828 & -18.3911 \end{bmatrix}$$

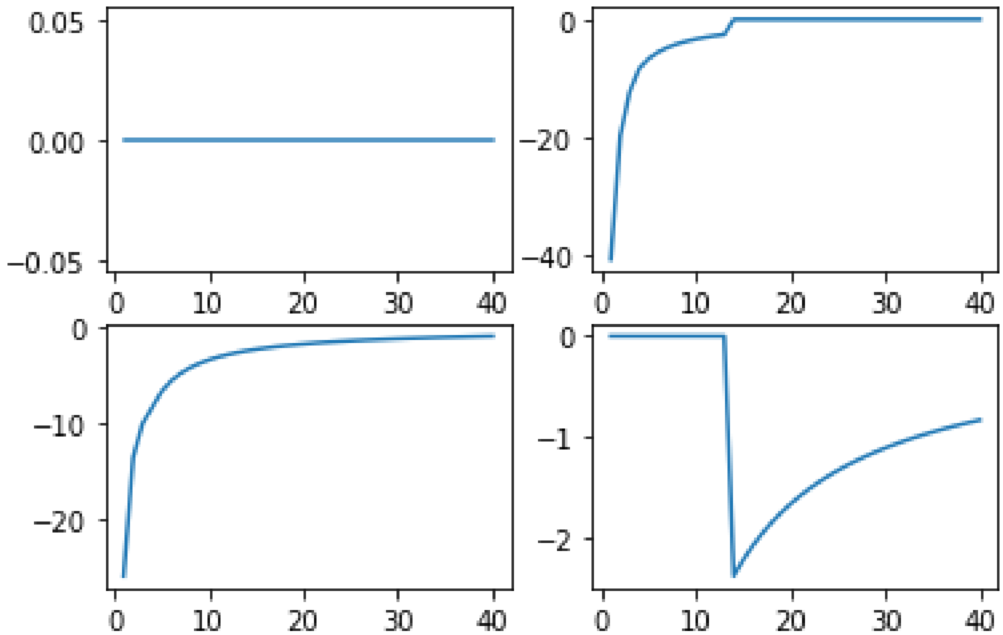
$$\dot{x}=5: \quad A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -6 & 30 & 1.8 \\ 0 & 0 & 0 & 1 \\ 0 & 1.0766 & -5.3828 & -7.3565 \end{bmatrix} \qquad B=\begin{bmatrix} 0 \\ 15 \\ 0 \\ 9.8684 \end{bmatrix}$$

$$\dot{x}=8: \quad A=\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -3.75 & 30 & 1.125 \\ 0 & 0 & 0 & 1 \\ 0 & 0.6728 & -5.3828 & -4.5978 \end{bmatrix}$$




Something that I don't understand occurred. During the test I had run time of 428s but when I ran it again I could only get a run time of 1100s….






We can observe that as the two singular value gets closer ( in the first graph the value gets closer to zero), the system becomes more controllable but less stable, as the poles become less negative.