

$$\ddot{y} = -\dot{\psi} \dot{x} + \frac{2Ca}{m} \left( \cos \delta \left( \delta - \frac{\dot{y} + l_f \dot{\psi}}{\dot{x}} \right) - \frac{\dot{y} - l_r \dot{\psi}}{\dot{x}} \right)$$

$$\ddot{x} = \dot{\psi} \dot{y} + \frac{1}{m} (F - fmg)$$

$$\ddot{\psi} = \frac{2l_f Ca}{I_z} \left( \delta - \frac{\dot{y} + l_f \dot{\psi}}{\dot{x}} \right) - \frac{2l_r Ca}{I_z} \left( -\frac{\dot{y} - l_r \dot{\psi}}{\dot{x}} \right)$$

$$S_1 = \begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix} \quad \dot{S}_1 = \begin{bmatrix} \dot{y} \\ \ddot{y} \\ \ddot{\psi} \end{bmatrix} \quad S_2 = \begin{bmatrix} x \\ \dot{x} \end{bmatrix} \quad \dot{S}_2 = \begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix}$$

$$\dot{S}_1 = \begin{bmatrix} \dot{y} \\ -\dot{\psi} \dot{x} + \frac{2Ca}{m} \left( \cos \delta \left( \delta - \frac{\dot{y} + l_f \dot{\psi}}{\dot{x}} \right) - \frac{\dot{y} - l_r \dot{\psi}}{\dot{x}} \right) \\ \dot{\psi} \\ \frac{2l_f Ca}{I_z} \left( \delta - \frac{\dot{y} + l_f \dot{\psi}}{\dot{x}} \right) - \frac{2l_r Ca}{I_z} \left( -\frac{\dot{y} - l_r \dot{\psi}}{\dot{x}} \right) \end{bmatrix}$$

$$S_2 = \begin{bmatrix} x \\ \dot{\psi} \dot{y} + \frac{1}{m} (F - fmg) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} S_2 + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m} \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix} + \begin{bmatrix} 0 \\ \dot{\psi} \dot{y} - f \dot{y} \end{bmatrix}$$

$$\begin{aligned} \ddot{y} &= -\frac{2Ca}{m} \left( \cos \delta \frac{1}{\dot{x}} + \frac{1}{\dot{x}} \right) \dot{y} + \left( -\dot{x} + \frac{2Ca}{m} \left( \cos \delta \frac{l_f}{\dot{x}} + \frac{l_r}{\dot{x}} \right) \right) \dot{\psi} + \frac{2Ca}{m} \cos \delta \cdot \delta \\ &= -\frac{2Ca(\cos \delta + 1)}{m \dot{x}} \dot{y} + \left( -\dot{x} - \frac{2Ca}{m \dot{x}} (\cos \delta l_f + l_r) \right) \dot{\psi} + 2 \frac{Ca}{m} \cos \delta \cdot \delta \end{aligned}$$

$$\ddot{\psi} = \left( \frac{2l_f Ca}{I_z} \left( -\frac{1}{\dot{x}} \right) - \frac{2l_r Ca}{I_z} \left( -\frac{1}{\dot{x}} \right) \right) \dot{y} + \left( \frac{2l_f Ca}{I_z} \left( -\frac{l_f}{\dot{x}} \right) - \frac{2l_r Ca}{I_z} \left( \frac{l_r}{\dot{x}} \right) \right) \dot{\psi} + \frac{2l_f Ca}{I_z} \delta$$

$$\therefore \dot{S}_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{2Ca(\cos \delta + 1)}{m \dot{x}} & 0 & -\dot{x} - \frac{2Ca}{m \dot{x}} (\cos \delta l_f + l_r) \\ 0 & 0 & 0 & -\dot{x} - \frac{2Ca}{m \dot{x}} (\cos \delta l_f + l_r) \\ 0 & \frac{2(l_f - l_r)Ca}{I_z \dot{x}} & 0 & -\frac{2(l_f^2 + l_r^2)Ca}{I_z \dot{x}} \end{bmatrix} \quad S_{11} = \begin{bmatrix} 0 & 0 \\ \frac{2Ca}{m} \cos \delta & 0 \\ 0 & 0 \\ -\frac{2l_f Ca}{I_z} & 0 \end{bmatrix} \begin{bmatrix} \delta \\ F \end{bmatrix}$$

