Penn State Abington
CMPEN 271
Lecture Set #13
Adders, Multiplication

R. Avanzato © 2014-2015

Topics:

- Half-adders, Full-adders — **Video part 1** ←
- MSI 4-bit ripple adder IC (7483)
- Binary adder applications

- Binary Multiplication — Video part 2

- Review Questions — Video part 3

Half-adder

Consider the addition of 2 bits: X + Y

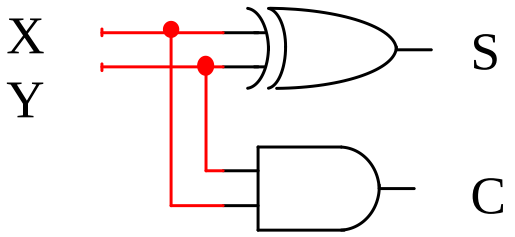
```
  X
+ Y
C S
```

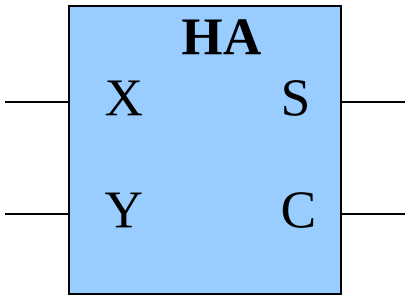
```
  1
+ 1
1 0
C S
```

X	Y	Sum	Carry
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

S (sum) = X'Y + XY' = X⊕Y

C (carry) = XY



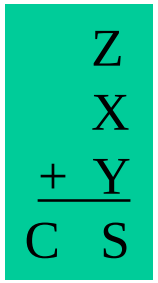


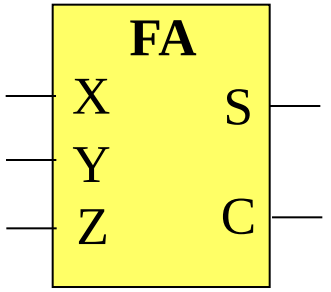
Full-adder

Consider the addition of 3 bits: X + Y + Z

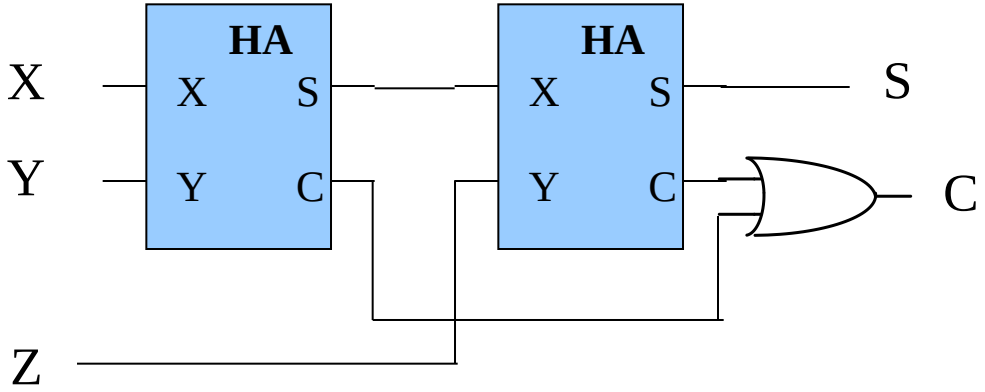
(inputs)			(outputs)	
X	**Y**	**Z**	**S(sum)**	**C(carry)**
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$$S\ (sum) = X \oplus Y \oplus Z$$
$$C\ (carry) = XY + Z(X \oplus Y)$$





Block diagram of full-adder



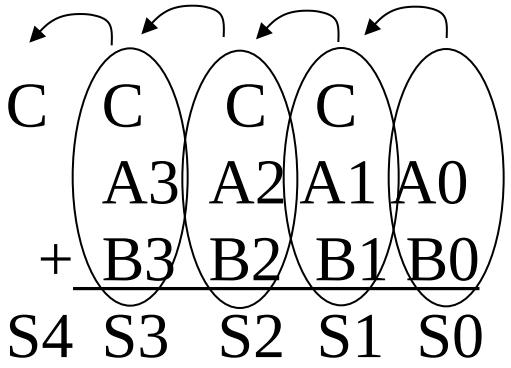
Full-adder from 2 half-adders

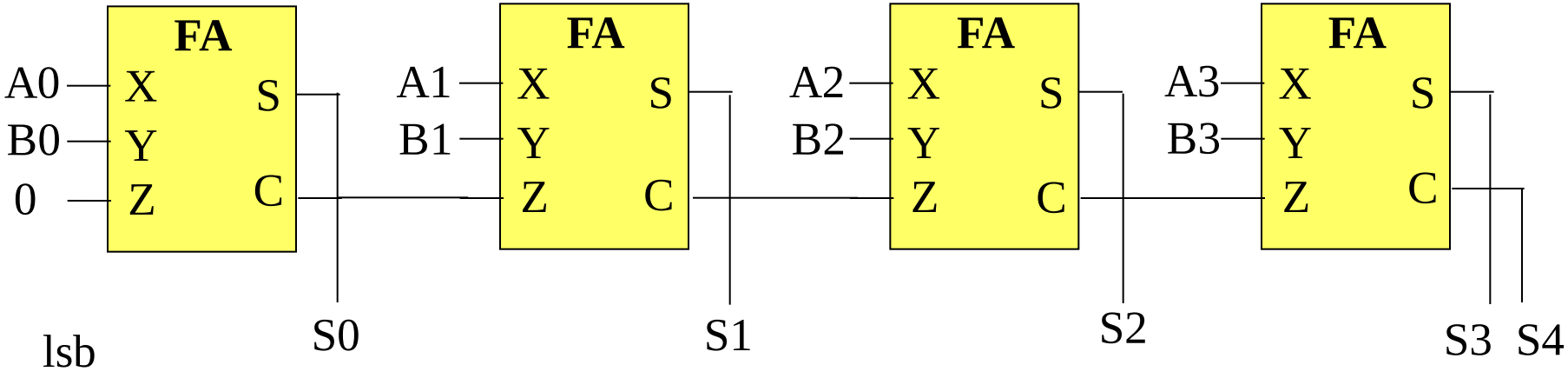
4-bit Binary Adder - 1

What is the best way to implement the following operation?
(8 inputs, 5 outputs -- could you use truth table and K-map?)

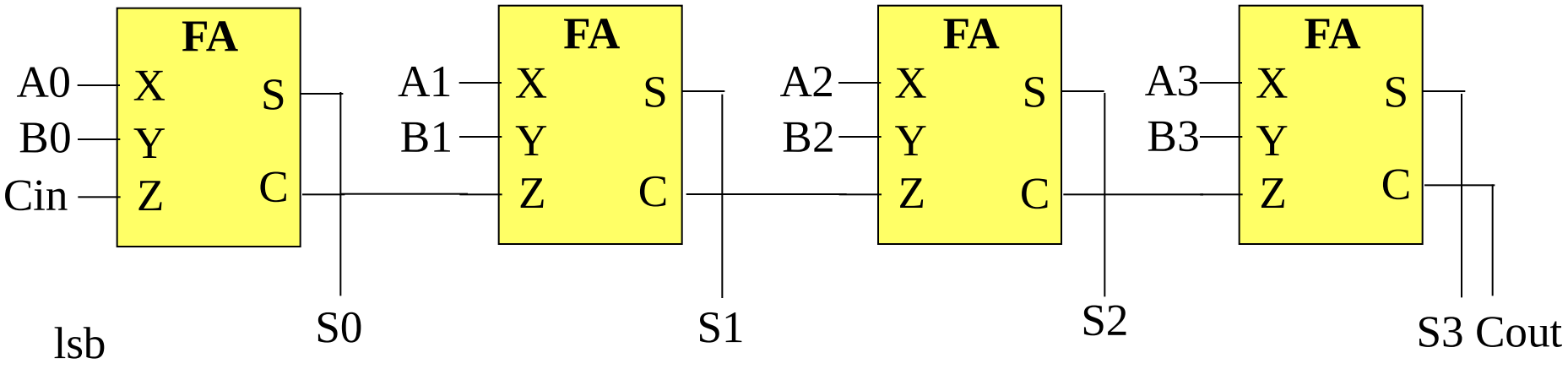
```
     A3  A2 A1 A0
+    B3  B2  B1 B0
S4   S3   S2  S1  S0
```

Add one column at a time with a FA. Send carry bit to left.

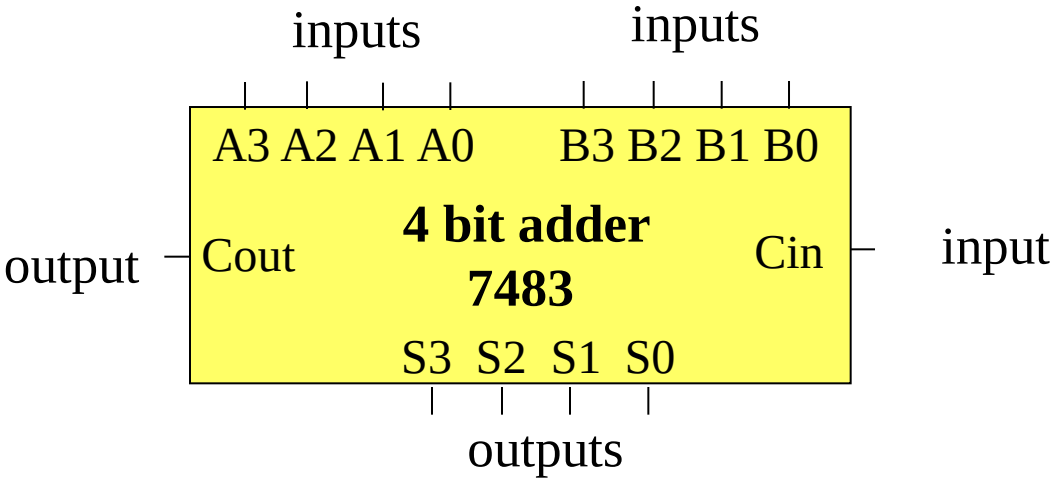




4-bit Binary Adder - 2



Design engineers created the 7483 4-bit adder MSI chip with 4 FAs

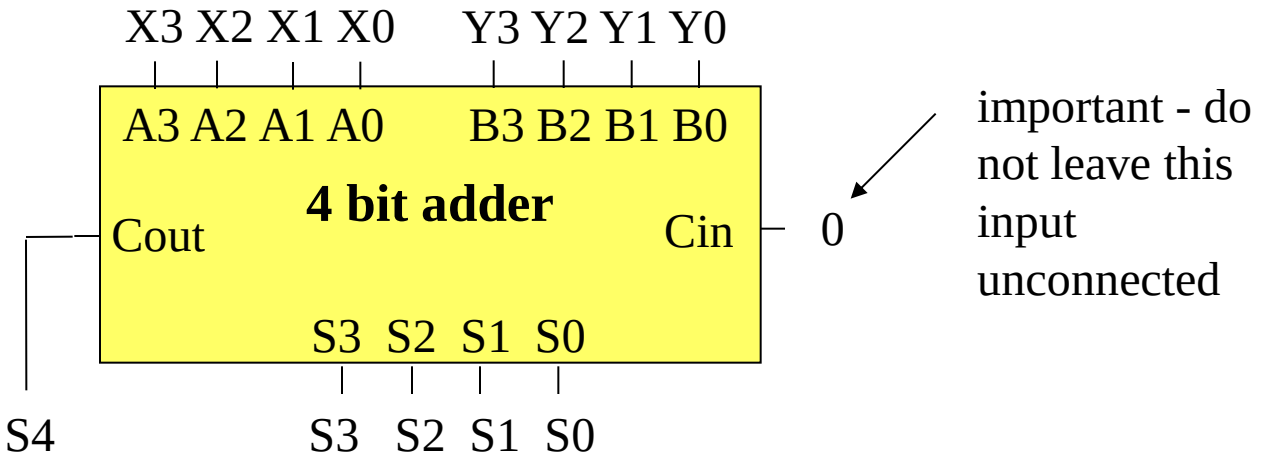


4-bit Binary Adder - 3

Example: Design a digital circuit which will **add the two, 4-bit numbers X3 X2 X1 X0 and Y3 Y2 Y1 Y0**. Use a 7483 4-bit adder in the solution. Determine the number of outputs. Label all inputs and outputs.

Define inputs & outputs:

X3 X2 X1 X0
+ Y3 Y2 Y1 Y0
S4 S3 S2 S1 S0



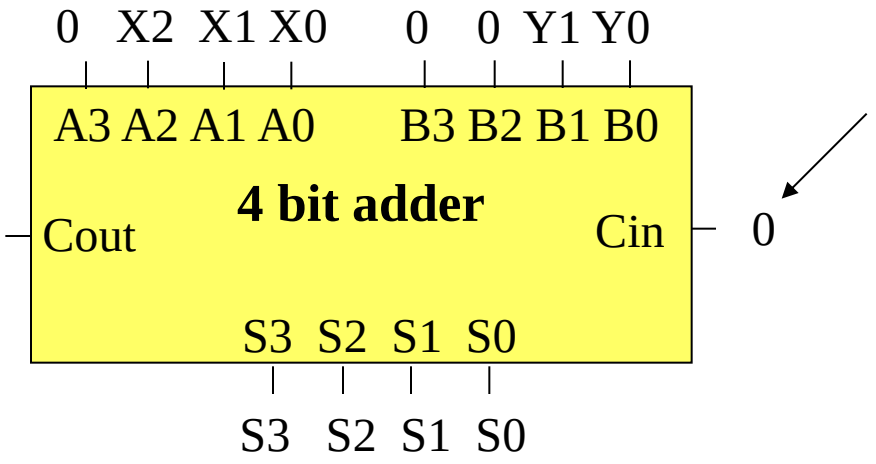
Note difference between internal labels and external labels.

4-bit Binary Adder - 4

Example: Design a digital circuit which will **add the two binary numbers X2 X1 X0 and Y1 Y0**. Use a 7483 4-bit adder in the solution. Determine the number of outputs. Label all inputs and outputs.

Define inputs & outputs:

```
   X2 X1 X0
+     Y1 Y0
-------------
S3 S2 S1 S0
```



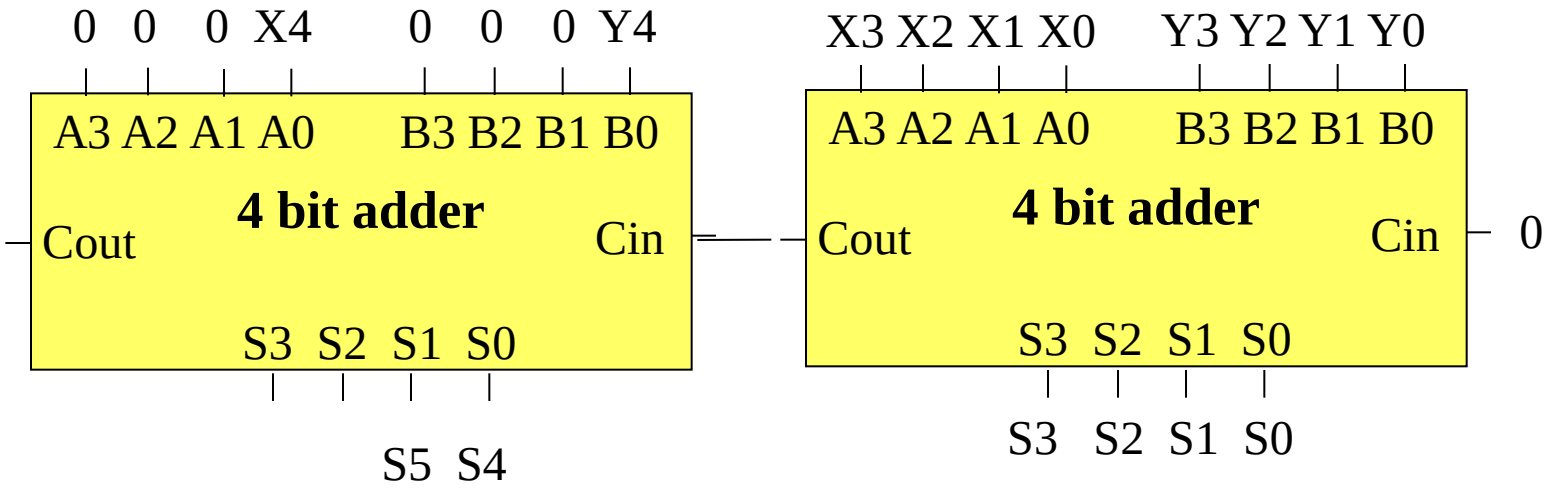
important - do not leave this input unconnected

4-bit Binary Adder - 5

Example: Design a digital circuit which will **add the two, 5-bit numbers X4 X3 X2 X1 X0 and Y4 Y3 Y2 Y1 Y0**. Use 7483 4-bit adders in the solution. Determine the number of outputs. Label all inputs and outputs.

Define inputs & outputs:

X4 X3 X2 X1 X0
+ Y4 Y3 Y2 Y1 Y0
S5 S4 S3 S2 S1 S0



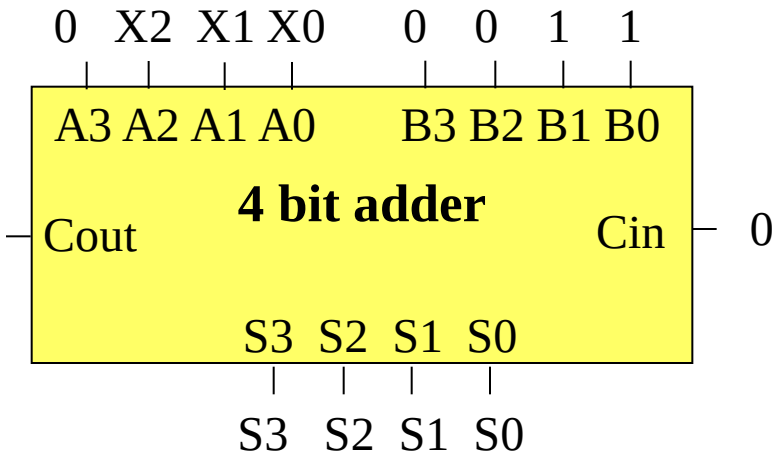
Note difference between internal labels and external labels.

4-bit Binary Adder - 6

Example: Design a digital circuit which will **add the value 3 (11) to a 3-bit number X2 X1 X0**. For example if the input is 4 (100) then the output would be 7 (111). Use a 7483 4-bit adder in the solution. Determine the number of outputs. Label all inputs and outputs.

Define inputs & outputs:

```
   X2 X1 X0
+     1  1
S3 S2 S1 S0
```

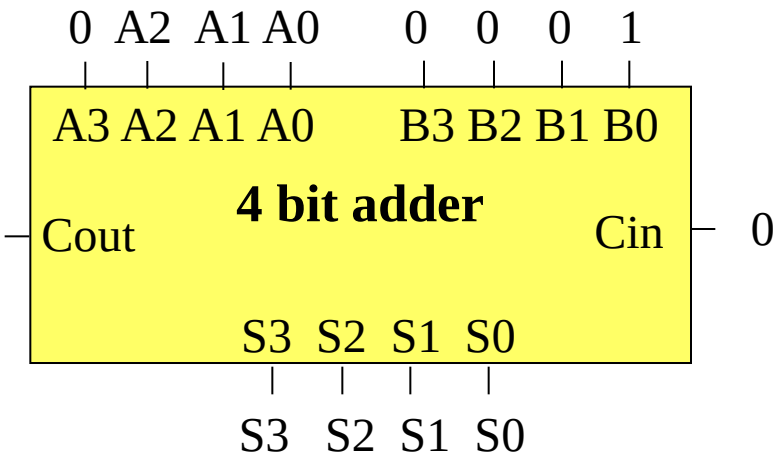


Generalize.

4-bit Binary Adder - 7

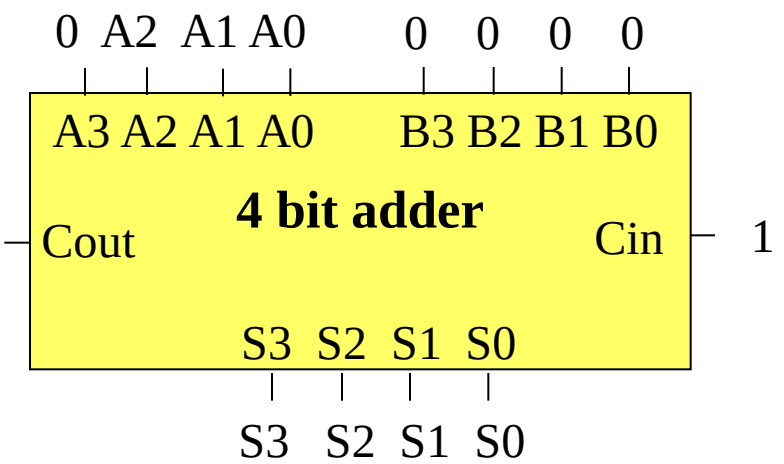
Example: Design a digital circuit which will **increment a 3-bit value, A2 A1 A0, by one**. For example, if the input is 4 (100) then the output would be 5 (101). Use a 7483 4-bit adder in the solution. Determine the number of outputs. Label all inputs and outputs. Show 2 solutions.

Solution #1



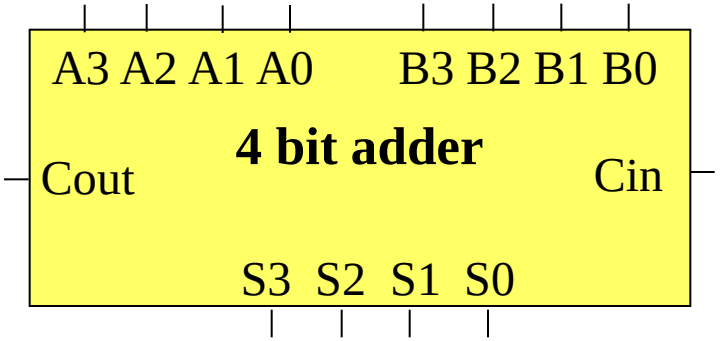
OR

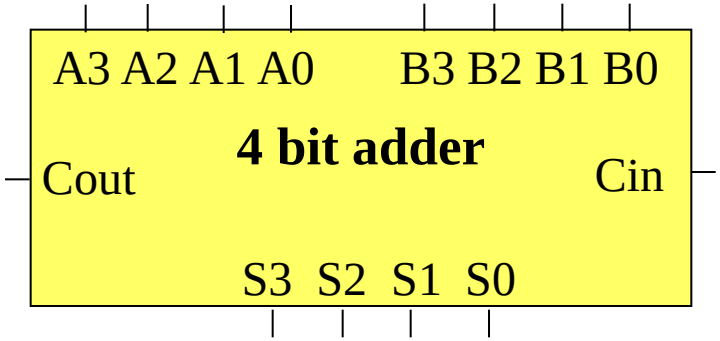
Solution #2



4-bit Binary Adder - 8

Exercise: Design a digital circuit which will **add together three, 3-bit numbers** X2 X1 X0, Y2 Y1 Y0, and W2 W1 W0. Use one or more 7483 4-bit adders in the solution. Determine the number of outputs. Label all inputs and outputs.






```
      X2  X1  X0
      Y2  Y1  Y0
  T3  T2  T1  T0
      W2  W1  W0
S4 S3  S2  S1  S0
```

- Add 2 numbers at a time; create a temporary result (T3 T2 T1 T0)
- Max output is 21, so 5 bits needed.

Binary Adders - Summary

• Addition of binary numbers can be accomplished **using truth tables and K-maps OR by using a more modular approach with full adders and 4-bit adder MSI devices**. The truth table approach can be very cumbersome if the number of bits is large. Consider adding 2, 4-bit numbers. The truth table would consist of 256 lines and require minimization of 5, 8-variable functions:

A3	A2	A1	A0	B3	B2	B1	B0	S4	S3	S2	S1	S0	
0	0	0	0	0	0	0	0	0	0	0	0	0	0 + 0 = 0
0	0	0	0	0	0	0	1	0	0	0	0	0	0 + 1 = 1
0	0	0	0	0	0	1	0	0	0	0	1	0	0 + 2 = 2
…								…					
1	1	1	1	1	1	1	1	1	1	1	1	0	15 + 15 = 30

• What would be the results if we needed to add 2, 32-bit binary numbers?

• The 4-bit binary adder offers a great deal of flexibility and modularity, and simplifies design.

Binary Adders - Summary

• 4-bit binary adders (7483 or 74283) can be **cascaded together** to perform addition operations on binary numbers that are greater than 4 bits.

• Binary adders can be relatively **slow** due to the **"ripple" nature** of the carry propagation. For example, in a circuit to add two, 16-bit numbers together, it takes some time for the carry information to propagate through the 16 full adders (from the lsb to the msb). In order to accelerate the addition process, a technique called "lookahead carry" can be used. This circuitry is added to the standard 4-bit binary adder to reduce the delay. Project: investigate and simulate "**lookahead carry**". Determine the reduction in time to add 2 binary numbers using lookahead carry. Compare a 4-bit adder with lookahead carry with a 4-bit adder without lookeahead carry. [Reference Mano, Kime ,or Wakerly.]

What you should know about Adders

- Be able to construct truth table and circuit for **half-adder** and **full-adder**.
- Understand and describe **operation and block diagram** (with internal labels) of 7483 4-bit MSI adder.
- Be able to apply 7483 MSI adder to **add any two n-bit binary numbers**. Use appropriate **external labels and internal labels**.
- Be able to **cascade** two or more 7483 MSI adders to solve binary addition problems.

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Topics:

- Half-adders, Full-adders — Video part 1
- MSI 4-bit ripple adder IC (7483)
- Binary adder applications

- Binary Multiplication — **Video part 2 ←**

- Review Questions — Video part 3

Binary Multiplication - 1

Consider a 2-bit x 2-bit multiplication operation.

```
                A1  A0
            x   B1  B0
          ---------------
          A1B0  A0B0
      A1B1  A0B1
    ---------------------
M3    M2    M1    M0
```

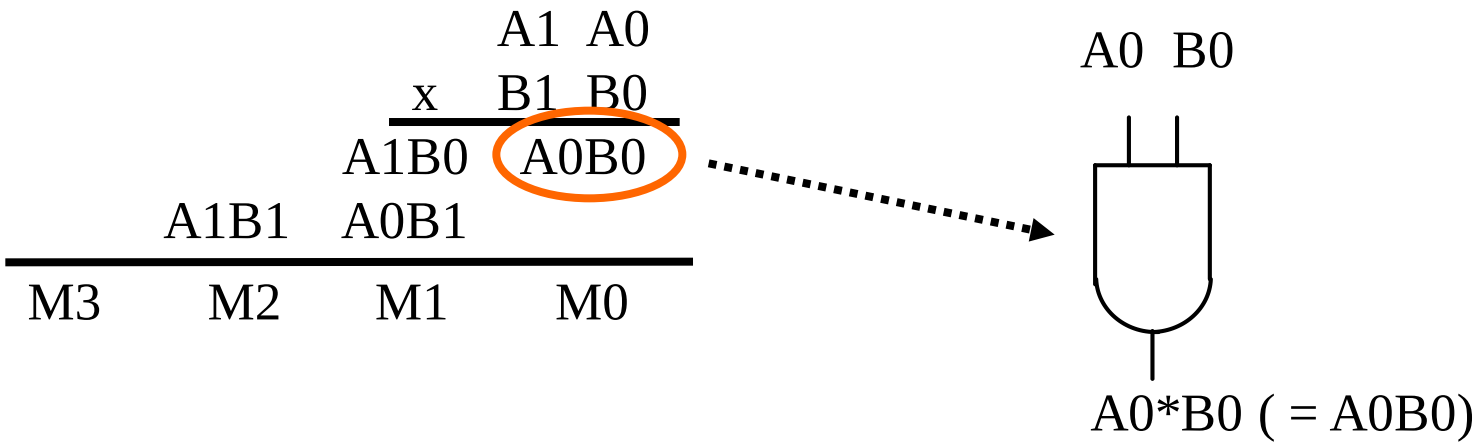
- 4 inputs A1, A0, B1, B0
- 4 outputs needed.
- What is the max output value?
- Shift and add
- How tp produce A0B0 term?
- How do we build a circuit to perform this calculation?

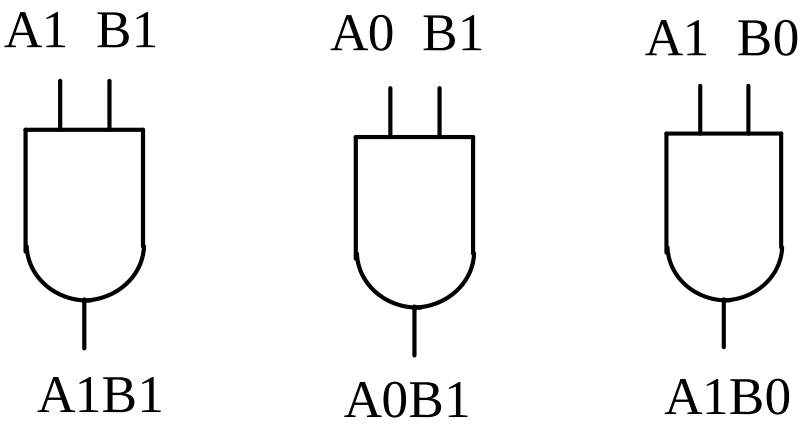
Try some examples.

```
   1 0
x  1 1
-------
   1 0
 1 0
-------
 1 1 0
```

Binary Multiplication - 2

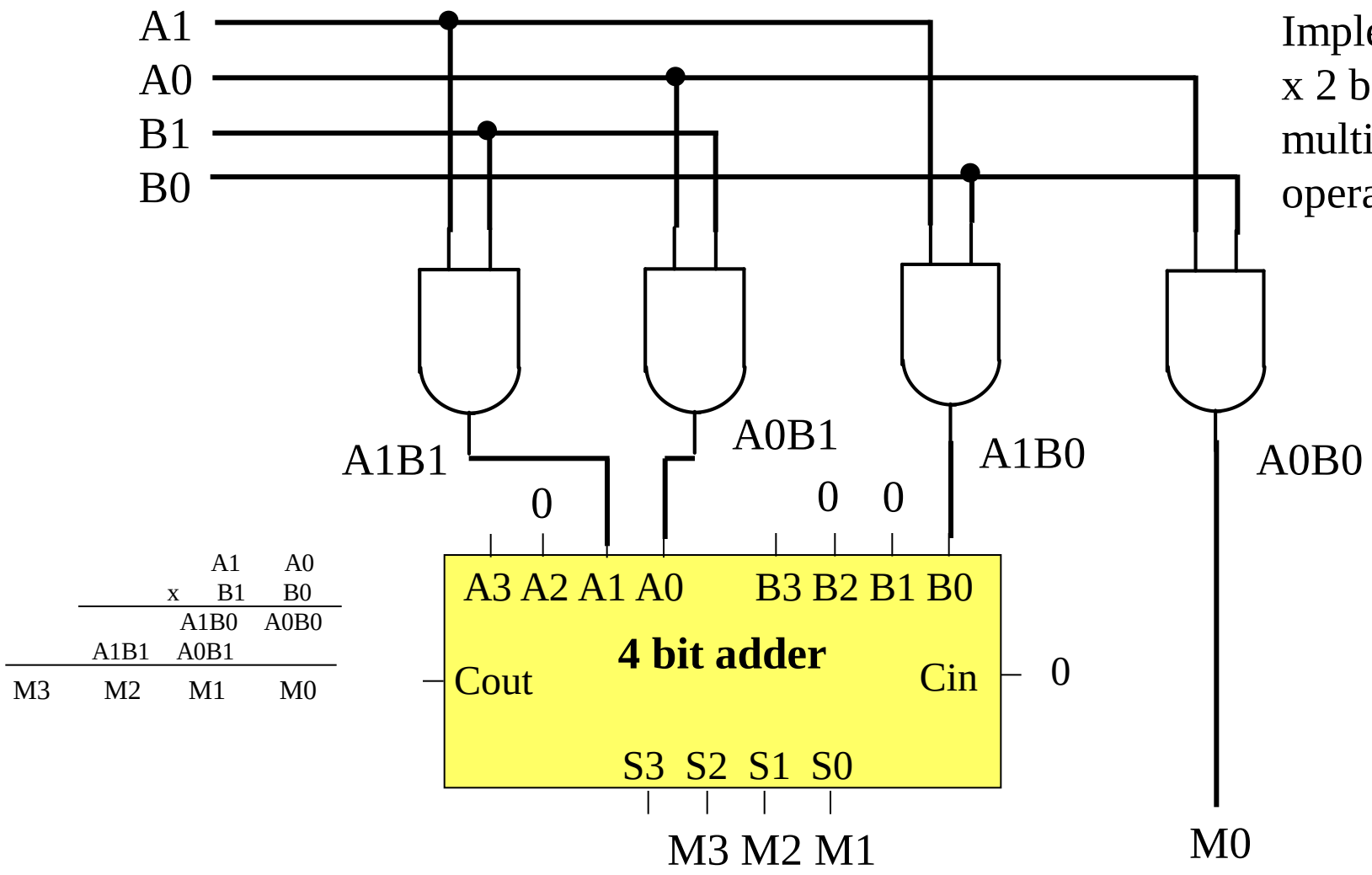
Consider a 2-bit x 2-bit multiplication operation.





Note: use an AND gate to implement each product term.
- Next step: add terms together using adder circuit.

Binary Multiplication - 3



Implement 2 bit x 2 bit multiplication operation.

		A1	A0	
	x	B1	B0	
		A1B0	A0B0	
	A1B1	A0B1		
M3	M2	M1	M0	

Binary Multiplication - 4

Consider a 3 bit x 3 bit multiplication operation.

```
                    A2  A1  A0
                x   B2  B1  B0
            ---------------------
                A2B0  A1B0  A0B0
            A2B1  A1B1  A0B1
        A2B2  A1B2  A0B2
    ---------------------------------
M5    M4    M3    M2    M1    M0
```

- 6 inputs
- 6 outputs needed.
- What is the max output?
- Shift and add
- How to produce A0B0 term?

Try some examples.

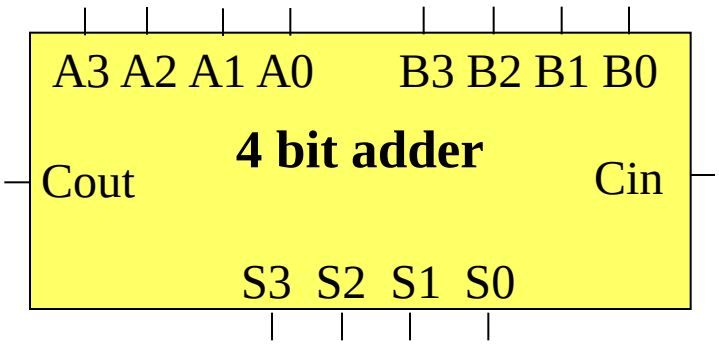
```
  1 1 0
x 1 0 1
```

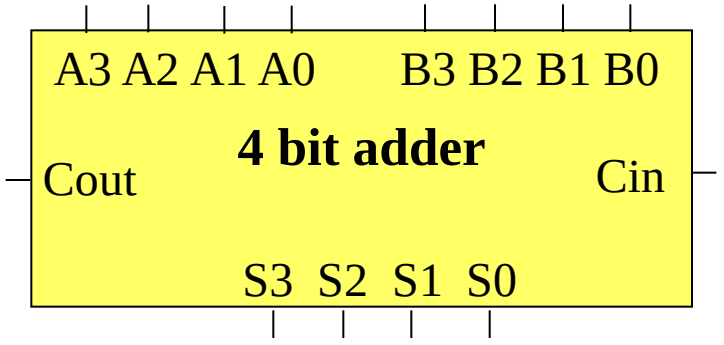
```
  1 1 1
x 1 1 1
```

Binary Multiplication - 5

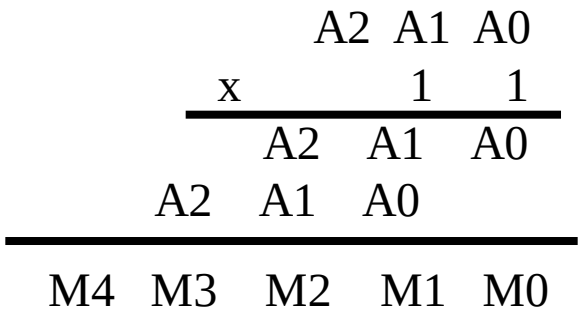
```
                          A2  A1  A0
                      x   B2  B1  B0
                  ---------------------
                  A2B0  A1B0  A0B0
              A2B1  A1B1  A0B1
          A2B2  A1B2  A0B2
      ---------------------------------
M5    M4    M3    M2    M1    M0
```

Exercise: Design a circuit to perform a 3 bit x 3 bit multiplication operation.
- show calculation
- determine # of outputs
- label all inputs and outputs
- draw circuit.

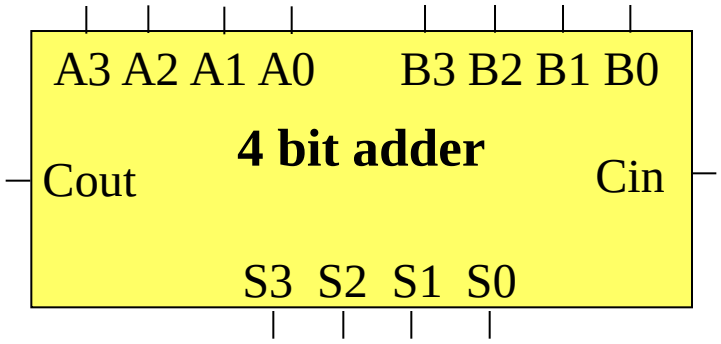


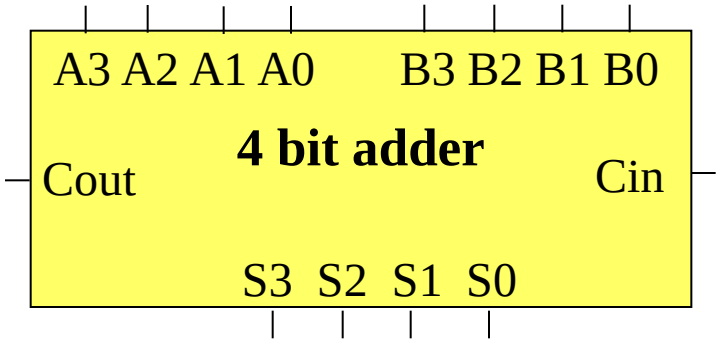


Binary Multiplication - 6



Exercise: Design a circuit to multiply a 3-bit number by the constant value of 3 (011)
- show calculation
- 3 inputs only (A2 A1 A0)
- determine # of outputs
- label all inputs and outputs
- draw circuit.
- keep internal labels and external labels separate





What you should know about binary multiplication

- Be able to design any n-bit by m-bit binary multiplier. Show operation, outputs, product terms, label all inputs and outputs, show lsb, circuit, use 4-bit adders where appropriate (Example: 2 by 2, 2 by 3, 3 by 3, 3 by 4, 4 by 4, 4 by 5, etc.)

- Be able to design any n-bit by constant multiplier. Example: 3-bit (or 4-bit) input multiplied by value of 3 (11) or 5(101) etc. Generalize from examples shown in previous slides.

- Clear documentation, problem statement, sample operation, input labels, output labels for every component are required.

- Can you design a circuit that multiplies a 3-bit number by a constant of 4 (100) without using any gates or components?

- Setup the truth table for a 3-bit by 3-bit multiplier. What is the disadvantage of this approach? How many IC's are required?

Practice Exercises

- Design circuit to **add** two, 3-bit numbers (A2,A1,A0; B2; B1, B0) with a 7483 adder chip. Label all inputs and outputs.
- Design circuit to **add** two 6-bit binary numbers with one or more 7483 adder chip(s). Label all inputs and outputs.
- Design circuit to **multiply** a 3-bit number by a 2 bit number (future HW problem)
- Design a circuit to **multiply** a 4-bit binary number by the constant 5.
- Design a circuit to **multiply** a 4-bit binary number by the constant 4. (do not use any gates!)

Summary

- The **4-bit binary adder** (7483) is an MSI device and it incorporates 4 full adders internally. It can be used as a building block to design complex circuits.
- The 4-bit adder can **add two, 4-bit binary numbers** (8 data inputs). There are 4 outputs plus a carry out. There is also a carry-in input.
- The **7483 IC can be cascaded** to perform addition of larger binary numbers. For example, one 7483 can add two, 4-bit numbers; two 7483 chips can add two, 8-bit numbers and so on. The carry-out of one 7483 must be connected to the carry-in of the next 7483 chip in the appropriate fashion.
- A 4-bit adder (such as the 7483) is generally represented by a **block diagram showing internal labels** for all inputs and outputs.
- **Binary multiplication** is implemented using AND gates to perform the bit multiplication, and one or more MSI adder chips to add the results.
- When designing digital circuits to perform addition and multiplication of binary numbers, it is important to **clearly define all inputs and all outputs**. Generally the inputs will be defined in the problem, but you must determine how many outputs will be generated and provide the outputs with appropriate labels/names (and also clearly define lsb, msb)

Further Reading

- Mano,Kime, Logic and Computer Design Fundamentals, 2nd edition, Prentice Hall (Chapter 3).

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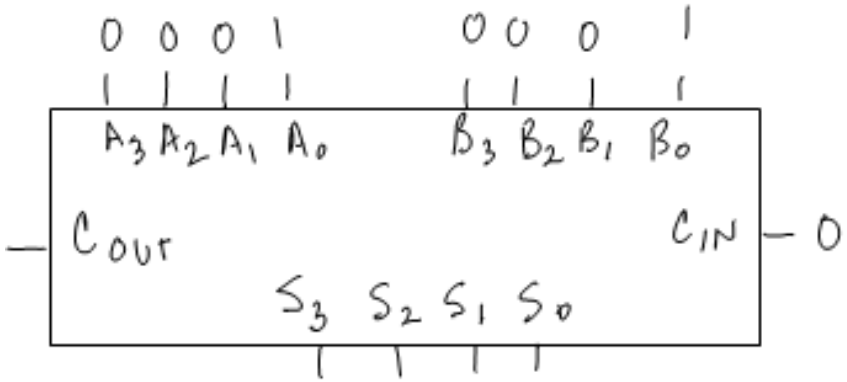
- Binary Multiplication — Video part 2

- Review Questions — **Video part 3 ←**

Sample Questions

#1.- What are the outputs of the following 4-bit adder circuit?



a) Cout = 0; S3=0; S2=0;S1=1; S0=1

b) Cout = 0; S3=0; S2=0;S1=1; S0=0

c) Cout = 1; S3=0; S2=0;S1=1; S0=0

d) Cout = 1; S3=0; S2=0;S1=1; S0=1

Sample Questions

#2.- What are the outputs of the following 4-bit adder circuit?

a) Cout = 1; S3=1; S2=1;S1=1; S0=1

b) Cout = 1; S3=0; S2=0;S1=1; S0=0

c) Cout = 1; S3=0; S2=0;S1=1; S0=1

d) Cout = 1; S3=1; S2=1;S1=1; S0=0

Sample Questions

#3.- How many outputs are required for a circuit that multiplies 2, 2-bit numbers?

a) 2 b) 3 c) 4 d) 5 e) 6

#4.- How many outputs are required for a circuit that multiplies 2, 3-bit numbers?

a) 2 b) 3 c) 4 d) 5 e) 6