Penn State Abington
CMPEN 271
Lecture Set #7
K-Maps

Topics:

- Karnaugh Maps (K-Maps) I — **Video part 1 of 3 ←**
- Karnaugh Maps II — Video part 2 of 3
- Sample Exercises — Video part 3 of 3

What is a Karnaugh Map (K-MAP)?

- The **Karnaugh Map** is a technique invented by Maurice Karnaugh in the 1950s to simply digital logic minimization.
- A K-map is a **2-d, graphical representation** of the 1's and 0's from a truth table
- Using graphical methods of **circling groups of 1's**, K-maps result in the minimization of a Boolean function
- The K-map method is generally **quicker and more reliable** than using algebra by hand
- If a K-map is used correctly, **no Boolean algebra is necessary** -- the function will be completely minimized
- In order to use a K-map you must first **start with a truth table** or a Boolean function to be minimized
- K-maps are useful for **small tasks**, and for demonstrating minimization techniques.
- **Computer programs** exist for the minimization of complex Boolean function (e.g. Espresso II). Software such as Multisim can also minimize Boolean functions.

Review of Algebraic Approach

Example:

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

$A'BC' = \mathbf{m_2}$

$A'BC = \mathbf{m_3}$

$F = \Sigma\, m\,(2, 3)$

$= A'BC' + A'BC$

$= A'B\,(C + C')$

$= A'B$ (final answer)

Logical Adjacency (discuss)! Can we leverage?

K-map -1

Same Example:

Start with truth table:

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

$A'BC' = \mathbf{m_2}$

$A'BC = \mathbf{m_3}$

Step 1: Construct and fill K-map

	B'C'	B'C	BC	BC'
A'	m_0	m_1	m_3 1	m_2 1
A	m_4	m_5	m_7	m_6

3-var. K-map

1) K-map - same contents as truth table
2) normally show 1's; assume empty squares are 0's
3) notice labels and ordering (based on log. adj.)

K-map -2

Step 2: Group 1's according to rules and
Step 3: Obtain product terms to be OR'd

	B'C'	B'C	BC	BC'
A'	m_0	m_1	m_3 1	m_2 1
A	m_4	m_5	m_7	m_6

3-var. K-map

Directly from K-map: F = A'B (now we are done!)

K-map -3

Step #1: **Draw K-map**, label, and fill with 1's and 0's (optionally leave 0's blank/empty) from truth table.

Step #2: **Group (circle) all 1's** in "rectangular" sets of 1, 2, 4, or 8. Each group corresponds to a product term in answer. **LARGER** groups are better. **FEWER** groups are better. Groups can **OVERLAP**. All 1's must be circled. (We will discuss more formal methods later).

Step #3: Write down a **product term** for each group of 1's. Any variable that changes (from inverted to non-inverted) throughout the group will vanish in product term. **"OR" all product terms together** to create final simplified function. More variables will vanish in a product term with the larger groups.

K-map -4

Given: $F(A,B,C) = \Sigma\, m\,(2, 3, 7)$

	B'C'	B'C	BC	BC'
A'			1	1
A			1	

2 groups

Answer: F = A'B + BC

K-map - 5

Given arbitrary truth-table, simplify F(A,B,C):

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

	B'C'	B'C	BC	BC'
A'				
A				

F = ?

K-map - 6

<u>Given an arbitrary truth-table, simplify F(X,Y,Z):</u>

X	**Y**	**Z**	**F**
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

	Y'Z'	Y'Z	YZ	YZ'
X'				
X				

F = ?

K-map - 7

Given an arbitrary truth-table, simplify F(X,Y,Z):

X	Y	Z	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

	Y'Z'	Y'Z	YZ	YZ'
X'				
X				

F = ?

(Note: edges of K-map connect!)

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4-variable K-map (16 squares)

Notice the ordering; groups of 1, 2, 4, 8, 16 possible; edges are connected

A	B	C	D	F	
0	0	0	0		m_0
0	0	0	1		m_1
0	0	1	0		m_2
0	0	1	1		m_3
0	1	0	0		m_4
0	1	0	1		m_5
0	1	1	0		m_6
0	1	1	1		m_7
1	0	0	0		m_8
1	0	0	1		m_9
1	0	1	0		m_{10}
1	0	1	1		m_{11}
1	1	0	0		m_{12}
1	1	0	1		m_{13}
1	1	1	0		m_{14}
1	1	1	1		m_{15}

	C'D'	C'D	CD	CD'
A'B'	m_0	m_1	m_3	m_2
A'B	m_4	m_5	m_7	m_6
AB	m_{12}	m_{13}	m_{15}	m_{14}
AB'	m_8	m_9	m_{11}	m_{10}

K-map - 8

4-variable K-map -- 16 squares

A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	1

	C'D'	C'D	CD	CD'
A'B'	m_0	m_1 1	m_3 1	m_2
A'B	m_4 1	m_5	m_7 1	m_6 1
AB	m_{12} 1	m_{13}	m_{15} 1	m_{14} 1
AB'	m_8	m_9	m_{11}	m_{10} 1

F = ?

edges are connected

K-map - 9

	C'D'	C'D	CD	CD'
A'B'		**1**	**1**	**1**
A'B	**1**	**1**	**1**	**1**
AB	**1**	**1**	**1**	**1**
AB'				

F = ?

K-map - 10

	C'D'	C'D	CD	CD'
A'B'	**1**		**1**	**1**
A'B	**1**	**1**		
AB	**1**	**1**		
AB'	**1**			**1**

F = ?

K-map - 11

	C'D'	C'D	CD	CD'
A'B'		**1**	**1**	**1**
A'B	**1**	**1**		
AB	**1**	**1**		
AB'		**1**		**1**

F = ?

K-map - 12

	C'D'	C'D	CD	CD'
A'B'	**1**			
A'B		**1**		
AB			**1**	
AB'				**1**

F = ?

K-map - 13

	C'D'	C'D	CD	CD'
A'B'	**1**			
A'B	**1**	**1**		
AB	**1**			
AB'				

F = ?

K-map - 14

	C'D'	C'D	CD	CD'
A'B'				
A'B	**1**	**1**		
AB	**1**			
AB'	**1**	**1**		

F = ?

K-map - 15

	Y'Z'	Y'Z	YZ	YZ'
W'X'	1	1	1	1
W'X		1		
WX		1		
WX'	1	1	1	1

F = ?

K-map - 16

	Y'Z'	Y'Z	YZ	YZ'
W'X'	**1**	**1**	**1**	**1**
W'X	**1**	**1**	**1**	**1**
WX	**1**	**1**	**1**	**1**
WX'	**1**	**1**	**1**	**1**

F = ?

K-map - 17

	Y'Z'	Y'Z	YZ	YZ'
W'X'	1	1	1	1
W'X	1		1	1
WX	1	1	1	1
WX'	1	1	1	1

F = ?

K-map Summary

- K-map is a **graphical method** for minimizing Boolean functions. It is an alternative to algebraic methods.
- If a K-map is performed correctly, then the resulting function is minimum -- **no further reduction** (by algebra or any other method) can reduce it further.
- When grouping 1's in a K-map, **maximize** the size of the group, **minimize** the number of groups. Groups can overlap.
- In a K-map, you are permitted to group 1's in **group sizes** of 1, 2, 4, 8, 16. Groups can be square or rectangular
- **Edges** of a K-map are connected (also the **4 -corners** are in a group)
- If you group the 0's in a K-map (instead of the 1's), then you will obtain the **complement** (inverse) of F. Sometimes this is useful if you want F' on SOP or if F has very few zero entries in the K-map.
- We will examine more **formal methods** for K-maps in a future lecture.
- There are **computer programs** which can automate the minimization of Boolean functions, especially when there are a large number of variables (example: Quine-McKluskey, expresso s/w)

What you should know...

- Be able to explain to **purpose** of the K-map method.
- Understand the **pros and cons** of K-map versus algebra for Boolean function minimization
- Be able to **minimize** any 3 or 4-variable function with 2 methods: 1) algebra, and 2) K-map. Start with truth table or Boolean expression in SOP or Sum of minterms form.
- Be able to **construct and label** a 3 and 4-variable K-map. Follow correct ordering and labeling of rows and columns. Use variable defined in problem statement.
- Be able to determine minimized F' (**inverse of F**) from a K-map (circle 0's in K-map instead of 1's)

Further Reading

- Mano Kime, Logic and Computer Design Fundamentals, Prentice Hall, Chapter 2.

- Tocci R., Digital Systems, Prentice Hall, Chapter 3&4.

- www.howstuffworks.com
  "How Boolean Logic Works"

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Sample Exercises

#1.- What is the minimal function described by this K-map?

a) F = AC + B’C’

b) F = C’D’ + CD

c) F = A’B’ + C’D’

d) F = A’C’ + BC’

e) F = A’B’C’D’ + ABC’D

	C'D'	C'D	CD	CD'
A'B'				
A'B				
AB				
AB'				

#2.- Based on the K-Map above, what is F as a sum of minterms?

a) $F = \sum m\,(0, 1, 2, 3)$
b) $F = \sum m\,(0, 1, 2, 3, 4, 5)$
c) $F = \sum m\,(0, 1, 4, 5, 8, 9)$
d) $F = \sum m\,(0, 1, 4, 5, 12, 13)$

Sample Exercises

#3.- What is the minimal function described by this K-map?

a) F = B’ + C’D’
b) F = A’B’ + B’CD’
c) F = AC + B’C’
d) F = B’D’ + C’D’
e) F = A’B’C’D’ + C’D’

	C'D'	C'D	CD	CD'
A'B'				
A'B				
AB				
AB'				

#4 Based on the K-Map above, what is F(A,B,C,D) as a sum of minterms?

a) $F = \sum m\,(0, 2, 4, 8, 10, 12)$
b) $F = \sum m\,(0, 3, 4, 9, 12, 15)$
c) $F = \sum m\,(0, 2, 4, 8, 12, 15)$
d) $F = \sum m\,(0, 1, 4, 5, 8, 9)$