Penn State Abington
CMPEN 271
Lecture Set #4
Logic Gates, Boolean Algebra

R. Avanzato © 2017-2018

Topics:

- Boolean Algebra (Switching Algebra)
- Logic Gates - AND, OR, NOT

Video part 1 of 4 ←

- Boolean Functions
- Circuit Diagrams
- Practice Exercises

Video part 2 of 4

- HW #2 assignment & Multisim Demo

Video part 3 of 4

- Review Questions

Video part 4 of 4

Design Problem

You have been hired at a **nuclear power plant** to design a **logic circuit** that will activate an **alarm** in the event that dangerous conditions are present. There are a series of sensors throughout the plant that monitor conditions such as core temperature (high or normal), fuel rod placement (up or down), control rod placement (up or down), cooling system status (normal or failed). These sensor values (5 volts or 0 volts) will be the **inputs** to your circuit. The **output** of your circuit will be the alarm (on or off).

IF core temperature = **HIGH** **and** fuel rods = **DOWN** **and** control rods = **UP**

or if cooling system status = **FAILED,**

THEN The alarm should be **ACTIVATED (turned on)**

Is the solution software or hardware? We are looking for a hardware (circuit) solution in this course

Boolean Algebra & Logic Gates

- **Boolean Algebra** and **logic gates** are the basis for all the fundamental operations in any digital computer

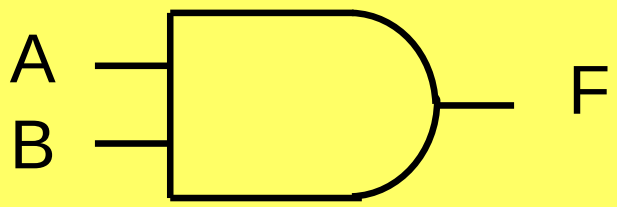
- Boolean Algebra is a **mathematical system** (based on values of "true" and "false") developed by George Boole in 1854. (There were no computers in 1854!)

- **Claude Shannon (Bell Labs) in 1938** applied Boolean Algebra to logic circuits that could be used to build digital computers

- **Boolean algebra is based on 3 operations**: AND, OR, and NOT. Boolean variables can have only one of 2 values: either true (1) or false (0)

AND Gate

If input A is true (1) **AND** input B is true (1), then output F is true (1)



Logic circuit symbol

inputs		output
A	B	F
0	0	0
0	1	0
1	0	0
1	1	1

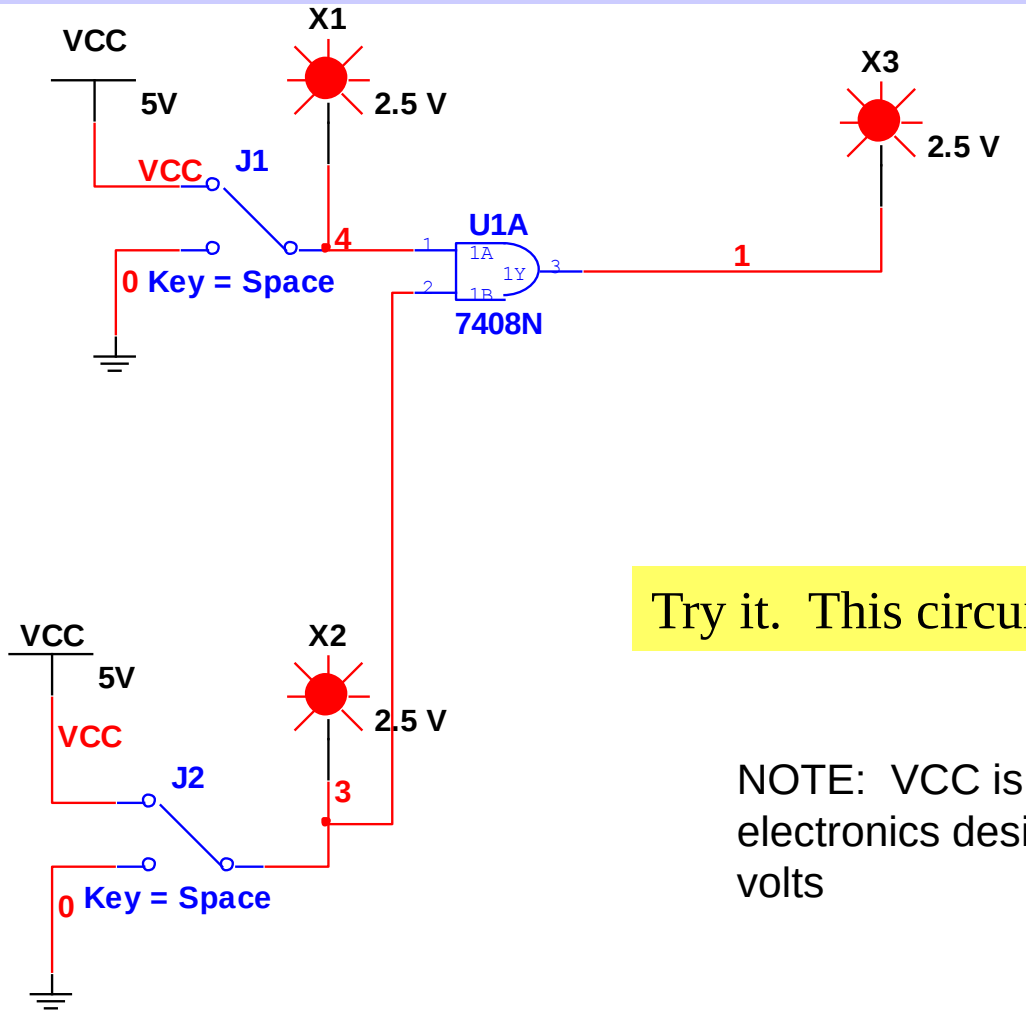
Truth table

$F = A \bullet B = AB$

Algebraic expression

"1" = "high" = "true" = 5v (TTL)
"0" = "low" = "false" = 0v

AND Gate in Multisim



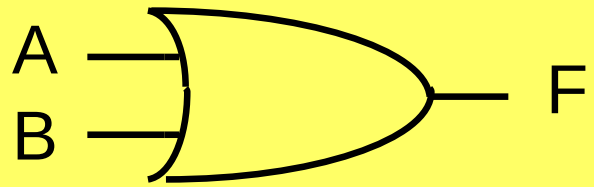
Try it. This circuit is on Canvas

NOTE: VCC is common electronics designation for +5 volts

- Describe inputs and outputs; why are simulation packages important?

OR Gate

If input A is true (1) **OR** input B is true (1), then output F is true (1)



Logic circuit symbol

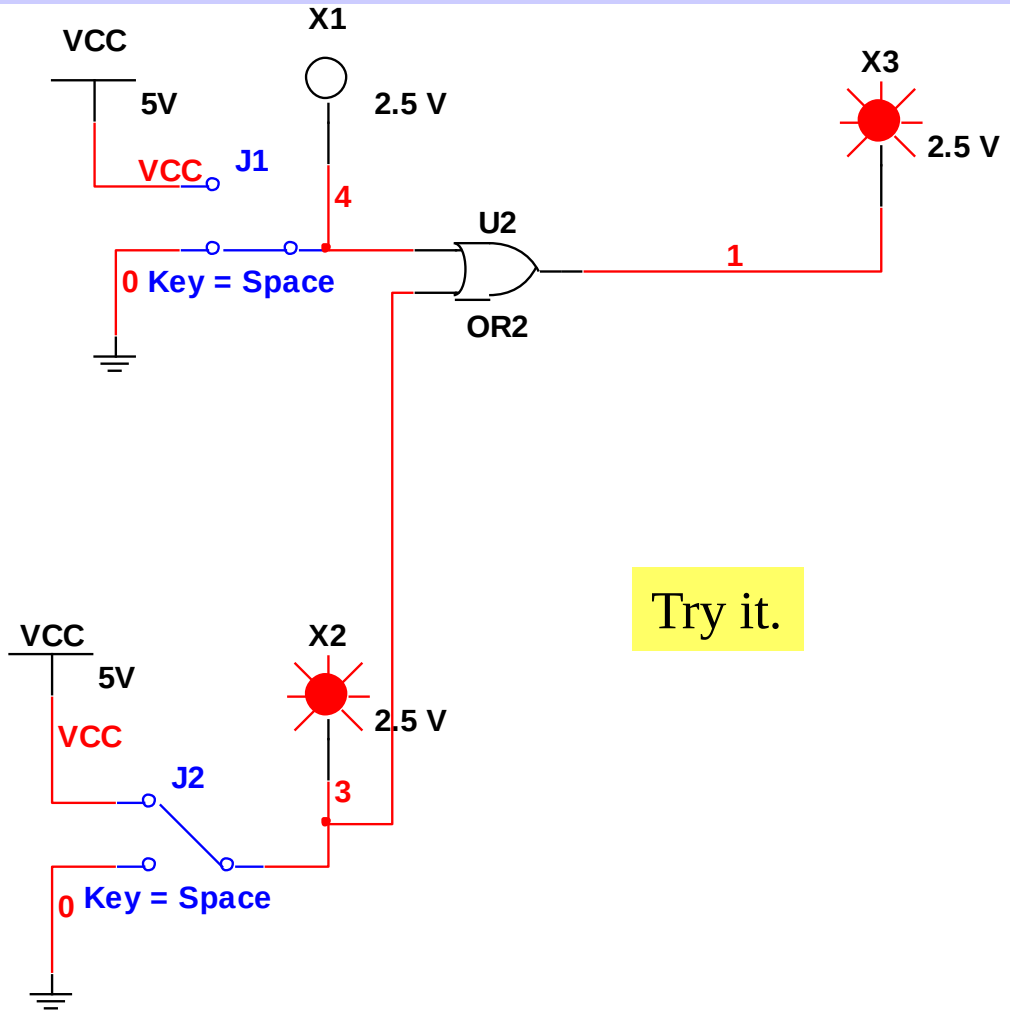
inputs		output
A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

Truth table

$$F = A + B$$

Algebraic expression

OR Gate in Multisim

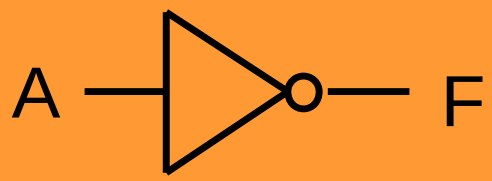


Try it.

- Describe inputs and outputs; why are simulation packages important?

NOT (Inverter) Gate

If input A is true (1) then output F is false(0)
If input A is false (0) then output F is true (1)



Logic circuit symbol

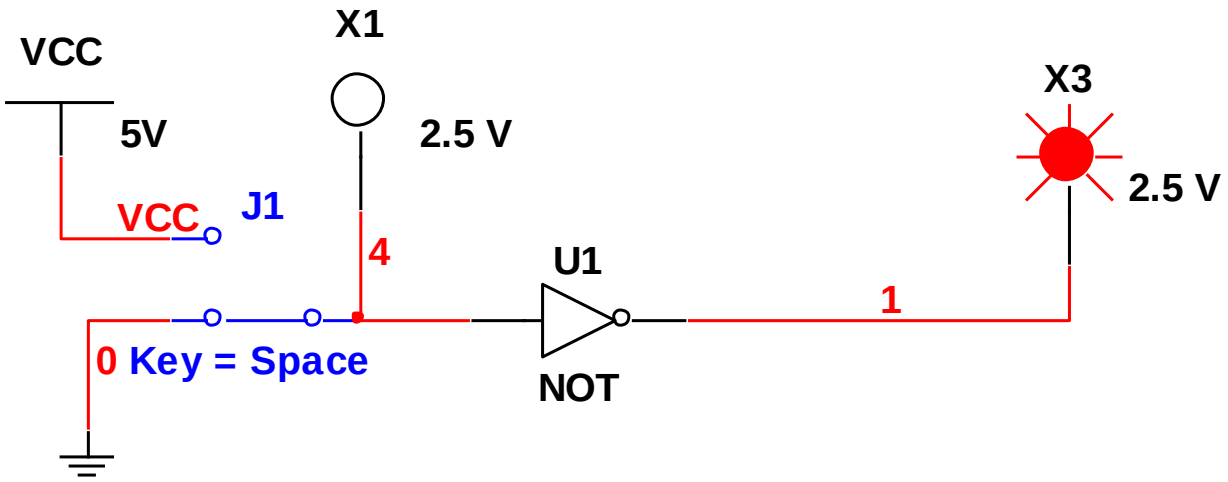
$F = A' = \overline{A}$

Algebraic expression

input	output
A	F
0	1
1	0

Truth table

NOT Gate in Multisim



NOTE: VCC is common electronics designation for +5 volts

Try it.

- Describe inputs and outputs; why are simulation packages important?

Boolean Algebra (0 = false; 1 = true)

$0 \cdot 0 = 0$

$0 \cdot 1 = 0$

$1 \cdot 0 = 0$

$1 \cdot 1 = 1$

$0 + 0 = 0$

$0 + 1 = 1$

$1 + 0 = 1$

$1 + 1 = 1$

$1' = 0$

$0' = 1$

same as

$\overline{1} = 0$

$\overline{0} = 1$

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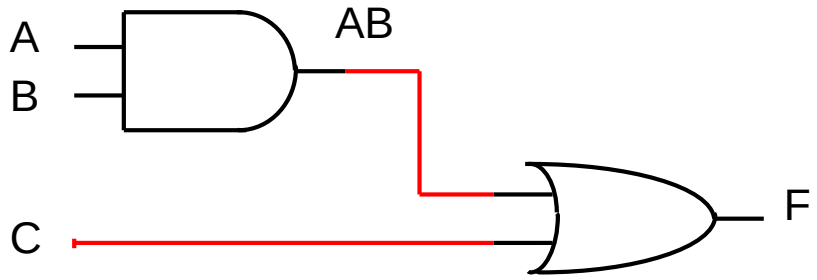
Boolean Functions - 1

Example: **F = AB + C**



inputs			output
A	**B**	**C**	**F**
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

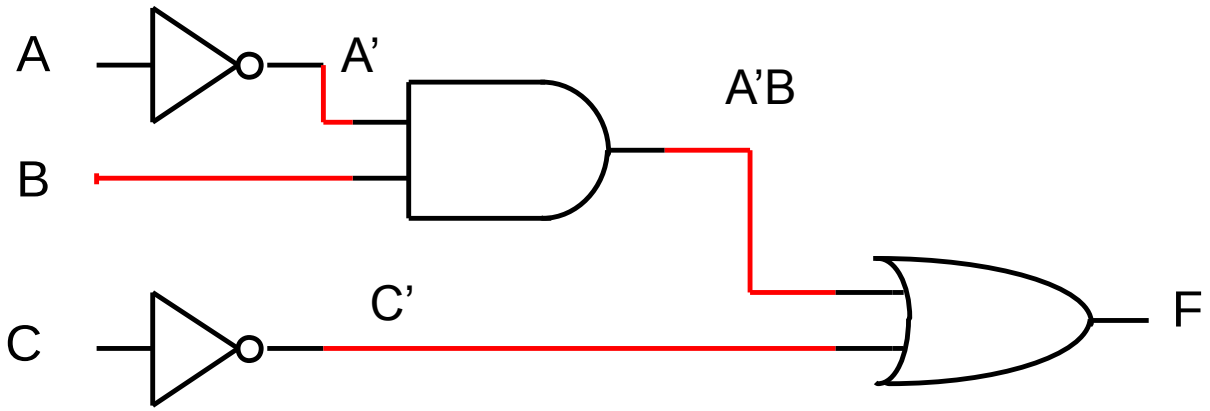
Truth Table

-How many inputs?
-How many outputs?
-Verify truth table with simulation.
-What is **precedence** (priority) of Boolean operators? 1)NOT, 2) AND, 3)OR
- Use **parentheses** to force priority

Try it

Boolean Functions - 2

Example: **F = A'B + C' = $\overline{A}$ B + $\overline{C}$**



inputs			output
A	**B**	**C**	**F**
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Truth Table

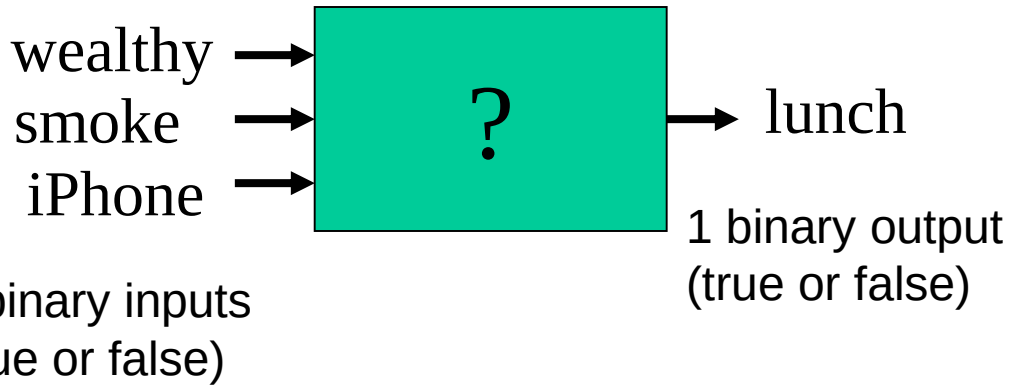
-Given Boolean function, find
- a) logic circuit
- b) truth table
- c) English description

Try it in Multisim

Boolean Functions - 3

Start with a word problem.
Generate: a) Boolean function, b) logic circuit, c) truth table

Example: Design a lunch date assistant logic circuit to help students decide when it is appropriate to ask another individual out to lunch. Design a circuit with **3 binary inputs** and **1 binary output** with the following logic -- if the person is wealthy and does not smoke, or if the person owns an iPhone, then ask to lunch. Design and build a circuit which accomplishes this task.



Boolean Functions - 4

Consider the 2 separate Boolean functions….

#1) friend1 = drink' * smoke' = $\overline{\text{drink}} * \overline{\text{smoke}}$

(output)

#2) friend2 = (drink * smoke)' = $\overline{\text{drink} * \text{smoke}}$

(output)

1) What are the inputs? Outputs?
2) Are the 2 functions the same?
3) Do they have the same truth table?
4) Do they have the same logic circuit?

Boolean Algebra Exercises -1

Practice exercises: For each Boolean expression, generate
a) Logic circuit using AND, OR, and NOT gates
b) Truth table (How many inputs? How many outputs?)
c) Simulation in MultiSim (try on your own)
Note: consider operator precedence (priority): 1) NOT, 2) AND, 3) OR
(Perform operations inside in parentheses first)
Note: AND operation may be denoted by * or · or no symbol; A*B = A · B = AB

#1 F = A + B'

#2 F = (X' + X*Y)'

#3 F = (A + B)' + C

#4 F = (A' + B' * C)'

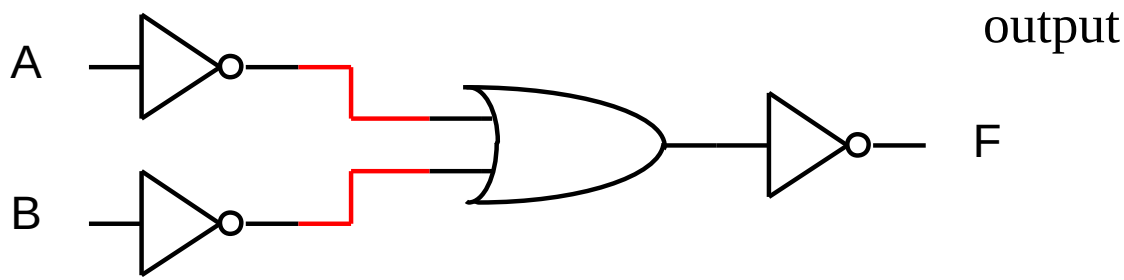
#5 F = X + Y + Z'

#6 F = (A' * B')'

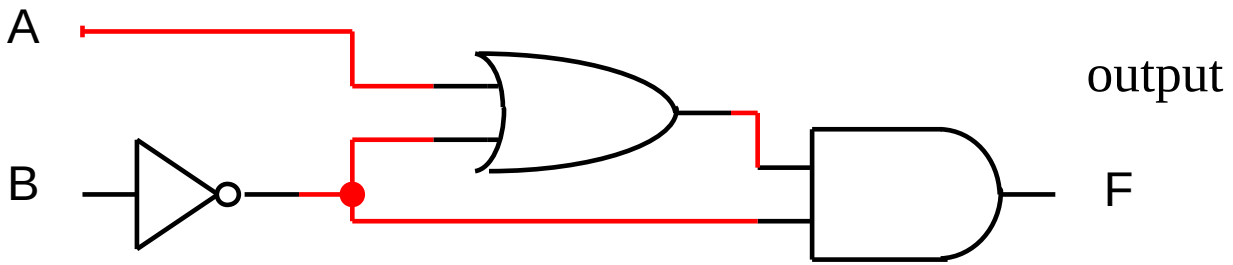
Boolean Algebra Exercises - 2

Practice Exercises: For each logic circuit below, a) generate the Boolean algebraic expression and b) truth table

Circuit #1:



Circuit #2:



Boolean Algebra Summary

- Boolean functions are based on AND, OR, NOT logical operations
- The AND Boolean operation is often represented by the symbol for multiplication, and the OR operation is often represented by a "+" sign. A NOT operation can be represented as an apostrophe (') or a horizontal bar.
- Logic Circuit Diagrams (logic gates) can represent any Boolean function.
- Truth Table (list all inputs and outputs) also represents a Boolean function.
- An English description of a circuit operation can define a Boolean function. (This is often the starting point in a problem.)
- AND, OR, NOT gates can be physically built using transistors (and purchased at electronics suppliers) and are available on integrated circuits (ICs).
- All modern computer circuits (including microprocessors) are based on Boolean logic (AND, OR, NOT gates)
- Circuit simulation software can be used to verify operation of a logic circuit before it is constructed. (Examples: LogicWorks, MultiSim, PSpice).

Boolean Algebra Applications

- How is Boolean Algebra used in WWW search engines? Explain. Provide examples.

- Does C/C++ programming language support AND, OR, NOT operations? Give examples.

- What is the difference between logical AND/OR and bitwise AND/OR in the C/C++ language?

```
x  = 2 && 3;    // what is value of x?
y  = 2 & 3;     // what is value of y?
```

What you should know...

- Know the truth tables and circuit symbols for each of the Boolean operations -- AND, OR, and NOT.
- Given an arbitrary Boolean function, be able to 1) construct a truth table, and 2) draw a circuit diagram with appropriate labels.
- Given an arbitrary logic diagram (circuit), be able to 1) construct the Boolean function, and 2) construct the truth table.
- Be able to determine if 2 Boolean functions are equal by comparing results of truth table.
- Be able to construct and simulate a simple circuit in a digital circuit simulation package such as LogicWorks, CircuitMaker, or MultiSim
- NOTE: See sample exam practice questions on Canvas

Further Reading

- Mano Kime, Logic and Computer Design Fundamentals, Prentice Hall, 2001, Chapter 2.
- Tocci R., Digital Systems, Prentice Hall, Chapter 3.
- www.howstuffworks.com
  "How Boolean Logic Works"
- www.play-hookey.com/digital
- http://www.youtube.com/user/billkleitz

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Homework #2 (see due date)

1. Design a circuit for **nuclear power plant alarm (slide #2)**. Construct a) truth table, b) Boolean expression, c) draw logic circuit, and d) simulate with **Multisim** (or equivalent) circuit simulation software. Clearly label all inputs and outputs. Follow criteria and format for homeworks. **Submit one Word document file** – copy all computer simulation circuits and paste into Word document. (Do not submit MultiSim files; copy Multisim circuits and paste circuits into Word.) Must be well organized. Label and title all diagrams and figures. (Note: You can use Multisim circuit to draw circuit for part c) – you do not need to draw circuit by hand. So, parts c) and d) are same. Test you circuit in Multisim so that it agrees with truth table. **Make sure your name, course name, date , HW number and title of problem are at top of page. Include original problem statement.**

Hints: Create meaningful Boolean variables (whose values are either true (1) or false(0). These variables must relate directly to the given problem. For example, one good Boolean variable name might be: **coreTempHigh**. That is, when coreTempHigh = 1 (true), then the core temperature is high. When coreTempHigh = 0 (false), then the core temperature is not high (it's low). Create other Boolean variables in similar fashion. Remember that **Boolean variables** can only have the values of either True (represented by 1) or False (represented by 0)

Homework #2 (see due date)

This is an **individual** effort. Verbal communication only. If you have questions, ask instructor during class or via email. For a grade of A, all work must be technically correct and complete and also presented in a clear and organized fashion with all relevant documentation.

Label input and output wires in Multisim circuit with same meaningful names as in problem. For example, do not use A,B C for input names.

Use proper functioning inputs with switches in Multisim – see notes or ask questions.

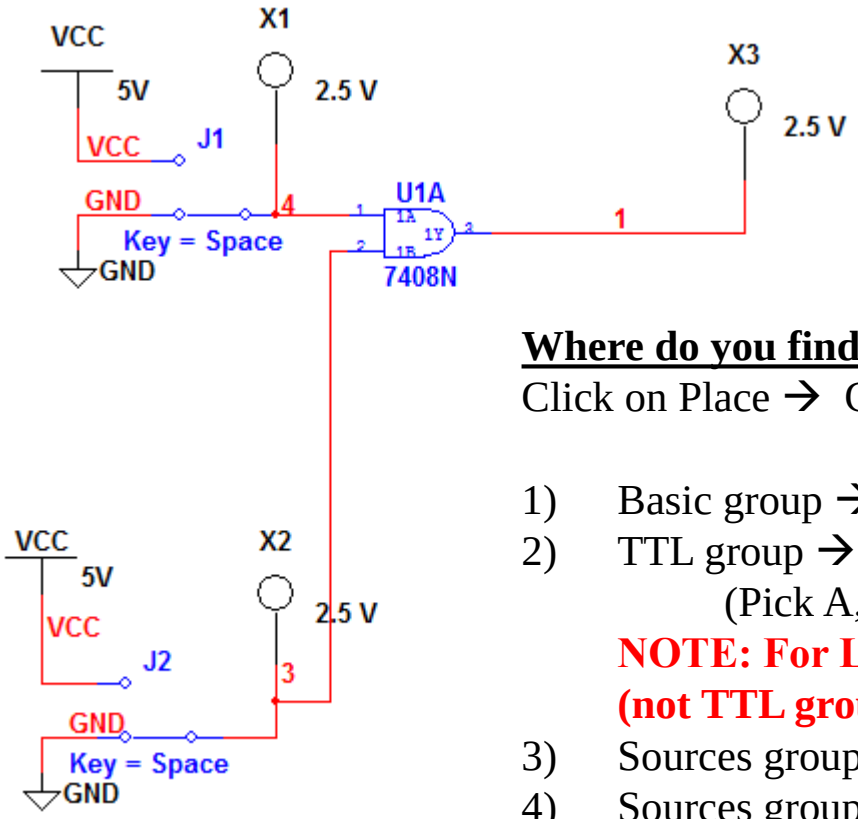
Include original problem statement in the solution – you may copy and paste.

Ask questions in the next class or make appointment to discuss with instructor, or email instructor.

Homeworks in CMPEN 271 are "design" problems and include careful analysis and circuit simulation. These problems may take 3 to 5+ hours to complete successfully. **HW and projects are the most important part of this course!!!**

MultiSIM software

Note: to add labels in Multisim, select Place→ Text



What components do you need to build this circuit?

- SPDT switches
- 7408N AND Gate IC
- Vcc (+5 volts)
- Digital Ground (0 volts)
- Probes

Demo!!

Where do you find components in MultiSim?

Click on Place → Component

Or use search

1) Basic group → Switch family → SPDT
2) TTL group → TTLSTD family → 7408N
   (Pick A, B, C, or D. There are 4 AND gates on the 7408 IC)
   NOTE: For Lecture course, use "Misc Digital" Group -→ TIL (not TTL group)
3) Sources group → Sources → Vcc
4) Sources group → Sources → DGND (digital ground)
5) Indicators group → Probe → probe

Notes: 1) to connect wires: click once at start point, then click once at endpoint; don't drag.
2) 7404N is a NOT gate IC; 7432N is an OR gate IC; 7400N is a NAND gate IC.
3) Turn "on" to simulate circuit.

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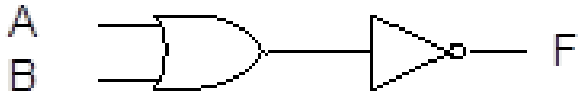
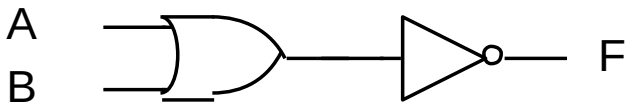
Review Questions

#1. **What Boolean Function corresponds to logic circuit below?**



a) F = A+B b) F = (A+B)' c) F = A' + B' d) F = A + B'

#2. **What truth table corresponds to logic circuit below**



a)

A	B	F
0	0	1
0	1	1
1	0	1
1	1	0

b)

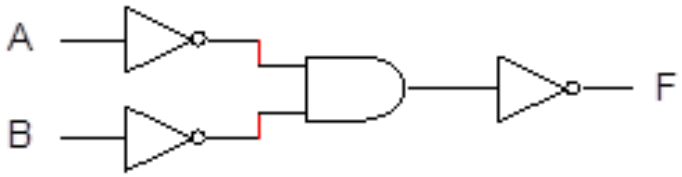
A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

c)

A	B	F
0	0	1
0	1	0
1	0	0
1	1	0

Review Questions

#3. **What truth table corresponds to logic circuit below?**



a)

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

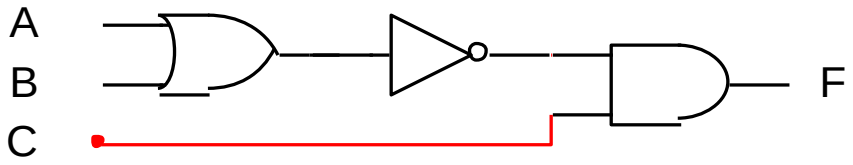
b)

A	B	F
0	0	0
0	1	1
1	0	1
1	1	0

c)

A	B	F
0	0	1
0	1	1
1	0	1
1	1	0

#4. **What Boolean Function corresponds to logic circuit below?**



a) F = (A'+B')C' b) F= (A + B)' C c) F = (AB)' + C d) F = (A + B)' + C