String Searching

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(edits by M. Hudachek-Buswell)

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- There are many applications for these algorithms: test editors, search engines, and DNA and image sequences.

Brute Force

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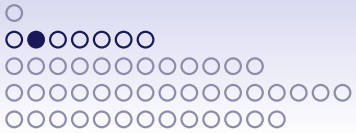
- Align the pattern at the beginning of the text.
- Compare the first character in the text with the first character in the pattern.
- If they don't match, then shift the pattern to the right by one character and compare the first character in the pattern with the current character in the text. If they do match, compare the next character in the pattern with the next character in the text. Repeat this step until you find a match or there can be no more matches.



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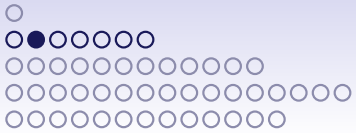
- Once a match is found, to search for additional matches, shift the pattern to the right by one, and compare the first character in the pattern to the current character in the text.



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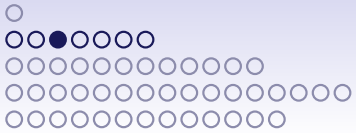
- Once a match is found, to search for additional matches, shift the pattern to the right by one, and compare the first character in the pattern to the current character in the text.
- If any part of the pattern is "hanging off" of the end of the text, then there are no more matches.



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For example, if the text is “abbacacca” and the pattern is “acc”:

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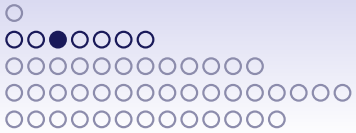

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If you were looking for more matches, then the pattern would be shifted over by one character.

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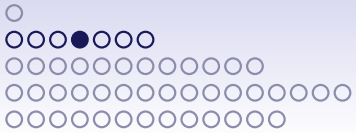

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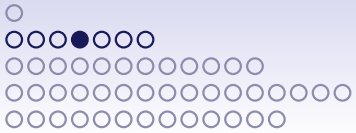

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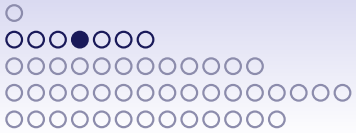

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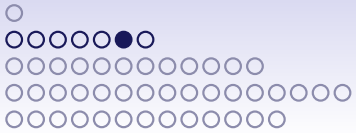
(Note: the pseudocode below returns the first match.)

```
procedure BruteForce(text, pattern)
    i ← 0
    while i <= length of text − length of pattern do
        j ← 0
        while j < length of pattern and text[i + j] = pattern[j]
do
            j ← j + 1
        end while
```



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(Note: the pseudocode below returns the first match.)

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        if j = length of pattern then
            return j
        else
            i ← i + 1
        end if
    end while
    return -1
end procedure
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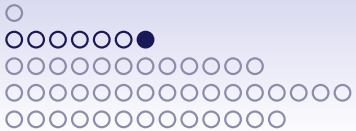
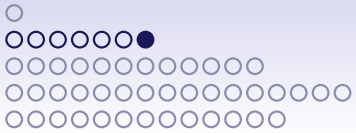


Performance

- In the best case, when searching for only the first match, brute force is $O(m)$, where m is the length of the pattern.



Performance

- In the best case, when searching for only the first match, brute force is $O(m)$, where m is the length of the pattern.
- In all other cases (best case when searching for all matches, as well as average and worst cases), brute force is $O(mn)$, where m is the length of the pattern and n is the length of the text, because there could be a mismatch on the very last character of the pattern at each alignment in the text.

Boyer-Moore

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- In addition, the algorithm starts from the back of the pattern instead of the front.

Boyer-Moore Last Table

- The last table used by Boyer-Moore is a mapping from each character in the alphabet (the set of all characters that may be in the pattern or the text) to the last index the character appears in the pattern.

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- If the character isn't in the pattern, then -1 is used as the value.
- When writing out the last table, instead of writing potentially hundreds of entries for characters that aren't in the pattern, * can be used to represent all other characters.

Boyer-Moore Last Table

For example, the last table for “dog” is:

0	1	2
d	o	g

*
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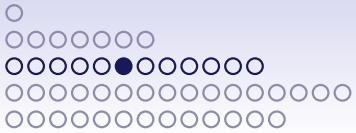
procedure BoyerMooreLastTable(*pattern*)
    lastTable ← mapping from letter to integer
    for *i* ← 0, length of *pattern* **do**
        lastTable[*pattern*[*i*]] ← *i*
    end for
    return *lastTable*
end procedure



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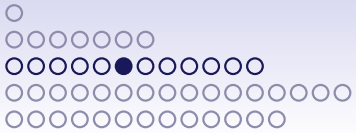

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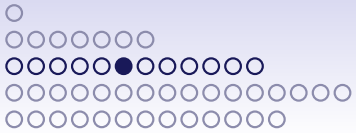
- Align the pattern at the beginning of the text.



Boyer-Moore

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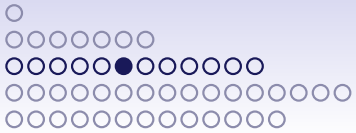
- Compare the last character in the pattern with the character in the text at that index.



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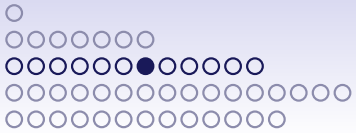
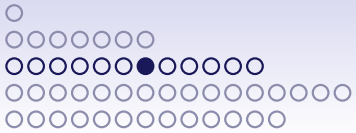
- Align the pattern at the beginning of the text.
- Compare the last character in the pattern with the character in the text at that index.
- If they do match, then go to the previous character in the pattern and the text, and repeat the previous step. If you've compared all of the characters in the pattern, you've found a match.



Boyer-Moore

- If they don't match, then take the character in the text, and look up the value in the last table to get the next alignment of the pattern. For example, if the value in the last table is 2, align the pattern so that index 2 of the pattern is at the mismatching character in the text. Compare again with the last character in the pattern and the current character in the text.



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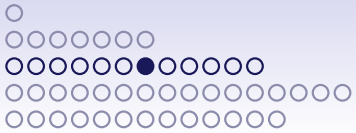

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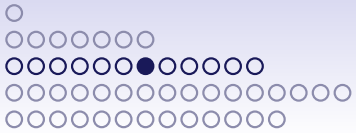
- Once a match is found, to search for additional matches, shift the pattern to the right by one, and compare the last character in the pattern to the current character in the text.



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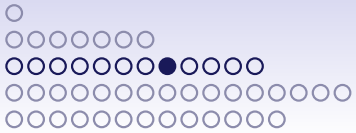
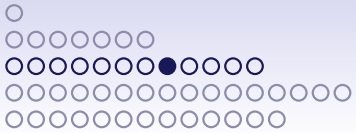
- Once a match is found, to search for additional matches, shift the pattern to the right by one, and compare the last character in the pattern to the current character in the text.
- If any part of the pattern is "hanging off" of the end of the text, then there are no more matches.



Boyer-Moore

For example, if the text is “abdckckcmd” and the pattern is “ckcm”:

a	b	d	c	k	c	k	c	m	d
c	k	c	m						



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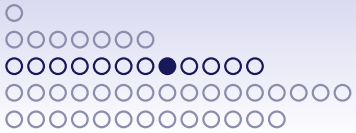
a	b	d	c	k	c	k	c	m	d
c	k	c	m						

c	k	m	*
2	1	3	-1



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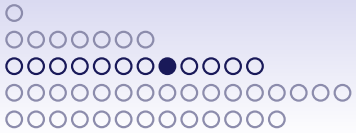

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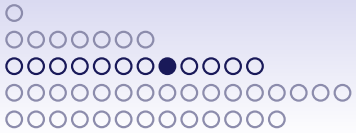

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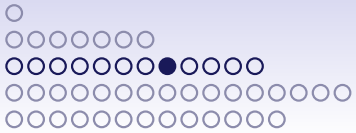

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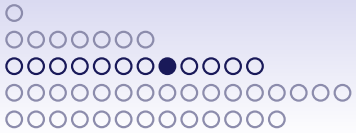

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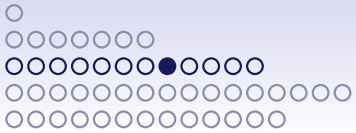

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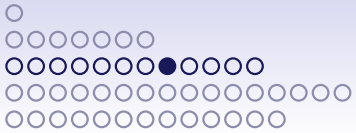

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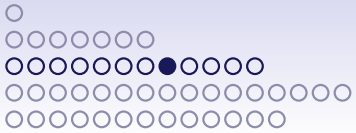

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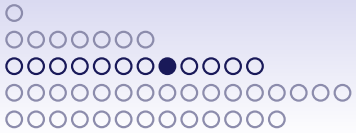

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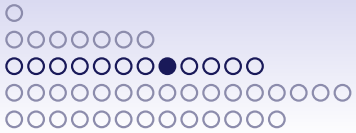

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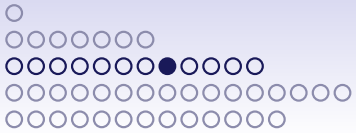

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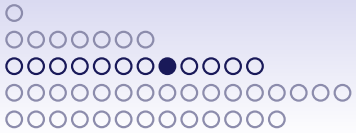

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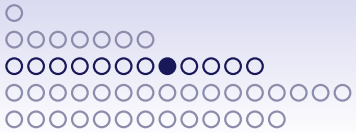

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					c	k	c	m	
						c	k	c	m

c	k	m	*
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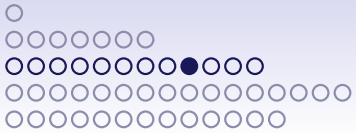
Next alignment is beyond the end of the text.



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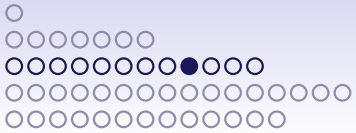
c	k	c	m									



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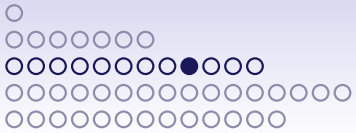

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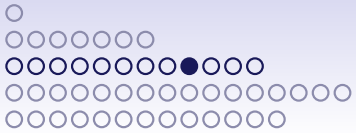

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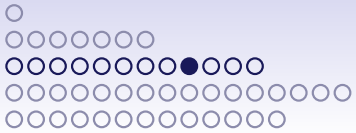

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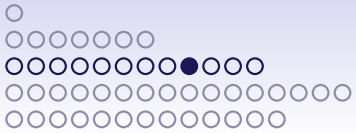

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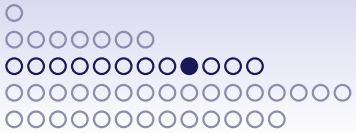

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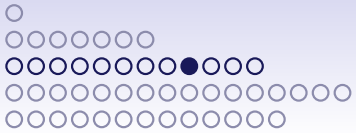

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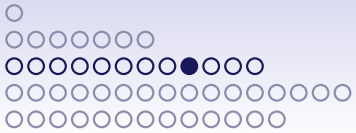
c	k	m	*
2	1	3	-1



Boyer-Moore

For example, if the text is “abdacccmkckcm” and the pattern is “ckcm”:

a	b	d	a	c	c	c	m	k	c	k	c	m
c	k	c	m									
				c	k	c	m					
			c	k	c	m						

c	k	m	*
2	1	3	-1

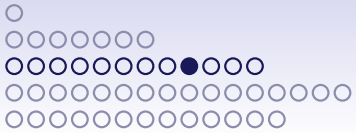
Never shift the pattern back.

Boyer-Moore

For example, if the text is “abdacccmkckcm” and the pattern is “ckcm”:

a	b	d	a	c	c	c	m	k	c	k	c	m
c	k	c	m									
				c	k	c	m					
					c	k	c	m				

c	k	m	*
2	1	3	-1



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				c	k	c	m					
					c	k	c	m				
							c	k	c	m		
									c	k	c	m

c	k	m	*
2	1	3	-1

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				c	k	c	m					
					c	k	c	m				
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					c	k	c	m				
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				c	k	c	m					
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					c	k	c	m				
							c	k	c	m		
									c	k	c	m

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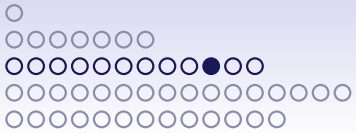
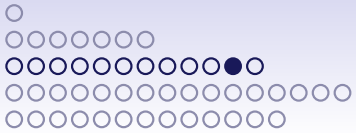


Boyer-Moore

(Note: the pseudocode below returns the first match.)

```
procedure BoyerMoore(text, pattern)
    lastTable ← BoyerMooreLastTable(pattern)
    i ← 0
    while i <= length of text − length of pattern do
        j ← length of pattern − 1
        while j >= 0 and text[i + j] = pattern[j] do
            j ← j − 1
        end while
```



Boyer-Moore

(Note: the pseudocode below returns the first match.)

```
            if j = −1 then
                return i
            else
                shiftedIndex ← lastTable[text[i + j]]
                if shiftedIndex < j then
                    i ← i + (j − shiftedIndex)
                else
                    i ← i + 1
                end if
            end if
        end while
        return -1
    end procedure
```

Performance

- In the best case, when searching for only the first match, Boyer-Moore is $O(m)$, where m is the length of the pattern.

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- In the average case, Boyer-Moore is $O(m+n)$, where m is the length of the pattern and n is the length of the text.

KMP

- Knuth-Morris-Pratt (KMP) string searching constructs a **failure table** (also known as a failure function) to determine how much to shift the pattern by on a mismatch.

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- KMP initially starts searching from the beginning of the pattern. However, when the pattern is shifted over, it may or may not restart from the beginning of the pattern

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- In other words, given the word "revararev", the longest suffix here that is also a prefix is "rev" (notice how the word begins and ends with "rev", and that if we were to add another letter, the prefix "reva" wouldn't match the suffix "arev").

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- For the word “ababaaababbabab”, the longest suffix here that is also a prefix is “abab”. Also note that there is also a shorter suffix that is also a prefix (“ab”), but the longest one should be used.

KMP Failure Table

- KMP needs to know the length of the longest suffix for the first 2 to m characters of the pattern (where m is the length of the pattern). In other words, for the word "revararev", this would be the following:

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  - "rev"

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  - "re"
  - "rev"
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  - "reva"
  - "revar"

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  - "rev"
  - "reva"
  - "revar"
  - "revara"

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  - "reva"
  - "revar"
  - "revara"
  - "revarar"

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  - "revar"
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  - "revarar"
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  - "rev"
  - "reva"
  - "revar"
  - "revara"
  - "revarar"
  - "revarare"
  - "revararev"
- Fortunately, there is a efficient way to do this in code.

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  - If they are different, and `i` is at the first character of the pattern, then write 0 into entry `j` of the table, and move `j` forward by one character. Do not move `i`.
- Repeat the previous step until `j` goes past the end of the string, and all of the entries in the table have a value.

KMP Failure Table

For example, the failure table for “amanama” is:

0	1	2	3	4	5	6
a	m	a	n	a	m	a

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i	j					
0	1	2	3	4	5	6
a	m	a	n	a	m	a
0						

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i		j				
0	1	2	3	4	5	6
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0	0					

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		i				j
0	1	2	3	4	5	6
a	m	a	n	a	m	a
0	0	1	0	1	2	

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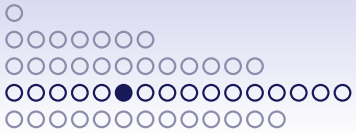
For example, the failure table for “amanama” is:

			i			
0	1	2	3	4	5	6
a	m	a	n	a	m	a
0	0	1	0	1	2	3

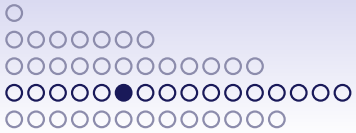
j would be pointing to index 7, which doesn’t exist.



KMP Failure Table

For example, the failure table for “andanandandan” is:

0	1	2	3	4	5	6	7	8	9	10	11	12
a	n	d	a	n	a	n	d	a	n	d	a	n



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i	j											
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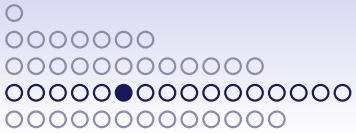
a	n	d	a	n	a	n	d	a	n	d	a	n
0												



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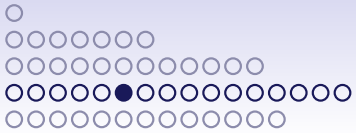
0	0											



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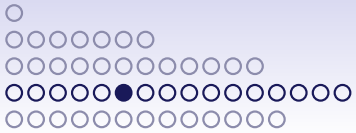
0	0	0										



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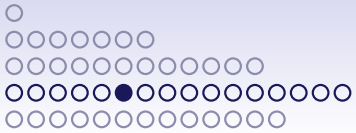
a	n	d	a	n	a	n	d	a	n	d	a	n
0	0	0	1									



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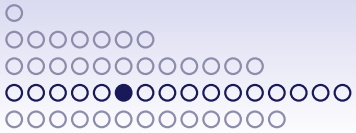

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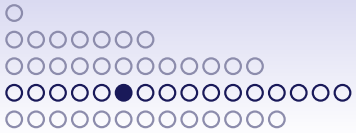
a	n	d	a	n	a	n	d	a	n	d	a	n
0	0	0	1	2								



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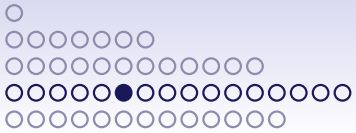
0	0	0	1	2	1							



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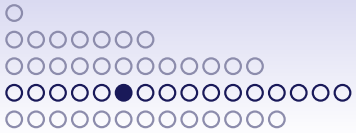
0	1	2	3	4	5	6	7	8	9	10	11	12
a	n	d	a	n	a	n	d	a	n	d	a	n
0	0	0	1	2	1	2						



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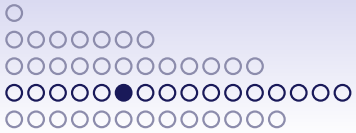
For example, the failure table for “andanandandan” is:

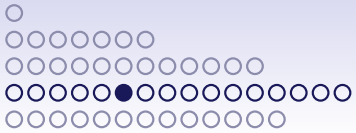
			i					j				
0	1	2	3	4	5	6	7	8	9	10	11	12
a	n	d	a	n	a	n	d	a	n	d	a	n
0	0	0	1	2	1	2	3					



KMP Failure Table

For example, the failure table for “andanandandan” is:

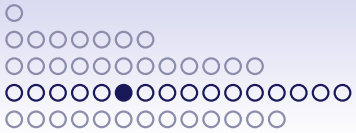
				i					j			
0	1	2	3	4	5	6	7	8	9	10	11	12
a	n	d	a	n	a	n	d	a	n	d	a	n
0	0	0	1	2	1	2	3	4				



KMP Failure Table

For example, the failure table for “andanandandan” is:

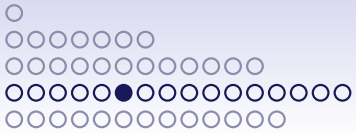
					i					j		
0	1	2	3	4	5	6	7	8	9	10	11	12
a	n	d	a	n	a	n	d	a	n	d	a	n
0	0	0	1	2	1	2	3	4	5			



KMP Failure Table

For example, the failure table for “andanandandan” is:

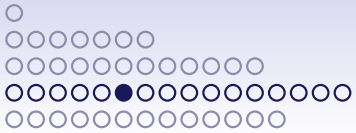
		i								j		
0	1	2	3	4	5	6	7	8	9	10	11	12
a	n	d	a	n	a	n	d	a	n	d	a	n
0	0	0	1	2	1	2	3	4	5			



KMP Failure Table

For example, the failure table for “andanandandan” is:

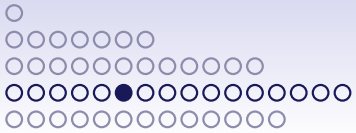
			i								j	
0	1	2	3	4	5	6	7	8	9	10	11	12
a	n	d	a	n	a	n	d	a	n	d	a	n
0	0	0	1	2	1	2	3	4	5	3		



KMP Failure Table

For example, the failure table for “andanandandan” is:

				i								j
0	1	2	3	4	5	6	7	8	9	10	11	12
a	n	d	a	n	a	n	d	a	n	d	a	n
0	0	0	1	2	1	2	3	4	5	3	4	

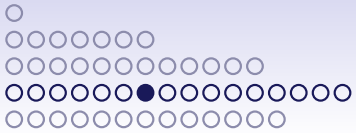


KMP Failure Table

For example, the failure table for “andanandandan” is:

					i							
0	1	2	3	4	5	6	7	8	9	10	11	12
a	n	d	a	n	a	n	d	a	n	d	a	n
0	0	0	1	2	1	2	3	4	5	3	4	5

j points to index 13.

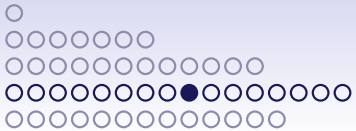


KMP Failure Table

```
procedure KMPFailureTable(pattern)
    failureTable ← array that is the same length as pattern
    i ← 0
    j ← 1
    failureTable[0] ← 0
    while j < length of pattern do
        if pattern[i] = pattern[j] then
            i ← i + 1
            failureTable[j] ← i
            j ← j + 1
        else
```

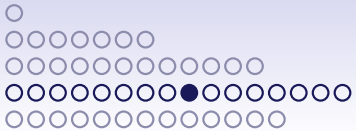
KMP Failure Table

```
            if i = 0 then
                failureTable[j] ← 0
                j ← j + 1
            else
                i ← failureTable[i − 1]
            end if
        end if
    end while
    return failureTable
end procedure
```



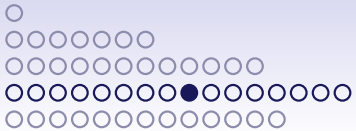
KMP

- First, construct a failure table for the pattern. The table should contain the length of the longest prefix that is the same as the suffix (of the same length) up to each character.



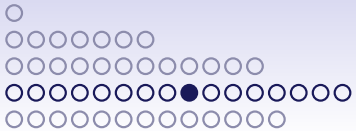
KMP

- First, construct a failure table for the pattern. The table should contain the length of the longest prefix that is the same as the suffix (of the same length) up to each character.
- Align the pattern at the beginning of the text.



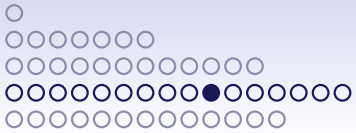
KMP

- First, construct a failure table for the pattern. The table should contain the length of the longest prefix that is the same as the suffix (of the same length) up to each character.
- Align the pattern at the beginning of the text.
- Compare the first character in the pattern with the character in the text at that index.



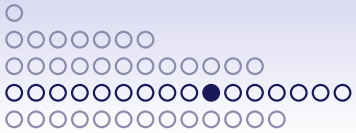
KMP

- First, construct a failure table for the pattern. The table should contain the length of the longest prefix that is the same as the suffix (of the same length) up to each character.
- Align the pattern at the beginning of the text.
- Compare the first character in the pattern with the character in the text at that index.
- If they do match, then go to the next character in the pattern and the text, and repeat the previous step. If you've compared all of the characters in the pattern, you've found a match.



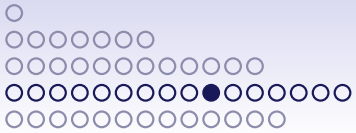
KMP

- If they don't match:



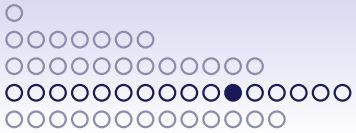
KMP

- If they don't match:
  - and the mismatch is on the first letter of the pattern, then shift the pattern to the right by one and restart comparing the pattern and the text from the first letter of the pattern.



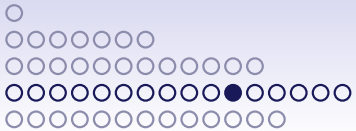
KMP

- If they don't match:
  - and the mismatch is on the first letter of the pattern, then shift the pattern to the right by one and restart comparing the pattern and the text from the first letter of the pattern.
  - and the mismatch is *not* on the first letter of the pattern, then, use the failure table to determine how much to shift the pattern by. Assuming the mismatch was on index j of the pattern, look at index $j-1$ of the failure table. This value tells you the next alignment of the pattern with the text. Align the pattern such that index $failureTable[j-1]$ of the pattern is aligned with the mismatching character in the text. Then, continue comparing from index $failureTable[j-1]$ of the pattern. Do not restart from the beginning.



KMP

- Once a match is found, to search for additional matches, look at the last index of the failure table. This value tells you the next alignment of the pattern with the text. Align the pattern such that the letter represented by this value is aligned **after** the last letter in the text. Continue comparing from this place in the pattern.



KMP

- Once a match is found, to search for additional matches, look at the last index of the failure table. This value tells you the next alignment of the pattern with the text. Align the pattern such that the letter represented by this value is aligned **after** the last letter in the text. Continue comparing from this place in the pattern.
- If any part of the pattern is "hanging off" of the end of the text, then there are no more matches.

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							

e	a	s	e	e
0	0	0	1	1

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							

e	a	s	e	e
0	0	0	1	1

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							

e	a	s	e	e
0	0	0	1	1

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							

e	a	s	e	e
0	0	0	1	1

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							

e	a	s	e	e
0	0	0	1	1

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							
		e	a	s	e	e					

e	a	s	e	e
0	0	0	1	1

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							
		e	a	s	e	e					

e	a	s	e	e
0	0	0	1	1

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							
		e	a	s	e	e					

e	a	s	e	e
0	0	0	1	1

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							
		e	a	s	e	e					
			e	a	s	e	e				

e	a	s	e	e
0	0	0	1	1

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							
		e	a	s	e	e					
			e	a	s	e	e				

e	a	s	e	e
0	0	0	1	1

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							
		e	a	s	e	e					
			e	a	s	e	e				

e	a	s	e	e
0	0	0	1	1

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							
		e	a	s	e	e					
			e	a	s	e	e				

e	a	s	e	e
0	0	0	1	1

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							
		e	a	s	e	e					
			e	a	s	e	e				

e	a	s	e	e
0	0	0	1	1

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							
		e	a	s	e	e					
			e	a	s	e	e				

e	a	s	e	e
0	0	0	1	1

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							
		e	a	s	e	e					
			e	a	s	e	e				

e	a	s	e	e
0	0	0	1	1

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							
		e	a	s	e	e					
			e	a	s	e	e				

e	a	s	e	e
0	0	0	1	1

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							
		e	a	s	e	e					
			e	a	s	e	e				
							e	a	s	e	e

e	a	s	e	e
0	0	0	1	1

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							
		e	a	s	e	e					
			e	a	s	e	e				
							e	a	s	e	e

e	a	s	e	e
0	0	0	1	1

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							
		e	a	s	e	e					
			e	a	s	e	e				
							e	a	s	e	e

e	a	s	e	e
0	0	0	1	1

KMP

For example, if the text is “ealeaseealle” and the pattern is “easee”:

e	a	l	e	a	s	e	e	a	l	l	e
e	a	s	e	e							
		e	a	s	e	e					
			e	a	s	e	e				
							e	a	s	e	e

e	a	s	e	e
0	0	0	1	1

Next alignment causes the pattern to hang off of the end of the text.

KMP

For example, if the text is “abababababccababccabab” and the pattern is “ababccabab”:

a	b	a	b	a	b	a	b	c	c	a	b	a	b	c	c	a	b	a	b
a	b	a	b	c	c	a	b	a	b										

KMP

For example, if the text is “abababababccababccabab” and the pattern is “ababccabab”:

a	b	a	b	a	b	a	b	c	c	a	b	a	b	c	c	a	b	a	b
a	b	a	b	c	c	a	b	a	b										

a	b	a	b	c	c	a	b	a	b
0	0	1	2	0	0	1	2	3	4

KMP

For example, if the text is “abababababccababccabab” and the pattern is “ababccabab”:

a	b	a	b	a	b	a	b	c	c	a	b	a	b	c	c	a	b	a	b
a	b	a	b	c	c	a	b	a	b										

a	b	a	b	c	c	a	b	a	b
0	0	1	2	0	0	1	2	3	4

KMP

For example, if the text is “abababababccababccabab” and the pattern is “ababccabab”:

a	b	a	b	a	b	a	b	c	c	a	b	a	b	c	c	a	b	a	b
a	b	a	b	c	c	a	b	a	b										

a	b	a	b	c	c	a	b	a	b
0	0	1	2	0	0	1	2	3	4

KMP

For example, if the text is “abababababccababccabab” and the pattern is “ababccabab”:

a	b	a	b	a	b	a	b	c	c	a	b	a	b	c	c	a	b	a	b
a	b	a	b	c	c	a	b	a	b										

a	b	a	b	c	c	a	b	a	b
0	0	1	2	0	0	1	2	3	4

KMP

For example, if the text is “abababababccababccabab” and the pattern is “ababccabab”:

a	b	a	b	a	b	a	b	c	c	a	b	a	b	c	c	a	b	a	b
a	b	a	b	c	c	a	b	a	b										

a	b	a	b	c	c	a	b	a	b
0	0	1	2	0	0	1	2	3	4

KMP

For example, if the text is “abababababccababccabab” and the pattern is “ababccabab”:

a	b	a	b	a	b	a	b	c	c	a	b	a	b	c	c	a	b	a	b
a	b	a	b	c	c	a	b	a	b										

a	b	a	b	c	c	a	b	a	b
0	0	1	2	0	0	1	2	3	4

KMP

For example, if the text is “abababababccababccabab” and the pattern is “ababccabab”:

a	b	a	b	a	b	a	b	c	c	a	b	a	b	c	c	a	b	a	b
a	b	a	b	c	c	a	b	a	b										

a	b	a	b	c	c	a	b	a	b
0	0	1	2	0	0	1	2	3	4

KMP

For example, if the text is “abababababccababccabab” and the pattern is “ababccabab”:

a	b	a	b	a	b	a	b	c	c	a	b	a	b	c	c	a	b	a	b
a	b	a	b	c	c	a	b	a	b										
		a	b	a	b	c	c	a	b	a	b								

a	b	a	b	c	c	a	b	a	b
0	0	1	2	0	0	1	2	3	4

KMP

For example, if the text is “abababababccababccabab” and the pattern is “ababccabab”:

a	b	a	b	a	b	a	b	c	c	a	b	a	b	c	c	a	b	a	b
a	b	a	b	c	c	a	b	a	b										
		a	b	a	b	c	c	a	b	a	b								

a	b	a	b	c	c	a	b	a	b
0	0	1	2	0	0	1	2	3	4

KMP

For example, if the text is “abababababccababccabab” and the pattern is “ababccabab”:

a	b	a	b	a	b	a	b	c	c	a	b	a	b	c	c	a	b	a	b
a	b	a	b	c	c	a	b	a	b										
		a	b	a	b	c	c	a	b	a	b								

a	b	a	b	c	c	a	b	a	b
0	0	1	2	0	0	1	2	3	4

KMP

For example, if the text is “abababababccababccabab” and the pattern is “ababccabab”:

a	b	a	b	a	b	a	b	c	c	a	b	a	b	c	c	a	b	a	b
a	b	a	b	c	c	a	b	a	b										
		a	b	a	b	c	c	a	b	a	b								

a	b	a	b	c	c	a	b	a	b
0	0	1	2	0	0	1	2	3	4

KMP

For example, if the text is “abababababccababccabab” and the pattern is “ababccabab”:

a	b	a	b	a	b	a	b	c	c	a	b	a	b	c	c	a	b	a	b
a	b	a	b	c	c	a	b	a	b										
		a	b	a	b	c	c	a	b	a	b								

a	b	a	b	c	c	a	b	a	b
0	0	1	2	0	0	1	2	3	4

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a	b	a	b	c	c	a	b	a	b										
		a	b	a	b	c	c	a	b	a	b								
				a	b	a	b	c	c	a	b	a	b						

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		a	b	a	b	c	c	a	b	a	b								
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		a	b	a	b	c	c	a	b	a	b								
				a	b	a	b	c	c	a	b	a	b						

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		a	b	a	b	c	c	a	b	a	b								
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		a	b	a	b	c	c	a	b	a	b								
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		a	b	a	b	c	c	a	b	a	b								
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		a	b	a	b	c	c	a	b	a	b								
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		a	b	a	b	c	c	a	b	a	b								
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		a	b	a	b	c	c	a	b	a	b								
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		a	b	a	b	c	c	a	b	a	b								
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		a	b	a	b	c	c	a	b	a	b								
				a	b	a	b	c	c	a	b	a	b						
										a	b	a	b	c	c	a	b	a	b

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		a	b	a	b	c	c	a	b	a	b								
				a	b	a	b	c	c	a	b	a	b						
										a	b	a	b	c	c	a	b	a	b

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		a	b	a	b	c	c	a	b	a	b								
				a	b	a	b	c	c	a	b	a	b						
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				a	b	a	b	c	c	a	b	a	b						
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		a	b	a	b	c	c	a	b	a	b								
				a	b	a	b	c	c	a	b	a	b						
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		a	b	a	b	c	c	a	b	a	b								
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a	b	a	b	c	c	a	b	a	b
0	0	1	2	0	0	1	2	3	4

KMP

(Note: the pseudocode below returns **all** matches.)

```
procedure KMP(text, pattern)
    failureTable ← KMPFailureTable(pattern)
    matches ← array holding indices of matches
    i ← 0
    j ← 0
    while i <= length of text − length of pattern do
        while j < length of pattern and text[i + j] = pattern[j]
do
            j ← j + 1
        end while
```

KMP

(Note: the pseudocode below returns **all** matches.)

```
        if j = 0 then
            i ← i + 1
        else
            if j = length of pattern then
                Add i to matches
            end if
            nextAlignment ← failureTable[j − 1]
            i ← i + j − nextAlignment
            j ← nextAlignment
        end if
    end while
    return matches
end procedure
```

Performance

- In the best case, when searching for only the first match, KMP is $O(m)$, where m is the length of the pattern.

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- In the best case, when searching for only the first match, KMP is $O(m)$, where m is the length of the pattern.
- In all other cases, KMP is $O(m+n)$, where m is the length of the pattern and n is the length of the text.

Rabin-Karp

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- Rabin-Karp string searching doesn't directly compare characters (at least, initially).
- It instead compares the hashes of the pattern and a substring of the text.

Rabin-Karp Rolling Hash

- Rabin-Karp uses a **rolling hash** to calculate a hash of the pattern and a hash of the substring (that is the same length as the pattern) of the text.

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- If these hashes are equal, then there **may** be (but not guaranteed to be, because hashes can collide) a match starting at this index. At this point, each character is compared.

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- If these hashes are equal, then there **may** be (but not guaranteed to be, because hashes can collide) a match starting at this index. At this point, each character is compared.
- A special property of a rolling hash is that while it is $O(n)$ to calculate the initial hash, it is $O(1)$ to "slide" the hash window and calculate the new hash.

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- To calculate the hash, the following formula is used: $\sum_{i=0}^{j} text[i] \times BASE^{j-i}$, where j is the number of letters in $text$, and $BASE$ is a prime number.
- Note that, in computing, each letter can be represented as an integer. In Java, the integer representation of each character is its Unicode value (It's actually a little more complicated than this, but this is sufficient for this algorithm.).

Rabin-Karp Rolling Hash

- Formula: $\sum_{i=0}^{j} text[i] \times BASE^{j-i-1}$, where j is the number of letters in $text$, and $BASE$ is a prime number.

Rabin-Karp Rolling Hash

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- For the text (or the section of the text) you need to calculate the hash of, calculate the result of the formula. j is the number of letters that you need to hash, and *BASE* is a prime number that is usually provided.

Rabin-Karp Rolling Hash

For example, to calculate the hash of “crow”, given a base of 101: $\sum_{i=0}^{j} text[i] \times BASE^{j-i-1}$

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For example, to calculate the hash of “crow”, given a base of 101:
$\sum_{i=0}^{j} text[i] \times BASE^{j-i-1}$
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$99 \times 101^3 + 114 \times 101^2 + 111 \times 101 + 119$

$103,174,043$

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For example, to calculate the hash of the first four characters of "welcome", given a base of 157:
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$119 \times 157^3 + 101 \times 157^2 + 108 \times 157 + 99$

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$119 \times 157^3 + 101 \times 157^2 + 108 \times 157 + 99$

$463,023,871$

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- For example, in the previous example, the hash for the first four characters (indices 0-3) of "welcome" was calculated. Using this hash, the hash for indices 1-4 can be calculated in $O(1)$ time.
- Given a hash length of j, the character being removed from the hash at index a, and the character being added into the hash at index b, the new hash can be calculated using $BASE \times (oldHash - text[a] \times BASE^{j-1}) + text[b]$.

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- In other words, what is happening is that what the first letter contributed to the hash is being removed, all of the other letters are being shifted up by one slot, and the new letter is being added in.

Rabin-Karp Rolling Hash

For example, given the word “welcome”, the hash of 463,023,871 (this hash is for indices 0-3 of the word), and a base of 157, calculate the next hash:
$BASE \times (oldHash - text[a] \times BASE^{j-1}) + text[b]$

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$BASE \times (oldHash - text[a] \times BASE^{j-1}) + text[b]$

$157 \times (463,023,871 -' w' \times 157^3) +' c'$

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For example, given the word “welcome”, the hash of 463,023,871 (this hash is for indices 0-3 of the word), and a base of 157, calculate the next hash:

$BASE \times (oldHash - text[a] \times BASE^{j-1}) + text[b]$

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$157 \times (463,023,871 - 119 \times 157^3) + 99$

$393,536,927$

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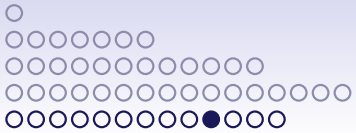
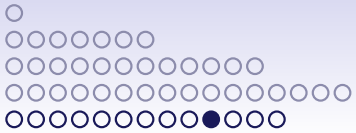
- If the hashes are the same, compare each character in the pattern along with the characters included in the hash of the text. If all of the characters are the same, then you've found a match. Remember that two different strings may give the same hash.
- If the hashes are different, or if the characters don't match, slide the hash window of the text by one character to the right. (The hash of the pattern remains the same), and repeat the previous step until you've reached the end of the text.



Rabin-Karp

Because this algorithm is more mathematical than the others, not all of the detail is shown. The yellow cells represent the hash window currently being used.
For example, if the text is “Surface tension” and the pattern is “tens” (assume that, for this example, different strings give different hashes):

S	u	r	f	a	c	e		t	e	n	s	i	o	n
t	e	n	s											



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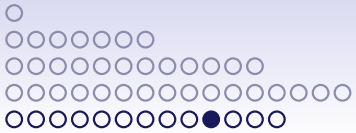

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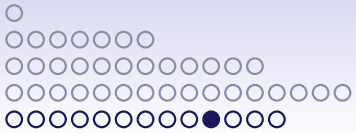

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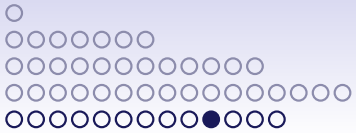

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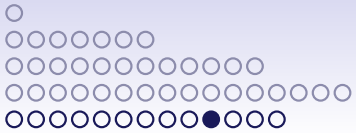

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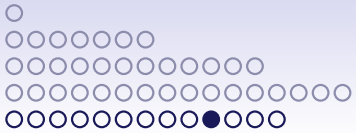

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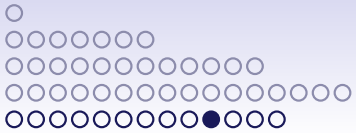

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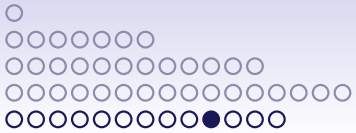

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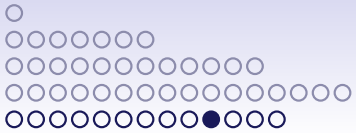

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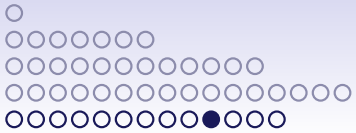

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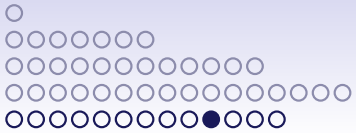

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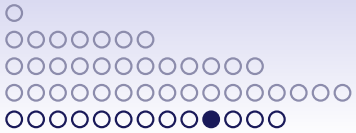

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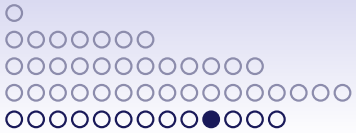

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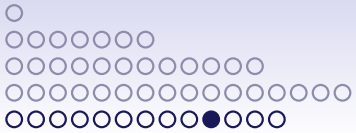

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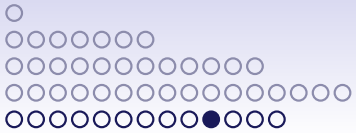

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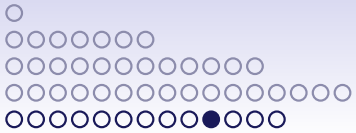

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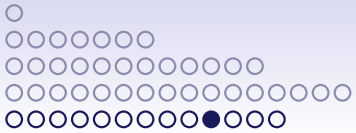

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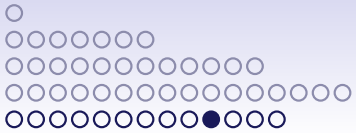

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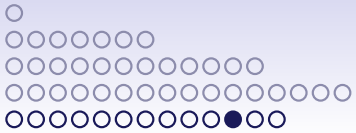

Rabin-Karp

(Note: the pseudocode below returns only the first match.)

```
procedure RabinKarp(text, pattern)
    patternHash ← rolling hash of pattern
    textHash ← rolling hash of first pattern.length characters of
text
    i ← 0
    while i <= length of text − length of pattern do
        if patternHash = textHash then
            j ← 0
            while j < length of pattern and
text[i + j] = pattern[j] do
                j ← j + 1
            end while
```

Rabin-Karp

(Note: the pseudocode below returns only the first match.)

```
            if j = length of pattern then
                return i
            end if
        end if
        i ← i + 1
        if i <= length of text − length of pattern then
            textHash ← new hash of text, with the hash window
shifted over
        end if
    end while
    return -1
end procedure
```

Performance

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- In all other cases, Rabin-Karp is $O(m + n)$, where m is the length of the pattern and n is the length of the text.