Penn State Abington
CMPEN 271
Lecture Set #5
Boolean Algebra II

Topics:

- Minimizing a Boolean Function
- Boolean Algebra Identities

Video part 1 of 4 ←

- Complement of a Function
- Truth Table to Boolean Function
- Summary

Video part 2 of 4

- Practice Design Exercises

Video part 3 of 4

- Review Questions

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Real-world Problem – Car Alarm

You are a **junior engineer** at an engineering design consulting firm. You have just designed a circuit for a **car alarm circuit**. Your boss evaluated your design but handed it back to you and said, "The circuit operation is correct, but it requires **too many integrated circuits**. There is no room on the circuit board for your alarm circuit. **Redesign the circuit** if possible with less components -- but it must have the same operation. Have the new solution on my desk at 10am tomorrow."

Here is your original design based on your conversation with the alarm design team:

Alarm = doorOpen • alarmEnabled + doorOpen • alarmEnabled • motionSensorActive + alarmEnabled • motionSensorActive

(Note: identify inputs and outputs)

Boolean Function and Circuit Minimization (Reduction)

- **Start with engineering design problem** description, then generate Boolean Function directly or from truth table.

- **Minimize** (or reduce) Boolean function using Boolean identities and theorems (if possible)

- A minimized Boolean function has **identical truth table**.

- A minimized Boolean function is **less complex, uses less logic gates, fewer wires, less expensive to build**.

Boolean Algebra Identities

$$A + 0 = A$$
$$A \bullet 0 = 0$$

$$A + 1 = 1$$
$$A \bullet 1 = A$$

$$A + A = A$$
$$A \bullet A = A$$

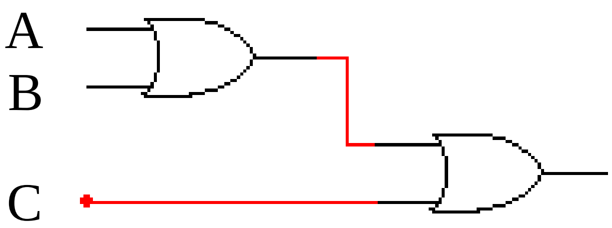
$$A + A' = 1$$
$$A \bullet A' = 0$$

$$A'' = A$$

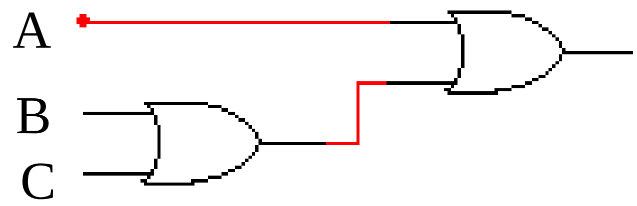
$$\overline{\overline{A}} = A$$

$A + B = B + A$ $A \bullet B = B \bullet A$	Commutative
$A + (B + C) = (A + B) + C$ $A \bullet (B \bullet C) = (A \bullet B) \bullet C$	Associative
$A \bullet (B + C) = A \bullet B + A \bullet C$ $A + B \bullet C = (A + B) \bullet (A + C)$	Distributive !!!!!!
$(A + B)' = A' \bullet B'$ $(A \bullet B)' = A' + B'$	DeMorgan's
$\overline{A + B} = \overline{A} \bullet \overline{B}$ $\overline{A \bullet B} = \overline{A} + \overline{B}$	DeMorgan's (with bar notation)

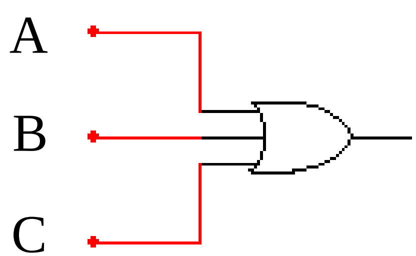
How to implement F = A + B + C



F = (A + B) + C



F = A + (B + C)



F = A + B + C using 3-input OR gate

- All 3 circuits above are equivalent
- Use ideas from above to implement F = ABC

Prove DeMorgan's Theorems

<u>Example</u>: (X+Y)' = X' • Y'   Use truth tables!

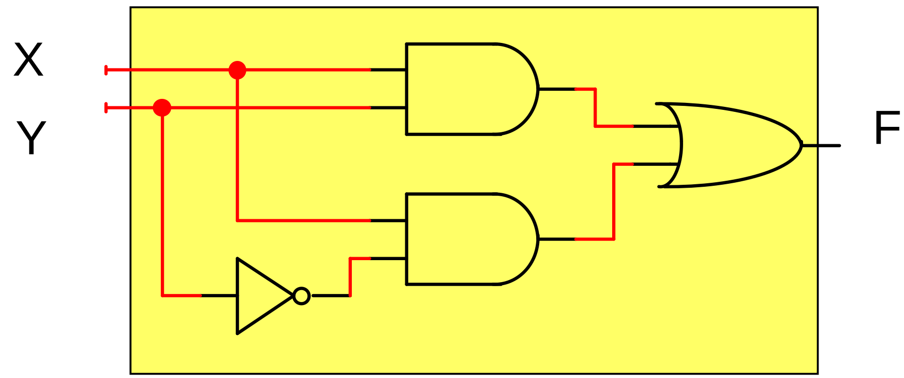
X	Y	X+Y	**(X+Y)'**	X'	Y'	**X'•Y'**
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

compare

(Prove other theorems as an exercise – use truth tables.)

Boolean Algebra Minimization - 1

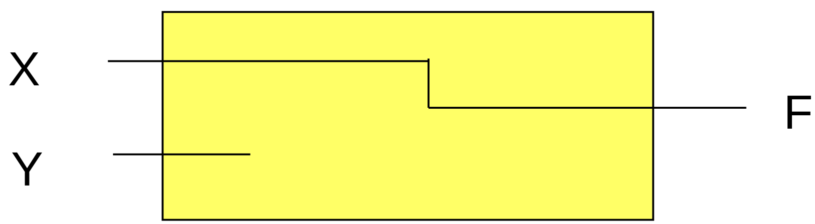
Example: F = XY + XY’



inputs output

X	Y	F
0	0	0
0	1	0
1	0	1
1	1	1

Minimize function: F = XY + XY’ = X (Y + Y’) = X • 1 = X



Comments?
Observations?
Try in Multisim

Boolean Algebra Minimization - 2

Example: Minimize function below:

$F = A'BC + AC + A'BC'$

$= A'BC + A'BC' + AC$

$= A'B\,(C + C') + AC$

$= A'B \bullet 1 + AC$

$= A'B + AC$

Boolean Algebra Minimization - 3

I successfully complete (pass) a course when I study while it is sunny, or I pass a class when I study and it is not sunny.

Pass = study • sunny + study • sunny’

Minimize (reduce)?

NOTE: Be able to identify inputs and outputs.

Boolean Algebra Minimization - 4

Minimize each function below using Boolean algebra

1) $F = X' + X + Y'Z'$

2) AllergicReaction = Apple + Apple • water + Apple • sugar

3) $F = AC + A'A + 0$

4) $F = 1 \bullet BB' + A + A'$

Boolean Algebra Minimization - 5

Minimize each function below using Boolean algebra

5) $F = (X + Y) \cdot (X' + Y')$

6) $F = (A' + B)\,(A'B)'$

7) $F = XY'Z + X(Y'Z)'$

Boolean Algebra Minimization - 6

Minimize each function below using Boolean algebra

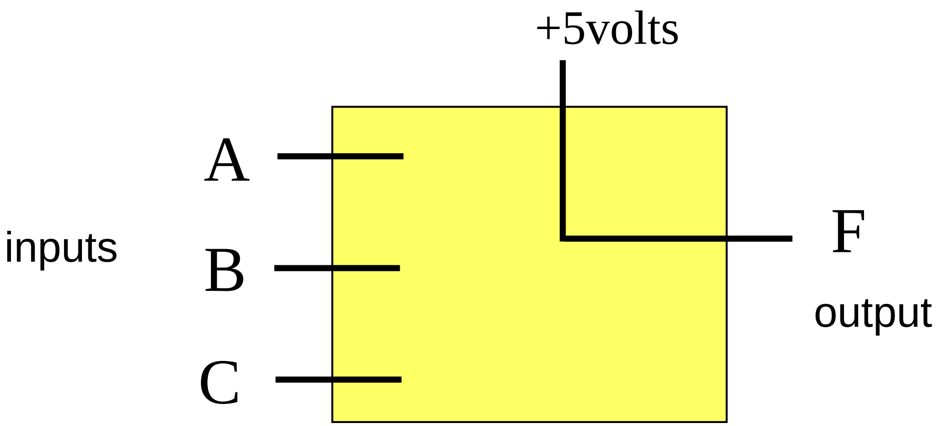
8) G = AB’ + A + AB’C + ABC’D

9) P = A'B + AB + 1

10) P = A'B + AB + 0

Boolean Algebra Minimization - 7

What does F(A,B,C) = 1 mean?



inputs			output
A	**B**	**C**	**F**
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

What does F(A,B,C) = 0 mean?

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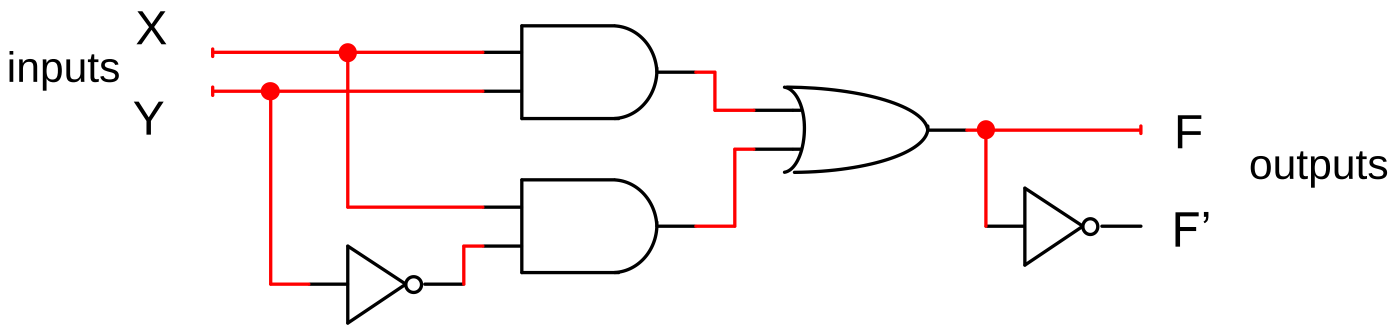
Complement of a Function - 1

[Complement = inverse]

<u>Example</u>: Given F = XY + XY', find F' (complement of F)



X	Y	F	F'
0	0	0	1
0	1	0	1
1	0	1	0
1	1	1	0

F' = (XY + XY')' = ? (minimize)

Compare word "compl<u>e</u>ment" with "compl<u>i</u>ment"

Complement of a Function - 2

<u>Example</u>: Find the complement of $F = (X+Y') \cdot Z$ using Boolean algebra.

$F' = ((X + Y') \cdot Z)'$ $\quad = \overline{(X + \overline{Y}) \cdot Z}$

$= (X + Y')' + Z'$ $\quad = \overline{X + \overline{Y}} + \overline{Z}$

$= X' \cdot Y + Z'$ $\quad = \overline{X} \cdot Y + \overline{Z}$

If function F already existed as a circuit, you could add an inverter (NOT gate) to the output (not to the inputs) to generate F'. Algebraic complementation does not guarantee a simpler circuit.

Boolean Function from Truth Table

(Given truth table, identify terms for each output of "1" and "or" the terms together)

Example: **Step #1**

A	B	C	F	
0	0	0	0	
0	0	1	0	
0	1	0	1	→ A'BC'
0	1	1	0	
1	0	0	0	
1	0	1	1	→ AB'C
1	1	0	0	
1	1	1	1	→ ABC

Step #2 Combine terms for F

F = A'BC' + AB'C + ABC

Step #3 Reduce using algebra

F = A'BC' + AB'C + ABC
= AC (B + B') + A'BC'
= AC + A'BC'

Step #4 Draw circuit diagram ?

Generalize!

Summary of Key Concepts

- Boolean functions can be **minimized (simplified)** algebraically using the theorems of Boolean Algebra.
- A minimized (simplified) Boolean function has the **exact same truth table** (inputs and outputs) as the original Boolean function.
- **DeMorgan's theorem** are an example of an important Boolean theorem to help minimize a Boolean function.
- The advantage of a minimized (simplified) Boolean function is that the minimized function **uses less gates** to implement so that the costs and size are less.
- Some Boolean functions **cannot be minimized**, but many can be minimized.
- Engineers are always looking for ways to simplify circuits to **save time and money** (while keeping the operation of the circuit the same).

What you should know…

- Given a Boolean function, draw circuit diagram and determine truth table.
- Given a circuit diagram, determine Boolean function and truth table.
- Given a Boolean function, minimize (reduce) the function (truth table stays the same) - simplifies design. Know all identities & theorems.
- Given a Boolean function, find the complement. (Use DeMorgan’s theorems)
- Given a truth table find the Boolean function (find 1’s in truth table.)
- Given a word problem, construct truth table

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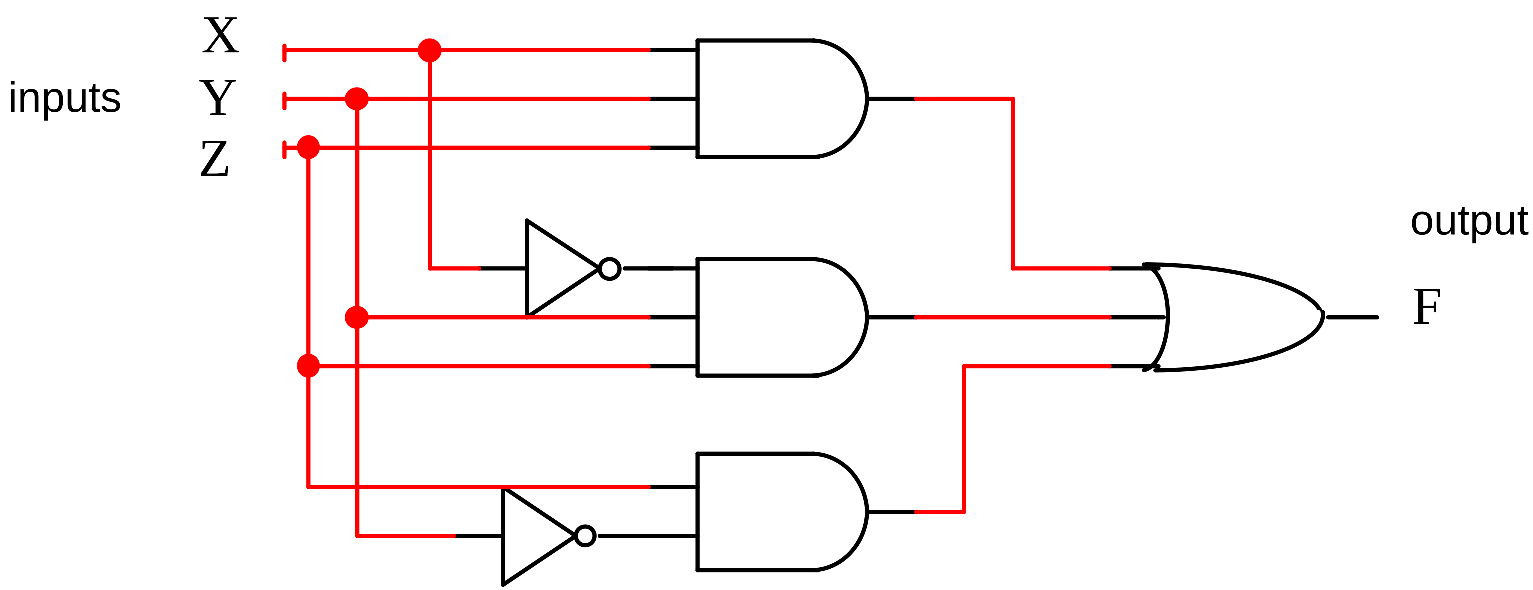
Design Exercise – 1 (in-class)

Determine if the circuit below can be simplified (reduced in complexity). If so, show reduced F, and new circuit.



Design Exercise – 2 (HW)

(**Majority Voting Circuit**) Design a circuit with 3 inputs and 1 output. The output will be high when 2 or more of the inputs are high, otherwise the output will be low.

Step 1 - Generate truth table (label all inputs and outputs)
Step 2 - Determine Boolean function
Step 3 - Minimize function using Boolean algebra (show steps)
Step 4 - Draw circuit diagram

Design Exercise – 3 (HW)

Car alarm circuit. Generate truth table and circuit for original car alarm circuit. Attempt to minimize.

Step 1 - Generate truth table (label all inputs and outputs)
Step 2 - Determine Boolean function
Step 3 - Minimize function using Boolean algebra (show steps)
Step 4 - Draw circuit diagram

Alarm = doorOpen • alarmEnabled + doorOpen • alarmEnabled • motionSensorActive + alarmEnabled • motionSensorActive

IEEE Spectrum Magazine

- Join IEEE (Approx $30 per year)

- Become member of IEEE Abington Student Chapter

- We can elect officers

- Visit IEEE Spectrum Magazine web site (free access)
  http://spectrum.ieee.org/

  • Review articles on professional and industrial developments in EE and CMPEN
  • Includes salary and job trends articles

Further Reading

- Mano Kime, Logic and Computer Design Fundamentals, Prentice Hall.
- Tocci R., Digital Systems, Prentice Hall, Chapter 3&4.
- www.howstuffworks.com
  "How Boolean Logic Works"
- www.play-hookey.com (digital section)

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Sample Questions

#1. Expression A + AB is equal to

a) A’ b) B c) A d) A+B

#2. Expression A + A'B is equal to

a) A b) A'+B c) A+B d) A‘

#3. Expression A + 1 is equal to

a) 1 b) 0 c) A d) A‘

#4. Expression A + 0 is equal to

a) 0 b) 1 c) A d) A‘

#5. Expression A + A' is equal to

a) 0 b) 1 c) A d) A'

Sample Questions

#6. DeMorgans' theorem states (AB)' is equal to

a) A+B   b) A'B'   c) A’+B’   d) AB

#7. Expression A + BC is equal to

a) A(A + C)   b) (A + B) (B + C)   c) (A + B) (A + C)   d) ABC

#8. Expression ABC + (ABC)' is equal to

a) 1   b) 0   c) ABC   d) A'B'C‘

#9. Expression X'YZ(X'YZ)'

a) 1   b) 0   c) XYZ   d) X + Y + Z