Higher Order Functions

Thursday, 6/25

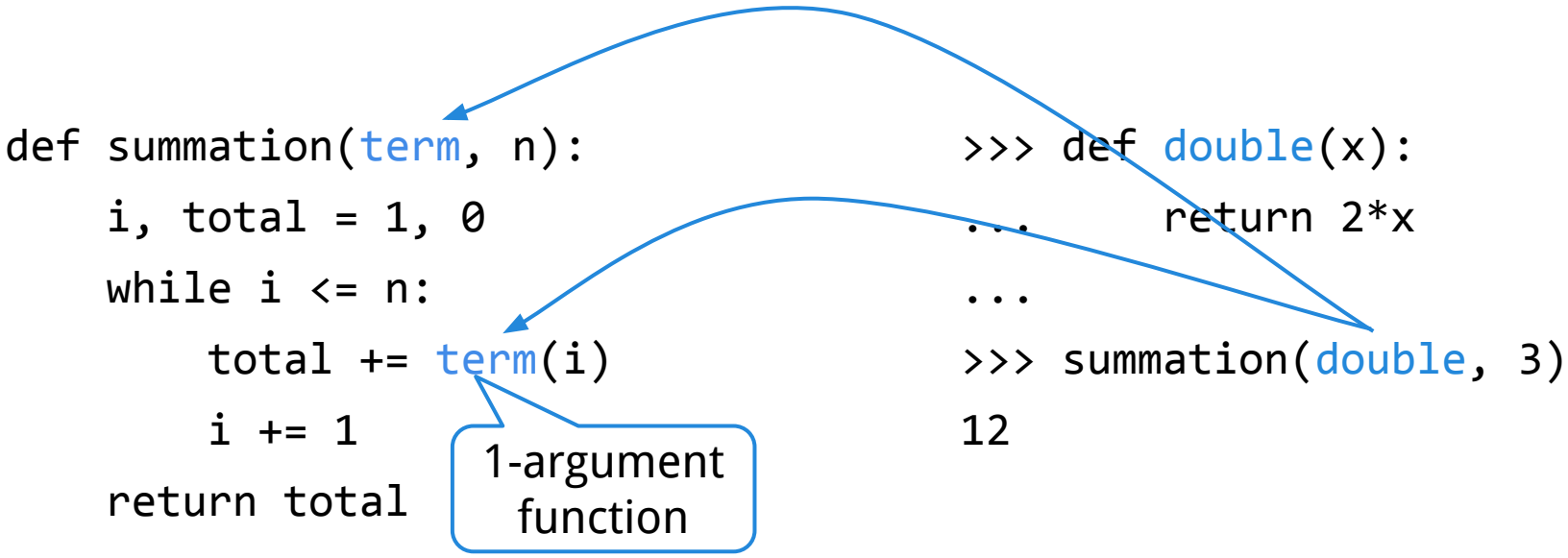
Announcements

- Fill out OK Issues survey
- Tutoring
  - Signups available today
  - Starts this Saturday
  - Tutoring sessions next week (up to Friday) will review week 1 material
- Homework 1 due Friday, June 26
- Homework 2 due Monday, June 29
- Project 1: Hog will be released this afternoon
  - Due next Thursday, July 2
  - Work with at most 1 partner
    - Must declare partner on ok!
  - Start early!

Higher Order Functions

1. Review: functions as arguments
2. Nested functions
3. Functions as return values
4. Lambda functions

Review: Functions as Arguments



Higher Order Functions

1. Review: functions as arguments
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Nested functions

Functions can be defined inside functions.

This is useful for helper functions.

$$f(a,b,c) = \frac{ab + \frac{b}{a} + a^b}{ac + \frac{c}{a} + a^c}$$

```
def f(a, b, c):
    numer_ab = a*b + b/a + a**b
    denom_ac = a*c + c/a + a**c
    return numer_ab / denom_ac
```

DRY

numer_ab & denom_ac are very similar!

Nested functions

Helper functions can simplify code!

$$f(a,b,c) = \frac{ab + \frac{b}{a} + a^b}{ac + \frac{c}{a} + a^c}$$

How?

```
def f(a, b, c):
    def g(d):
        return a*d + d/a + a**d
    return g(b) / g(c)

f(2, 3, 4)
```

g is defined inside the body of f!

Nested functions

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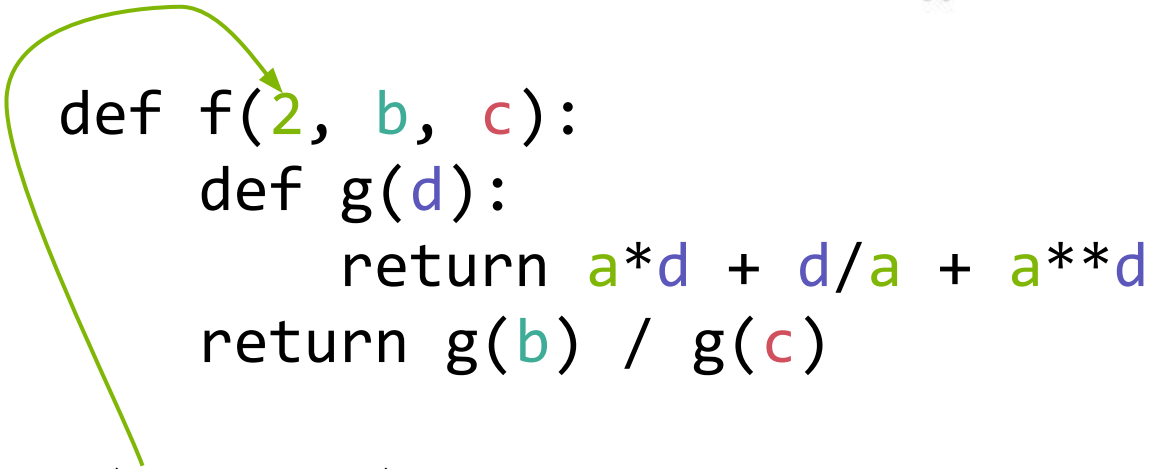
$$f(a,b,c) = \frac{ab + \frac{b}{a} + a^b}{ac + \frac{c}{a} + a^c}$$



```
def f(2, b, c):
    def g(d):
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    return g(b) / g(c)

f(2, 3, 4)
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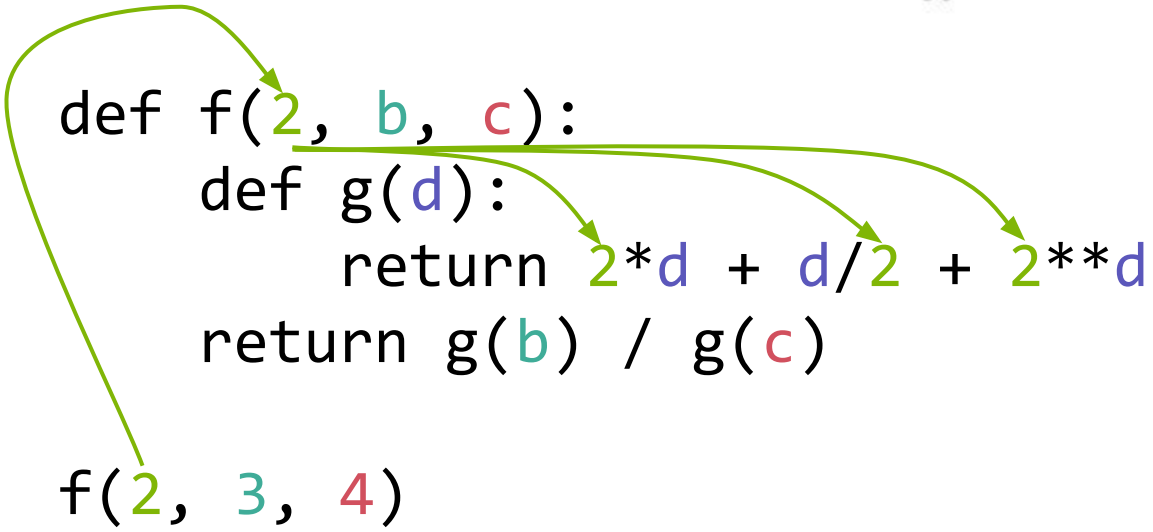

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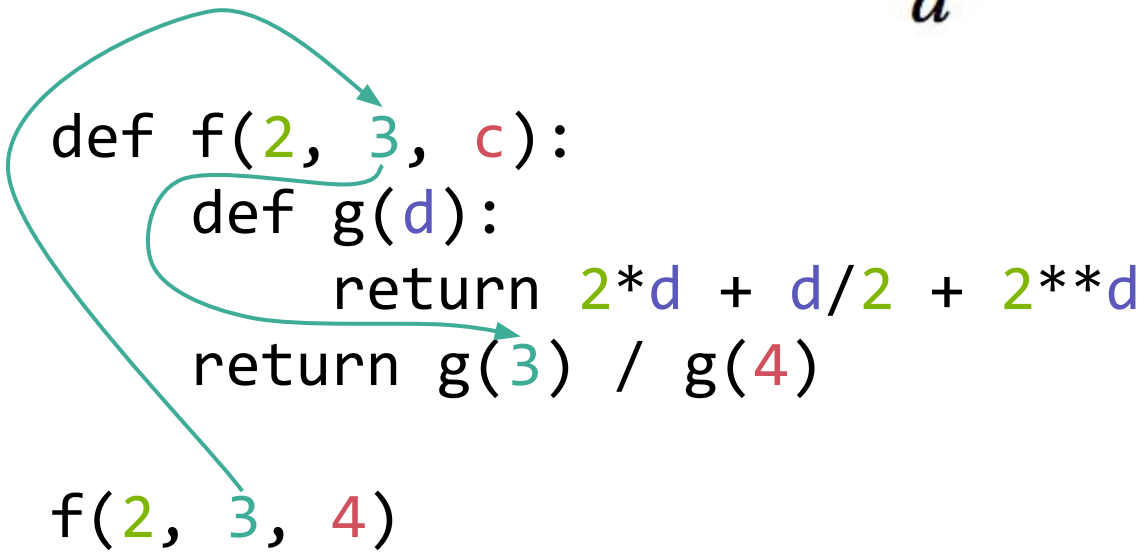

```
def f(2, 3, c):
    def g(d):
        return 2*d + d/2 + 2**d
    return g(3) / g(4)

f(2, 3, 4)
```

Nested functions

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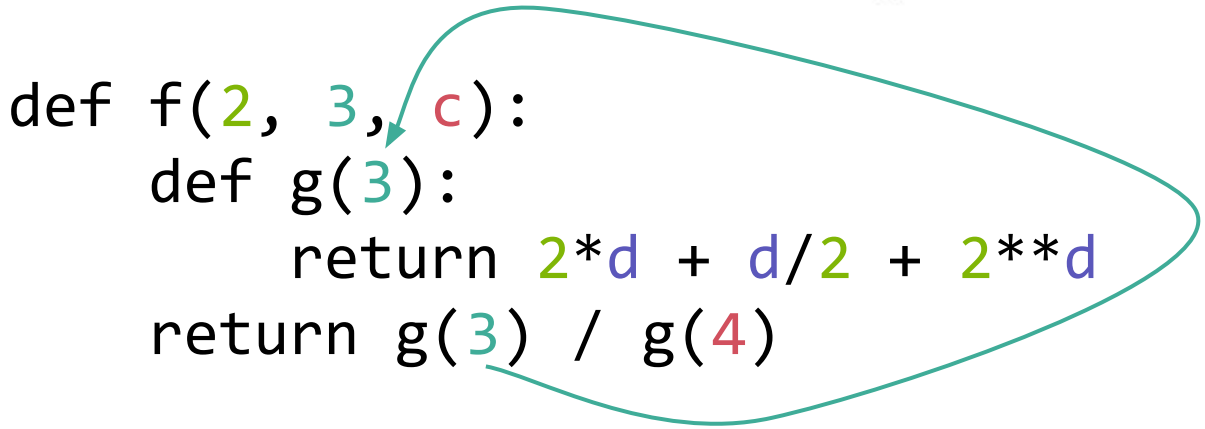

```
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f(2, 3, 4)
```

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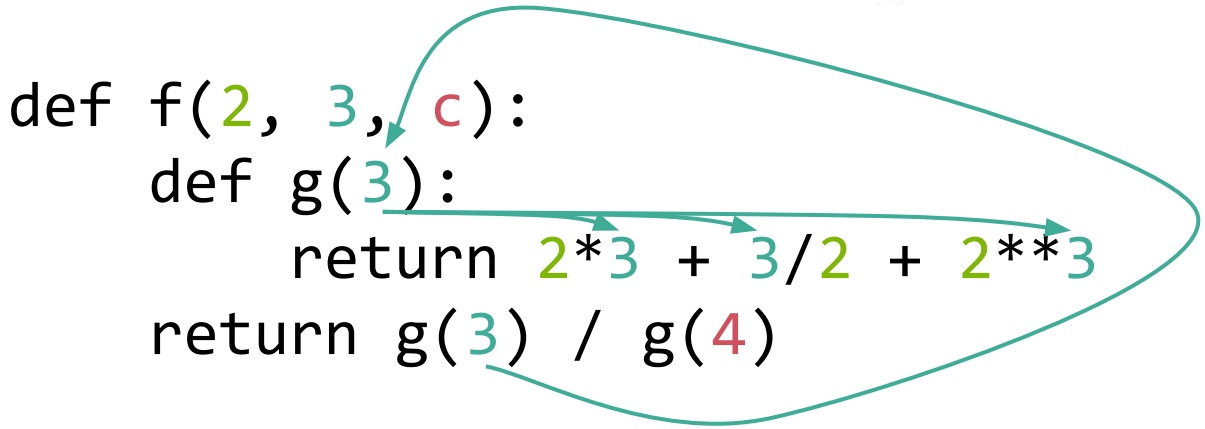


```
def f(2, 3, c):
    def g(3):
        return 2*3 + 3/2 + 2**3
    return g(3) / g(4)

f(2, 3, 4)
```

Nested functions

Helper functions can simplify code!

$$f(a,b,c) = \frac{ab + \frac{b}{a} + a^b}{ac + \frac{c}{a} + a^c}$$

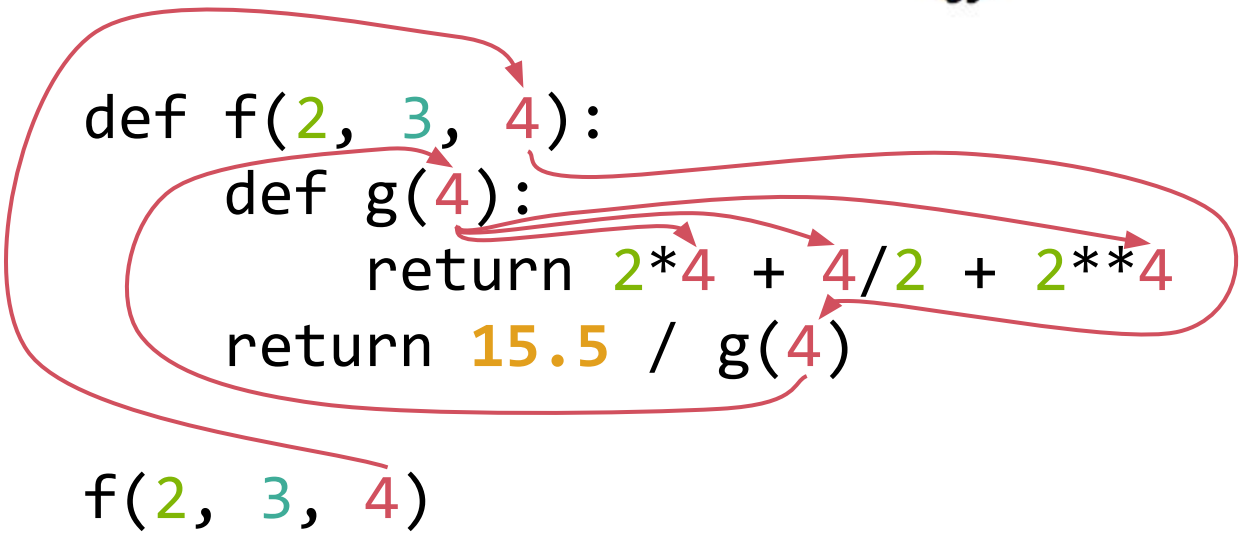
```
def f(2, 3, c):
    def g(3):
        return 2*3 + 3/2 + 2**3
    return 15.5 / g(4)

f(2, 3, 4)
```

Nested functions

Helper functions can simplify code!

$$f(a,b,c) = \frac{ab + \frac{b}{a} + a^b}{ac + \frac{c}{a} + a^c}$$



Nested functions

Helper functions can simplify code!

$$f(a,b,c) = \frac{ab + \frac{b}{a} + a^b}{ac + \frac{c}{a} + a^c}$$

```
def f(2, 3, 4):
    def g(4):
        return 2*4 + 4/2 + 2**4
    return 15.5 / 26.0

f(2, 3, 4)
```

Nested functions

Helper functions can simplify code!

$$f(a,b,c)=\frac{ab+\frac{b}{a}+a^b}{ac+\frac{c}{a}+a^c}$$

```
def f(2, 3, 4):
    def g(4):
        return 2*4 + 4/2 + 2**4
    return 15.5 / 26.0
```

```
1.6774193548387097
```

Higher Order Functions

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Adders

Functions that add some number to another number

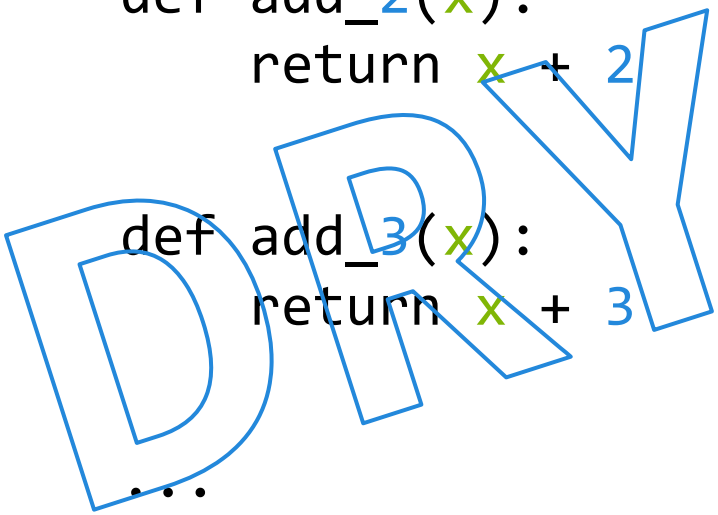
(Demo)

```
def add_1(x):
    return x + 1

def add_2(x):
    return x + 2

def add_3(x):
    return x + 3
...

def add_128(x):
    return x + 128
```



make_adder

function that returns a function

```
def make_adder(n):
    """Return func that takes one arg k and returns k + n

    >>> add_three = make_adder(3)
    >>> add_three(4)
    7
    """
    def adder(k):
        return k + n
    return adder
```

name add_three is bound to a function

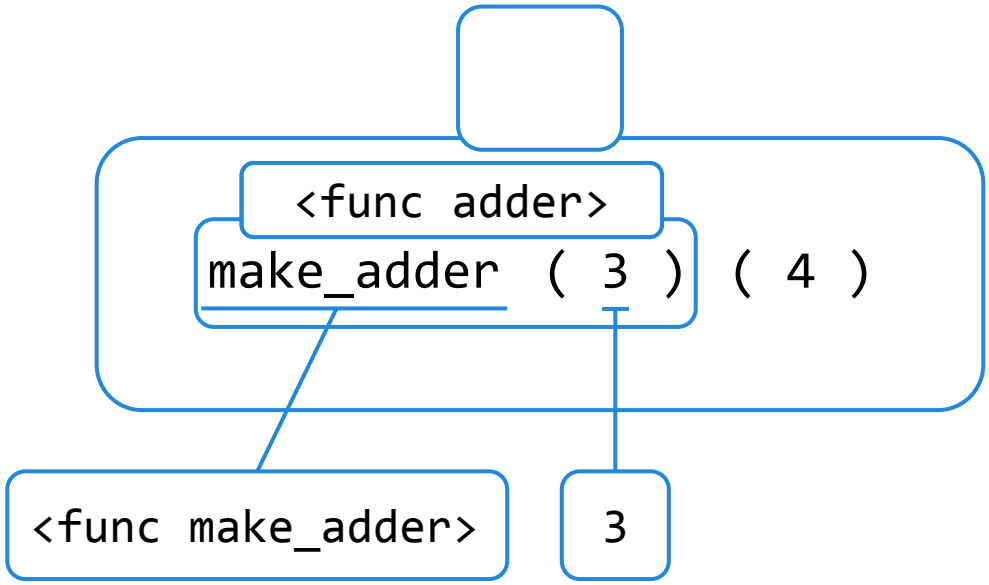
nested function

can refer to names in enclosing functions

make_adder

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
```

For compound call expressions, read left to right



make_adder

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def make_adder(n):
    def adder(k):
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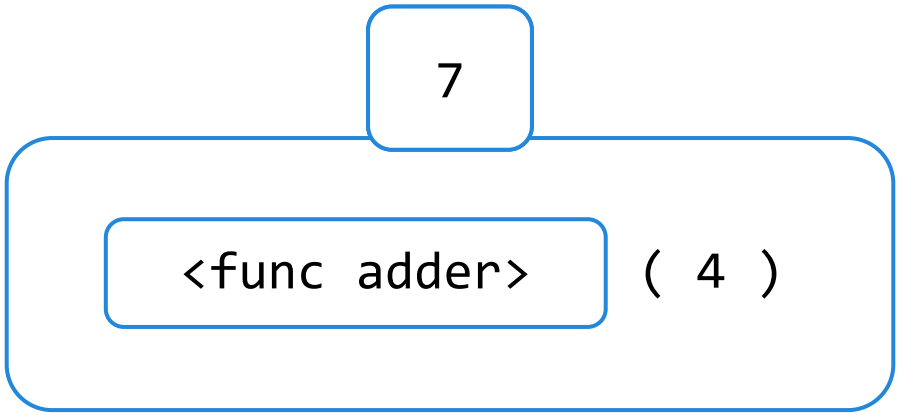
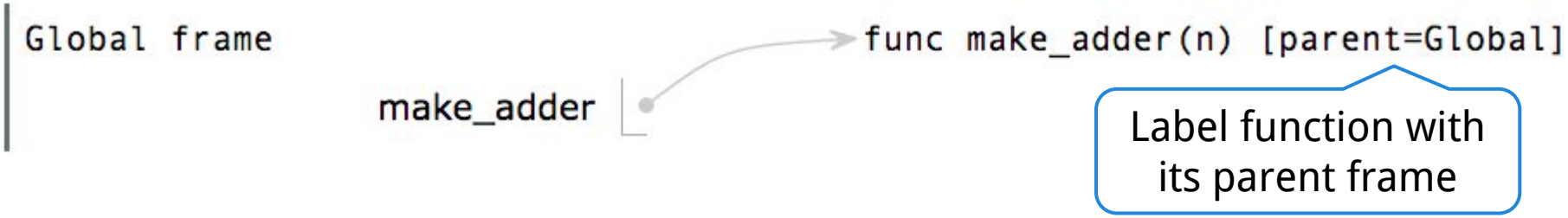


Environment diagram: make_adder

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder

make_adder(3)(4)
```

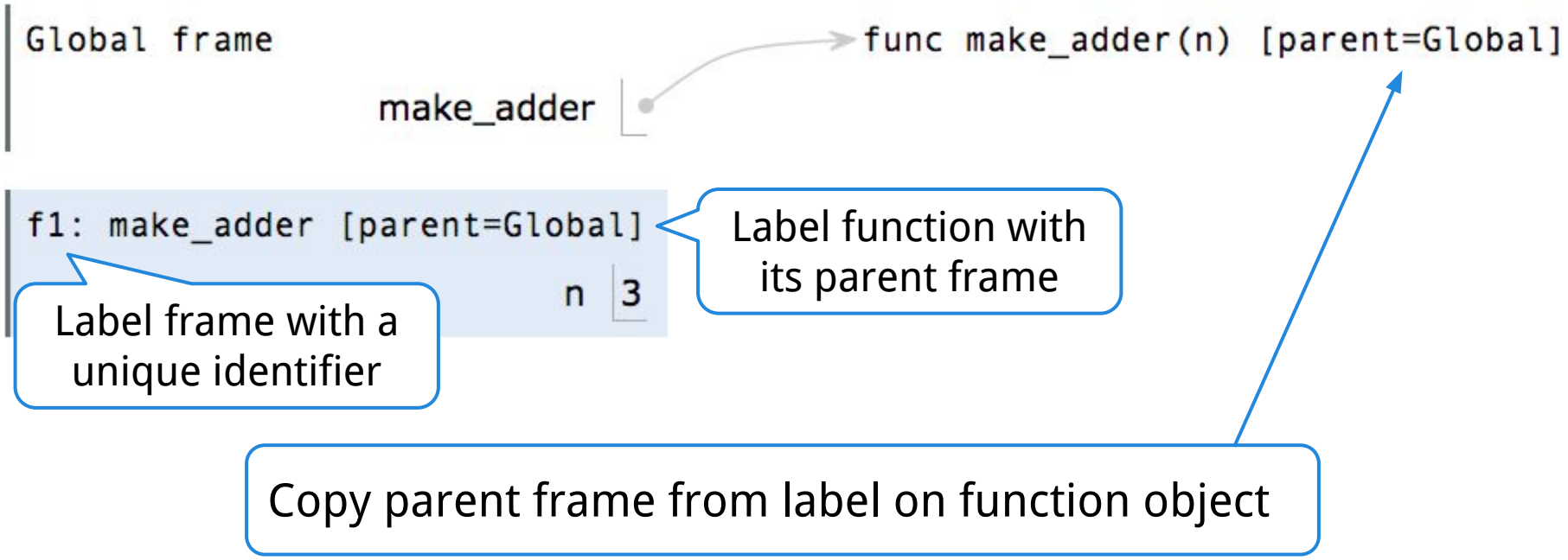


Parent frame: frame in which the function is defined

Environment diagram: make_adder

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
```

```
make_adder(3)(4)
```



Environment diagram: make_adder

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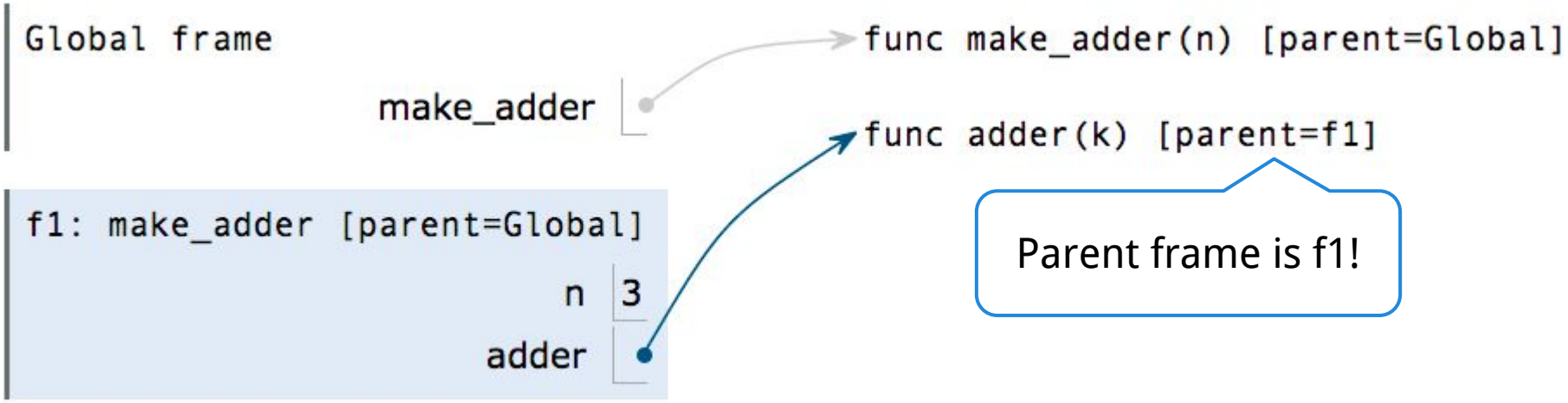

Environment diagram: make_adder

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def make_adder(n):
    def adder(k):
        return k + n
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```

```
make_adder(3)(4)
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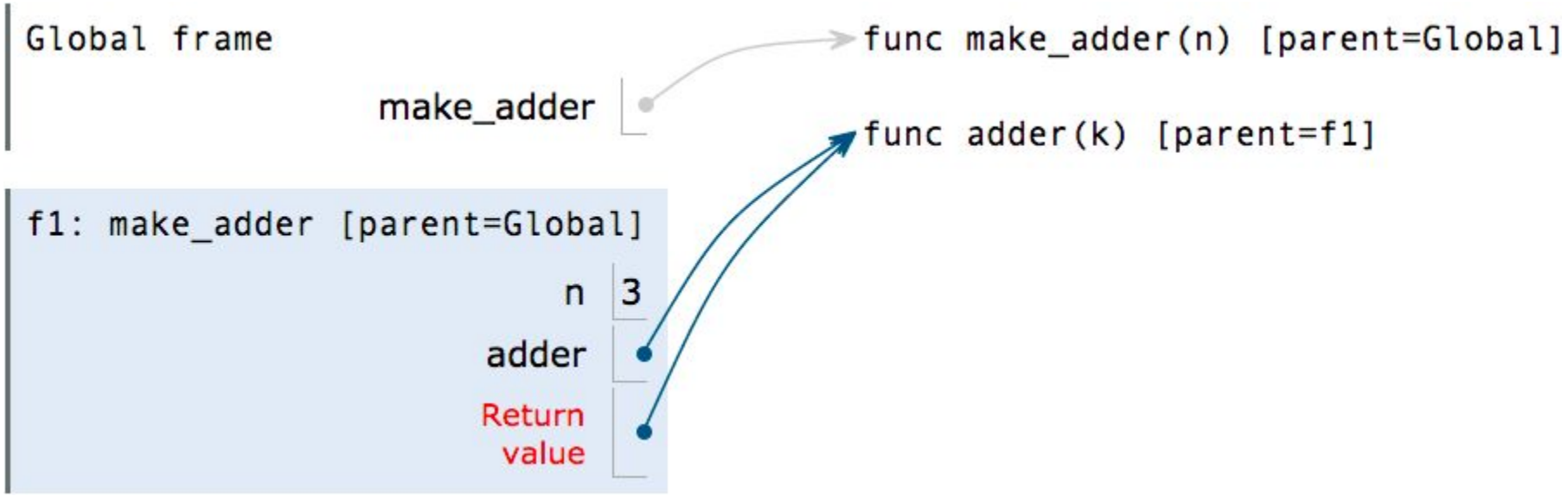

Environment diagram: make_adder

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def make_adder(n):
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    return adder
```
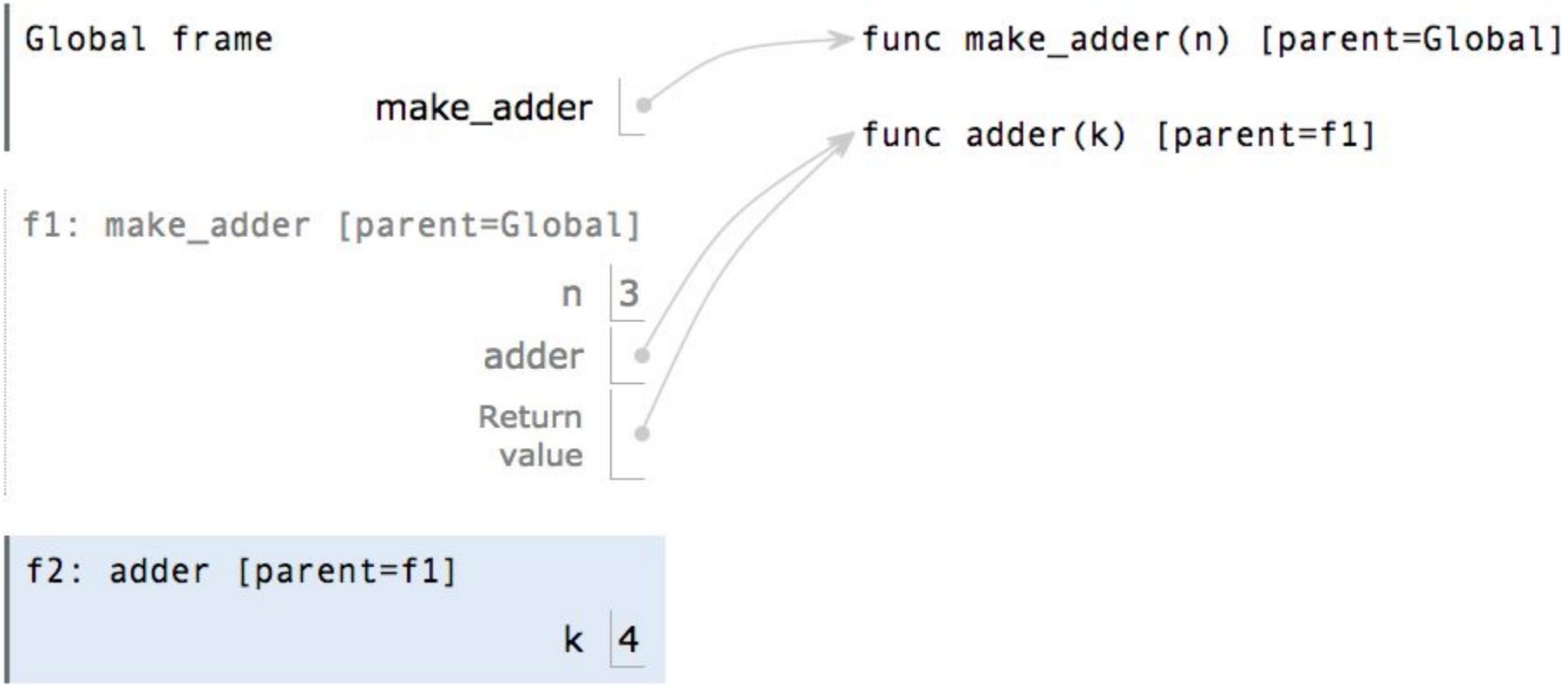

▶ `make_adder(3)(4)`



Environment diagram: make_adder

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def make_adder(n):
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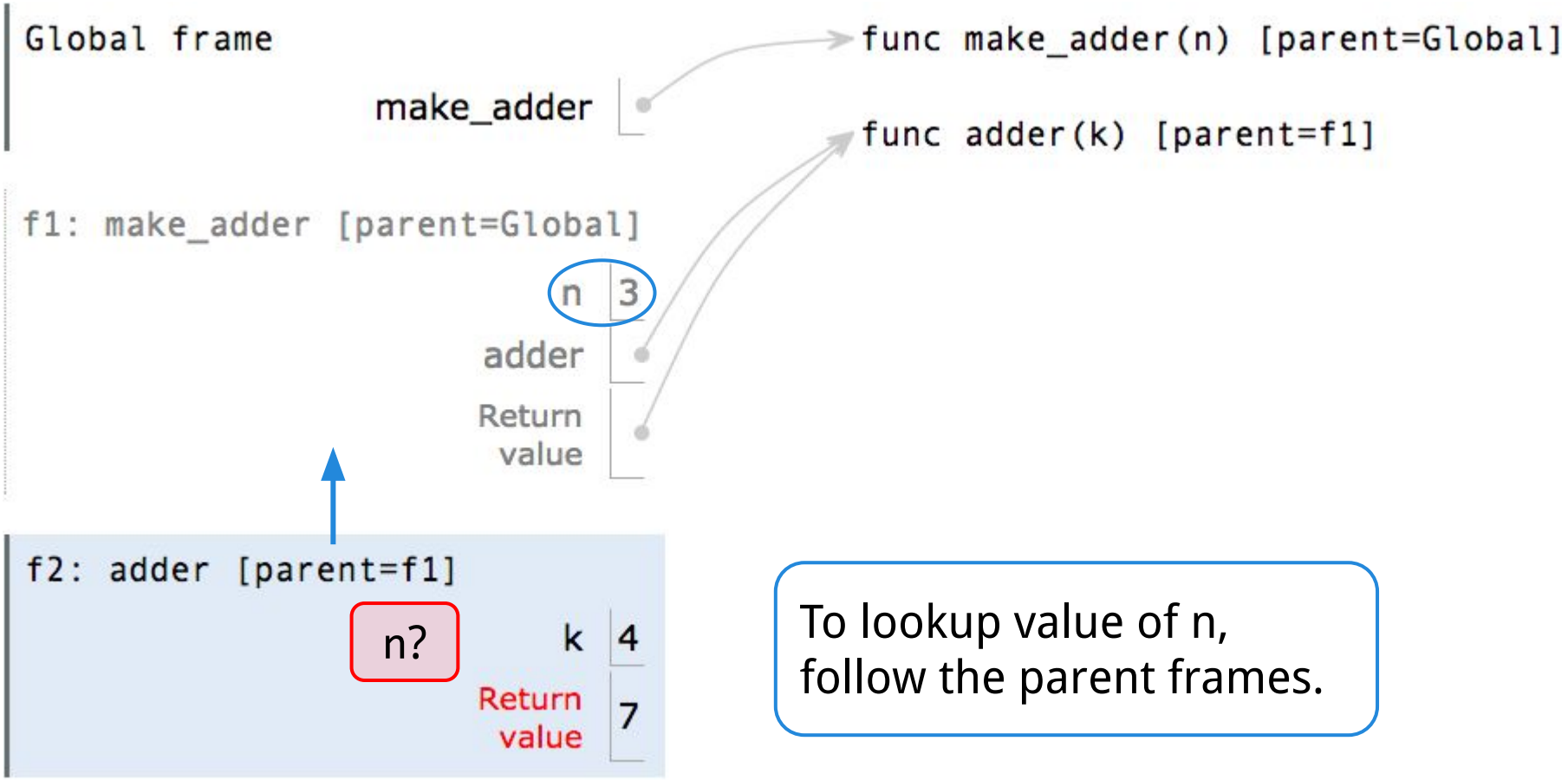

To lookup value of n, follow the parent frames.

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make_adder

```
def make_adder(n):
    def adder(k):
        return k + n
    return adder
```

Why bother naming `adder` if we only use the name once?

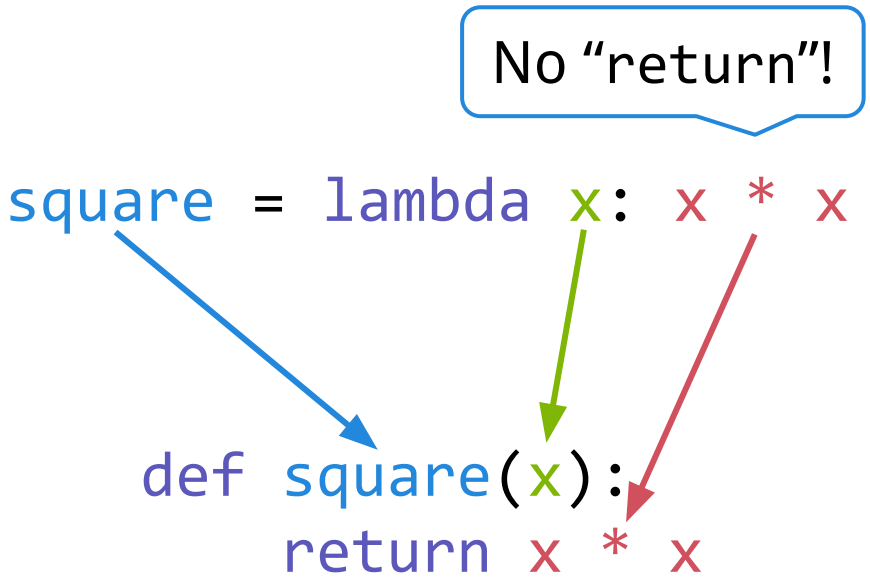
Lambda functions

```
def make_adder(n):
    return lambda k: n + k
```

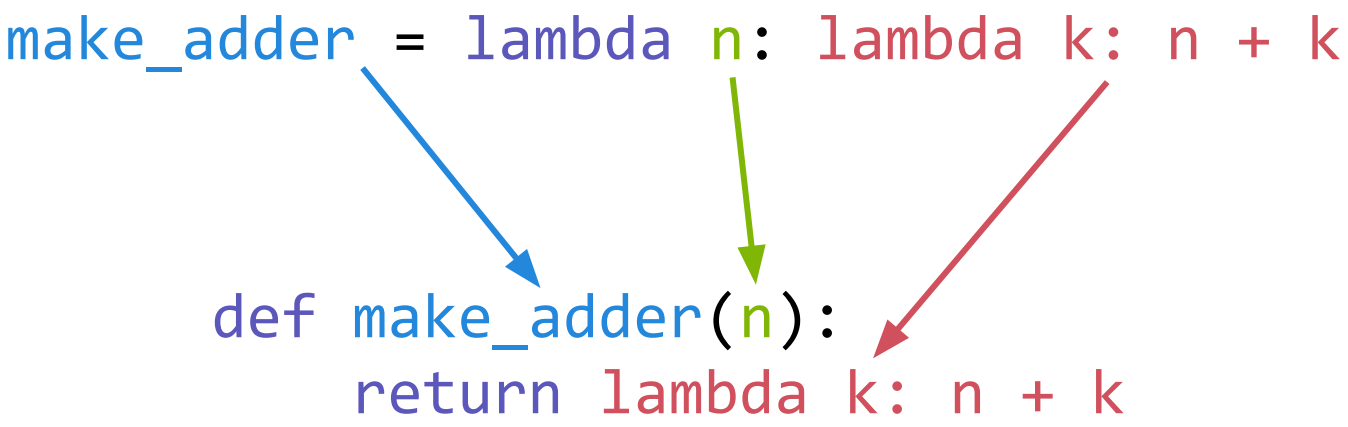
The lambda expression creates an anonymous function!

```
lambda <parameters>: <return expression>
```

Translating between named and anonymous functions



Translating between named and anonymous functions



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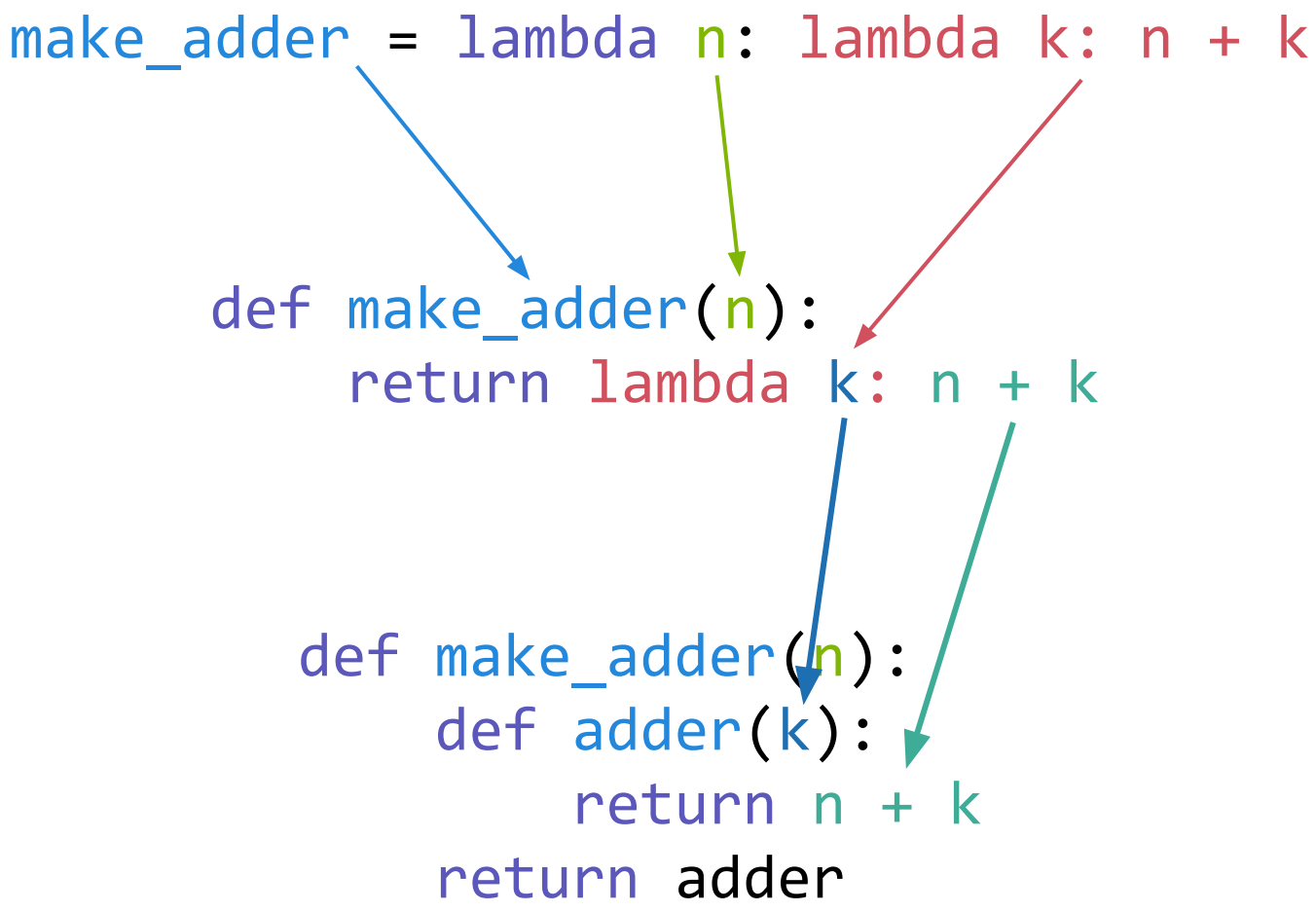


Lambda functions

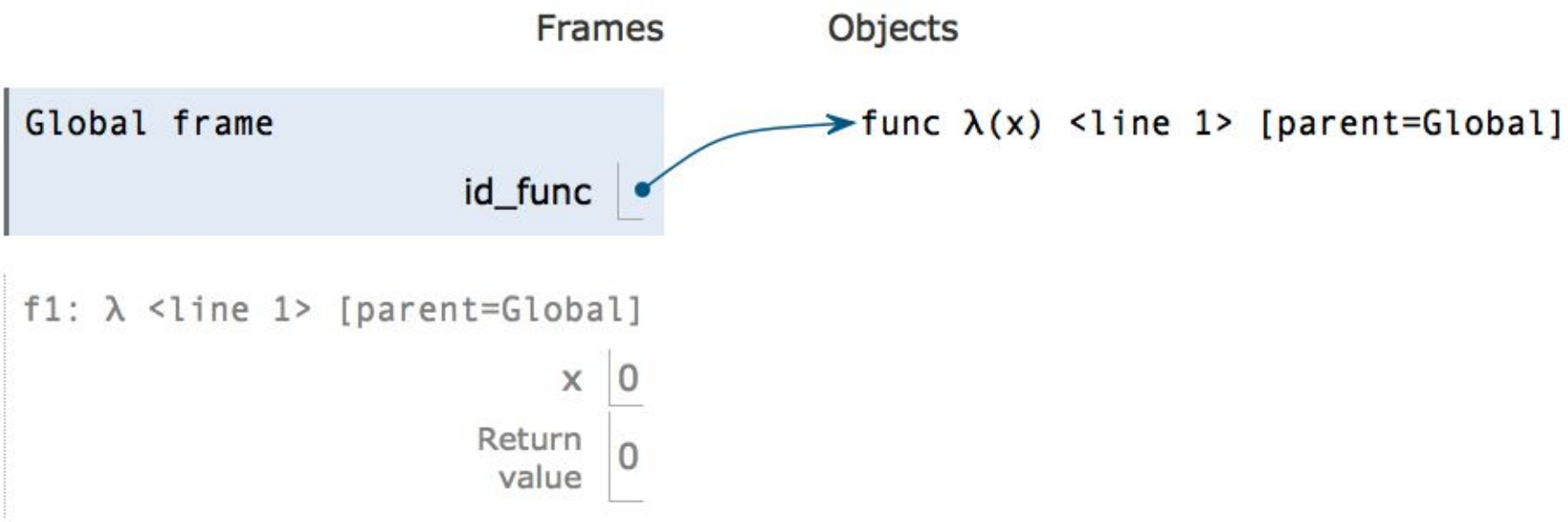
Lambdas are expressions! You can use them anywhere you can use any other expression.

A lambda's body can only be a *single* expression. This is the return expression.

Use sparingly! Better to have clear names.

Environment diagram: lambdas

```
id_func = lambda x: x
id_func(0)
```



Environment diagram: `make_adder`

```
def make_adder(n):
    return lambda k: n + k

add_3 = make_adder(3)
add_3(4)
```

(Interactive environment diagram)

Lambda functions: practice

```
>>> def summation(term, n):
...     i, total = 1, 0
...     while i <= n:
...         total += term(i)
...         i += 1
...     return total
...
>>> summation(lambda x: x**2, 3)
14
```

Lambda functions: practice

```
def repeat(f, x):
    while f(x) != x:
        x = f(x)
    return x

def g(y):
    return (y + 5) // 3

result = repeat(g, 5)
```

Hog demo!