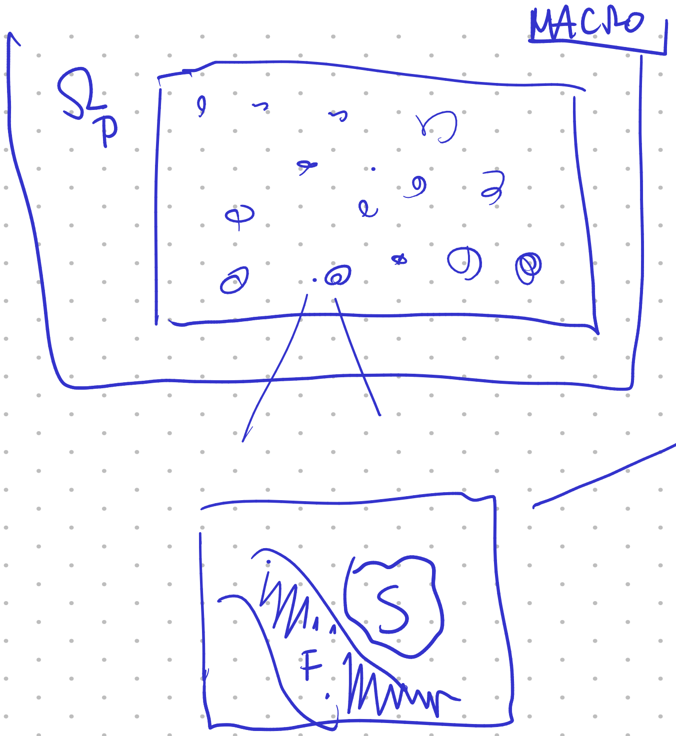

DARCY :

$$\begin{cases} \underbrace{K^{-1}u}_{\text{drag}} \cancel{\left(-\nu\Delta u\right)}^{\to 0} + \nabla p = g \\ \operatorname{div} u = 0 \end{cases}$$

BRINKMAN : STOKES-DARCY

BALANCE OF LINEAR MOMENTUM

u : velocity
p : pressure

$$\text{NS}\begin{cases} \rho\,\cancel{\left(\partial_t u + \underbrace{u\cdot\nabla u}\right)}^{\to 0\ \text{stokes}} - \operatorname{div}\left(\nu\nabla u - pI\right) = f \\ \operatorname{div} u = 0 \end{cases} \quad \text{in } \Omega$$

MASS

porous

$$f = g - \boxed{K^{-1}u}$$

$$\Rightarrow \boxed{\begin{cases} \underbrace{K^{-1}}\,u - \nu\Delta u + \nabla p = g \\ \operatorname{div} u = 0 \end{cases}}$$

BRINKMAN

$$\int_{\Omega_K} \operatorname{div} u = 0$$

$$\underbrace{\vec{w} = \operatorname{curl}\vec{u}} \quad \text{in 3D}$$

vector

$$\underbrace{w} = \operatorname{curl}\vec{u} \quad \text{in 2D}$$

scalar

$$\Delta u = \text{curl}\,\underbrace{\text{curl}\,u}_{w}$$

$$= \text{curl}\,w$$

Find u, w, p

$$\begin{cases} \int_\Omega K^{-1} u \cdot v + \int_\Omega \sqrt{\nu}\,\text{curl}\,w \cdot v + \int_\Omega \nabla p \cdot v = \int_\Omega g \cdot v \\ \int_\Omega q\,\text{div}\,u = 0 \\ \int_\Omega \vec{\theta} \cdot w - \int_\Omega \sqrt{\nu}\,\text{curl}\,u \cdot \vec{\theta} = 0 \end{cases}$$

in Ω_p

$\tilde{w} = \sqrt{\nu}\,\text{curl}\,u$ → rescaled vorticity

2016 Numer Math.

Gauss theorem

$$\begin{pmatrix} \Delta u & , \nabla u \\ \text{curl}\,u & , \text{curl}\,v \end{pmatrix}$$

C : concentration of particles.

$$\partial_t c + \underbrace{u \cdot \nabla c}_{\text{advection}} - \underbrace{\text{div}(D \nabla c)}_{\text{diffusion}} = \underbrace{f(c)}_{\text{reaction}}$$

Momentum :

$$-\operatorname{div}\big(\underbrace{\nu \nabla u - pI}_{\text{Cauchy stress}}\big)$$

$$\begin{cases} \cancel{\boxed{\nu = \nu(c)}} \\ \boxed{g = g(c)} \end{cases}$$

external 'forces'

$$c \circlearrowleft \begin{cases} \text{Brinkmann} \\ \updownarrow \\ \text{ADR} \end{cases} \circlearrowright u$$

sedimentation + consolidation of suspensions

(mining industry wastewater treatment)

Bioconvection of bacteria in porous media