Public Economics

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Contents

4 Social Insurance

- Social insurance: government intervention in providing insurance against adverse shocks to individuals.
  - Examples
    * Health insurance;
    * Retirement and disability insurance (social security);
    * Unemployment insurance.
    *
- Insurance premium ( ): **Money paid** to an insurer so that an individual will be insured against adverse events.

$$\text{Insurance} \Longleftarrow \text{Uncertainty}$$

4.1 Model Uncertainty

Suppose there are multiple $n \geq 2$ outcomes, and each outcome i is assigned with a monetary value $x_i \in \mathbb{R}$,

$$\vec{x} = (x_1, ..., x_n) \in \mathbb{R}^n$$

and is realized with probability

$$\vec{p} = (p_1, ..., p_n) \in [0,1]^n, p_i \in [0,1], \sum_{i=1}^{n} p_i = 1$$

Note that one method to evaluate lottery $(\vec{p}, \vec{x})$ is through its expected payoff, $\mathbb{E}[\tilde{x}] = \sum_{i=1}^{n} p_i x_i$.

4.1.1 Expected Utility Model

The Expected Utility (EU) model: people **maximize expected utility**, instead of maximizing expected payoff.

$$\mathbb{E}[u(\tilde{x})] = \sum_{i=1}^{n} p_i u(x_i)$$

where $u : \mathbb{R} \to \mathbb{R}$, a Bernoulli utility function mapping from monetary outcome to utility.

$\forall \mathcal{L} = (\vec{p}, \vec{x}), \mathcal{L}' = (\vec{p}\,', \vec{x}\,'), \mathcal{L} \succeq \mathcal{L}'$ if and only if

$$\mathbb{E}[u(\mathcal{L})] = \sum_{i=1}^{n} p_i u(x_i) \geq \sum_{i=1}^{n} p_i' u(x_i') = \mathbb{E}[u(\mathcal{L}')]$$

Axioms for Expected Utility Theorem

- Completeness: $\forall \mathcal{L}, \mathcal{L}'$, either $\mathcal{L} \succeq \mathcal{L}'$ or $\mathcal{L}' \succeq \mathcal{L}$.
- Transitivity: $\forall \mathcal{L}, \mathcal{L}', \mathcal{L}''$, if $\mathcal{L} \succeq \mathcal{L}'$ & $\mathcal{L}' \succeq \mathcal{L}''$, then $\mathcal{L} \succeq \mathcal{L}''$.

- Continuity: If $\mathcal{L} \succeq \mathcal{L}' \succeq \mathcal{L}''$, then $\exists \alpha \in [0,1]$, s.t. $\alpha\mathcal{L} + (1-\alpha)\mathcal{L}'' \sim \mathcal{L}'$
- Independence: $\forall \mathcal{L}''$ and $\alpha \in [0,1]$, $\mathcal{L} \succeq \mathcal{L}' \Leftrightarrow \alpha\mathcal{L} + (1-\alpha)\mathcal{L}'' \succeq \alpha\mathcal{L}' + (1-\alpha)\mathcal{L}''$

Expected Utility Theorem

vNM expected utility representation

Suppose that "$\succeq$ "satisfies Axioms 1~4, there must exist a function $U(.)$, s.t.

$$\mathcal{L} \succeq \mathcal{L}' \iff U(\mathcal{L}) \succeq U(\mathcal{L}')$$

Remarks

- $U(.)$ is mapping from a lottery to a utility; $u(.)$ is mapping from a monetary outcome to a utility.
- The theorem holds for continuous outcome spaces as well. If $F : \mathbb{R} \to [0,1]$ is the c.d.f. representing the lottery, $U(\mathcal{L}) = \int u(x)\mathrm{d}F(x)$.

4.1.2 Certainty Equivalent & Risk Attitude

For a utility function u and a lottery $\mathcal{L}$ with c.d.f. F, the certainty equivalent is the amount of money such that

$$u(c(F,u)) = \int u(x)\mathrm{d}F(x)$$

Moreover, risk attitude is defined through $c(F,u)$,

- Risk-Neutral: $c(F,u) = \int x\mathrm{d}F(x)$;
- Risk-Loving: $c(F,u) > \int x\mathrm{d}F(x)$;
- Risk-Averse: $c(F,u) < \int x\mathrm{d}F(x)$.
  - Risk-averse is assumed most of the time.

For any strictly increasing utility function u, the following two statements are equivalent.

1. $c(F,u) < \int x\mathrm{d}F(x)$, for all non-degenerate distribution F.
2. u is strictly concave.

Remarks

Shape of utility function is a convenient way to categorize risk attitude.

- Strictly Convex $\Leftrightarrow$ Risk-Averse
- Strictly Concave $\Leftrightarrow$ Risk-Loving
- Linear $\Leftrightarrow$ Risk-Neutral

4.2 Demand for Insurance

Consider an individual with initial wealth $w > 0$. She is sick with probability $q > 0$, which incurs a loss $d > 0$. To buy an amount α (in percentage of the loss) of insurance contracts, the individual pays premium of $p\alpha$ always, and receives payout αd only if sick. Individual will choose α to maximize $U(\alpha)$, expected utility would be

$$U(\alpha) = (1-q) \cdot u(w - p\alpha) + q \cdot u(w - p\alpha - d + \alpha d)$$

where $u' > 0, u'' < 0$.

For the insurer, the (pre-unit) expected profit is $\Pi(p) = p - qd$. And if $\Pi(p) = 0 \Leftrightarrow p = qd$, the insurance is **actuarially fair** ( ). Such concept can be extended that a game is actuarially fair if its entry cost equals to its expected payoff. Also note that since the individual is risk-averse, she is at least willing to pay qd to insure her wealth against the lost.

One way to think about individual's choice on α is think of such insurance as a contingent plan. Correspondingly, the person has two state in the future, and has w_s of wealth if sick, w_h otherwise. Then, the problem is what we have been quite familiar with - to weight your choice between two options, subject to a budget constraint.

$$U(\alpha) = U(w_s, w_h) = (1-q) \cdot u(w_h) + q \cdot u(w_s) \text{with } w_h = w - p\alpha, w_s = w - p\alpha - d + \alpha d$$

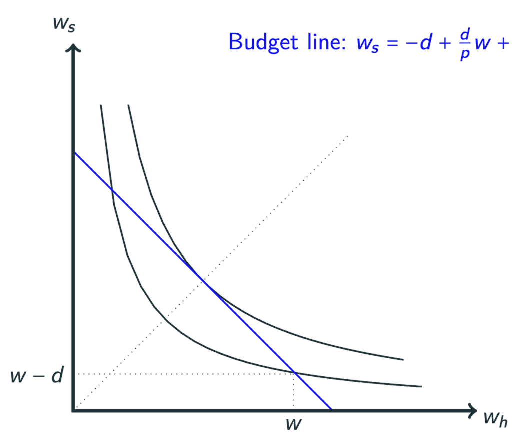
where we have to further work out the relationship of w_h and w_s to represent the budget constraint. Since α is unknown and endogenous, eliminate α in expressions above and get that

$$w_s = -d + \frac{d}{p} w + \frac{p-d}{p} w_h$$

Another way to view the choice on α is to consider α as the unique unknown parameter in $U(\alpha)$, and solve the optimal α^* **without** constraint.

Theorem

> If the premium is actuarially fair, it's optimal for a risk-averse individual to choose $\alpha^* = 1$, i.e. buy the full insurance.



Intuition is that, with concave utility, it's always desirable to reallocate consumption from good states to bad states, in order to wipe out uncertainty and risks.

4.3 Market Mechanism for Insurance

There are three mechanisms by which insurance can operate:

- Risk pooling

- Risk spreading
- Risk transfer

4.3.1 Risk Pooling

Risk Pooling is the main mechanism underlying most private insurance markets, which relies on the Law of Large Numbers. If $x_1, ..., x_n$ are i.i.d. trails with $\mathbb{E}[x_i] = \mu$,

$$\text{plim } \bar{X}_n = \mu$$

Therefore, by pooling amounts of independent risks, insurance company can effectively make risks ”disappear” (vanishingly small).

4.3.2 Risk Spread

When risks are *not* independent, LLN does not hold any more, and the risk-pooling mechanism does not work.

- Example: catastrophes are likely to affect many people simultaneously, and the risks are non-diversifiable and correlated.
- This explains that many insurance policies has an escape clause regarding catastrophic events.

Risk Spread works because some of the population are likely to be unaffected, though some co-affected. Since u is concave, if the burden is shared by the body, it is still much better than shouldered by a few. In all kinds of disaster relief, the government is effectively spreading risks to increase social welfare.

4.3.3 Risk Transfer

The key idea of risk transfer is to exploit the heterogeneity in people's risk attitude. For example, if the poor are more risk-averse, it's Pareto efficient to pay the rich to bear risks. The extent to which an individual is risk averse is measured by absolute risk aversion,

$$A(w) = -\frac{u''(w)}{u'(w)}$$

which is decreasing in w. The wealthier you are, the lower the utility cost of risk. Intuitively, a concave utility function is more ”linear” when wealth is larger, and exhibits more risk-averse when wealth is smaller.

Take a concrete example. Suppose each indivial has a utility function u over wealth w, $u(w) = \ln w$, which satisfies decreasing absolute risk aversion (DARA). Each person has a 50% chance of losing \$100,000. For someone who is poor and has an initial wealth of \$200,00, $EU = \frac{1}{2}\ln(200000) + \frac{1}{2}\ln(100000) \approx 11.86, \text{Expected Wealth} = \$150000, c(F,u) = \$141421$; and she would be willing to pay \$8579 to defray the risk. For someone who is rich with an initial wealth of \$2,000,000, $EU = \frac{1}{2}\ln(2000000) + \frac{1}{2}\ln(1900000) \approx 14.45, \text{Expected Wealth} = \$1950000, c(F,u) = \$1949359$; and she would be willing to pay only \$641 to avoid the risk. Hence, the poor can pay the rich to bear the risk at any price p, such that $p \in (\$641, \$8570)$, which is a Pareto improvement.

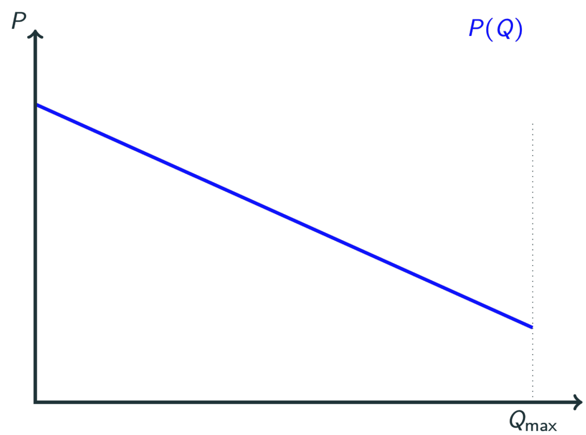
4.4 Adverse Selection

Previously, a rationale for publicly-provided insurance is to spread correlated risks. However, there are government intervention in markets where risks do not seem to be highly correlated, such as health insurance and auto insurance.

- Paternalism and individual optimization failures (e.g. myopia).
- Physical externalities, especially infectious diseases (e.g. mandatory vaccines).
- Redistribution (ex ante insurance behind the "veil of ignorance").
- Samaritan's dilemma.
  - Government cannot commit not to help out uninsured individuals, a form of government failure.

Apart from all those, government intervention may also increase welfare when insurance markets are affected by adverse selection, which comes down to asymmetric information. Asymmetric information can have serious ramifications for market outcomes.

Consider a single insurance contract that covers some probabilistic loss. For the consumer side, consumers are equally risk-averse, different only in their ***privately*** known probabilities of incurring the loss, $\pi \in [0, 1]$. Each consumer only makes a binary choice of whether or not to purchase the insurance contract. Hence, for a given price P of the contract, only individuals with sufficiently high risks will buy it, i.e., $\pi \geq \bar{\pi}(P)$, where as $\bar{\pi}(P)$ is increasing in P. So, the demand function is $Q(P) = 1 - \bar{\pi}(P)$. Let $P(Q) = \bar{\pi}^{-1}(1 - Q)$ be the inverse demand function. Apparently, $P(Q)$ is decreasing in Q.



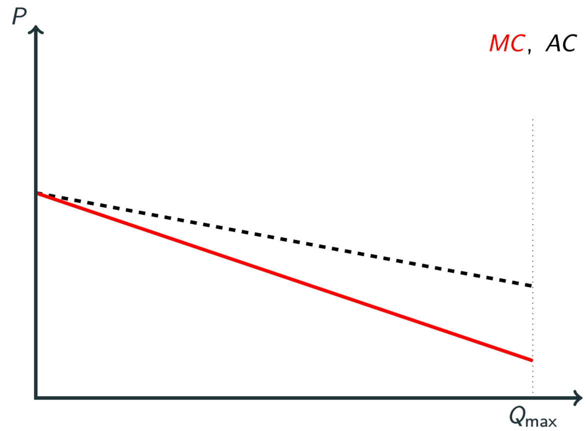
For the insurance companies, they compete only over what price to charge for the market. Let $C(\pi)$ be the firms' costs associated with prividing insurance to the type-π individual. Since firms are risk-neutral, $C(\pi)$ is equal to the expected insurance claim, and is increasing in π. Because the consumers who are most likely to incur the loss have the highest willingness-to-pay, we assume the insurance will be first bought by those will highest willingness. (Even if not, the individuals can trade with each other and reach Pareto improvement.) Therefore, the firms' marginal cost is

$$MC(Q) = C(\bar{\pi}(P(Q)))$$

and the average cost is

$$AC(Q) = \mathbb{E}[C(\pi)|\pi \geq \bar{\pi}(P(Q))]$$

Clearly, Marginal Cost is lower than Average Cost.



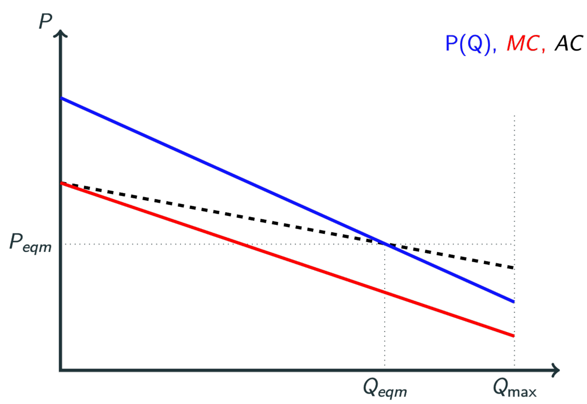
The *downward sloping* MC has represented the adverse-selection property of the insurance market, which is driven by the demand-side customer selection. And this is what makes selection markets special, as in most other contexts, demand curves and cost curves are independent objects.

In a competitive equilibrium, firms' expected profits are zero.

$$P(Q) = AC(Q)$$

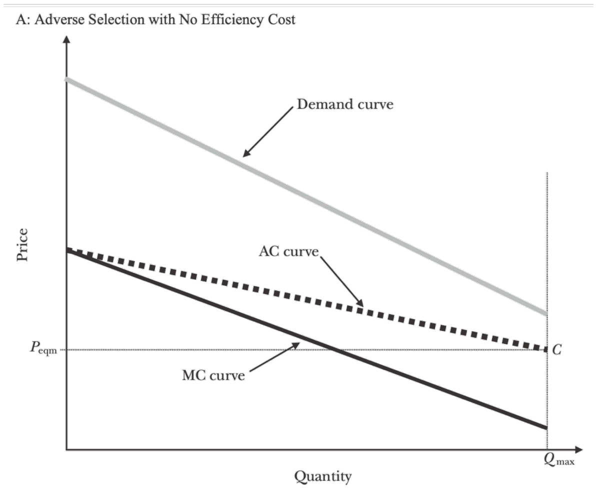
which implies the following equilibrium price

$$P = \mathbb{E}[C(\pi)|\pi \geq \bar{\pi}(P)]$$

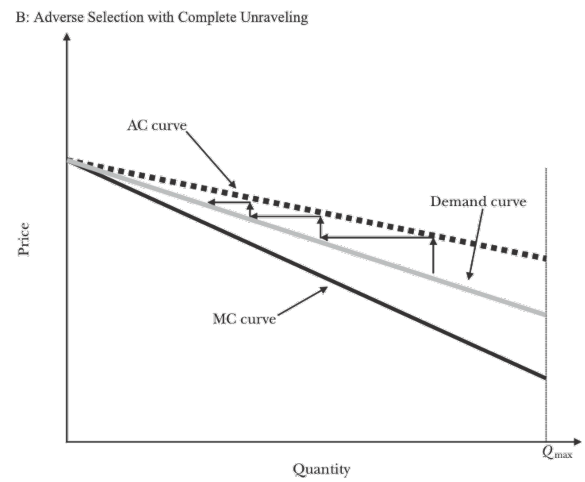


If we assume all consumers are risk-averse, and no other market frictions. Explicitly, the demand curve is always above the MC curve. For the society, it's efficient for all individuals to be insured, say $Q_{eff} = Q_{max}$. However, the equilibrium allocation is determined by the relationship between average cost and demand, but the AC curve is not always below the demand curve! This is the fundamental inefficiency created by the asymmetric information between the two sides of the market.

The amount of under-insurance and its associated welfare loss of adverse selection can vary greatly. For one extreme, the efficient outcome ($Q_{eff} = Q_{eqm} = Q_{max}$) may arise despite adverse selection, if the heterogeneity in unobserved risks is small.



For another extreme, a competitive equilibrium may involve no insurance at all, i.e., the market unravels.



Critique of such model of insurance is that, the model just focus on one single contract and price competition. In fact, firms can compete over both price and coverage. This makes it possible for insurance companies to screen the risk type of consumer by offering different contracts.

4.5 Risk Types and Insurance Contracts

4.5.1 Settings

There is a unit mass of consumers, a fraction $\lambda \in [0, 1]$ of whom are low risk (denoted by L), and the remaining $1 - \lambda$ are high risk (denoted by H). Denote the probability of monetary loss as π_H and π_L, respectively.

The market is competitive and firms are risk-neutral. Each firm can offer one insurance contract, specifying coverage $C \in [0, D]$ as premium P. All consumers are risk-averse expected-utility maximizesrs. Competition makes all insurance contracts actuarially fair, say $P = \bar{\pi}C$, where $\bar{\pi}$ is the ”average risk” of the insurance buyers.

A set of contracts S is a Rothschild-Stiglitz (RS) equilibrium if

- Each contract in S makes non-negative profits;

- There does not exists a contract (C, P) outside of S that earns strictly positive profits when offered in addition to S.

For each risk type $k = H, L$, the expected utility is

$$EU = (1 - \pi_k) \cdot u(W_1) + \pi_k \cdot u(W_2)$$

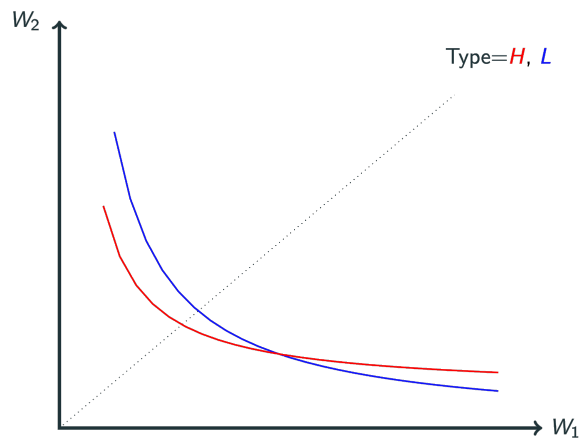
where $W_1 \equiv W_0 - P, W_2 \equiv W_0 - P - D + C$.

Let $\mathrm{d}EU = 0$, we obtain the slope of the indifference curve (or, the marginal rate of substitution) in the (W_1, W_2)-space.

$$\frac{\mathrm{d}W_2}{\mathrm{d}W_1} = -\frac{1 - \pi_k}{\pi_k} \cdot \frac{u'(W_1)}{u'(W_2)}$$

Every point in the space of (W_1, W_2) implicitly corresponds to a pair of (C, P).

Since $\pi_L < \pi_H$, the slope of type-L's indifference curve is **steeper** than that of type-H at any (W_1, W_2), which induces the single-crossing property of the indifference curves.



The two indifference curves will cross and cross only once.

The expected profit from a contract is

$$\begin{aligned}\Pi &= P - \bar{\pi}C = P - \bar{\pi}(W_2 - W_0 + P + D) \\ &= (W_0 - W_1) - \bar{\pi}(W_2 - W_0 + W_0 - W_1 + D) \\ &= -(1 - \bar{\pi})W_1 - \bar{\pi}W_2 + W_0 - \bar{\pi}D\end{aligned}$$

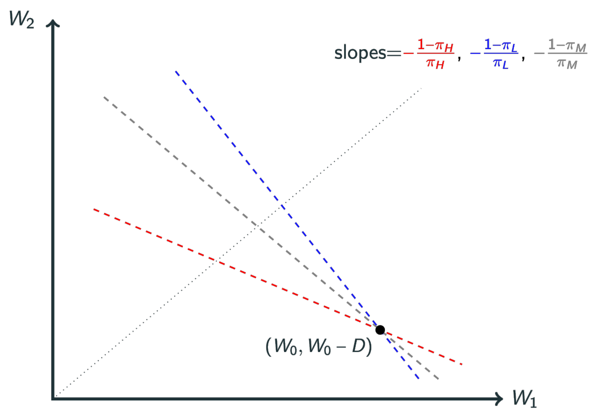
where $W_1 \equiv W_0 - P, W_2 \equiv W_0 - P - D + C$.

So the slope of the isoprofit curves in the (W_1, W_2)-space is

$$\frac{\mathrm{d}W_2}{\mathrm{d}W_1} = -\frac{1 - \bar{\pi}}{\bar{\pi}}$$

Note that the isoprofit curve is also the line defining the *budget constraint* for type-$\bar{\pi}$ individuals if actuarially fair insurance is offered.

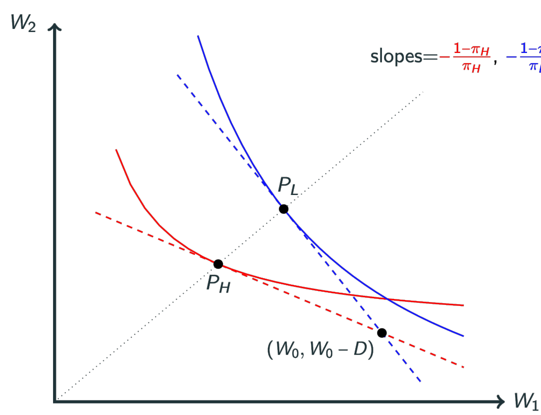
A key observation is that, any zero-isoprofit curve must pass through $(W_0, W_0 - D)$, which corresponds to consumers buying no fraction of insurance and the firm earning zero profit.



4.5.2 Complete Information

Consider the case of complete information as a benchmarke. If the risk-types of the consumers are **publicly known**, then each type can get separate contract with fair premium. In addition, the **equilibrium** contracts must provide full coverage to the buyers (with zero-profit), which is the first-best allocation.

$$P_k = \pi_k D, \quad C_k = D, \quad \forall k = H, L$$



However, with asymmetric information about individual risk types, $\{(\pi_k D, D)\}_{k=H,L}$ is no longer an equilibrium. The problem is that the high-risk type would clearly have the incentive to claim that they are low-risk. Hence, the contract $(P_L, C_L) = (\pi_L D, D)$ would actually attract both risk types, resulting a loss in expectation.

4.5.3 Incomplete Information

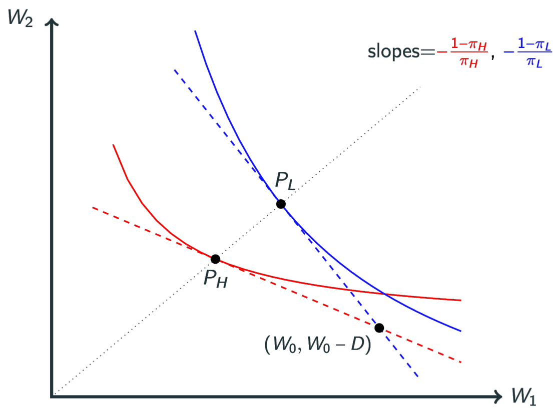
With incomplete information, we are interested in pooling equilibria and separating equilibria.

Separating Equilibria To obtain a separating equilibrium, it's necessary that H-types do not want to take up the contract offered to L-types. With fair premium, the incentive compatibility (IC) of H can only be achieved by restricting the coverage of available to L. Moreover, the single-crossing property of the indifference curves implies that the incentive campatibility (IC) of the$L$-type follows from the H-type. Therefore, the insurance for L-type should be designed to be less atractive so that the H-type do not prefer it anymore and stick with H-type customized insurance. If this is realized, we achieve the separating equilibrium.

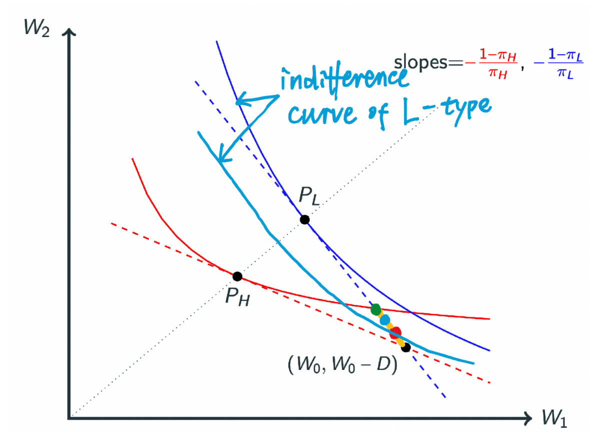
The redesigned constract selling to L, denoted as P'_L, should be located such that

1. Contract at P'_L is purchased by L only. Zero-profit condition implies that P'_L must be located on the zero-isoprofit curve for L.
2. For the H-type, it must be that $P_H \succeq P'_L$. So P'_L should be located on or below the indifference curve of H.

The only region satisfying these two conditions is the yellow-highlighted line in the graph below.



However, not all points in the yellow line hold in equilibrium. Look at the *red* point in the graph below. If the re-designed insurance is provided at the red point, another company will instead sell at the *blue* point because it's above the indifference curve of L crossing the red point so L will purchase blue instead of red. Then P'_L cannot be located at the red point. By the same logic, P'_L cannot sustain at the blue point, until the market competition pushes the insurance product to the green point, which is the intersection between the zero-isoprofit line for L and the indifference curve of H crossing P_H.



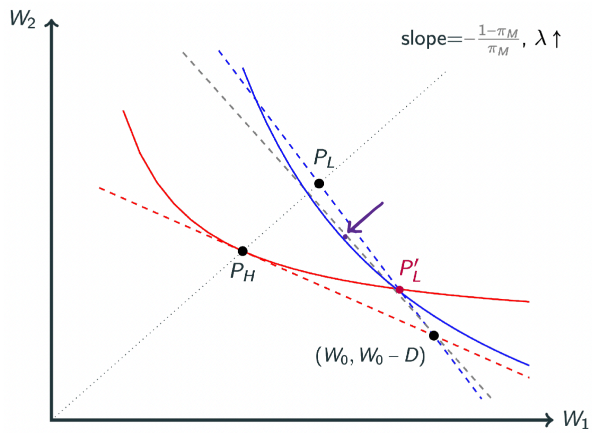
So now (P_H, green) is a candidate for separating equilibrium. The last thing we need to verify is that L will prefer the insurance at the green point over P_H. The single-crossing property of the indifference curves in fact implies that the IC of -types is automatically met if the IC of -types holds. Look at the indifference curve of L crossing the green point. Since it's steeper everywhere than the indifference curve of H and cross only once, it must be located above the P_H, so L prefer the green over P_H. Thus, we find the P'_L at the green point, and (P_H, P'_L) is the only separating equilibrium in this case. At this equilibrium:

1. Insurace contracts P_H and P_L' are sold in the market;
2. Type-H purchases P_H and type-L purchases P_L';
3. Both contracts generate zero profit for insurance companies.

Let's look at some properties of this separating equilibrium. First, *the high risk types are fully insured*, while the *low risk types are only partly insured*, which is especially ironic since they should be easier to insure. But if a company offered a contract that fully insured L, it would also attract the H and cannot sustain. Second, notice that H are not better off for the hard they do to L. The externality is entirely *dissipative*, meaning that one group loses but on group gains. This is the opposite of Pareto improvement and potentially a large social cost.

Separating Equilibrium May NOT Exist

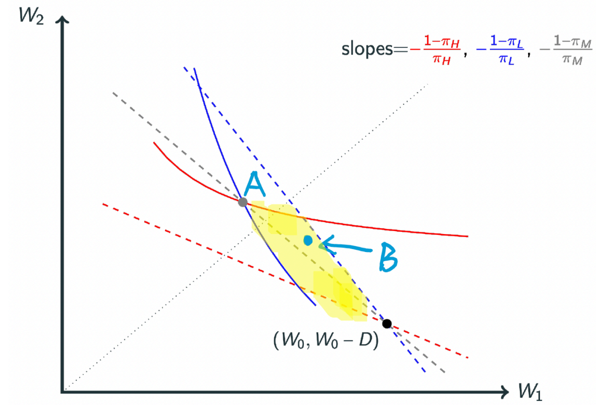
The next thing we consider is whether the derived separating equilibrium above always exist, and the answer is **not**. Consider the case where the share of the low-risk type among the population, , increases. Then if selling insurance to both types, the zero-isoprofit line (the grey dashed line in the graph below) moves closer to the one for L.
Now consider the purple point in the graph below. If a new company offers insurance in this point, who would deviate from (P_H, P_L') and buy this insurance? Both types would buy the purple one because it lies above each of their indifference curves when originally purchasing P_H and P_L'. Will any new companies offer the purple contract? Yes because it attracts both types and lies below the grey zero-isoprofit curve so they can earn *positive* profit. So the sole candidate for separating equilibrium, (P_J, P_L'), breaks down and doesn't form an equilibrium anymore.



Thinking about this a bit more and we can get the condition for when the separating equilibrium exists. The condition is that is low enough so that the aggregate zero-isoprofit curve (grey one) is flat enough and never intersect with the L's indifference curve crossing P_L' anywhere.

Pooling Equilibira First, we can immediately see that the purple point cannot be an equilibrium because insurance company there makes positive profit, so new firm entry under market competition will break that down until profit is zero. In other words, any potential pooling equilibrium must lie on the grey zero-isoprofit line, as shown in the graph above.
Consider one (any) point on that line and draw indifference curves for both types in the graph below. We know it satisfies the zero-profit condition, but how about the no market entry condition for equilibrium exist? Consider a new company sells another insurance at point B. Given its location relative to two indifference curves, this new insurance will be preferred by the L-type but not by the H-type. So the L-types all move to B while all H-types stay with A. But when only H-types purchase the A insurance, companies will lose money since A lies above the H's zero-isoprofit line (red dashed

line), so no matter where the original pooling insurance contract A is located (within the aggregate zero-isoprofit line), it cannot sustain. As a result, pooling equilibrium never exist.



Why is that? Well, for any pooling insurance at A, it lies above the H's zero-isoprofit line and below the L's zero-isoprofit line. This means the insurance company loses money from H and earns profit from L when both purchases this insurance. In other words, insurance company uses the profit from L to subsidize H and keep the zero profit. This is really not optimal for any profit-maximizing firms; they will want to abandon the unprofitable H and only serve the L, so pooling equilibrium doesn't exist.

4.5.4 Design of Separating Equilibirum in Reality

Insurance companies have designed two features of annuity contracts:

- Degree of back-loading. (higher payments in later periods v.s. constant)
- Payment to estate in the event of death. (e.g., repay beneficiary if die too early)

Competitive markets can function very poorly in the presence of adverse selection. In extreme cases, the market may even unravel completely. In reality, insurance companies may also use **statistic discrimination** to limit who can buy.

- Discrimination itself is neutral, just a description of screening and classifying.
- The public policy, in order to support the market, statistic discrimination is allowed.
- This provides a rationale for public policy interventions in insurance markets.

4.6 Public Policy Intervention

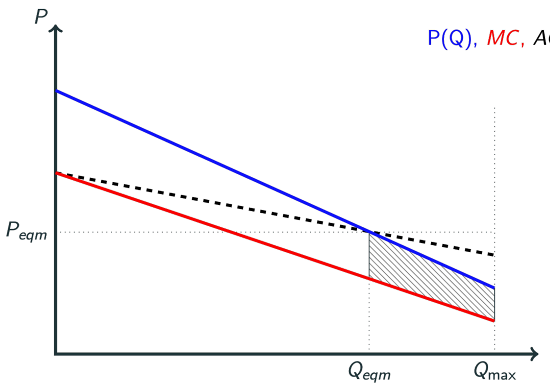
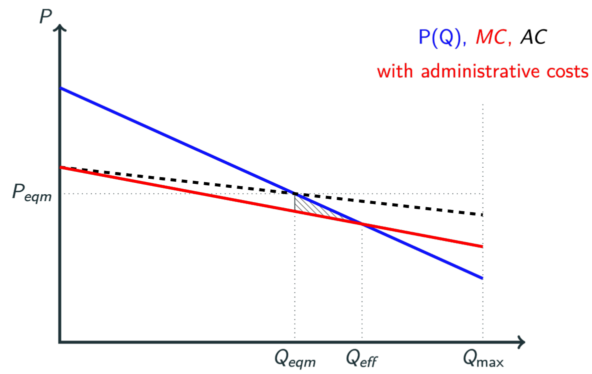
- Theoretically, the assumptions are that
  - Everyone is equally risk-averse;
  - No other market frictions exist.

  And a mandatory insurance policy is always efficient.
- The magnitude of the welfare gain, however, will depend on the specifics of the market.
  - Administrative costs
  - Preference heterogeneity
    - Preference heterogeneity may not be independent of risk types.

* People differ not only in their risk types.

4.6.1 Administrative Costs

Administrative costs, a form of market friction, will deviate the equilibrium and affect the social welfare level.

Without Administrative Costs	With Administrative Costs
	

4.6.2 Adverse Selection & Moral Hazard

One interesting and natrual question when it comes to social insurance is that, are those who buy more insurance more likely to file claims. Specifically, say

$$\text{Insurance} = \beta X + \varepsilon \text{Claims} = \Gamma X + \eta$$

where X defines the characteristics that determine individual insurance prices.

Hence, we are to carry a **positive correlation test**, see if $Cov(\varepsilon, \eta) > 0$. However, it is noteworthy that $Cov(\varepsilon, \eta)$ could be generated by both *adverse selection* and *moral hazard*.

- Adverse Selection: Individuals with private information that they have high expected cost (risks) self-select into insurance markets.
  - Ex-ante. Information before buying is unknown.
- Moral Hazard: Individuals with greater coverage have less incentive to take actions to reduce their expected costs (risks).
  - Ex-post. Behavior after buying is unknown.
  - Intuition suggests that moral hazard can be quite prevalent. For policy implications, government tends not to have comparative advantage with moral hazard.

In order to distinguish adverse selection from moral hazard, we need exogenous variation in contracts. For example, quasi-exogenous variation in premioms (over time, or regression discontinuity in geography), or field experiment.

4.7 Advantageous Selection

Advantageous Selection describes the phenomenon that those who are *less* risky than other observably similar people have *higher* demand for insurance.

Consider a population with two risk types, H and L, and two risk-preference types, C and R (for Cautious and Reckless, respectively). Risk types are private information, and insurance companies cannot tell them apart.

(Marginal) distribution of types are

$$\Pr(H) = \lambda_H, \Pr(L) = 1 - \lambda_H \Pr(C) = \delta_C, \Pr(R) = 1 - \delta_C$$

Assume that, on average, type-H individuals are morel likely to require mediacl service than the type-L.

$$\Pr(M|H) = p_H > p_L = \Pr(M|L)$$

Suppose the likelihood of buying insurance satisfy:

$$Pr(I|H,C) > Pr(I|L,C) \Pr(I|H,R) > \Pr(I|L,R) \Pr(I|H,C) > \Pr(I|H,R) \Pr(I|L,C) > \Pr(I|L,R)$$

In other words, we have both adverse selection on H/L (the first two inequalities) and advantageous selection on C/R (the last two inequalities) operating simultaneously.

Wr are then interested in whether the insured consumers will be more likely than average to require medical service. The overall probability of requiring medical service in the population is

$$\bar{p} = p_H \lambda_H + p_L \lambda_L$$

And the probability for an insured individual to require medical service is

$$p_I = p_H \Pr(H|I) + p_L \Pr(L|I)$$

Then if the insured consumers are more likely than average to require medical service, where adverse selection dominates advantageou selection, the condition is

$$p_I > \bar{p} \iff \Pr(H|I) > \lambda_H$$

where in general, the conditional probability $\Pr(H|I)$ is a function of both λ_H and δ_C and their correlation.

4.8 Exercise for Insurance

There are two states G and B, where G is the no-loss state and B the loss state. The customer has initial wealth $W_0 = 4$, and will suffer loss $\quad = 3$if state B occurs. The customer is an expected utility maximizer with log function over wealth, so her final wealth levels (consumption) in the two states are W_G and W_B, and the probability of loss is $\quad$, his expected utility is

$$EU = (1 - \pi) \ln W_G + \pi \ln W_B$$

The insurance company is risk-neutral and wants to maximize expected profit, but perfect competition in the insurance market keeps its expected profit equal to zero. All insurance contracts will be characterized by the amounts of wealth (W_G, W_B) that the consumer will end up with in the two states. Denote P as the premium and C as the coverage.

4.8.1 Moral Hazard

Luckily, the customer can exert effort to affect the probability of loss. A bad effort has zero cost to the consumer in utility ters, and then $\pi = 0.5$; a good effort has cost k to the customer in utility terms, and then $\pi = 0.25$.

> **Q1. Find the inequality linking W_G, W_B, k such that the customer will make good effort if the inequality holds, and bad efforts if the opposite inequality holds. (Fine to assume that the customer will make good effort in the borderline case where the relation holds with exact equality.)**

Simply compare the expected utilities net of effort cost in the two situations. The customer will make good efforts if

$$\frac{1}{4}\ln W_B + \frac{3}{4}\ln W_G - k \geq \frac{1}{2}\ln W_B + \frac{1}{2}\ln W_G \Longrightarrow W_G \geq W_B \cdot e^{4k}$$

Note that this is not only the incentive condition for the customer to make good effort, but also the constraint for insurance company's available zero-profit insurace products. On the zero-isoprofit line, if such inequality does not hold, the customers buying insurance designed for the good-effort and low-risk ones will commit moral hazard and only exert bad efforts, eventually making the company to earn negative profits.

> **Q2: Find the equation for the insurance company's zero-isoprofit line when the customer makes bad effort and when the customer makes good effort, respectively.**

When earning zero profit, it must be that

$$P = \pi C$$

The wealth of the customer in two states can be written as

$$\begin{cases} W_G = W_0 - P \\ W_B = W_0 - P - L + C \end{cases} \Longrightarrow \begin{cases} L = C + W_G - W_B \\ P = W_0 - W_G \end{cases} \stackrel{P=\pi C}{\Longrightarrow} \pi W_B + (1-\pi)W_G = W_0 - \pi L$$

Therefore, the zero-isoprofit lines for each type of customers are

$$\begin{cases} W_B + W_G = 5, & \text{Bad Effort} \\ W_B + 3W_G = 13, & \text{Good Effort} \end{cases}$$

And note that the two lines intersect at point $(4, 1)$, where the customer is not insured.

> **Q3: Find the local optimum, namely the values of (W_G, W_B) that maximizes the customer's expected utility subject to the insurance company's zero-profit constraint, when the customer is making bad and good effort respectively. Together, find the customer's expected utility under each scenario.**

- If making bad effort, based on the standard model, the customer will buy full insurance at the fair price. Therefore, $W_G = W_B$. Along with the zero-isoprofit line $W_G + W_B = 5$, we can get that $W_B = W_G = \frac{5}{2}$, corresponding utility is $\ln \frac{5}{2} = 0.916$.
- If making good effort, it must be primarily met that $W_G \geq W_B \cdot e^{4k}$. Combined with the zero-profit condition, $W_G = \frac{13 \cdot e^{4k}}{1+3 \cdot e^{4k}}, W_B = \frac{13}{1+3 \cdot e^{4k}}$. The utility level is $\ln 13 - \ln(1 + 3 \cdot e^{4k}) + 2k$.

Q4: Find the value range of k when making good effort is globally optimal.

Simply compare two utility level when making good or bad efforts,

$$\ln 13 - \ln(1 + 3 \cdot e^{4k}) + 2k \geq \ln \frac{5}{2} \Longrightarrow k \leq 0.207$$

Q5: Now suppose $k = 0.1$. Find the customer's expected utility in two situations.

1. **where she gets no insurance at all;**
2. **where effort is observable, and the company sells full insurance at the statistically fair price conditional on customer making good effort.**

1. Note that the non-insured point is located in the region where it is optimal for the customer to *exert good effort*:

   $$W_G = 4 > W_B \cdot e^{4k} = e^{0.4} = 1.49$$

   Thus, the customer's expected utility where she gets no insurance at all is

   $$\frac{1}{4} \ln 1 + \frac{3}{4} \ln 4 - 0.1 = 0.9397$$

2. This time, since effort can be observed, there is no room for moral hazard. The difference between this and the previous setting is that, since competitive firms have to keep zero profit, they will surely not sell those insurances that do not satisfy $W_G \geq W_B \cdot e^{4k}$, which is also the incentive compatibility for the customer to exert good effort. But if effort can be observed by the firm, such commitment will make *all* points on zero-isoprofit line of low-risk type available. Because good effort is ”committed”, the cost of effort is kind of ”sunk cost”, instead of marginal cost for the customer, and will not further influence her behavior like loafing or paying effort. Thus, the incentive compatibility (IC) does not play a role here.

   If the customer exert good effort and then buy the insurance, it is sure that she will buy full insurance to maximize her utility, $W_G = W_B$. Along with the zero-isoprofit line $W_B + 3W_G = 13$, we can get $W_B = W_G = \frac{13}{4}$, and the expected utility is

   $$\frac{1}{4} \ln \frac{13}{4} + \frac{3}{4} \ln \frac{13}{4} - 0.1 = 1.0787 > 0.9397$$

   which is greater than non-insured scenario. So she will buy the insurance

Q7: Quantify the social cost of the information asymmetry.

Fistly, calculate the expected utility of global optimum under information asymmetry:

$$\ln 13 - \ln(1 + 3e^{0.4}) =).2 = 1.0647$$

So the gain with insurance is $1.0647 - 0.9397 = 0.125$.

If the information asymmetry does not exist, the customer should receive full insurance and make good effort, gaining $1.0787 - 0.9397 = 0.139$.

Therefore, the social cost is $\frac{0.139-0.125}{0.139} \approx 10\%$.

4.8.2 Adverse Selection

The basic settings are the same as before, expect that there is no good or bad effort anymore, and the customer's type is exogenous and fixed, also privately known.

> **Q1: Find the equation for the insurance company's zero-isoprofit lines for the customer being the high risk type and the low risk type, respectively.**
>
> **Q2: What insurance contract intended for the high risk type will be offered in the equilibrium? What is the resulting expected utility?**
>
> **Q3: What insurance contract intended for the low risk type will be offered in the equilibrium? What is the resulting expected utility?**
>
> **Q4: Compare the low risk type customer's expected utility in three situations:**
>
> - **first where he gets no insurance at all**
> - **second where he gets the Rothschild-Stiglitz separating equilibrium contract**
> - **third, the hypothetical ideal optimum where type is observable, and the customer is given full insurance at the actuarially fair price for the low type.**
>
> **Q5: How much (in percentage of utility) does the H-type and L-type suffer from the information asymmetry in the insurance market, respectively?**

The body is the same with the previous part of moral hazard.

$$\begin{cases} W_B + W_G = 5, & \text{High Risk} \\ W_B + 3W_G = 13, & \text{Low Risk} \end{cases}$$

In the separating equilibrium, the insurance contract available to be selected by the high risk type will offer them *full* insurance at the fair price. Easy to get $W_G = W_B = \frac{5}{2}, EU_H = \ln\frac{5}{2} = 0.9163$.

The insurance contract available to be selected by the low risk type will be on the zero-isoprofit line $W_B + 3W_G = 13$, and such that the high risk type finds it exactly indifferent to take this contract or the previous one. The equation for this indifference is

$$\frac{1}{2}\ln W_B + \frac{1}{2}\ln W_G = \ln\frac{5}{2} \iff W_B W_G = 6.25 \overset{W_B+3W_G=13}{\Longrightarrow} (W_{G,L1}, W_{B,L1}) \Longrightarrow EU_{L1} = 1.1234$$

When the low-risk type customer gets no insurance,

$$EU_{L0} = \frac{1}{4}\ln 1 + \frac{3}{4}\ln 4 = 1.0397$$

When she gets the Rothschild-Stiglitz separating equilibrium contract, $EU_{L1} = 1.1234$.

Under observable risk type, the customer is given full insurance at the actuarially fair price, so

$$\begin{cases} W_B = W_G \\ W_B + 3W_G = 13 \end{cases} \implies W_{B,L2} = W_{G,L2} = \frac{13}{4} \implies EU_{L2} = 1.1787$$

Jointly speaking,

$$EU_{L0} < EU_{L1} < EU_{L2}$$

Compared to no insurance at all, insurance can improve the low-risk type customer's expected utility. But RS equilibrium contract is less favorable than the hypothetical ideal optimum without information asymmetry.

$$\begin{cases} \Delta EU_1 = EU_{L1} - EU_{L0} = 0.0837 \\ \Delta EU_2 = EU_{L2} - EU_{L0} = 0.139 \end{cases} \implies \frac{\Delta EU_2 - \Delta EU_1}{\Delta EU_2} = 40\%$$

Therefore, the low risk type suffer by 40% of utility compared to the hypothetical ideal optimum.

And for the high risk type, there's zero utility loss caused by information asymmetry.

4.9 Unemployment Insurance

The unemployment insurance aims to insure the risk of consumption loss when temporarily out of work and looking for a new job. Potential benefits are smooth path of consumption, and better job matches. Potential distortions are less job search, higher unemployment rate, and shrinking on the job.

The structure of UI (unemployment insurance) is typically three-fold.

- Eligibility: reason for being unemployed, and employment history.
- Coverage duration: waiting period and potential benefit duration.
- Benefit level: "replacement rate", typically proportional to previou earnings.

The government intervene into unemployment insurance possibly for

- Credit market failures
- Aggregate shocks and business cycle
- Adverse selection
  - Positive correlation test: correlation between probability of buying supplemental UI coverage in year t and unemployment outcome in year $t+1$.
- Moral hazard
  - Not very likely and difficult to actively become unemployed.
  - More of not actively searching for new jobs once covered by UI.

Baily-Chetty model depicts the tradeoff between insurance (of job loss) and incentive (of job search). It turns out that under a set of parameters in such a static model, an optimal benefit level can be determined, and this also provide a guidance to test the optimality of unemployment insurance.

4.9.1 Setup

Static model with two states: a risk-averse agent is either

- Employed and earns wage $w > 0$, or
- Unemployed and has no income.

The agent is initially employed, with an exogenous probability $p \in (0, 1)$ of becoming unemployed.

Once unemployed, she can exert effort $e \in [0, 1]$ to search for jobs (in current period).

- The probability of finding a job is e.
- The cost of effort is $c(e)$, where $c' > 0, c'' > 0$.

The UI system pays a constant benefit b to the unemployed.

- Financed via lump sum tax τ paid by the employed agents.
- For simplicity, assume the re-employed doesn't need to pay immediately.

Government's budget constraint is

$$p(1-e)b \leq (1-p)\tau$$

Assume that the government keeps its budget balanced.

Agent's expected utility over consumption is

$$U(e) = (1-p) \cdot u(w-\tau) + p\left[e \cdot u(w) + (1-e) \cdot u(b) - c(e)\right]$$

where $u' > 0, u'' < 0$. Note that if an unemployed person successfully finds a job at this period, she does not need to pay the tax τ to the government, and this is also consistent with our expression of government's budget constraint.

4.9.2 First-Best Optimal

Socially Optimal is achievable under perfect information. Under such assumption, there's no moral hazard. In this setting, e can be perfectly monitored.

The optimization problem of a benevolent government is

$$\max_{b,\tau,e} U(e) \text{ s.t. } (1-p)\tau = p(1-e)b$$

The first-best are characterized by

$$\begin{aligned}
\text{[F.O.C. - b]: } & u'(w-\tau) = u'(b) \\
\text{[F.O.C. - e]: } & bu'(w-\tau) + [u(w) - u(b)] = c'(e) \\
\text{[F.O.C. - } \tau\text{]: } & u'(w-\tau) = \lambda \\
\text{[BC]: } & (1-p)\tau = p(1-e)b
\end{aligned}$$

where

- [F.O.C. - e] internalizes the personal cost. The marginal input to find a job should equate the marginal utility of losing job plus those who are implicitly paying insurance for the unemployed.

- [F.O.C. - b] implies that in the first-best, the agent is fully insured.

$$w - \tau = b$$

which means perfect consumption smoothing. (kind of buying full insurance)

4.9.3 Second-Best with Moral Hazard

More realistically, suppose e is privately chosen by the agent.

Given any government UI policy (b, τ), the agent privately chooses e to maximize $U(e)$.

$$\text{[F.O.C. - e]: } \frac{\mathrm{d}}{\mathrm{d}e}U(e) = 0 \Leftrightarrow u(w) - u(b) = c'(e)$$

Note that both b and τ are exogenous here, and the agent is facing a maximizing problem without constraint.

Compared to the first-best's condition:

$$\text{[F.O.C. - e]: } bu'(w-\tau) + [u(w) - u(b)] = c'(e)$$

Since $c''(\cdot) > 0$, then $e < e^*$. This implies that the agent just searches too little, which is the source of inefficiency. More realistically, agents do not care about their previous co-workers once they are unemployed. The agents will simply get away with receiving b with all employers paying τ to fund it.

4.9.4 Government's Problem

Knowing individuals' reaction under UI policy (b, τ), the government can use *Backward Induction* to solve the maximization problem, taking into account agent's behavioral responses.

$$\begin{aligned}
\max_{b,\tau} \ & (1-p)\cdot u(w-\tau) + p\left[e\cdot u(w) + (1-e)\cdot u(b) - c(e)\right] \\
\text{s.t. } & (1-p)\tau = p(1-e)b \qquad (BC) \\
& e = \arg\max_{\tilde{e}} U(\tilde{e}) \qquad\quad (IC)
\end{aligned}$$

The last incentive compatibility (IC) follows the individual's problem solution:

$$\text{[F.O.C. - e]: } \frac{\mathrm{d}}{\mathrm{d}e}U(e) = 0 \Leftrightarrow u(w) - u(b) = c'(e)$$

Such problem can be solved, given the government's constrained optimization problem. The second-best (b^*, τ^*, e^*) must satisfy the condition:

$$\frac{u'(b) - u'(w-\tau^*)}{u'(w-\tau^*)} = \frac{\frac{\mathrm{d}(1-e^*)}{1-e^*}}{\frac{\mathrm{d}b^*}{b^*}} = \varepsilon_{1-e^*,b^*}$$

where each side of the equation has practical meanings that

- LHS: **social benefit** of transferring \$1 from high to low state due to increased insurance, which is decreasing in insurance coverage b.

- RHS: **social cost** of transferring $1 due to decreased search effort, which is non-decreasing with respect to b.

Especially, costs come from the *fiscal externality* from agent's behavioral response to policy on government budget. This would cost every else working agents to fund her longer UI benefits.

Formal derivation of second-best parameters:

The issue of second-best can be addressed by *Backward Induction*. Specifically, given a UI with (τ, b), agents will choose e to maximize her epected utility. Her effort level is determined by (as is shown previously)

$$u(w) - u(b) = c'(e) \Longrightarrow e = e(b)$$

where the effor e, as a response for the agent, is a function of b, say $e = e(b)$. Then, we should treat e as $e(b)$ to incorporate such behavioral response into government's choice.

$$\begin{aligned}
&\max_{b,\tau}\ (1-p)\cdot u(w-\tau) + p\left[e(b)\cdot u(w) + (1-e(b))\cdot u(b) - c(e(b))\right] \\
&\quad \text{s.t. } (1-p)\tau = p(1-e(b))b \qquad (BC) \\
&\qquad\quad u(w) - u(b) = c'(e) \qquad\quad (IC) \\
\Longrightarrow &\begin{cases}
\text{[F.O.C. - b]:} & \frac{\mathrm{d}e}{\mathrm{d}b}u(w) - \frac{\mathrm{d}e}{\mathrm{d}b}u(b) + u'(b)(1-e) - c'(e)\frac{\mathrm{d}e}{\mathrm{d}b} - \lambda(1-e) + \lambda\frac{\mathrm{d}e}{\mathrm{d}b}b = 0 \\
\text{[F.O.C. - } \tau\text{]:} & u'(w-\tau) = \lambda \\
\text{[BC]:} & (1-p)\tau = p(1-e)b
\end{cases}
\end{aligned}$$

where the behavioral response of the agent, $c'(e) = u(w) - u(b)$ should be plugged into [F.O.C. - b] and then have

$$\text{[F.O.C. - b]} \iff u'(b)(1-e) - u'(w-\tau)(1-e) + u'(w-\tau)\frac{\mathrm{d}e}{\mathrm{d}b}b = 0$$

To make this meaningful, do some equivalent transformation to the equation above and then have

$$\text{[F.O.C. - b]} \iff \frac{u'(b) - u'(w-\tau)}{u'(w-\tau)} = \frac{\frac{\mathrm{d}(1-e)}{1-e}}{\frac{\mathrm{d}b}{b}} \equiv \varepsilon_{1-e,b}$$

which is coincidentally linked with unemployment elasticity $\varepsilon_{1-e,b}$. Also, all parameters determined by F.O.C. are optimized ones. (Just for simplicity all parameters were not starred through the proof.)

4.9.5 Test Optimality of Social Insurance

To test the above optimality condition with data, rewrite the marginal utility gap of the LHS using Taylor expansion

$$u'(b) - u'(w-\tau^*) \approx -u''(w-\tau^*)\cdot(w-\tau^*-b^*)$$

Define the coefficient of relative risk aversion (CRRA)

$$\gamma = -\frac{u''(c)\cdot c}{u'(c)}$$

Then, the [F.O.C. - b] can be equivalently written as

$$\begin{aligned}\text{[F.O.C. - b]} \iff \frac{u'(b) - u'(w-\tau^*)}{u'(w-\tau^*)} &\approx -\frac{u''(w-\tau^*) \cdot (w-\tau^*)}{u'(w-\tau^*)} \cdot \frac{w-\tau^*-b^*}{w-\tau^*} \\ &= \gamma \frac{\Delta c}{c} = \varepsilon_{1-e^*,b^*}\end{aligned}$$

where $\frac{\Delta c}{c}$ measures the consumption drop during unemployment, and γ is the coefficient of relative risk aversion, and ε_{1-e^*,b^*} is the unemployment elasticity. All those can be estimated from data.

4.9.6 Behavioral Economics & Public Policy

If people fail to choose what is the best for themselves, the government may internene to improve welfare in principle. THere are two behavioral biases that may be particularly relevant for the public policy intervention.

- Exponential-Growth Bias (ERB)
  - e.g. pension, loans, etc.
- Present Bias
  - Time-inconsistency & Procrastination
  - Short-run self is extremely impatient: relative to the current period, all future periods are weighted much less.
  - Long-run self is extremely patient: all future periods are weighted equally.
  - Nudge might help: from opt-in to opt-out.