Advanced Microeconomics Theory

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Chapter 1

Choice Theory

The utility-maximization approach to choice has several characteristics that help account for its long and continuing dominance in economic analysis:

- Normative usefulness: policy-making; individuals' choices v.s. government's welfare criterion; modern democratic values.
- Positive predictions: comparative statics predictions.
- Wide scope.
- Compactness: make empirical predictions from a relatively sparse model of the choice problem, just a description of the chooser's objectives and constraints.

1.1 Preference-Based Approach

Rational choice theory starts with the idea that individuals have preferences and optimize their utilities according to preferences. The primary task is to formalize what that means and precisely what it implies about the pattern of decision-making we should observe.

1.1.1 Preference Relation

Definition 1.1.1: (Weak) Preference Relation

Let X be a set of possible choices. Consider a preference over the set X, as a binary relationship:

$$x \succeq y \iff \text{“}x \text{ is at least as good as } y\text{”}$$

Remark.

> The weak preference relation $\succeq$ implies the associated strict preference relation $\succ$ and indifference relation $\sim$:
>
> - x is *strictly preferred to* y, or $x \succ y$, if $x \succeq y$ but not $y \succeq x$.
> - x is indifferent to y, or $x \sim y$, if $x \succeq y$ and $y \succeq x$.

After the primary definition, we need to make assumptions about rationality, or definition of rational preference.

The first is *completeness*, which means an agent would never be clueless when faced with two choices.

Definition 1.1.2: Completeness

A preference relation $\succeq$ on X is *complete*, if for all $x, y \in X$, either $x \succeq y$, or $y \succeq x$, or both.

The second is *transitivity*, which means that an agent's weak preference cannot cycle unless among choices that are indifferent.

Definition 1.1.3: Transitivity

A preference relation $\succeq$ on X is *transitive*, if whenever $x \succeq y$ and $y \succeq z$, $x \succeq z$.

From completeness and transitivity, we have the following corollaries:

Corollary 1.1.4

1. While the definition of transitivity involves only 3-choice cycles, it also extends to all n-object cycles, i.e., that it implies that for any n choices $x_1, x_2, \cdots, x_n \in X$ such that $x_1 \succeq x_2$, $x_2 \succeq x_3$, ..., $x_{n-1} \succeq x_n$, we must have $x_1 \succeq x_n$. (Hint: use induction on n.)

2. Transitivity of a weak preference relation $\succeq$ implies transitivity of the associated strict preference relation $\succ$ and the indifference relation $\sim$.

Remark.

1. The assumption of transitivity is inconsistent with certain ”framing effects”.

2. Preference relation is defined upon $\succeq$ for simplicity.

Completeness and transitivity allow us to formalize what we mean by rationality.

Definition 1.1.5: Rationality

A preference relation $\succeq$ on X is *rational* if it is both complete and transitive.

1.1.2 Choice Rule

Given preferences, we should define the agent's *choice rule* (given an "opportunity set" $B \subseteq X$) induced by preference relation $\succeq$ as

$$C_{\succeq}(B) = \{x \in B | x \succeq y, \forall y \in B\},$$

which is the set of items in B the agent likes at least as much as any of the other alternatives in B; or say, most preferred.

Remark.

1. $C_{\succeq}(B)$ may contain more than one element.
2. $C_{\succeq}(B)$ might be empty. If B is finite and non-empty, then $C_{\succeq}(B)$ is non-empty.
   - As an example for $C_{\succeq}(B)$ to be empty, define a preference relation $x \succeq y \iff x \geq y$. Take $X = [0, +\infty)$ and $B = (0, 1)$. Then clearly there is no "most preferred" option.
   - To guarantee the existence of choices from infinite choice sets, we will later make some technical assumptions (such as compactness of choice sets and continuity of preference relation).

Proposition 1.1.6

Suppose $\succeq$ is complete and transitive. Then, for every finite non-empty set B,

$$C_{\succeq}(B) \neq \varnothing.$$

Proof for Proposition.

Proceed by mathematical induction on the number of elements of B.

- $|B| = 1$, say $B = \{x\}$.
  - By completeness, $x \succeq x$, so $x \in C_{\succeq}(B)$. $C_{\succeq}(B) \neq \varnothing, \forall |B| = 1$.
- Fix $n \geq 1$ and suppose that for all sets B with exactly n elements, $C_{\succeq}(B) \neq \varnothing$. Next we move on to examine the case of $|B| = n + 1$.
  - Consider any B_n such that $|B_n| = n$. Since $C_{\succeq}(B_n) \neq \varnothing$, say $C_{\succeq}(B_n) = x^*$. Let $B = B_n \cup \{x_{n+1}\}$.
  - By completeness, we have only two (not mutually-exclusive) possibilities:
    * If $x^* \succeq x_{n+1}$, then by definition $x^* \in C_{\succeq}(B)$, so $C_{\succeq}(B) \neq \varnothing$.
    * If $x_{n+1} \succeq x^*$. Since $x^* \in C_{\succeq}(B_n)$, by definition, $x^* \succeq y, \forall y \in B_n$. By transitivity, this implies $x_{n+1} \succeq y, \forall y \in B$. Therefore, $x_{n+1} \in C_{\succeq}(B)$, so $C_{\succeq}(B) \neq \varnothing$.
- Hence, for every set B with exact $n + 1$ elements, $C_{\succeq}(B) \neq \varnothing$. By the principle of mathematical induction, it follows that for every finite set B that is non-empty, $C_{\succeq}(B) \neq \varnothing$.

1.2 Choice-Based Approach

Much empirical work reasons in the reverse way contrary to economic theories (in preference-based approach): it looks at individuals' choices and tries to "rationalize" those choices. That is, to figure out whether the choices are compatible with preference maximization and, if so, what they imply about those preferences. Under choice-based approach, choice rule is the primitive object of the theory.

Definition 1.2.1: Choice Rule

Let $\mathcal{B}$ be the set of all nonempty subsets of X ($\mathcal{B} = 2^X \backslash \varnothing = \{B \neq \varnothing : B \subset X\}$). A *choice rule* is a function $C : \mathcal{B} \to \mathcal{B}$ with the property that for all $B \in \mathcal{B}$, $C(B) \subseteq B$.

Remark.

- $\mathcal{B}$ is the set of all nonempty **subsets** of X, which means all possible set of available choice(s) the agent is facing. And the choice rule C is a mapping from $\mathcal{B}$ to $\mathcal{B}$, which means that the agent is choosing from a set of available choice(s) to pick his most preferred choice(s), also a subset of X.
- Here we assume that we can see the agent choose from *all* possible subsets of X, and that the agent reports *all* of his optimal choices from a given opportunity set.

By definition of choice rule, we do not impose any assumptions or restrictions on the rule or underlying preference relation. We are then interested in two questions:

- If this rule comes from maximizing some underlying preferences, what can we infer about these preferences?
- Is this choice rule consistent with the maximization of *some* complete and transitive preference relation (i.e., *rationlizable*)?

Consider the first question first. Suppose that choice rule C is consistent with the maximization of some preference relation $\succeq$, i.e., $C(\cdot) = C_{\succeq}(\cdot)$. Then, observing for some $A \subseteq X$ that $y \in A$ and $x \in C(A)$ (i.e., x is chosen when y is available) allows us to infer that $x \succeq y$. This implies that for any $B \subseteq X$ such that $x \in B$ and $y \in C(B)$, we must also have $x \in C(B)$ (Indeed, we have $x \succeq y$ and $y \succeq z$ for all $z \in B$, and so by transitivity $x \succeq z$ for all $z \in B$). By a symmetric argument, we should then have $y \in C(A)$. Thus, any rationalizable choice rule must have the following property (as a *necessary* condition):

Definition 1.2.2: HARP

A choice function $C : \mathcal{B} \to \mathcal{B}$ satisfies *Houthaker's Axiom of Revealed Preference (HARP)* if, whenever $x, y \in A \cap B$, and $x \in C(A)$ and $y \in C(B)$, we have $x \in C(B)$ and $y \in C(A)$.

In words, HARP says that if choices x, y are both available in two choice experiments, and x is chosen in one experiment and y in the other, then both x and y must be chosen in both experiments. HARP guarantees that there is no (obvious) inconsistency in the agent's choices.

It has been argued that HARP is a necessary condition for a choice rule to be rationalizable. Turns out that HARP is also sufficient for rationalizability:

Proposition 1.2.3

Suppose $C : \mathcal{B} \to \mathcal{B}$ is nonempty-valued. Then there exists a rational (complete and transitive) preference relation $\succeq$ on X such that $C(\cdot) = C_{\succeq}(\cdot)$ if and only if C satisfies HARP.

Proof for Proposition.

- "Only if" part: As argued before, HARP is a necessary condition for rational relation's existence.

- "If" part

  - Definition of revealed preference relation (by choice rule).

    Suppose choice rule C satisfies HARP. Construct the "revealed preference relation" $\succeq_C$ as follows: say that $x \succeq_C y$ if and only if there exists some $A \subseteq X$ such that $y \in A$ and $x \in C(A)$.

  - Prove that $\succeq_C$ is complete.

    Since C is nonempty-valued, pick any $x, y \in X$, we have either $x \in C(\{x, y\})$, in which case $x \succeq_C y$, or $y \in C(\{x, y\})$, in which case $y \succeq_C x$ (or both, in which case $x \sim_C y$).

  - Prove that $\succeq_C$ is transitive.

    Suppose $x \succeq_C y$ and $y \succeq_C z$, and consider $C(\{x, y, z\})$, which by hypothesis is nonempty. There are three possibilities (which are though not mutually exclusive):

    * $x \in C(\{x, y, z\})$. Then by construction of $\succeq_C$, $x \succeq_C z$.
    * $y \in C(\{x, y, z\})$. Then by HARP, since $x \succeq_C y$, we must also have $x \in C(\{x, y, z\})$, and so by case 1 we have $x \succeq_C z$.
    * $z \in C(\{x, y, z\})$. Then by HARP, since $y \succeq_C z$, we also have $y \in C(\{x, y, z\})$, and so by case 2 $x \succeq_C z$.

  - Prove that $C(\cdot) = C_{\succeq_C}(\cdot)$.

    It is equivalent to show that, for all $x \in X$ and $A \in \mathcal{B}$, $x \in C(A)$ if and only if $x \in A$ and $x \succeq_C y$ for all $y \in A$.

    * "Only if" part holds by construction of $\succeq_C$.
    * "If" part: Take any $x \in C_{\succeq_C}(A)$. Then, $x \succeq_C y$ for all $y \in A$. Since $C(\cdot)$ is nonempty-valued, $\exists y_0 \in A$ such that $y_0 \in C(A)$. By the definition of $\succeq_C$, $\exists A_0$ such that $x, y_0 \in A_0$ and $x \in C(A_0)$. It follows that $x, y_0 \in A_0 \cap A$ and $x \in C(A_0)$ and $y_0 \in C(A)$. By HARP, $x \in C(A)$.

■

Remark.

1. It is *HARP* that plays a fundamental role in the proof of "if" part, that is, to endow the revealed preference relation $\succeq_C$ with characteristics of "rationality".

2. The properties of rational choice are guaranteed only when the entire choice function $C(A)$ is observed, i.e., (i) for any given choice set, all of the agent's optimal choices are observed, not just some of them; (ii) the agent's optimal choices over all choice sets are observed. However,

   - Real data is commonly less comprehensive like that. For example, in consumer choice problems, the relevant sets A may be only budget sets that consist of affordable choices given income and prices, which is a particular subcollection of $\mathcal{B}$.
   - The first aspect of incomplete observation is virtually insolvable. For the second, to develop a theory based on more limited observations, other "axioms of revealed preference" have been developed. For example, the *weak axiom of revealed preference (WARP)* is an equivalent of HARP for choice sets restricted to budget sets described by linear prices and for choice rules restricted to be single-valued (so the conclusion of HARP can be strengthened to $x = y$). WARP is necessary for rationalizability but proves insufficient. A stronger version, *Generalized Axiom of Revealed Preference*, has been shown both necessary and sufficient for rationalizability.