Advanced Econometrics

Professor: Julie Shi

Timekeeper: Rui Zhou

Fall 2023

Contents

6 Panel Data

6.1 Basic Idea for Panel Data

The basic framework of a regression model for panel data is

$$y_{it} = x_{it}'\beta + z_i'\alpha + \varepsilon_{it}$$

- Pooled regression: If z_i contains only a constant term;
- Fixed effects: If z_i is unobserved, but correlated with x_{it}, the model becomes $y_{it} = x_{it}'\beta + \alpha_i + \varepsilon_{it}$;
- Random effects: If z_i is unobserved, and uncorrelated with x_{it}, the model becomes $y_{it} = x_{it}'\beta + \alpha + u_i + \varepsilon_{it}$;
- Random parameters: Coefficients vary randomly across individuals, i.e., $y_{it} = x_{it}(\beta + h_i) + (\alpha + u_i) + \varepsilon_{it}$.

We begin the analysis by assuming the simplest version of the model, the pooled model.

$$\begin{aligned} y_{it} =& \alpha + x_{it}'\beta + \varepsilon_{it}, \quad \text{for } i = 1, \cdots, n; t = 1, \cdots, T_i \\ \text{with } & \begin{cases} \mathrm{E}\left[\varepsilon_{it} | x_{i1}, x_{i2}, \cdots, x_{iT_i}\right] = 0 \\ \mathrm{Var}\left[\varepsilon | X\right] = \sigma^2 \mathbf{I} \end{cases} \end{aligned}$$

Remarks: In the panel data context, this is also called the population averaged model under the assumption that any latent heterogeneity has been averaged out.

In this form, if the remaining assumptions of the classical model are met (zero conditional mean of ε_{it}, homoskedasticity, independence across observations, i, and strict exogeneityof x_{it}), then no further analysis beyond the chapter of classical linear model is needed. Ordinary least squares is the efficient estimator and inference can reliably proceed along the lines developed before.

The crux of the panel data analysis in this chapter is that the assumptions underlying ordinary least squares estimation of the pooled model are unlikely to be met. The question, then, is what can be expected of the estimator when the heterogeneity does differ across individuals?

The fixed effects case is obvious. As we will examine later, omitting (or ignoring) the heterogeneity when the fixed effects model is appropriate renders the least squares estimator inconsistent—sometimes wildly so. In the random effects case, in which the true model is $y_{it} = c_i + x_{it}'\beta + \varepsilon_{it}$, where $\mathrm{E}[c_i|X_i] = \alpha$, we can rewrite the model

$$\begin{aligned}
y_{it} &= \alpha + x_{it}'\beta + \varepsilon_{it} \\
&= \alpha + x_{it}'\beta + \varepsilon_{it} + (c_i - \mathrm{E}[c_i|X_i]) \\
&= \alpha + x_{it}'\beta + \varepsilon_{it} + u_i \\
&= \alpha + x_{it}'\beta + w_{it}
\end{aligned}$$

In this form, we can see that the unobserved heterogeneity induces autocorrelation, that is, $\mathrm{E}[w_{it}w_{is}] = \sigma_u^2 \neq 0$ when $t \neq s$.

6.1.1 Robust Covariance Matrix Estimation

Here we stack the T_i observations for individual i in a single equation:

$$y_i = X_i\beta + w_i$$

where β now includes the constant term.

In this setting, there may be heteroskedasticity across individuals. However, in a panel data set, the more substantive problem is crossobservation correlation, or autocorrelation. In a longitudinal data set, the group of observations may all pertain to the same individual, so any latent effects left out of the model will carry across all periods. Suppose, then, we assume that the disturbance vector consists of ε_{it} plus these omitted components. Then,

$$\mathrm{Var}[w_i|X_i] = \sigma_\varepsilon^2 \mathbf{I}_{T_i} + \Sigma_i = \Omega_i$$

The OLS estimator of β is

$$\begin{aligned}
b &= (X'X)^{-1}X'y \\
&= \left[\sum_{i=1}^n X_i'X_i\right]^{-1} \left[\sum_{i=1}^n X_i'y_i\right] \\
&= \left[\sum_{i=1}^n X_i'X_i\right]^{-1} \left[\sum_{i=1}^n X_i'\left(X_i\beta + w_i\right)\right] \\
&= \beta + \left[\sum_{i=1}^n X_i'X_i\right]^{-1} \sum_{i=1}^n X_i'w_i
\end{aligned}$$

Consistency can be established along the lines developed as before. The true asymptotic covariance matrix would take the form we saw for the generalized regression model:

$$\begin{aligned}
\text{Asy.Var}\,[b] &= \frac{1}{n}\text{plim}\ \left[\frac{1}{n}\sum_{i=1}^{n} X_i' X_i\right]^{-1}\ \text{plim}\ \left[\sum_{i=1}^{n} X_i' w_i w_i' X_i\right]\ \text{plim}\ \left[\frac{1}{n}\sum_{i=1}^{n} X_i' X_i\right]^{-1} \\
&= \frac{1}{n}\text{plim}\ \left[\frac{1}{n}\sum_{i=1}^{n} X_i' X_i\right]^{-1}\ \text{plim}\ \left[\sum_{i=1}^{n} X_i' \Omega_i X_i\right]\ \text{plim}\ \left[\frac{1}{n}\sum_{i=1}^{n} X_i' X_i\right]^{-1}
\end{aligned}$$

It is quite likely that the more important issue for appropriate estimation of the asymptotic covariance matrix is the correlation across observations, not heteroscedasticity. As such, it is quite likely that the White estimator is not the solution to the inference problem here.

6.1.2 Robust Estimation Using Group Means

The pooled regression model can be estimated using the sample means of the data. The implied regression model is obtained by premultiplying each group by $\left(\frac{1}{T}\right)\mathbf{1}'$ where $\mathbf{1}'$ is a row vector of 1s:

$$\begin{aligned}
\left(\frac{1}{T}\right)\mathbf{1}' y_i &= \left(\frac{1}{T}\right)\mathbf{1}' X_i\beta + \left(\frac{1}{T}\right)\mathbf{1}' w_i \\
\iff \qquad \bar{y}_i &= \bar{x}_i'\beta + \bar{w}_i
\end{aligned}$$

In the transformed linear regression, the disturbances continue to have zero conditional means but heteroskedastic variances $\sigma_i^2 = \frac{1}{T^2}\mathbf{1}'\Omega_i\mathbf{1}$. With Ω_i unspecified, this is a heteroskedastic regression for which we would use the White estimator for appropriate inference.

The first difference model is,

$$\begin{aligned}
\Delta y_{it} &= \Delta\alpha + (\Delta x_{it})'\beta + \Delta\varepsilon_{it} \\
&= \Delta\alpha + (\Delta x_{it})'\beta + \left(\varepsilon_{it} - \varepsilon_{i,t-1}\right) \\
&= (\Delta x_{it})'\beta + u_{it}
\end{aligned}$$

6.1.3 The Within- And Between-Groups Estimators

We can formulate the pooled regression model in three ways. First, the general regression is:

$$y_{it} = \alpha + x_{it}'\beta + \varepsilon_{it}$$

In terms of the group means

$$\bar{y}_i = \alpha + \bar{x}_i'\beta + \bar{\varepsilon}_i$$

While in terms of deviations from the group means:

$$(y_{it} - \bar{y}) = (x_{it} - \bar{x}_i)' \beta + (\varepsilon_{it} - \bar{\varepsilon}_i)$$

We are assuming there are no time-invariant variables.

$$S_{xx}^{Total} = \sum_{i=1}^{n} \sum_{t=1}^{T} (x_{it} - \bar{\bar{x}}) (x_{it} - \bar{\bar{x}})'$$
$$S_{xy}^{Total} = \sum_{i=1}^{n} \sum_{t=1}^{T} (x_{it} - \bar{\bar{x}}) (y_{it} - \bar{\bar{y}})'$$
$$S_{xx}^{Within} = \sum_{i=1}^{n} \sum_{t=1}^{T} (x_{it} - \bar{x}) (x_{it} - \bar{x})'$$
$$S_{xy}^{Within} = \sum_{i=1}^{n} \sum_{t=1}^{T} (x_{it} - \bar{x}) (y_{it} - \bar{y})'$$
$$S_{xx}^{Between} = \sum_{i=1}^{n} T (\bar{x}_i - \bar{\bar{x}}) (\bar{x}_i - \bar{\bar{x}})'$$
$$S_{xy}^{Between} = \sum_{i=1}^{n} T (\bar{x}_i - \bar{\bar{x}}) (\bar{y}_i - \bar{\bar{y}})'$$

It is easy to verify that

$$S_{xx}^{Total} = S_{xx}^{Within} + S_{xx}^{Between}$$
$$S_{xy}^{Total} = S_{xy}^{Within} + S_{xy}^{Between}$$

Therefore, there are three possible least squares estimators of β corresponding to the decomposition. By FWL theorem, we see that the least squares estimator is

$$b^{Total} = \left(S_{xx}^{Total}\right)^{-1} S_{xy}^{Total} = \left(S_{xx}^{Within} + S_{xx}^{Between}\right)^{-1} \left(S_{xy}^{Within} + S_{xy}^{Between}\right)$$

The within-groups estimator is

$$b^{Within} = \left(S_{xx}^{Within}\right)^{-1} S_{xy}^{Within}$$

An alternative estimator would be the between-groups estimator

$$b^{Between} = \left(S_{xx}^{Between}\right)^{-1} S_{xy}^{Between}$$

This is the group means estimator.

From the preceding expressions, we can rewrite as

$$\begin{cases} b^{Within} = \left(S_{xx}^{Within}\right)^{-1} S_{xy}^{Within} \\ b^{Between} = \left(S_{xx}^{Between}\right)^{-1} S_{xy}^{Between} \end{cases} \Longrightarrow \begin{cases} S_{xy}^{Within} = S_{xx}^{Within} b^{Within} \\ S_{xy}^{Between} = S_{xx}^{Between} b^{Between} \end{cases}$$

Inserting these into the expression of b^{Total}, we see that the least squares estimator is a matrix weighted average of the within- and between-groups estimators:

$$b^{Total} = F^{Within} b^{Within} + F^{Between} b^{Between}$$

where $F^{Within} = \left(S_{xx}^{Within} + S_{xx}^{Between}\right)^{-1} S_{xx}^{Within} = \mathbf{I} - F^{Between}$.

6.2 Fixed Effects Model

Recall that the general model is $y_{it} = x_{it}'\beta + c_i + \varepsilon_{it}$. The fixed effects model arises from the assumption that the omitted effects, c_i, are correlated with the included variables. In a general form, we assume $\mathrm{E}\left[c_i | X_i\right] = h\left(X_i\right)$.

The fixed effects model has the form

$$y_{it} = x_{it}'\beta + \alpha_i + \varepsilon_{it}$$

Remarks: "Fixed effects" does NOT mean any variable is "fixed" in this context and random elsewhere. It means differences across groups can be captured in differences in the constant term. α_i is allowed to be correlated with x_{it}, and each α_i is treated as an unknown parameter to be estimated. As discussed, the coefficients on the time-invariant variables cannot be estimated.

For each group, i.e., for individual i, we have

$$y_i = X_i\beta + \mathbf{1}\alpha_i + \varepsilon_i$$

where y_i and X_i are the T observations for the i-th unit, $\mathbf{1}$ is a $T \times 1$ ones, and ε_i is the associated $T \times 1$ vector of disturbances.

For all the groups,

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \beta + \begin{bmatrix} \mathbf{1} & 0 & \cdots & 0 \\ 0 & \mathbf{1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{1} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_n \end{bmatrix} + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{bmatrix}$$

The model can be written as

$$y = \begin{bmatrix} X & d_1 & d_2 & \cdots & d_n \end{bmatrix} \begin{bmatrix} \beta \\ \alpha \end{bmatrix} + \varepsilon$$

Let $D = \begin{bmatrix} d_1 & d_2 & \cdots & d_n \end{bmatrix}$, which is a $nT \times n$ matrix, the model becomes

$$y = X\beta + D\alpha + \varepsilon$$

which is called *Least Squares Dummy Variables (LSDV)* model.

6.2.1 Estimating LSDV Model

$$\begin{aligned}
D'D &= \begin{bmatrix} d_1' \\ d_2' \\ \vdots \\ d_n' \end{bmatrix} \begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix} \\
&= \begin{bmatrix} d_1'd_1 & d_1'd_2 & \cdots & d_1'd_n \\ d_2'd_1 & d_2'd_2 & \cdots & d_2'd_n \\ \vdots & \vdots & \ddots & \vdots \\ d_n'd_1 & d_n'd_2 & \cdots & d_n'd_n \end{bmatrix} \\
&= \begin{bmatrix} T & 0 & \cdots & 0 \\ 0 & T & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & T \end{bmatrix} \\
&= T\mathbf{I}_n \\
D(D'D)^{-1}D' &= \begin{bmatrix} b_1 & b_2 & \cdots & b_n \end{bmatrix} \left[T\mathbf{I}_n\right]^{-1} \begin{bmatrix} d_1' \\ d_2' \\ \vdots \\ d_n' \end{bmatrix} \\
&= \frac{1}{T} \begin{bmatrix} \mathbf{1} & 0 & \cdots & 0 \\ 0 & \mathbf{1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{1} \end{bmatrix} \begin{bmatrix} \mathbf{1}' & 0 & \cdots & 0 \\ 0 & \mathbf{1}' & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{1}' \end{bmatrix} \\
&= \frac{1}{T} \begin{bmatrix} \mathbf{1}\mathbf{1}' & 0 & \cdots & 0 \\ 0 & \mathbf{1}\mathbf{1}' & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \mathbf{1}\mathbf{1}' \end{bmatrix}
\end{aligned}$$

$$M_0 := \mathbf{I} - \frac{1}{T}\mathbf{1}\mathbf{1}'$$

$$M_0 z_i = \left(\mathbf{I} - \frac{1}{T}\mathbf{1}\mathbf{1}'\right) z_i = z_i - \mathbf{1}\left(\frac{1}{T}\left(\mathbf{1}' z_i\right)\right) = z_i - \mathbf{1}\left(\frac{1}{T}\sum_{i=1}^{T} z_i\right) = z_i - \bar{z}\mathbf{1}$$

$$M_D = \mathbf{I} - D(D'D)^{-1}D' = \begin{bmatrix} M_0 & 0 & \cdots & 0 \\ 0 & M_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & M_0 \end{bmatrix}$$

$$M_D X = \begin{bmatrix} M_0 & 0 & \cdots & 0 \\ 0 & M_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & M_0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} M_0 x_1 \\ M_0 x_2 \\ \vdots \\ M_0 x_n \end{bmatrix} = \begin{bmatrix} x_1 - \bar{x}_1\mathbf{1} \\ x_2 - \bar{x}_2\mathbf{1} \\ \vdots \\ x_n - \bar{x}_n\mathbf{1} \end{bmatrix}$$

$$M_D y = \begin{bmatrix} M_0 & 0 & \cdots & 0 \\ 0 & M_0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & M_0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} M_0 y_1 \\ M_0 y_2 \\ \vdots \\ M_0 y_n \end{bmatrix} = \begin{bmatrix} y_1 - \bar{y}_1\mathbf{1} \\ y_2 - \bar{y}_2\mathbf{1} \\ \vdots \\ y_n - \bar{y}_n\mathbf{1} \end{bmatrix}$$

$$b = \left[X' M_D X\right]^{-1}\left[X' M_D Y\right] = b^{Within}$$

$$\begin{aligned} y &= Xb + Da + e \\ D'y &= D'Xb + D'Da + D'e \\ \iff \left(D'D\right)a &= D'\left(y - Xb\right) \\ \alpha &= (D'D)^{-1}D\left(y - Xb\right) \end{aligned}$$

$$\begin{aligned} (D'D)^{-1}D'y &= \left(T\mathbf{I}_n\right)^{-1} \begin{bmatrix} d_1' \\ d_2' \\ \vdots \\ d_n' \end{bmatrix} y \\ &= \left(T\mathbf{I}_n\right)^{-1} \begin{bmatrix} \sum_{t=1}^{T} y_{1t} \\ \sum_{t=1}^{T} y_{2t} \\ \vdots \\ \sum_{t=1}^{T} y_{nt} \end{bmatrix} \\ &= \left(T\mathbf{I}_n\right)^{-1} \begin{bmatrix} T\bar{y}_1 \\ T\bar{y}_2 \\ \vdots \\ T\bar{y}_n \end{bmatrix} \\ &= \begin{bmatrix} \bar{y}_1 \\ \bar{y}_2 \\ \vdots \\ \bar{y}_n \end{bmatrix} \end{aligned}$$

Similarly, it follows that

$$
\begin{aligned}
(D'D)^{-1}DXb &= (T\mathbf{I}_n)^{-1}\begin{bmatrix} d_1' \\ d_2' \\ \vdots \\ d_n' \end{bmatrix} Xb \\
&= (T\mathbf{I}_n)^{-1}\begin{bmatrix} \sum_{t=1}^{T} x_{1t}b \\ \sum_{t=1}^{T} x_{2t}b \\ \vdots \\ \sum_{t=1}^{T} x_{nt}b \end{bmatrix} \\
&= (T\mathbf{I}_n)^{-1}\begin{bmatrix} T\bar{x}_1 b \\ T\bar{x}_2 b \\ \vdots \\ T\bar{x}_n b \end{bmatrix} \\
&= \begin{bmatrix} \bar{x}_1 b \\ \bar{x}_2 b \\ \vdots \\ \bar{x}_n b \end{bmatrix}
\end{aligned}
$$

Jointly, so we have

$$\alpha_i = \bar{y}_i - \bar{x}_i' b$$

Covariance matrix for b is

$$
\begin{aligned}
\text{Est.Asy.Var}\,[b] &= s^2\left[X'M_D X\right]^{-1} = s^2\left[S_{XX}^{Within}\right]^{-1} \\
\text{where } s^2 &= \frac{\sum_{i=1}^{n}\sum_{t=1}^{T}\left(y_{it} - x_{it}'\beta - \alpha_i\right)}{nT-n-k}
\end{aligned}
$$

6.2.2 Testing the Significance of Group Effects

Hypothesis

$$
\begin{aligned}
H_0 &: \text{All } \alpha_i \text{ are equal.} \\
H_1 &: \text{NOT all } \alpha_i \text{ are equal.}
\end{aligned}
$$

Statistic:

$$F\left[n-1, nT-n-K\right] = \frac{\left(R_{LSDV}^2 - R_{Pooled}^2\right)/\left(n-1\right)}{\left(1-R_{LSDV}^2\right)/\left(nT-n-K\right)}$$

$\ddot{X}$ can be treated as an instrument of X:

- Exogeneity: We have assumed that plim $\left(\frac{1}{nT}\right)X'\varepsilon = \mathbf{0}$, then we must have plim $\left(\frac{1}{nT}\right)X'M_D\varepsilon =$ plim $\left(\frac{1}{nT}\right)X'\left(M_D\varepsilon\right) = \mathbf{0}$. That is, if X is uncorrelated with ε, it will be uncorrelated with ε in deviations from its group means.

- Relevance:

6.3 Random Effects Model

The random effects model is

$$y_{it} = \alpha + x_{it}'\beta + u_t + \varepsilon_{it}$$

with assumptions

$$\begin{cases} \mathrm{E}\left[\varepsilon_{it}|X\right] = \mathrm{E}\left[u_i|X\right] = 0 \\ \mathrm{E}\left[\varepsilon_{it}^2|X\right] = \sigma_\varepsilon^2, \mathrm{E}\left[u_i^2|X\right] = \sigma_u^2 \\ \mathrm{E}\left[\varepsilon_{it}\varepsilon_{js}|X\right] = 0 \text{ for } t \neq s \text{ or } i \neq j \\ \mathrm{E}\left[u_i u_j\right] = 0 \text{ for } i \neq j \\ \mathrm{E}\left[\varepsilon_{it} u_j\right] = 0 \text{ for all } i, t, j \end{cases}$$

For each individual (or viewed as a group) i,

$$\begin{aligned} \Sigma &= \begin{bmatrix} \sigma_\varepsilon^2 + \sigma_u^2 & \sigma_u^2 & \cdots & \sigma_u^2 \\ \sigma_u^2 & \sigma_\varepsilon^2 + \sigma_u^2 & \cdots & \sigma_u^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_u^2 & \sigma_u^2 & \cdots & \sigma_\varepsilon^2 + \sigma_u^2 \end{bmatrix} \\ &= \sigma_\varepsilon^2 \mathbf{I}_T + \sigma_u^2 \mathbf{1}_T \mathbf{1}_T' \end{aligned}$$

and for the nT observations,

$$\Omega = \begin{bmatrix} \Sigma & 0 & \cdots & 0 \\ 0 & \Sigma & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix} = \mathbf{I}_n \otimes \Sigma$$

Four models

$$\begin{cases} y_{it} = \alpha + x_{it}'\beta + u_i + \varepsilon_{it} \\ y_{it} - \bar{y}_i = \left(x_{it} - \bar{x}_i\right)'\beta + \varepsilon_{it} - \bar{\varepsilon}_i \\ y_{it} - y_{i,t-1} = \left(x_{it} - x_{i,t-1}\right)'\beta + \varepsilon_{it} - \varepsilon_{i,t-1} \\ \bar{y}_i = \alpha + \bar{x}_i'\beta + u_i + \bar{\varepsilon}_i \end{cases} \overset{\text{Est.Var}}{\Longrightarrow} \begin{cases} \sigma_\varepsilon^2 + \sigma_u^2 \\ \sigma_{\varepsilon^2}\left(1 - \frac{1}{T}\right) \\ 2\sigma_\varepsilon^2 \\ \frac{1}{T}\sigma_\varepsilon^2 + \sigma_u^2 \end{cases}$$

$$\begin{aligned} \Omega^{-1/2} &= \left[\mathbf{I}_n \otimes \Sigma\right]^{-1/2} = \mathbf{I}_n \otimes \Sigma^{-1/2} \\ \text{with } \Sigma^{-1/2} &= \frac{1}{\sigma_\varepsilon}\left[\mathbf{I}_T - \frac{\theta_i}{T}\mathbf{1}_T\mathbf{1}_T'\right] \\ \theta &= 1 - \frac{\sigma_\varepsilon}{\sqrt{\sigma_\varepsilon^2 + T\sigma_u^2}} = 1 - \frac{1}{\sqrt{1 + T\left(\frac{\sigma_u}{\sigma_\varepsilon}\right)^2}} \end{aligned}$$

The transformation of y_i and X_i for GLS is

$$\Sigma^{-1/2} y_i = \frac{1}{\sigma_\varepsilon} \begin{bmatrix} y_{i1} - \theta \bar{y}_i \\ y_{i2} - \theta \bar{y}_i \\ \vdots \\ y_{in} - \theta \bar{y}_i \end{bmatrix}$$

The GLS estimator is

$$b^{Total} = \hat{F}$$

$$\hat{\sigma}_\varepsilon^2 = s_{LSDV}^2 = \frac{\sum_{i=1}^n \sum_{t=1}^T (e_{it} - \bar{e}_i)^2}{nT - n - K}$$

Lagrange multiplier test:

$$H_0 : \sigma_u^2 = 0$$
$$H_1 : \sigma_u^2 \neq 0$$

The test statistic is

$$LM = \frac{nT}{2(T-1)} \cdot \left[\frac{\sum_{i=1}^n \left[\sum_{t=1}^T e_{it}^2 \right]}{\sum_{i=1}^n \sum_{t=1}^T e_{it}^2} \right]^2 \sim \chi$$

6.4 Hausman's Specification Test

Hausman's test is used to decide which model, fixed or random effects,

The model is

$$y_{it} = \alpha + x_{it}' \beta + u_i + \varepsilon_{it}$$
$$H_0 : x_{it} \text{ is uncorrelated with } u_i$$
$$H_1 : x_{it} \text{ is correlated with } u_i$$