Public Economics

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3 Taxation

Typically, there are two types of sales taxes:

- Ad-valorem tax: in percentage terms
- Unit tax: in dollar terms

  where ad-valorem tax are more common, but we are going to go through the machanics of unit tax, because it is easier.

3.1 Tax Incidence

Tax incidence: Analysis of how taxes affect the prices paid for goods; Critical for knowing who actually bears the burden of the tax.

Distinction: **statutory** incidence v.s. **economic** incidence

- Statutory (or legislative): who by law has to remit the tax to govt
- Economic: how does the tax affect prices, analysing how tax is shared among producers and consumers.

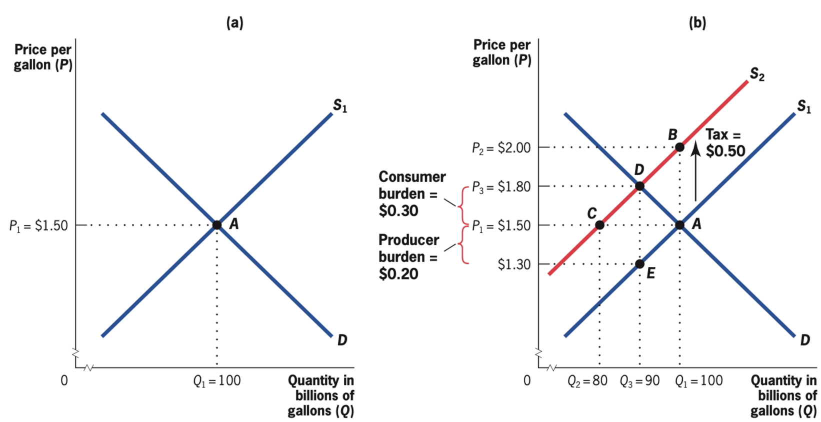
Key insight: we need to keep track of two prices, seller price and consumer price:

$$p_B - t = p_S$$

where p_B is the price buyers receive, and p_S is the price sellers receive.

3.1.1 Tax on Sellers

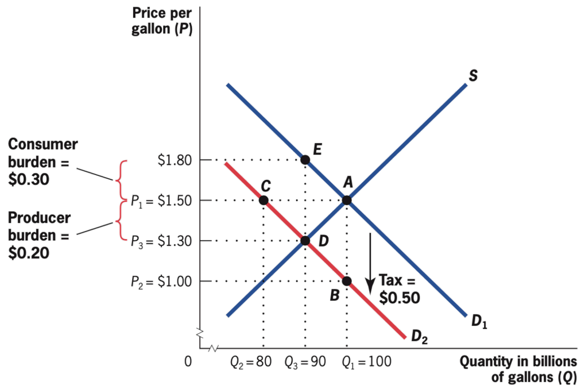
If tax is levied directly on sellers, the supply curve itself has not changed. But we have to track the S curve to track the two prices (p_B, p_S). In this case, p_B is the price consumers paid, and correpsondingly, the sellers are going to get $p_B - t$ as p_S. On graph, the tax will result the S curve to move up by the amount of tax per unit.



In equilibrium, the quantity declines; the price paid by buyers p_B increases, and the price received by the seller $p_S = p_B - t$ decreases. Here, the incidence of the tax is shared by both sellers and buyers, and both parties are "worse off", if we just limit our analysis on price paid.

3.1.2 Tax on Buyers

If tax is levied on buyers, similarly, the demand curve itself has not changed, but hte D curve will shift due to the inconsistency of p_B and p_S. Suppose sellers receive a price p_S, considering the tax, the price paid by the consumer is $p_S + t$ as p_B. On graph, the tax will result the D curve to move down by the amount of tax per unit.



The equilibrium analysis is as before, the equilibrium quantity falls, with lower seller price and higher buyer price. The incidence of tax is also shared by both sellers and buyers, and they are made "worse off" on prices received.

Interestingly, the two results with tax either levied on buyers or sellers will reach the same result: same equilibrium quantity, same incidence of tax. We can safely conclude that statutory incidence does not tell us about economic incidence; but under either scenario, the incidence analysis is identical. Moreover, in both cases, the exact incidence of tax is shared among sellers and buyers according to their relative elasticity. The party with less elasticity will suffer more.

3.1.3 Math of Taxation

Assume unit tax of t on suppliers. Note that it does not matter statutory incidence is attributed to which party. Then, the consumer pay p_B and producers receive $p_S = p_B - t$.

In equilibrium, the intersection is set by $S(p_S)$ and $D(p_B)$, which satisfies

$$S(p_B - t) = D(p_B)$$

Notice that p_B is a function of t, since the tax will change the equilibrium price, so the equilibrium condition can be further specified as

$$D\left(p_B(t)\right)) = S\left((p_B(t) - t\right))$$

Take derivative with regard to t,

$$\begin{aligned}
&\frac{\mathrm{d}D}{\mathrm{d}p}\frac{\mathrm{d}p_B}{\mathrm{d}t} = \frac{\mathrm{d}S}{\mathrm{d}p}\frac{\mathrm{d}(p_B(t) - t)}{\mathrm{d}t} = \frac{\mathrm{d}S}{\mathrm{d}p}\left(\frac{\mathrm{d}p_B}{\mathrm{d}p} - 1\right) \\
\Longleftrightarrow\ &\frac{\mathrm{d}p_B}{\mathrm{d}t} = \frac{\frac{\mathrm{d}S}{\mathrm{d}p}}{\frac{\mathrm{d}S}{\mathrm{d}p} - \frac{\mathrm{d}D}{\mathrm{d}p}} \\
\Longleftrightarrow\ &\frac{\mathrm{d}p_B}{\mathrm{d}t} = \frac{\frac{\mathrm{d}S}{\mathrm{d}p}\frac{P}{Q}}{\frac{\mathrm{d}S}{\mathrm{d}p}\frac{P}{Q} - \frac{\mathrm{d}D}{\mathrm{d}p}\frac{P}{Q}} = \frac{\varepsilon_S}{\varepsilon_S - \varepsilon_D}
\end{aligned}$$

Note that $\varepsilon = \frac{\frac{dQ}{Q}}{\frac{dP}{P}} = \frac{dQ}{dP}\frac{P}{Q}$, and $\varepsilon_S > 0, \varepsilon_D < 0$.

Similarly, we can write the equlibrium condition as

$$D(p_S(t) + t) = S(p_S(t))$$

Take derivative with regard to t, and get

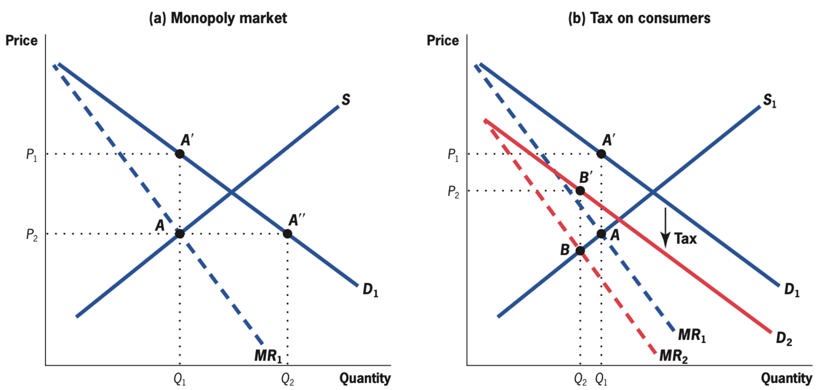
$$\frac{dp_S}{dt} = \frac{\varepsilon_D}{\varepsilon_S - \varepsilon_D}$$

Therefore, tax incidence depends on relative elasticities.

- The relatively smaller the demand elasticity [in absolute value] (i.e., more inelastic) the larger the change in p_B, the smaller the change in p_S.
  - $\varepsilon_D = 0$, i.e., inelastic demand
  - $\varepsilon_S = +\infty$, i.e., perfectly elastic supply
- The smaller the supply elasticity (i.e., more inelastic) the smaller the change in p_B, the greater the change in p_S.
  - $\varepsilon_S = 0$, i.e., inelastic supply
  - $\varepsilon_D = -\infty$, i.e., perfectly elastic supply

In sum, statutory incidence is not equal to economic incidence, and equilibrium is independent of who nominally pays the tax. The bottomline for economic incidence is that, larger burden of tax goes to less elastic side of the market.

We have discussed the taxation in perfectly competitive market, which is the simplest case to analyse. In extension, the tax incidence in the monopioly market would be more complicated. In monopoly market, the optimum output level for the producer is determined via the intersection of MC and MR curves, instead of simply the S and D curves.



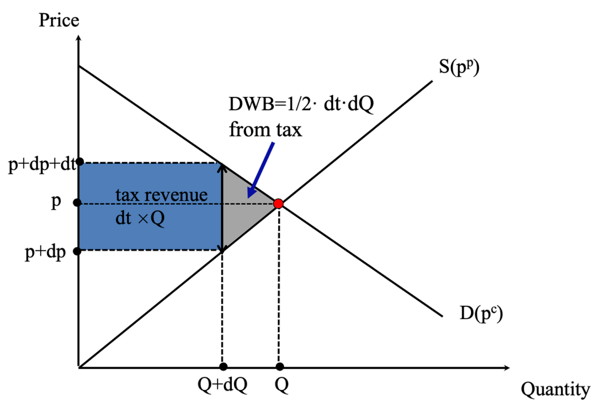
3.2 Efficiency Costs of Taxation

The inefficiency of any tax is determined by the extent to which consumers and producers change their behavior to avoid the tax. Or on the other hand, the inefficiency is caused because taxation will distort the free market mechanism and some trades that could have possibly deal are now ”prohibited” due to taxation. Such inefficiency, or distortion, will cause deadweight loss. But note that if there is no change in quantities consumed, the tax has no efficiency costs, which means no such trades are "limited".

The taxes are levied on sellers and buyers, and then collected by the government as revenue for socially welfare-improving activities. Thus, from the perspective of society, the definition of surplus should be modified as

$$\begin{aligned}\text{Total Surplus} &= \text{Consumer Surplus} + \text{Producer Surplus} + \text{Government Revenue}\\ &= \text{Consumer Surplus} + \text{Producer Surplus} + \text{Tax}\end{aligned}$$

Deadweight loss, or excess burden of taxation, is the welfare loss created by tax over and above the tax revenue generated by the tax. Note that welfare consists of consumer surplus and producer surplus.



Consider a small tax dt starting from tax-free equilibrium, the DWL of such tax dt is measured by the Harberger Triangle.

$$DWL = \frac{1}{2}\mathrm{d}Q\mathrm{d}t$$

where $\mathrm{d}Q < 0, DWL < 0$.

In equilibrium, $Q = S(p_S)$, and hence $\mathrm{d}Q = \frac{\mathrm{d}S}{\mathrm{d}p_S}\mathrm{d}p_S$.

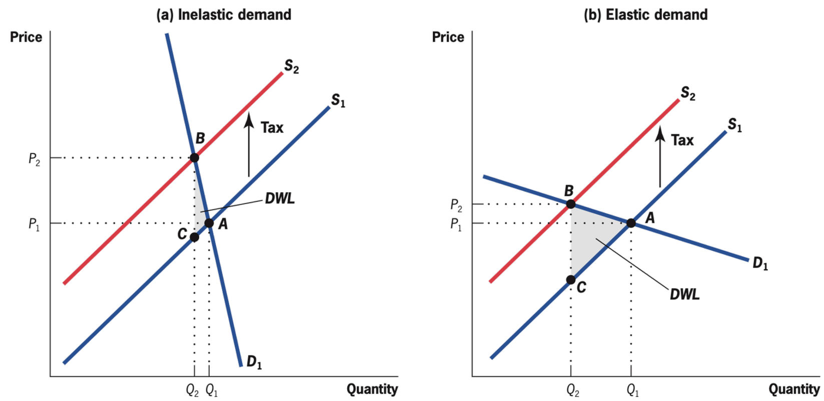
$$DWL = \frac{1}{2}\mathrm{d}Q\mathrm{d}t = \frac{1}{2}\frac{\mathrm{d}S}{\mathrm{d}p_S}\mathrm{d}p_S\mathrm{d}t = \frac{1}{2}\frac{\mathrm{d}S}{\mathrm{d}p_S}\frac{p_S}{S}\frac{Q}{P}\frac{\mathrm{d}p_S}{\mathrm{d}t}(\mathrm{d}t)^2$$

where $\varepsilon_S = \frac{\mathrm{d}S}{\mathrm{d}p_S}\frac{p_S}{S}$, and $\frac{\mathrm{d}p_S}{\mathrm{d}t} = \frac{\varepsilon_D}{\varepsilon_S - \varepsilon_D}$. Therefore

$$DWL = \frac{1}{2}\mathrm{d}Q\mathrm{d}t = \frac{1}{2}\frac{Q}{P}\frac{\varepsilon_S\varepsilon_D}{\varepsilon_S - \varepsilon_D}(\mathrm{d}t)^2 = \frac{1}{2}\frac{Q}{P}\frac{1}{\frac{1}{\varepsilon_D} - \frac{1}{\varepsilon_S}}(\mathrm{d}t)^2$$

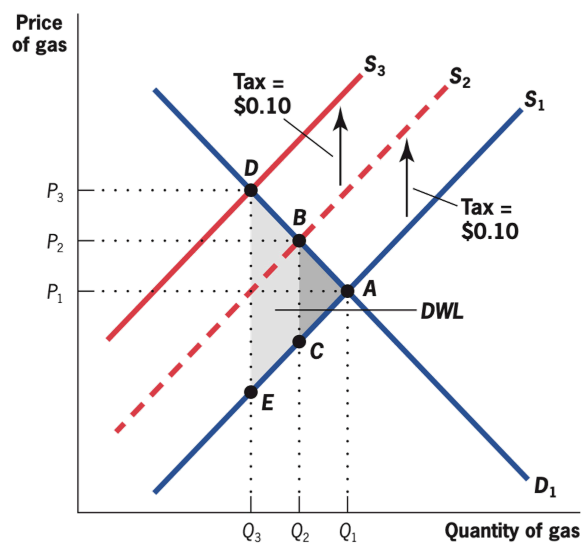
DWL's expression enlightens us that

- DWL increases with the absolute size of elasticities. ($\varepsilon_S > 0, \varepsilon_D < 0$)
  - More efficient to tax relatively inelastic goods.

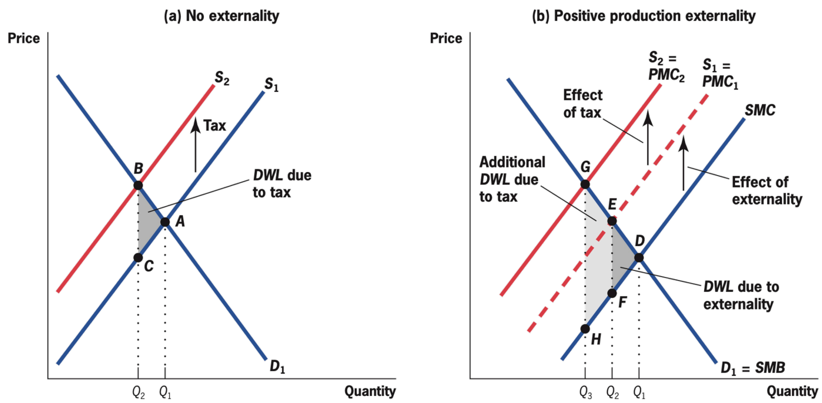


- DWB increases with the square of the tax rate.
  - Small taxes have relatively small efficiency costs; large taxes have relatively large efficiency costs.
  - Better to spread taxes across all goods to keep each tax rate low.
  - Better to fund large one time govt expense (such as a war) with debt and repay slowly afterwards than have very high taxes only during war.

Note that our induction of DWL starts from tax-free equilibrium. Pre-existing distortions (e.g. an existing tax) makes the cost of taxation higher. On graph, the effect of tax (DWL) moves from the triangle to trapezoid. Marginal DWL rises with tax rate.



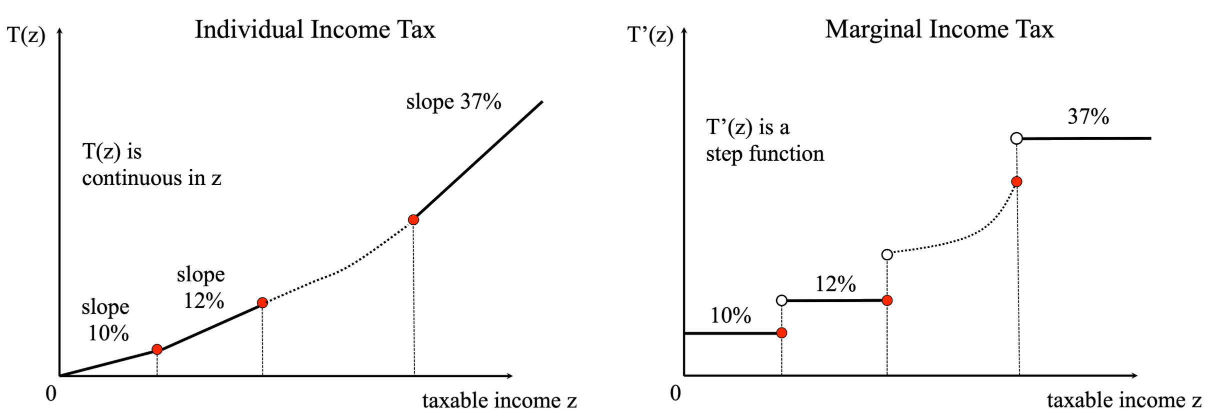
Similarly, the externality is a form of latent tax. Sometimes, the effect of externalities and taxes may overlap and create an even greater effect, like the case of marginal DWL with increasing tax rate.



3.3 Taxation and Behavior

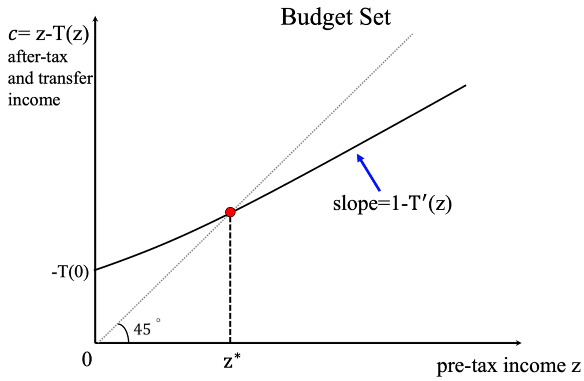
Generally, the government use taxes and transfers to reduce inequity. Specifically, taxes are collected by the government to raise revenue, and then this revenue funds transfer programs. Moreover, the transfers can be categorized into two kinds.

- Universal Transfers
  - Public education, health care benefits, retirement and disability benefits, unemployment benefits.
- Means-tested Transfers
  - In-kind (public housing, food stamps in the US) & in-cash benefits (minimum income).



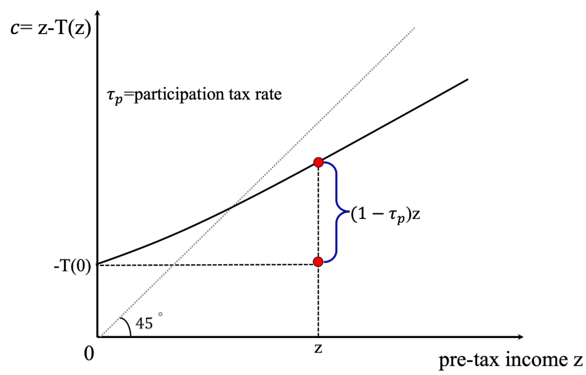
Individual income tax is the amount of tax the person is levied. The tax rates in different ranges may differ, taking derivatives for individual income tax and get the marginal income tax (i.e., tax rates for each income range).

Suppose an individual has the pre-tax income z, and $T(z)$ denotes net tax for her, which integrates taxes and transfers. For individuals with zero earnings, the after-tax income is $0-T(0)$. The break-even earnings point z^* is set at $T(z^*) = 0$, where tax equals to transfer.



When it comes to tax rate, distinguish from two concepts.

- Marginal tax rate $T'(z)$: individual keeps $1 - T'(z)$ for an additional \$1 of earnings from z.
  - This describes "intensive margin".
- Participation tax rate $\tau_p(z) = \frac{T(z)-T(0)}{z}$: individual keeps fraction $1-\tau_p$ of earnings when moving form not working (and then has zero earnings) to working (and then has earnings of z).
  - This describes "extensive margin".



Suppose in an economy, there are N individuals with fixed income $z_1 < z_2 < ... < z_N$. where income z is fixed for each individual, exogenous to $T(\cdot)$. $T(z)$ as before, is the net tax on income z, which means tax if $T(\cdot) > 0$, transfer if $T(\cdot) < 0$. After considering the net tax, the after-tax income c is therefore $c = z - T(z)$. The individual's utility $u(\cdot)$ is built upon her after-tax income c, which is strictly increasing and concave, and same for everyone in the economy. The government in the economy hopes to maximize Utilitarian objective, the social welfare function:

$$SWF = \sum_{i=1}^{N} u(c_i) = \sum_{i=1}^{N} u(z_i - T(z_i))$$

subject to government's budget constraint

$$\sum_{i=1}^{N} T(z_i) = 0$$

i.e, taxes need to fund transfers.

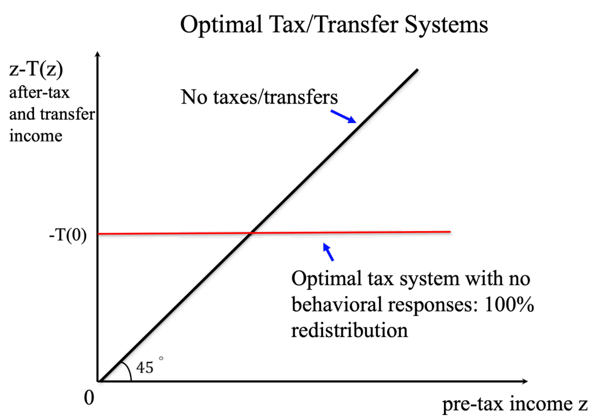
Start from focusing on individual $i = 1$. Replace $T(z_1) = -\sum_{i=2}^{N} T(z_i)$ from government's budget constraint, the social welfare function is like

$$SWF = u(c_1) + \sum_{i=2}^{N} u(c_2) = u\left(z_i + \sum_{i=2}^{N} T(z_i)\right) + \sum_{i=2}^{N} u(z_i - T(z_i))$$

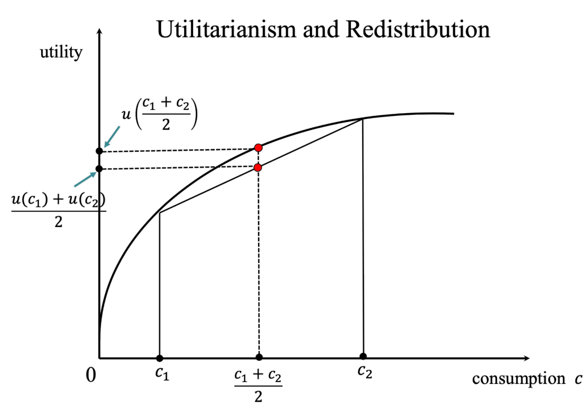
F.O.C. in $T(z_j)$ for a given $j = 2, ..., N$:

$$\frac{\partial SWF}{\partial T(z_j)} = u'\left(z_i + \sum_{i=2}^{N} T(z_i)\right) - u'(z_j - T(z_j)) = 0$$
$$\Longleftrightarrow \frac{\partial SWF}{\partial T(z_j)} = u'(z_1 - T(z_1)) - u'(z_j - T(z_j)) = 0$$
$$\Longrightarrow u'(z_1 - T(z_1)) = u'(z_j - T(z_j)), \forall j = 2, ..., N$$

Therefore, $z_j - T(z_j)$ is constant for all $j = 1, 2, ..., N$, since $u(\cdot)$ is strictly increasing and concave. This result of Utilitarian object indicates a perfect equalization of after-tax income; in other words, 100% tax rate and redistribution. Utilitatianism with decreasing marginal utility leads to perfect egalitarianism.



This egalitarianism result seems surprising at first glance, however, it is also straightforward from a two-person economy. All that mechanism goes to the strictly increasing and concave form of utility function.



Issues with the simple model:

1. No behavioral responses:
    - Obvious missing piece: 100% redistribution would destroy incentives to work and thus the assumption that is exogenous is unrealistic.
    - Optimal income tax theory incorporates behavioral responses.
2. Issue with Utilitarianism:
    - Even absent behavioral responses, many people would object to 100% redistribution.
    - Citizens' views on fairness impose bounds on redistribution government can do.

3.3.1 Behavior Response

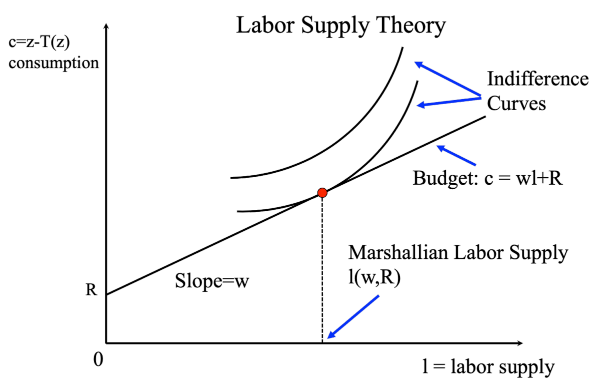
After considering the behavior responses to the taxes or transfers, the "appropriate" amount of taxes or transfers is a matter of equity and efficiency tradeoff.

If the society feels that inequality is so severe that taxes should be used to raise revenue for transfer programs to reduce inequality in disposable income, then taxes are more desirable. However, taxes and transfers would form a distortion to people's incentives to work. High tax rates will probably create economic inefficiency, because the poor can ease with the free-meal transfer, and the rich would step back from work due to shrinked profits. Thus, size of behavioral response limits the ability of government to redistribute with taxes and transfers, which generates an equity-efficiency tradeoff.

Labor Supply Theory Suppose individual has utility $u(c, l)$ over labor supply l (in hour) and consumption c, and her wage is $\bar{w}$ per hour. The utility function behaves well that $u(c, l)$ is increasing in c and decreasing in l (equivalent to increasing in leisure). For convenience, denote $w = (1-\tau)\bar{w}$ as the after-tax wage rate. Additionally, the individual has a non-labor income of R. Her objective for maximizing her utility is

$$\max_{c,l} u(c, l) \text{s.t. } c = wl + R \Longrightarrow \text{F.O.C } [l]: \quad \frac{\partial u}{\partial c} w + \frac{\partial u}{\partial l} = 0$$

where the F.O.C. of l defines the Marshallian labor supply $l = l(w, R)$.



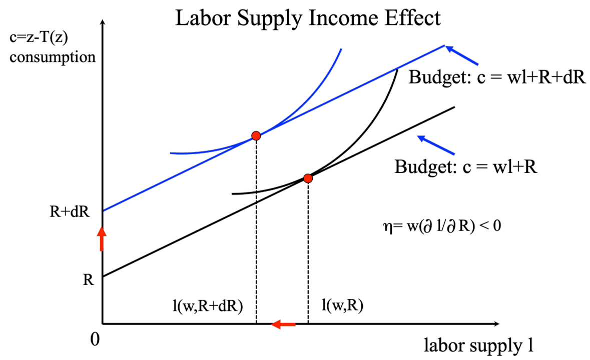
Uncompensated labor supply elasticity (with regard to after-tax wage rate w) is defined as

$$\varepsilon^u = \frac{\partial l}{\partial w} \frac{w}{l}$$

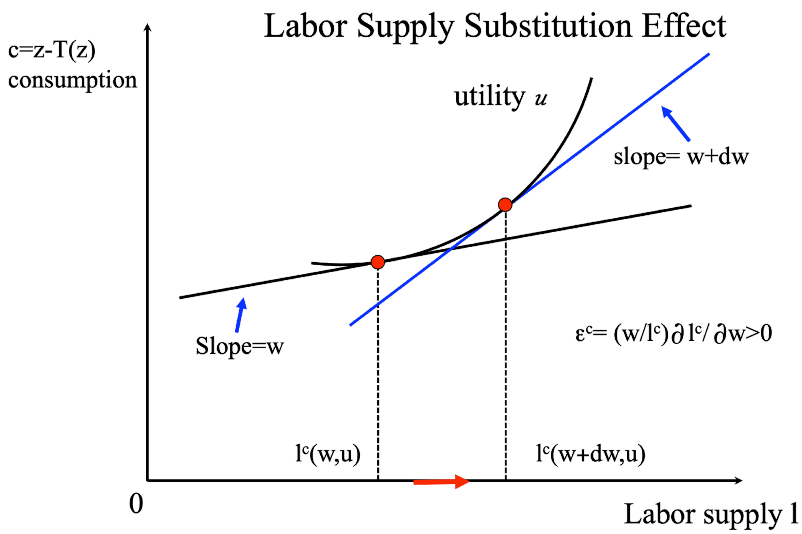
Income effect is defined as

$$\eta = \frac{\partial l}{\partial R} w$$

where leisure is assumed to be a normal good, and $\eta < 0$.



For substitution effects, consider Hicksian labor supply: $l^c(w, u)$ minimizes cost dR needed to reach u given slope w.



In other words, keeping the same utility level, find the tangent budget constraint for the indifference curve.

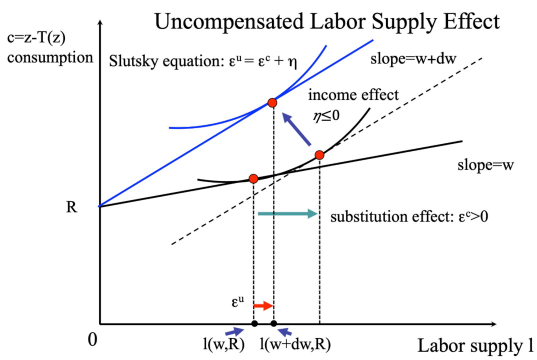
Compensated labor supply elasticity is defined as

$$\varepsilon^c = \frac{\partial l^c}{\partial w} \frac{w}{l}$$

where $\varepsilon^c > 0$.

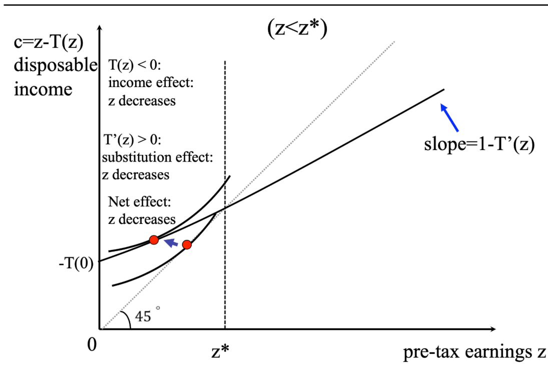
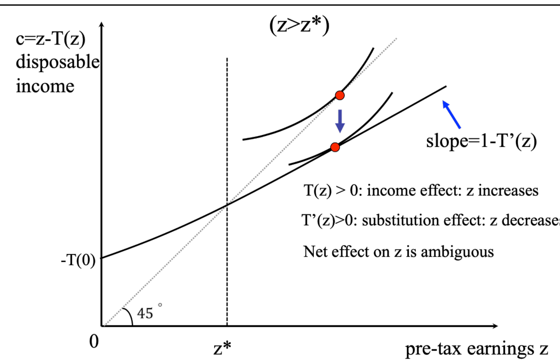
Combine the income and substitution effect together, Slusky decomposition on the effect of w on labor supply is as follows,

$$\frac{\partial l}{\partial w} = \frac{\partial l^c}{\partial w} + \frac{\partial l}{\partial R} l \iff \varepsilon^u = \varepsilon^c + \eta$$



Application of Labor Supply Theory to Tax Starting from no tax or transfer at all, i.e., $T(\cdot) = 0$.

- Income effects
  - Tax $T(z) > 0$ reduces disposable income and thus increases labor supply;
  - Transfer $T(z) < 0$ increases disposable income and decreases labor supply.
- Substitution effects
  - Marginal tax $T'(z) > 0$ reduces net wage rate and reduces labor supply.

$z < z^*$	$z > z^*$
	

- When $z < z^*$,
  - $T(z) < 0$: income effect causes z to decrease;
  - $T'(z) > 0$: substitution effect causes z to decrease either.
  - Net effect: z is sure to decrease.
- When $z > z^*$,
  - $T(z) > 0$: income effect causes z to increase;
  - $T'(z) > 0$: substitution effect causes z to decrease.
  - Net effect on z is ambiguous.

Interestingly, from the labor supply theory, the relatively poor who receive transfer from the government loaf from work.

Tax Revenue Suppose individual has disposable income $c = (1-\tau)z + R$, where z is her pre-tax income with τ as the linear tax rate, and R is the fixed universal transfer, fully funded by taxes, i.e., $R = \tau\bar{z}$. Based on all that, individual i chooses l_i to maximize $u_i((1-\tau)w_i l_i + R, l_i)$, where labor supply choices l_i will then determine individual earnings $z_i = w_i l_i$. The average earnings $Z = \frac{\sum_i z_i}{N}$ will respond to the net-of-tax rate $1-\tau$. Therefore, the average earnings Z can be written as $Z = Z(1-\tau)$.

Tax revenue TR collected from each individual is

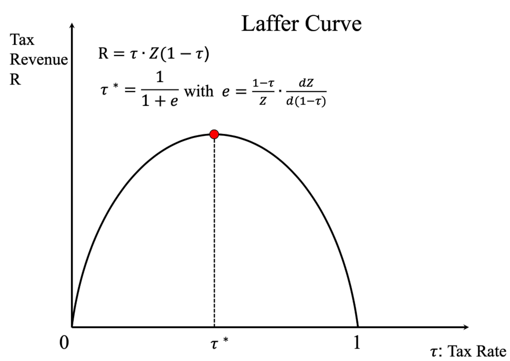
$$TR = \tau \cdot Z(1-\tau)$$

Laffer rate τ^* is set to maximize tax revenue TR, the F.O.C. is

$$TR'(\tau) = Z - \tau \cdot \frac{\mathrm{d}Z}{\mathrm{d}(1-\tau)} = 0 \iff \frac{\tau}{1-\tau}\left(\frac{1-\tau}{Z}\frac{\mathrm{d}Z}{\mathrm{d}(1-\tau)}\right) = 1$$

Define $e = \frac{1-\tau}{Z}\frac{\mathrm{d}Z}{\mathrm{d}(1-\tau)}$ with practical meanings as the elasticity of average income Z with respect to the net-of-tax rate $1-\tau$. Then

$$\tau^* = \frac{1}{1+e}$$



Tax Revenue per person $TR = \tau Z$ is hump-shaped (inverse U-shape) with τ, which is the so-called Laffer Curve. Laffer Curve enlightens us that, it is inefficient to have $\tau > \tau^*$, because decreasing τ would make taxpayers better off, with increasing tax revenue for the government. All in all, the Laffer Curve depicts the tradeoff between tax base (determined by labor supply) and tax rate (proportion of the base).

Considering behavioral responses, the government again chooses τ to maximize utilitarian social welfare

$$SWF = \sum_{i=1}^{N} u_i\left((1-\tau)w_i l_i + \tau \cdot Z(1-\tau), l_i\right)$$

The difference lies in the utility function here has taken labor supply l_i into account, i.e., behavioral responses to tax rate. And the response will affect the tax revenue per person $\tau \cdot Z(1-\tau)$ that is redistributed back as transfer to everyone.

Government's F.O.C. with respect to τ is

$$\frac{\mathrm{d}SWF}{\mathrm{d}\tau} = \sum_{i=1}^{N} \frac{\partial u_i}{\partial c} \cdot \left(-z_i + Z - \tau \frac{\mathrm{d}Z}{\mathrm{d}(1-\tau)} \right) = 0$$

where the envelope theorem is used since l_i has already maximized u_i. (deduction of Laffer Curve)

Hence, we have the following optimal linear income tax formula

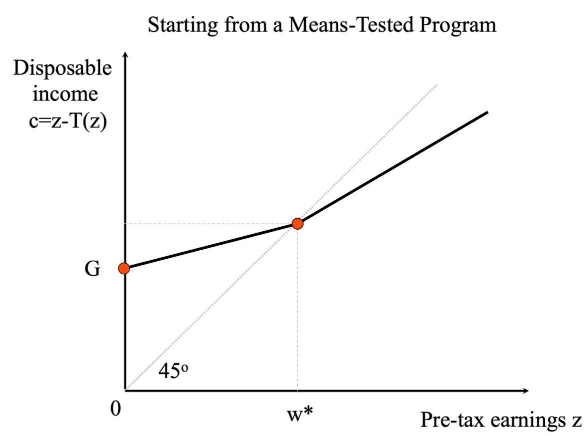
$$\tau = \frac{1-\bar{g}}{1-\bar{g}+e}, \text{with } \bar{g} = \frac{\sum_i z_i \frac{\partial u_i}{\partial c}}{Z \sum_i \frac{\partial u_i}{\partial c}}, e = \frac{1-\tau}{Z} \frac{\mathrm{d}Z}{\mathrm{d}(1-\tau)}$$

where $\bar{g}$ is kind of a measure of equity, and e, the elasticity, is a symbol of efficiency.

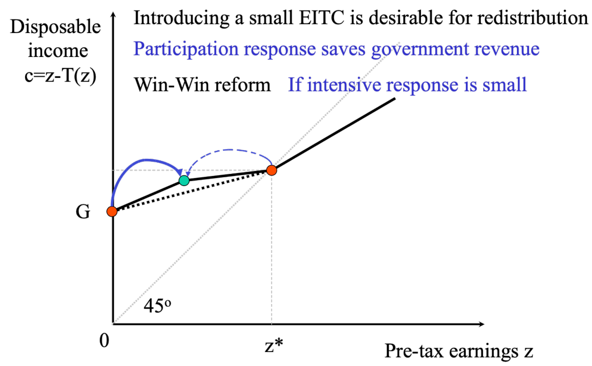
τ decreases with elasticity e and $\bar{g}$. This relationship further describes the equity-efficiency tradeoff. When $\bar{g}$ is low, τ should increase for redistribution, but that may hurt efficiency.

τ is close to Laffer rate $\tau^* = \frac{1}{1+e}$ when $\bar{g}$ is low, which means inequality is high, and marginal utility decreases fast with income.

3.3.2 Behavioral Responses & Labor Participation



Starting from a means-tested program, where the marginal tax rate before the break-even point is lower, and higher after break-even. If \$1 to low-paid workers values more than \$1 distributed to all, then introducint a small EITC (Earned Income Tax Credit, a tax break) is desirable for redistribution. Moreover, it is a win-win reform, since the participation response of the low-income workers will further in return save the government revenue. However, those medium-income workers may jump back and quit working due to the increased intensive margin. Therefore, the win-win scenario is reached when intensive response is small.


Introducing a small EITC is desirable for redistribution
Participation response saves government revenue
Win-Win reform
If intensive response is small
Disposable
income
c=z-T(z)
G
45°
0
z*
Pre-tax earnings z