Public Economics

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Spring 2023

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1.1 Background Knowledge

First Fundamental Theorem of Welfare Economics

- The market equilibrium will be Pareto optimal (i.e., reach a Pareto efficient allocation of goods) if,
  - Perfect competition. (All producers and consumers are price takers.)
  - Each sector has complete information.
  - There is no externalities.
- Implications
  - There is little role for government in economy.
  - Nevertheless, efficiency may not be the only goal for society, and we may also have a desire for some sort of equitable distribution of goods.

Different starting points lead to different Pareto optima.

Second Fundamental Theorem of Welfare Economics

- Any Pareto optimum can be achieved in a competitive market equilibrium for some initial set of endowments.
- Implications:
  - Altering the initial endowments of resources will change the final Pareto optimum.
  - The government plays its role by redistributing initial wealth.
  - After redistribution, market will do the rest, no further government interventions needed.
  - In sum, efficiency and equity issues can be addressed separately.

A framework for deciding a trade-off the society should make is needed.

Define a Social Welfare Function (SWF), which behaves as a utility function for society. SWF, combined with the ”Utility Possibility Curve” directs us toward the social optimum. (However, mind that the Utility Possibility Curve may not be strictly concave. Therefore, not all ”tangent” solutions are maximized ones.)

Government Intervention

Aside from distributional concerns, other reasons, called market failures, rationales government interventions.

- Market Power
  - Monopoly
    - If monopoly, the first Welfare Economics Theorem doesn't hold.
- Nonexistence of Markets
  - Asymmetric Information
  - Externality
  - Public Good

1.2 Public Goods

1.2.1 Definition

Pure public goods are perfectly **non-rival in consumption** and **nonexcludable** goods.

- Non-Rival in Consumption
  - One individual's consumption of a good doesn't affect another's opportunity to consume the good.
- Non-Excludable
  - Individuals cannot deny each other the opportunity to consume a good.
- Impure Public Goods
  - Goods that satisfy the two public-goods conditions just to some extent, not fully.

		Is the good rival in consumption?	
		Yes	No
Is the good excludable?	Yes	Private good (ice cream)	Impure public good (Cable TV)
	No	Impure public good (crowded sidewalk)	Public good (defense)

1.2.2 Optimal Provision for Private Goods

Suppose there are two goods, ice-cream (denoted as ic) with price P_{ic} and cookies (denoted as c) with price P_c, and P_c is normalized to one (i.e., numeraire good). Two individuals, Ben and Jerry, will demand different quantities of the good at the same competitive market price. Let $MRS_{ic.c} = \frac{MU_{ic}}{MU_c}$ denote how many cookies the consumer is willing to give up for 1 ice-cream.

The optimality condition for the consumption of private goods is written as

$$MRS^B_{ic,c} = \frac{P_{ic}}{P_c} = P_{ic} MRS^J_{ic,c} = \frac{P_{ic}}{P_c} = P_{ic}$$

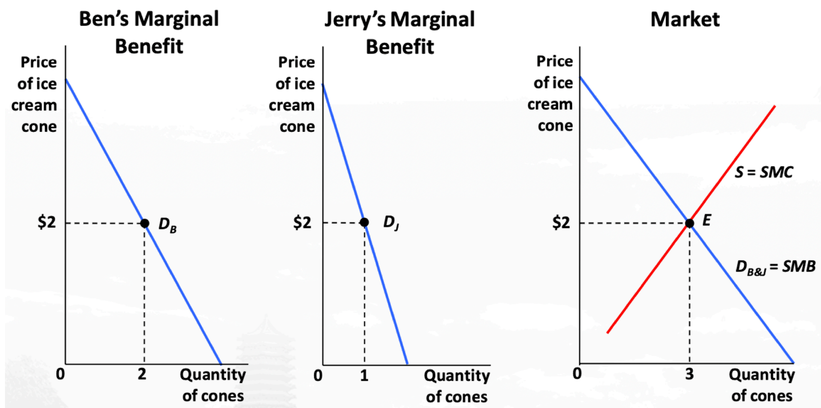
In general equilibrium, the supply side requires

$$MC_{ic} = P_{ic}$$

In equilibrium, it holds that

$$MRS^B_{ic,c} = MRS^J_{ic,c} = MC_{ic}$$

For each individual, she would find the optimization for herself; and from the perspective of the society, the equilibrium should be presented using horizontal summation.



To find social demand curve, add quantity at each price, which is equivalent to sum horizontally.

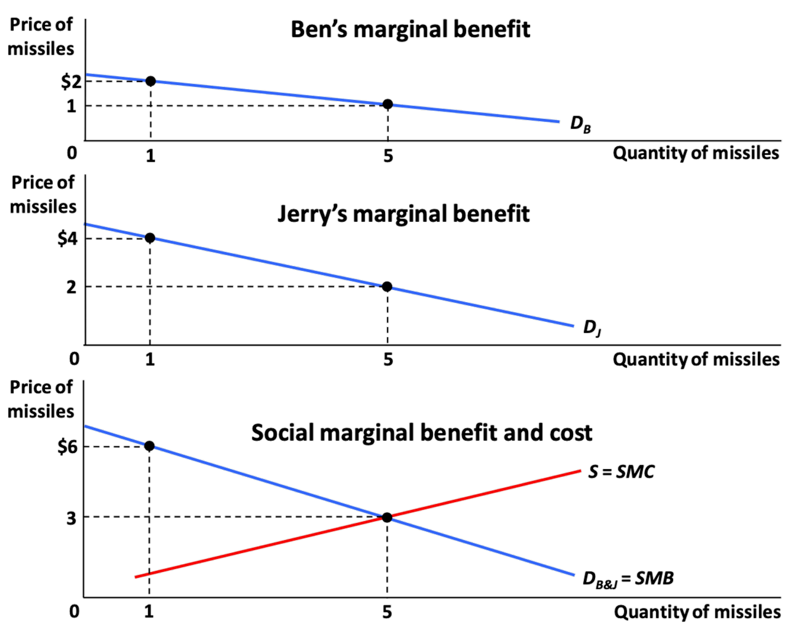
1.2.3 Optimal Provision for Public Goods

This time, we consider the tradeoff between cookies (denoted as c), the private good, and missiles (denoted as m), the public good. Let $MRS_{m,c}$ Denote how many cookies an individual is willing to give up for 1 missile.

In net, the society is willing to give up $(MRS^B_{m,c} + MRS^J_{m,c})$ cookies for 1 missle. Social-efficiency-maximizing condition for the public good is

$$MRS^B_{m,c} + MRS^J_{m,c} = MC_m$$

which means social efficiency is maximized when the marginal cost is set equal to the sum of the MRSs, rather than being set equal to each individual's MRS. (which is called the Samuelson rule)



Private sector provision implies $MRS^i_{m,c} = MC_m$ for each individual i, while the provision for public goods requires $\sum_i MRS^i_{m,c} = MC_m$ for optimization. Apparently, under private sector provision,

$$\sum_i MRS^i_{m,c} > MC_m$$

which implies that the outcome is not efficient, and could be improved in welfare by having more public goods and less private goods.

Free rider Problem describes the situation that, when an investment has a personal cost but a common benefit, selfish individuals will underinvest. Private sector provision induces an inefficient outcome in terms of public goods. Because of the free rider problem, the private market under-supplies public goods. (Private provision of a public good creates a positive externality. However, that part of positive externality isn't traded in the market.)

The free-rider problem is partially remedied by compulsory finance.

- Taxation
- Payment for public survices

The free-rider problem does not lead to a complete absence of private provision of public goods. Private provision works better when:

- Some individuals care more than others.
  - Private provision is particularly likely to surmount the free-rider problem when individuals are not identical, and when some individuals have an especially high demand for the public good.
- Altruism
  - When inidviduals value the benefits and costs to others in making their consumption choices.
- Warm glow
  - The emotional reward of giving to others; so individuals may care about their particular contributions to the public goods.

1.3 Lindahl Tax

Lindahl tax is a decentralized mechanism to achieve Pareto efficiency. Suppose each individual i with income Y hopes to maximize $U_i(X_i, G)$, and both private and public goods have price of 1. And suppose individual i has to pay a personalized share t_i (Lindahl tax) of the public good and can pick her favorite level of the public goods, G_i. The G_i should be set to maximize her utility, $U_i(Y_i - t_i G_i, G_i)$. Apparently, the F.O.C. is that, $MRS_i = t_i$. From the social perspective, $\sum_i t_i = 1$ must hold so that the public good is fully financed.

In equilibrium, all individuals must demand the same quantity of public goods. And luckily, such equilibrium generically exists, since n equations will determine n unknowns (t_i, for $i = 1, 2, ..., n$). Efficiency can be achieved since $\sum_i MRS_i = \sum_i t_i = 1 = MC$.

Limitations of Lindahl equilibrium are, individual preferences must be known to set personalized prices, however, people will nt truthfully reveal their preferences. And there would exist a difference between Lindahl equilibria and standard equlibria, since no market forces that will generate the right price vector.

Lindahl tax is itself a decentralized mechanism to achieve Pareto efficiency, where the government only designs the rule of Lindahl taxes t_i, and the market do the rest to determine the level of G. Each individual i only has to maximize her own utility, wihout caring about others. Lindahl tax internalizes the social welfare $t_i = 1 - \sum_{-i} t_{-i} = 1 - \sum_{-i} MRS_{-i}$ into individual's decision making. It turns out that the chosen G is socially optimal. Yet, there is possibly an alternative way to think about Lindahl tax, where the *government* chooses G, the set of tax $\{t_i\}_{i \in N}$ and try to

$$\max_{\{t_i\}_{i\in N}, G, \lambda} \sum_{i=1}^{N} U_i(Y_i - t_i G, G) + \lambda(\sum_{i=1}^{N} t_i - 1)$$

With $n+2$ equations and $n+2$ unknowns, the maximization problem has its unique solution. This logic will reach the same result as the decentralized solution, but more of a centralized allocation.

1.4 VCG Mechanism

VCG mechanism is designed to make it each agent's dominant strategy to reveal her preference truthfully.

Assume that there are n individuals, each with $U_i(X_i, G) = X_i + V_i(G)$, where individual's valuation of G, $V(\cdot)$ is privately known, but it is crucial for optimal provision of G. All prices are set to 1 for simplicity.

VCG mechanism works as follows:

- Each individual reports $v_i(\cdot)$ to the social planner.
- Based on all individuals' reports of valuation function $v(\cdot)$, the social planner sets $\hat{G} = \arg\max_G \sum_i (v_i(G) - G)$ and $\hat{G}_{-i} = \arg\max_G (\sum\limits_{j \neq i} v_j(G) - G)$.
- Each individual is required to pay $t_i(v) = (\sum_{j\neq i} v_j(\hat{G}_{-i}) - \hat{G}_{-i}) - (\sum_i v_i(\hat{G}) - \hat{G})$.
  - Intuitively, each agent pays for her "harm" to the rest of the society.
  - Generally speaking, the more $v_i(\cdot)$ deviates from $V_i(\cdot)$. the more $\hat{G}$ deviates from $\hat{G}_{-i}$, making $t_i(v)$ larger.

Under VCG mechanism, the dominant strategy for each individual is to truthfully report her valuation function of the public good, i.e., $v_i(\cdot) = V_i(\cdot)$. (Formal prove is skipped.) Thus, VCG mechanism will achieve Pareto optimal level of G, since it is satisfied that $\sum_i MRS_i = MC_G = 1$.

Assume that all other individuals report truthfully, then for individual i, her payoff would be

$$U_i(X_i, G) = V_i(G) - Y_i - [(\sum_{j\neq i} v_j(\hat{G}_{-i}) - \hat{G}_{-i}) - (\sum_i v_i(\hat{G}) - \hat{G})]$$

where individual i can indirectly choose $\hat{G}$, simply by reporting a particular $v_i(\cdot)$. Thus, we can turn the table and convert the maximization problem of individual i's choosing G.

$$\max_G \; U_i(X_i, G) = V_i(G) - Y_i - [(\sum_{j\neq i} v_j(\hat{G}_{-i}) - \hat{G}_{-i}) - (\sum_i v_i(G) - G)]$$

$$\text{F.O.C.}: \sum_i V_i'(G) = 1 \Longrightarrow \sum_i MRS_i = 1$$

Therefore, i will choose the socially optimal G, which implies she will report truthfully with $v_i(\cdot) = V_i(\cdot)$.