Bivariate summaries

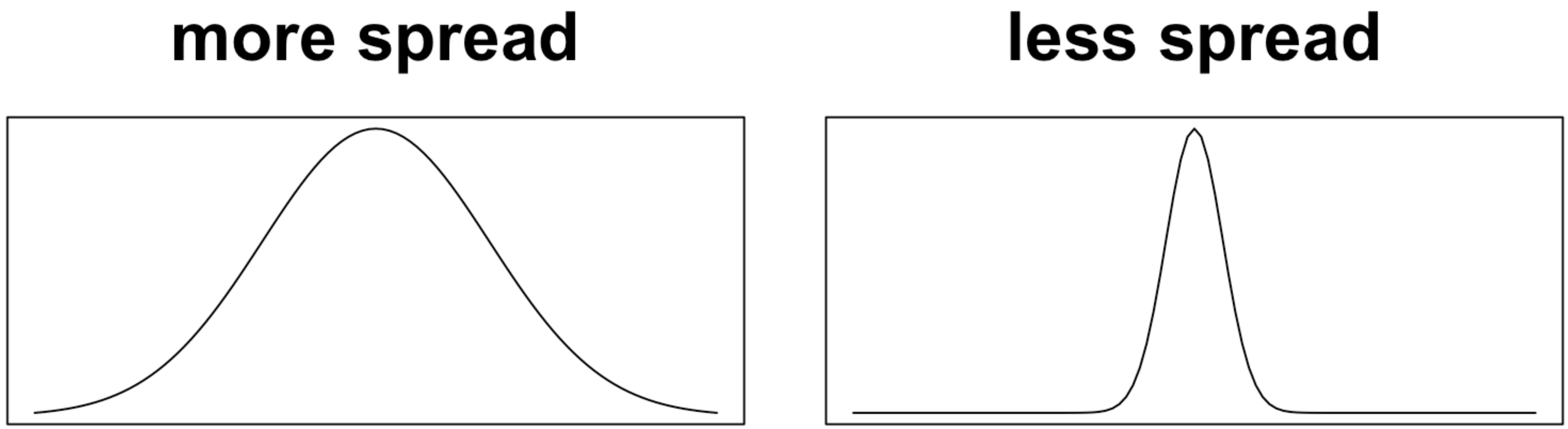
Quantitative and graphical techniques for summarizing two variables

Today’s agenda

1. HW discussion
2. Reading quiz [2pm section] [4pm section]
3. (Univariate) Measures of spread
4. Graphical summmaries for two variables
    - numeric/numeric
    - numeric/categorical
    - categorial/categorical
5. Lab: bivariate graphics in R

Measures of spread

The *spread* of observations refers to how concentrated or diffuse the values are.



Two ways to understand and measure spread:

- *ranges* of values capturing much of the distribution
- *deviations* of values from a central value

Range-based measures

A simple way to understand and measure spread is based on ranges. Consider more ages, sorted and ranked:

age	16	18	19	20	21	22	25	26	28	29	30	34
rank	1	2	3	4	5	6	7	8	9	10	11	12

- The **range** is the difference [maximum value] - [minimum value]

$$\text{range} = 34 - 16 = 18$$

- The **interquartile range** (IQR) is the difference [75th percentile] - [25th percentile]

$$\text{IQR} = 29 - 19 = 10$$

When might you prefer IQR to range? Can you think of an example?

Deviation-based measures

Another way is based on *deviations* from a central value. Continuing the example, the mean age is is 24. The deviations of each observation from the mean are:

age	16	18	19	20	21	22	25	26	28	29	30	34
deviation	-8	-6	-5	-4	-3	-2	1	2	4	5	6	10

The **average deviation** is defined as the average of the absolute values of the deviations from the mean:

$$\frac{8+6+5+4+3+2+1+2+4+5+6}{12}$$

The **standard deviation** is defined in terms of the squared deviations from the mean:

$$\sqrt{\frac{(-8)^2+(-6)^2+(-5)^2+(-4)^2+(-3)^2+(-2)^2+(1)^2+(2)^2+(4)^2+(5)^2+(6)^2}{12-1}}$$

Mathematical notations

Denote n observations of a variable x by $x_1, \dots, x_n$, so that x_i indicates the value of the ith observation. Our ages:

16, *18*, *19*, *20*, *21*, *22*, *25*, *26*, *28*, *29*, *30* and *34*

Applying the notation at right:

i	1	2	3	4	5	6	7	8	9	10	11	12
x_i												
$x_i - \bar{x}$												
$(x_i - \bar{x})^2$												

How would you write the formula for calculating average deviation using this notation?

The **mean** of the observations is written:

$$\bar{x} = \frac{1}{n} \sum_i x_i$$

The **standard deviation** is:

$$s_x = \sqrt{\frac{1}{n-1} \sum_i (x_i - \bar{x})^2}$$

Interpretations

Listed from largest to smallest, here are each of the measures of spread for the 12 ages:

range	iqr	st.dev	avg.dev
18	8.5	5.527	4.667

The interpretations differ between these statistics:

- [range] all of the data lies on an interval of 18 years
- [IQR] the middle half of the data lies on an interval of 8.5 years
- [average deviation] the average distance from the mean is 4.67 years
- [standard deviation] the average squared distance from the mean, rescaled to years, is 5.53 years

Robustness

The IQR is more robust than any of the other measures, because outliers only affect extreme percentiles.

Consider adding an observation of 94 to our 12 ages:

```
# initial range
range(ages)
```

```
[1] 16 34
```

```
# append an outlier
ages_add <- c(ages, 94)

# relative change in IQR
(IQR(ages_add) - IQR(ages))/IQR(ages)
```

```
[1] 0.05882353
```

```
# relative change in SD
(sd(ages_add) - sd(ages))/sd(ages)
```

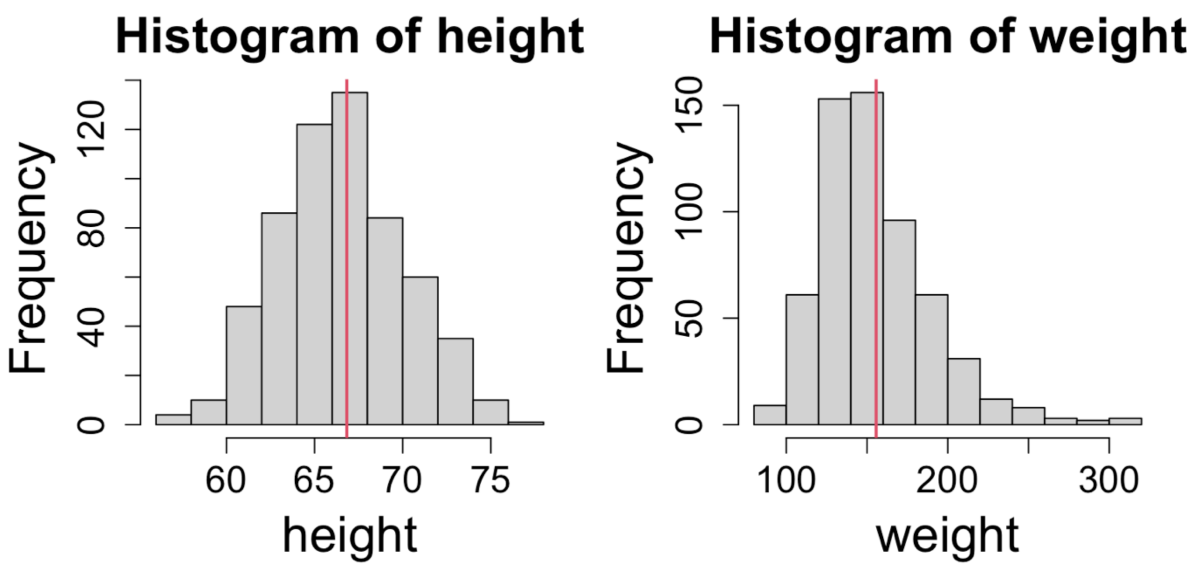
```
[1] 2.640935
```

The effect of the outlier on each measure is captured by the ratio $\frac{\text{measure with outlier}}{\text{measure without outlier}}$, which shows:

- the IQR increases by 5.88%
- the standard deviation increases by 264%

Limitations of univariate summaries

So far we have discussed **univariate** descriptive techniques — those that pertain to one variable at a time. Consider, for example, the height and weight of participants in the FAMuSS study:



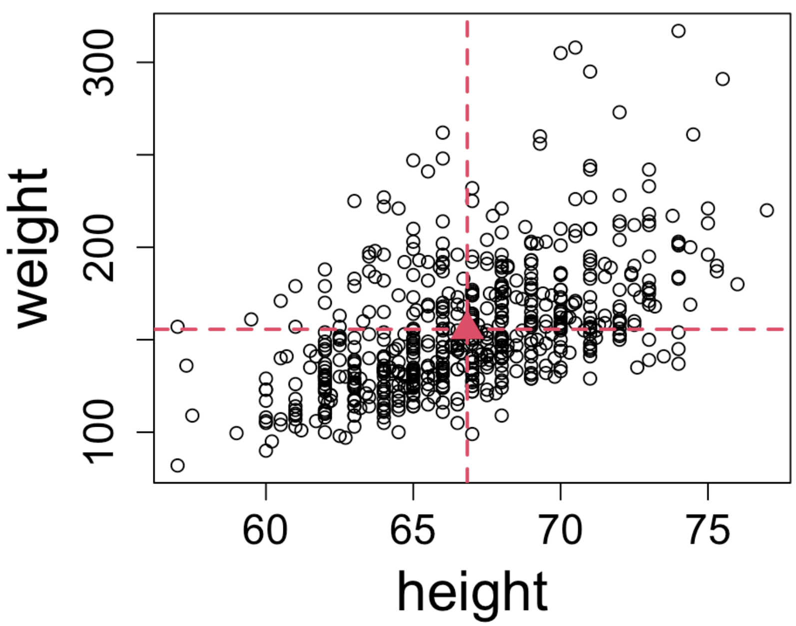
- both unimodal, no obvious outliers
- heights symmetric
- weights right-skewed
- but these observations actually come in *pairs*

The summaries we know how to make don't reflect how the variables might be related.

Bivariate summaries

Bivariate (and by extension multivariate) summaries are graphical or numerical descriptions that represent two (or more) variables *jointly*.

A simple example is a scatterplot:



Each point represents a pair of values (h, w) for one study participant.

- Reveals a relationship: taller participants tend to be heavier
- But no longer shows individual distributions clearly

Notice, though, that the marginal means (dashed red lines) still capture the center well.

Summary types

Bivariate summary techniques differ depending on the data types of the variables being compared. Some examples the context of the FAMuSS study:

Question	Comparison
Did genotype frequencies differ by race or sex among study participants?	categorical/categorical
Were differential changes in arm strength observed according to genotype?	numeric/categorical
Did change in arm strength appear related in any way to body size among study participants?	numeric/numeric
Did study participants experience similar or different changes in arm strength depending on arm dominance?	*??*

Categorical/categorical

A **contingency table** is a bivariate tabular summary of two categorical variables; it shows the frequency of each pair of values. Usually the marginal totals are also shown.

	CC	CT	TT	total
Female	106	149	98	353
Male	67	112	63	242
total	173	261	161	595

There are multiple ways to convert to proportions by using different denominators:

- grand total
- row total
- column total

Each has a different interpretation and should be chosen according to the question of interest.

Categorical/categorical

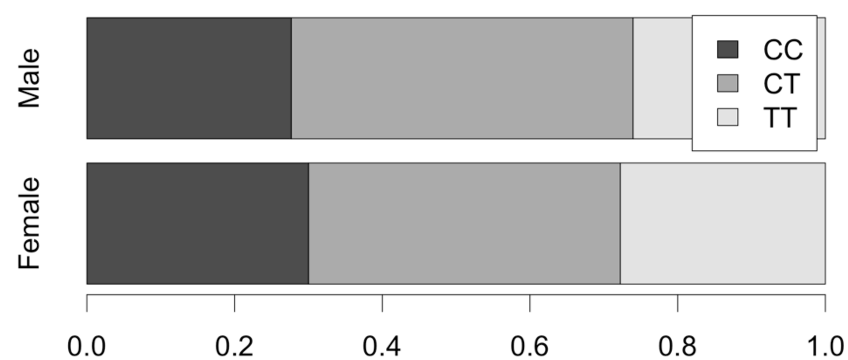
Did genotype frequencies differ by sex among study participants?

For this question, the **row totals** should be used to see the genotype composition of each sex.

As a table:

	CC	CT	TT
Female	0.3003	0.4221	0.2776
Male	0.2769	0.4628	0.2603

As a graphic:



The proportions are quite close, suggesting minimal sex differences.

Categorical/categorical

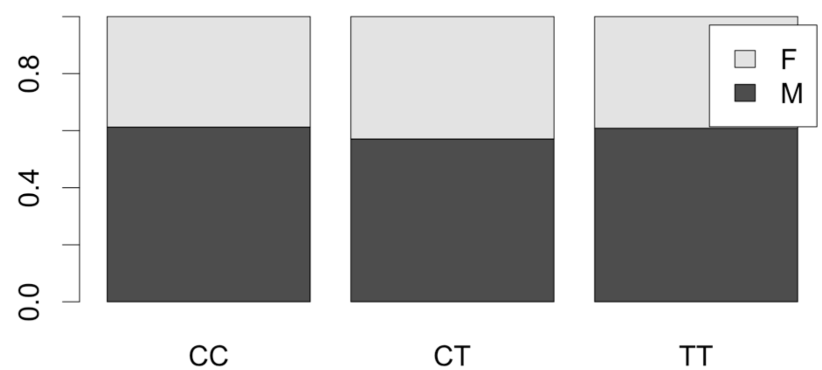
Did sex frequencies differ by genotype among study participants?

For this question, the **column totals** should be used to see the sex composition of each genotype.

As a table:

	CC	CT	TT
Female	0.6127	0.5709	0.6087
Male	0.3873	0.4291	0.3913

As a graphic:

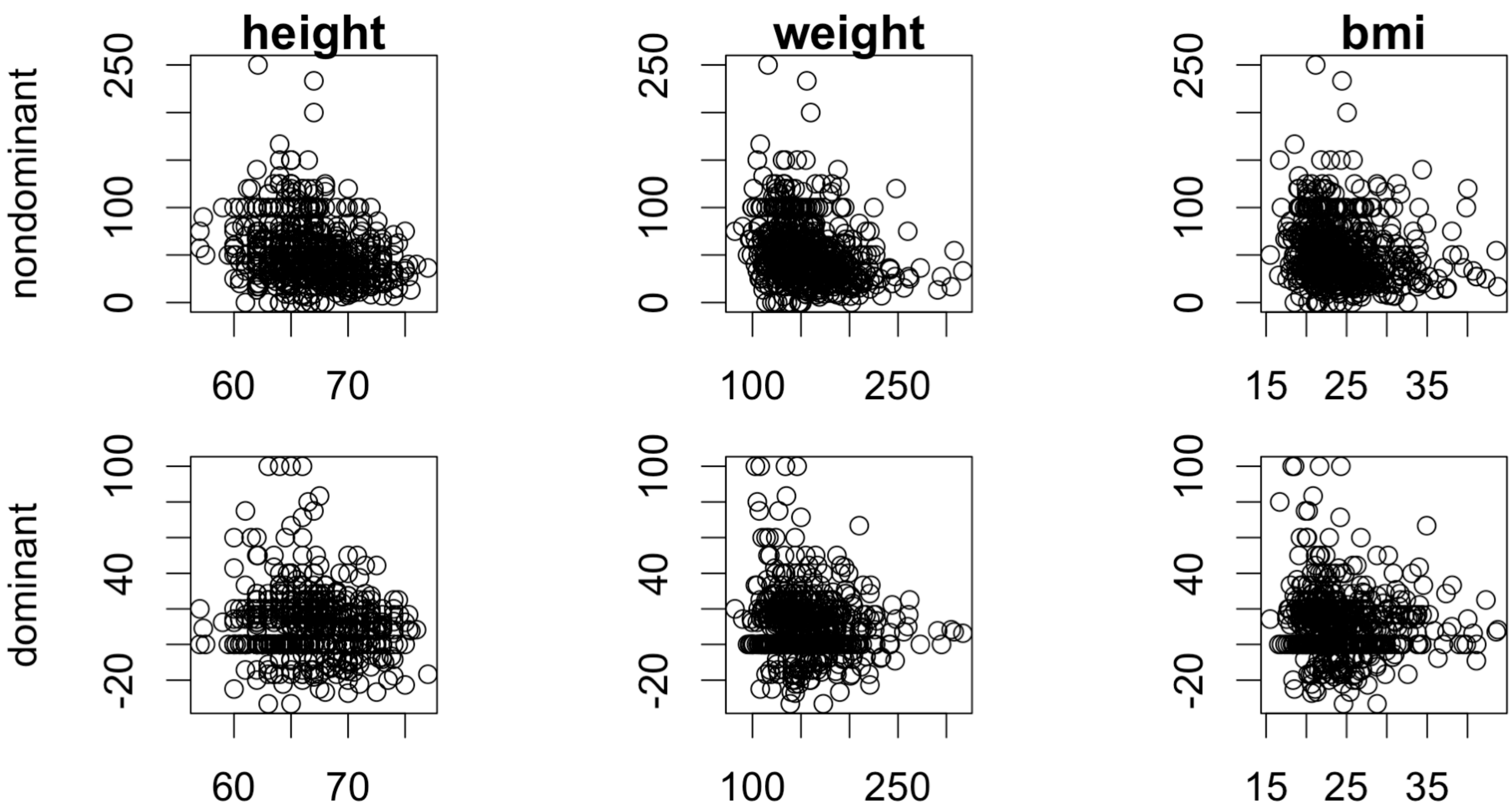


The proportions are close, suggesting minimal genotype differences.

Numeric/numeric

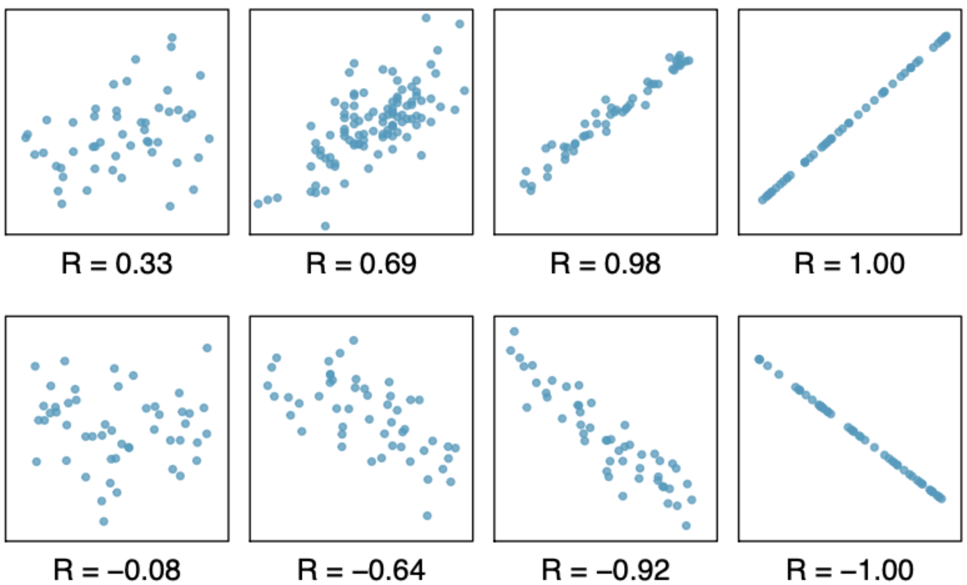
Did change in arm strength appear related in any way to body size among study participants?

Comparing numeric variables is easiest accomplished by scatterplots.



Correlation

In addition to graphical techniques, for numeric/numeric comparisons, there are also quantiative measures of relationship.



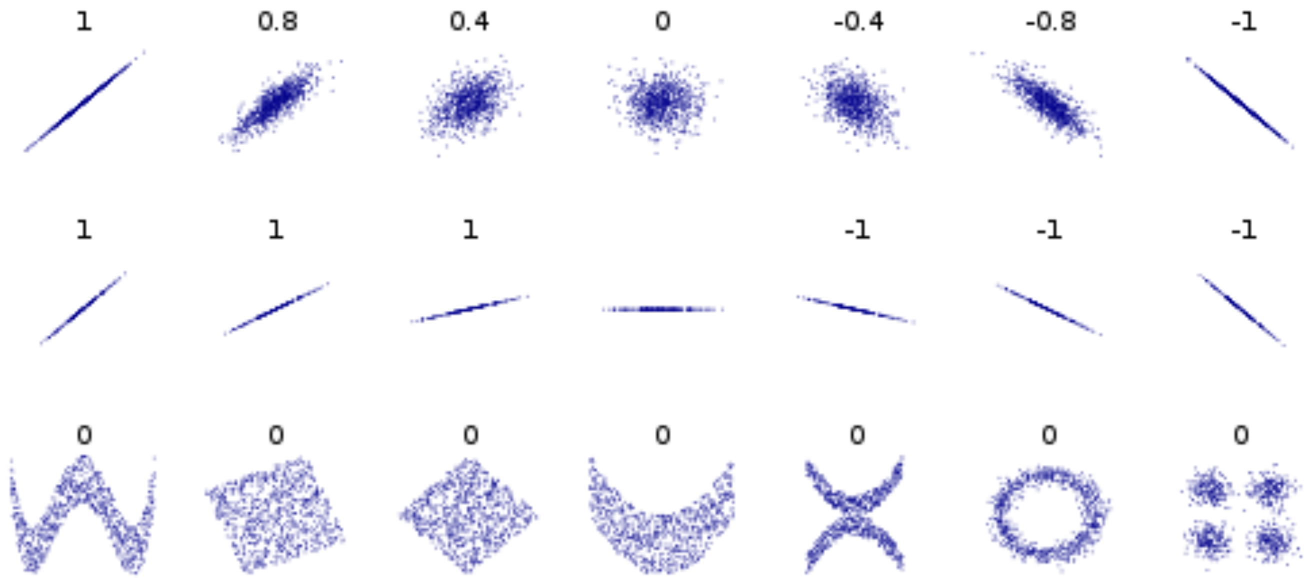
Correlation measures the strength of linear relationship, and is defined as:

$$r_{xy} = \frac{1}{n-1} \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{s_x s_y}$$

- $r \to 1$: *positive* relationship
- $r \to -1$: *negative* relationship
- $r \to 0$: no relationship

Uncorrelated $\neq$ no relationship

Correlation only captures *linear* relationships. Always do a graphical check.



Common misconceptions:

- stronger correlation $\longleftrightarrow$ greater slope
- weaker correlation $\longleftrightarrow$ no relationship

Interpreting correlations

Did change in arm strength appear related in any way to body size among study participants?

Here are the correlations corresponding to the plots we checked earlier.

	height	weight	bmi
drm.ch	-0.1104	-0.1159	-0.07267
ndrm.ch	-0.265	-0.2529	-0.1436

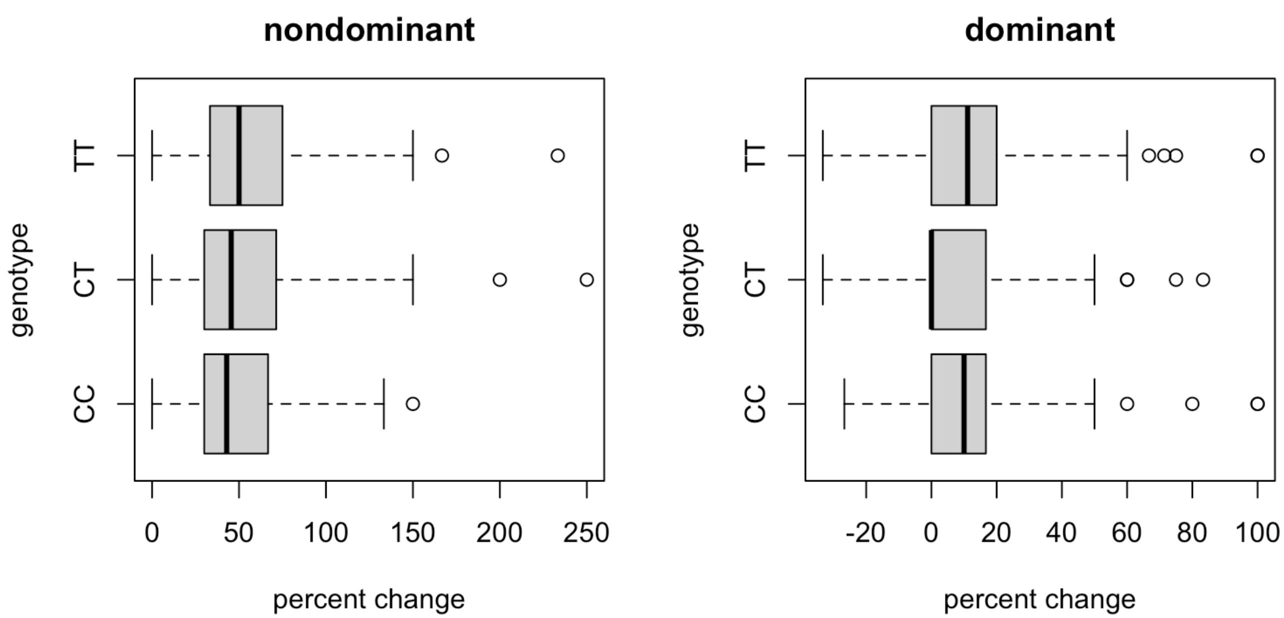
So there aren't any linear relationships here. A rule of thumb:

- $|r| < 0.3$: no relationship
- $0.3 \leq |r| < 0.6$: weak to moderate relationship
- $0.6 \leq |r| < 1$: moderate to strong relationship
- $|r| = 1$: either a mistake or not real data

Numeric/categorical

Side-by-side boxplots are usually a good option. Avoid stacked histograms.

Were differential changes in arm strength observed according to genotype?



Look for differences:

- location shift
- spread
- center

What do you think? Any notable relationships?