Name ____________________________________

Section ____________________________________

Instructions: read each problem carefully and provide solutions in the space below the prompt or on the reverse side of the page. You should provide as much justification and detail in calculation as needed to clearly display your thought process, but need not show or justify every step. If you use results from class in your solution, you can simply write, "by a theorem/problem/example from class…". If you are unable to obtain a complete solution, a solution sketch may receive partial credit if it reflects a clear understanding of the problem and a well-reasoned approach. Please feel free to ask any clarifying questions about the problems as they arise. Good luck!

1. Let X and Y be independent random variables and define $U = X - Y$ and $V = X + Y$. Find an expression for $\text{cov}(U, V)$. If $\text{var}(X) > \text{var}(Y)$, are U and V positively correlated, negatively correlated, or uncorrelated? Are U and V independent? Explain.

2. Suppose that for a particular gene of interest, the relative frequency of occurrence of genotypes CC, CT, and TT among the U.S. population by ethnicity is given by the table below.

	CC	CT	TT
African American	0.03	0.05	0.05
Asian	0.01	0.05	0.01
Hispanic/Latino	0.02	0.1	0.08
White	0.3	0.2	0.05
Other	0.01	0.02	0.03

Assume this table represents the probability that a randomly selected individual from the adult population has the indicated characteristics (ethnicity and genotype).

a) Are genotype and ethnicity independent? Why or why not?

b) Find the marginal probabilities that a randomly selected individual from the adult population has each genotype.

	CC	CT	TT

c) If the population is 50.4% female, fill in the joint probability table below assuming that sex and genotype are independent.

	CC	CT	TT
Female			
Male			

3. Mixture distributions are useful for modeling multimodal data. Let $Y_1 \sim \text{Poisson}(\lambda_1)$ and $Y_2 \sim \text{poisson}(\lambda_2)$ and $Z \sim \text{Bernoulli}(p)$. Assume all three are independent. let X be a random variable defined by the conditional distribution:

$$P(X = x|Z = 1) = P(Y_2 = x)$$
$$P(X = x|Z = 0) = P(Y_1 = x)$$

The distribution of X is a Poisson mixture. Find its PMF and expectation.

4. Let X_1, X_2 be independent standard Gaussian random variables. Find the MGF of $Y = X_1^2 + X_2^2$. Does Y belong to a common family of distributions? If so, which?

5. Let X_1, X_2 be jointly uniform on the unit square. Define $Y = X_1^2 + X_2^2$. Find the CDF of Y.

   Hint: Sketch the support set and draw the boundary $x_1^2 + x_2^2 = y$; find the CDF piecewise with cases where $0 \leq y \leq 1$ and $1 < y \leq 2$.

Name	PDF/PMF	Support	Parameters	Mean	Variance	MGF
Discrete uniform	$\frac{1}{n}$	$\{a_1, \ldots, a_n\}$	none	$\bar{a}$	$\frac{1}{n}\sum_{i=1}^{n}(a_i - \bar{a})^2$	$\frac{1}{n}\sum_{i=1}^{n} e^{ta_i}$
Bernoulli	$p^x(1-p)^{1-x}$	$\{0, 1\}$	$p \in (0,1)$	p	$p(1-p)$	$1 - p + pe^t$
Geometric	$(1-p)^x p$	$\mathbb{N}$	$p \in (0,1)$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$	$\frac{p}{1-(1-p)e^t}$, $t < -\log(1-p)$
Binomial	$\binom{n}{p}p^x(1-p)^{n-x}$	$\{0, 1, \ldots, n\}$	$n \in \mathbb{Z}^+$ $p \in (0,1)$	np	$np(1-p)$	$(1-p+pe^t)^n$
Negative binomial	$\binom{r+x-1}{x}p^r(1-p)^x$	$\mathbb{N}$	$r \in \mathbb{Z}^+$ $p \in (0,1)$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$	$\left(\frac{p}{1-(1-p)e^t}\right)^r$, $t < -\log(1-p)$
Poisson	$\frac{\lambda^x e^{-\lambda}}{x!}$	$\mathbb{N}$	$\lambda > 0$	λ	λ	$\exp\left\{-\lambda(1-e^t)\right\}$
Continuous uniform	$\frac{1}{b-a}$	(a, b)	$-\infty < a < b < \infty$	$\frac{b+a}{2}$	$\frac{(b-a)^2}{12}$	$\begin{cases} \frac{e^{ta}-e^{tb}}{t(b-a)} &, t \neq 0 \\ 1 &, t = 0 \end{cases}$
Gaussian	$\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$	$\mathbb{R}$	$\mu \in \mathbb{R}, \sigma > 0$	μ	σ^2	$\exp\left\{\mu t + \frac{1}{2}t^2\sigma^2\right\}$
Gamma	$\frac{1}{\Gamma(\alpha)\beta^\alpha}x^{\alpha-1}e^{-\frac{x}{\beta}}$	$x > 0$	$\alpha > 0, \beta > 0$	$\alpha\beta$	$\alpha\beta^2$	$(1-\beta t)^{-\alpha}$, $t < \frac{1}{\beta}$