1. (Random walk) Consider the natural numbers $\mathbb{N} = \{0, 1, 2, \dots\}$, and imagine a random process whereby given a particular location on the number line you move one step to the right with probability $p$ and one step to the left with probability $1 - p$. If you reach zero, the process stops.

   i. Find the probability that you make it to 50 if you start at 1 and $p = 0.6$.
   ii. Find the probability that you make it to 50 if you start at 1 and $p = 0.4$.
   iii. Find an expression for the smallest starting point required to make it to $n$ with probability $q$ if the odds favor moving to the right.
   iv. Find an expression for the smallest starting point required to make it to $n$ with probability $q$ if the odds favor moving to the left.
   v. Use your answers in (iii)-(iv) to find the smallest starting point for which the process will make it to 50 with probability 0.8 when $p = 0.45$ and when $p = 0.55$.
   vi. What is the minimum starting point for which the process is more likely to diverge than not when $p = 0.55$?

2. Show that if $X$ is a discrete random variable, then it has a countable support set.

3. Use the definition of the CDF to show that:

   i. If $X$ is a random variable with CDF $F$, then $P(a < X \leq b) = F(b) - F(a)$
   ii. If $X$ is a random variable with CDF $F$, then $P(X > a) = 1 - F(a)$

4. Let $X$ be a continuous random variable. Show that:

    i. for every $x \in \mathbb{R}$, $P(X = x) = 0$ (*Hint*: use the lemma defining $P(X = x)$ in terms of the CDF)

    ii. $P(X \leq x) = P(X < x)$ (*Hint*: use the result in (i))

5. (Triangular distribution) Let $X$ be a continuous random variable defined by the PDF:

$$f(x) = \begin{cases} 0 & x < -1 \\ x+1 & -1 \leq x < 0 \\ 1-x & 0 \leq x \leq 1 \\ 0 & x > 1 \end{cases}$$

Sketch the density and then find the CDF of $X$.

6. Let $X$ have a uniform distribution on the integers $\{-3, -2, -1, 0, 1, 2, 3\}$. If $Y = X^2$, find the CDF, PMF, and support of $Y$, and sketch the CDF.

7. Let $X$ have a uniform distribution on the interval $(0, 1)$ and let $Y = -\log(X)$. Find the distribution of $Y$.

8. (Negative binomial) Consider performing repeated independent trials in which each trial has a fixed probability of success $p$. Let $X$ denote the number of failures before $r$ successes are obtained. Find the PMF of $X$.