Name ____________________________

Section ____________________________

**Instructions:** read each problem carefully and provide solutionsin the space below the prompt or on the reverse side of the page. You should provide as muchjustifictation and detail in calculation as needed to clearly display yourthought process, but need not show or justify every step. If you use results fromclass in your solution, you can simply write, "by a theorem/problem/example fromclass. . . ' '. None of the problems require you to perform numerical calculations.If you are unable to obtain a complete solution, a solution sketch may receive partial creditif it reflects a clear understanding of the problem and a well-reasoned approach.Please feel free to ask any clarifying questions about the problems as they arise. Good luck!

1. Let $X$ be distributed according to the mass function below. Determine the value of $p$, sketch the CDF (a log scale is acceptable), and find $\mathbb{E}X$.

| $x$ | $P(X = x)$ |
|---|---|
| $\log(1)$ | 0.4 |
| $\log(2)$ | 0.1 |
| $\log(3)$ | 0.2 |
| $\log(4)$ | $p$ |
| $\log(5)$ | 0.2 |

2. Consider a random variable $Z \sim f(z)$ where $f(z) = ce^{-|z|}$ where $z \in \mathbb{R}$. Determine the value of $c$ that makes this a density and then find the moment generating function of $Z$.

   (*Hint*: $\int_{\mathbb{R}} g(|x|)dx = \int_{-\infty}^{0} g(-x)dx + \int_{0}^{\infty} g(x)dx$.)

3. Let $U \sim \text{uniform}(0,1)$. Find the distribution of $U^{\frac{1}{k}}$ and show that $\mathbb{E}\left[U^{\frac{1}{k}}\right] \to 1$ and $\text{var}\left[U^{\frac{1}{k}}\right] \to 0$ as $k \to \infty$.

| Name | PDF/PMF | Support | Parameters | Mean | Variance | MGF |
|---|---|---|---|---|---|---|
| Discrete uniform | $\frac{1}{n}$ | $\{a_1, \ldots, a_n\}$ | none | $\bar{a}$ | $\frac{1}{n}\sum_{i=1}^{n}(a_i-\bar{a})^2$ | $\frac{1}{n}\sum_{i=1}^{n} e^{ta_i}$ |
| Bernoulli | $p^x(1-p)^{1-x}$ | $\{0,1\}$ | $p \in (0,1)$ | $p$ | $p(1-p)$ | $1-p+pe^t$ |
| Geometric | $(1-p)^x p$ | $\mathbb{N}$ | $p \in (0,1)$ | $\frac{1-p}{p}$ | $\frac{1-p}{p^2}$ | $\frac{p}{1-(1-p)e^t}$<br>$t < -\log(1-p)$ |
| Binomial | $\binom{n}{p} p^x (1-p)^{n-x}$ | $\{0,1,\ldots,n\}$ | $n \in \mathbb{Z}^+$<br>$p \in (0,1)$ | $np$ | $np(1-p)$ | $\left(1-p+pe^t\right)^n$ |
| Negative binomial | $\binom{r+x-1}{x} p^r (1-p)^x$ | $\mathbb{N}$ | $r \in \mathbb{Z}^+$<br>$p \in (0,1)$ | $\frac{r(1-p)}{p}$ | $\frac{r(1-p)}{p^2}$ | $\left(\frac{p}{1-(1-p)e^t}\right)^r$<br>$t < -\log(1-p)$ |
| Poisson | $\frac{\lambda^x e^{-\lambda}}{x!}$ | $\mathbb{N}$ | $\lambda > 0$ | $\lambda$ | $\lambda$ | $\exp\left\{-\lambda(1-e^t)\right\}$ |
| Continuous uniform | $\frac{1}{b-a}$ | $(a,b)$ | $-\infty < a <$<br>$b < \infty$ | $\frac{b+a}{2}$ | $\frac{(b-a)^2}{12}$ | $\begin{cases} \frac{e^{ta}-e^{tb}}{t(b-a)} & , t \neq 0 \\ 1 & , t = 0 \end{cases}$ |
| Gaussian | $\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$ | $\mathbb{R}$ | $\mu \in \mathbb{R}, \sigma > 0$ | $\mu$ | $\sigma^2$ | $\exp\left\{\mu t + \frac{1}{2}t^2\sigma^2\right\}$ |
| Gamma | $\frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}}$ | $x > 0$ | $\alpha > 0, \beta > 0$ | $\alpha\beta$ | $\alpha\beta^2$ | $(1-\beta t)^{-\alpha}$ |