1. Reparametrize the uniform distribution in terms of center and length: propose an alternative PDF for the uniform (a, b) distribution with one parameter that indicates the center of the interval and one that indicates the length of the interval. Verify that your proposal is a PDF, and find the expectation and variance.

2. (Poisson approximation to the binomial.) In class it was noted that the Poisson distribution is a limiting case for the binomial with $np = \lambda$ fixed as $n \to \infty$. This fact can be leveraged to approximate binomial probabilities for large numbers of trials. Let $X_{n,p} \sim$ binomial(n, p) and $Y_{n,p} \sim$ Poisson(np). Then when n is large, $P(X_{n,p} = x) \approx P(Y_{n,p} = x)$. Fill in the following table:

n	p	$P(X_{n,p} = 100)$	$P(Y_{n,p} = 100)$	approximation error
1000	0.1			
10000	0.01			
100000	0.001			
1000000	0.0001			

3. The kth factorial moment of a random variable X is $\mathbb{E}\left(\frac{X!}{(X-k)!}\right)$, assuming the expectation exists.

   i. Find the second factorial moment of the Poisson distribution.
   ii. Find the second factorial moment of the binomial distribution.
   iii. Use your answers above to calculate the variance of each distribution.

4. (Exponential distribution) Let $X \sim F(x)$ where $F(x) = 1 - e^{-\alpha x}$. Find the mean and variance of X.

5. (Truncated Poisson) Let $X \sim \text{Poisson}(\lambda)$ and define Y by the probability mass function

$$P(Y = y) = \frac{P(X = y)}{P(X > 0)}\,, \qquad y = 1, 2, \ldots$$

Find an expression for the PMF of Y, and calculate its mean and variance.

6. With Y as in the previous problem, show that $\mathbb{E}\log(Y) \geq 0$.

7. (Probability integral transform) Let X be a continuous random variable with strictly increasing CDF F, and let $Y = F(X)$. Show that $Y \sim \text{uniform}(0, 1)$. (If F is strictly increasing, then $F^{-1}(x)$ is well-defined).