Name ______________________________

Section ______________________________

Instructions: read each problem carefully and provide solutions in the space below the prompt or on the reverse side of the page. You should provide as much justification and detail in calculation as needed to clearly display your thought process, but need not show or justify every step. If you use results from class in your solution, you can simply write, "by a theorem/problem/example from class…". None of the problems require you to perform numerical calculations. If you are unable to obtain a complete solution, a solution sketch may receive partial credit if it reflects a clear understanding of the problem and a well-reasoned approach. Please feel free to ask any clarifying questions about the problems as they arise. Good luck!

1. Let X be distributed according to the mass function below. Determine the value of p, sketch the CDF (a log scale is acceptable), and find an expression for $\mathbb{E}X$.

x	$P(X = x)$
$\log(1)$	0.4
$\log(2)$	0.1
$\log(3)$	0.2
$\log(4)$	p
$\log(5)$	0.2

2. Consider a random variable $Z \sim f(z)$ where $f(z) = ce^{-|z|}$ where $z \in \mathbb{R}$ and $c > 0$. Determine the value of c that makes this a density and then find the moment generating function of Z.

   (*Hints*: $\int_{\mathbb{R}} g(|x|)dx = \int_{-\infty}^{0} g(-x)dx + \int_{0}^{\infty} g(x)dx$.)

3. Let $U \sim \text{uniform}(0,1)$. Find the distribution of $U^{\frac{1}{k}}$ and show that $\mathbb{E}\left[U^{\frac{1}{k}}\right] \to 1$ and $\text{var}\left[U^{\frac{1}{k}}\right] \to 0$ as $k \to \infty$.

Name	PDF/PMF	Support	Parameters	Mean	Variance	MGF
Discrete uniform	$\frac{1}{n}$	$\{a_1, \dots, a_n\}$	none	$\bar{a}$	$\frac{1}{n}\sum_{i=1}^{n}(a_i - \bar{a})^2$	$\frac{1}{n}\sum_{i=1}^{n} e^{ta_i}$
Bernoulli	$p^x(1-p)^{1-x}$	$\{0, 1\}$	$p \in (0, 1)$	p	$p(1-p)$	$1 - p + pe^t$
Geometric	$(1-p)^x p$	$\mathbb{N}$	$p \in (0, 1)$	$\frac{1-p}{p}$	$\frac{1-p}{p^2}$	$\frac{p}{1-(1-p)e^t}$, $t < -\log(1-p)$
Binomial	$\binom{n}{p} p^x(1-p)^{n-x}$	$\{0, 1, \dots, n\}$	$n \in \mathbb{Z}^+$ $p \in (0, 1)$	np	$np(1-p)$	$(1 - p + pe^t)^n$
Negative binomial	$\binom{r+x-1}{x} p^r(1-p)^x$	$\mathbb{N}$	$r \in \mathbb{Z}^+$ $p \in (0, 1)$	$\frac{r(1-p)}{p}$	$\frac{r(1-p)}{p^2}$	$\left(\frac{p}{1-(1-p)e^t}\right)^r$, $t < -\log(1-p)$
Poisson	$\frac{\lambda^x e^{-\lambda}}{x!}$	$\mathbb{N}$	$\lambda > 0$	λ	λ	$\exp\{-\lambda(1 - e^t)\}$
Continuous uniform	$\frac{1}{b-a}$	(a, b)	$-\infty < a < b < \infty$	$\frac{b+a}{2}$	$\frac{(b-a)^2}{12}$	$\begin{cases} \frac{e^{ta}-e^{tb}}{t(b-a)} & , t \neq 0 \\ 1 & , t = 0 \end{cases}$
Gaussian	$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$	$\mathbb{R}$	$\mu \in \mathbb{R}, \sigma > 0$	μ	σ^2	$\exp\left\{\mu t + \frac{1}{2}t^2\sigma^2\right\}$
Gamma	$\frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-\frac{x}{\beta}}$	$x > 0$	$\alpha > 0, \beta > 0$	$\alpha\beta$	$\alpha\beta^2$	$(1 - \beta t)^{-\alpha}$, $t < \frac{1}{\beta}$