1. Show that if $X = (X_1, X_2)$ is a random vector, then $\mathbb{E}\left[aX_1 + bX_2 + c\right] = a\mathbb{E}X_1 + b\mathbb{E}X_2 + c$. Establish the result in both the discrete and continuous cases.

2. (Covariance formula) Show that $\mathbb{E}\left[(X-\mathbb{E}X)(Y-\mathbb{E}Y)\right]=\mathbb{E}(XY)-\mathbb{E}X\mathbb{E}Y$.

3. Suppose (X_1, X_2) are uniformly distributed on the unit square, *i.e.*,

$$f(x_1, x_2) = 1 \ , \qquad 0 < x_1 < 1, 0 < x_2 < 1$$

Find the distribution of $Y = X_1 + X_2$ by finding the CDF of Y.

4. Consider again the house hunting example from class where X_1 denotes the number of bedrooms and X_2 denotes the number of bathrooms, and for a randomly selected listing the vector (X_1, X_2) has joint distribution:

	$x_1 = 0$	$x_1 = 1$	$x_1 = 2$	$x_1 = 3$
$x_2 = 1$	0.1	0.1	0.2	0
$x_2 = 1.5$	0	0.1	0.2	0
$x_2 = 2$	0	0	0	0.3
$x_2 = 2.5$	0	0	0	0

Find the covariance and correlation of X_1 and X_2.

5. Let (X_1, X_2) be independent exponential random variables with parameter $\beta = 2$, so that they have joint distribution

$$f(x_1, x_2) = \frac{1}{4} \exp\left\{-\frac{1}{2}(x_1 + x_2)\right\} , \qquad x_1 > 0, x_2 > 0$$

Let Y_1, Y_2 denote the sum and difference, respectively, of X_1, X_2. Find the correlation $\mathrm{corr}(Y_1, Y_2)$.

6. Suppose that you arrive at work within a 2-minute window of your expected arrival time uniformly at random, and your expected arrival time may shift slightly depending on whether there are traffic delays. That is, if X denotes your arrival time where $X = 0$ indicates you are exactly on time, and Y denotes the traffic delay, then assume:

$$X|Y \sim \text{uniform}(Y - 1, Y + 1)$$

If $Y \sim$ exponential(1), find the mean arrival time $\mathbb{E}X$.