1. (Normal-χ^2 relationship) Show that if $Z \sim N(0,1)$ then $Z^2 \sim \chi_1^2$.

2. (Stochastic ordering) Two random variables X and Y are stochastically ordered if either $F_X(x) \leq F_Y(x)$ or the reverse inequality is true for every $x \in \mathbb{R}$. We say that:

$$X \geq_{st} Y \quad \text{if} \quad F_X(x) \leq F_Y(x) \quad \text{for every} \quad x \in \mathbb{R}$$

   Show that the exponential distribution is stochastically ordered in its parameter: that is, if $X \sim \text{exponential}(\alpha)$ and $Y \sim \text{exponential}(\alpha + c)$ where $c > 0$, then $X \geq_{st} Y$. (Use the 'rate' parametrization: $f(x) = \alpha e^{-\alpha x}, x > 0, \alpha > 0$).

3. Let X be a random variable with moment generating function $m_X(t)$. Define $s_X(t) = \log(m_X(t))$. Show that $s'_X(0) = \mathbb{E}X$ and $s''_X(0) = \mathrm{var}(X)$. Then use this approach to find the mean and variance of a random variable X when:

   i. $X \sim N(\mu, \sigma^2)$
   ii. $X \sim \Gamma(\alpha, \beta)$