Results summary

Model overview

Model:

$$Y_{ihdk} = \mu + \tau_i + f_i(h, d) + g_{ik}(h) + \epsilon_{ik}(h, d) \begin{cases} i = 1, 2 \\ h = 0, 1, \dots, 23 \\ d = 182, \dots, 237 \end{cases}$$
$$\sum_{i} \tau_i = 0$$
$$\epsilon_{ik}(h, d) \sim AR1$$
$$k = 1, \dots, n_i$$

The response Y_{itdk} represents body temperature T_b for pregnancy status i at time of day t on day d for snake k. μ represents the population mean body temperature. The fixed parameters τ_i represent deviations from the population mean body temperature corresponding to each pregnancy status, and are subject to a sum constraint, so that $\tau_1 = -\tau_2$.

The function f_i represents population-level temporal and seasonal deviations in mean body temperature for pregnancy status i, and is represented in the model by Fourier basis expansions $\left\{\phi_j^{(h)}\right\}$ and $\left\{\phi_l^{(d)}\right\}$ corresponding to time of day and day, respectively, plus all two- and three-way interaction terms between basis functions and group, yielding the representation:

$$f_{i} = \sum_{i} (\alpha_{j} + \tilde{\alpha}_{ij}) \cdot \phi_{j}^{(h)} + \sum_{l} (\beta_{l} + \tilde{\beta}_{il}) \cdot \phi_{l}^{(d)} + \sum_{i} \sum_{l} (\gamma_{jl} + \tilde{\gamma}_{ijl}) \cdot \phi_{j}^{(h)} \phi_{l}^{(d)}$$

All parameters in the above expression are fixed and the interaction terms are subject to a sum constraint: $\sum_i \tilde{\alpha}_{ij} = \sum_i \tilde{\beta}_{il} = \sum_j \sum_l \tilde{\gamma}_{ijl} = 0$. Three basis pairs spanning the period 0-23 are used for the time of day term, and a single basis pair spanning the annual period 1-365 is used for the day term.

The function g_{ik} gives individual-level temporal deviations from the group-level mean in group i for snake k, and is represented using the aforementioned basis expansion $\left\{\phi_j^{(h)}\right\}$ with random coefficients:

$$g_{ik} = \sum_{j} \delta_{jik} \cdot \phi_{j}^{(h)}$$
 where $\delta_{jik} \stackrel{iid}{\sim} N(0, \sigma_{j}^{2})$

Lastly, the error term $\epsilon_{ik}(h,d)$ is an autoregressive process of order 1. Expressed as a function of consecutive time-points t corresponding to each unique combination of hour and day arranged in sequence:

$$\epsilon_{ik}(t) = \phi \cdot \epsilon_{ik}(t-1) + w_t$$
 where $w_t \stackrel{iid}{\sim} N(0, \sigma^2)$

The model thus has, accounting for the sum constraints, a total of 42 fixed free parameters: the grand mean μ , τ_1 , 6 α_j and 6 $\tilde{\alpha}_{1j}$, 2 β_l and 2 $\tilde{\beta}_{1l}$, 12 γ_{jl} and 12 $\tilde{\gamma}_{1jl}$. The correlation structure for the random portion of the model comprises 8 parameters: 6 variance parameters σ_j^2 , the autoregressive parameter ϕ , and the residual variance σ^2 .

Parameters are estimated by REML.

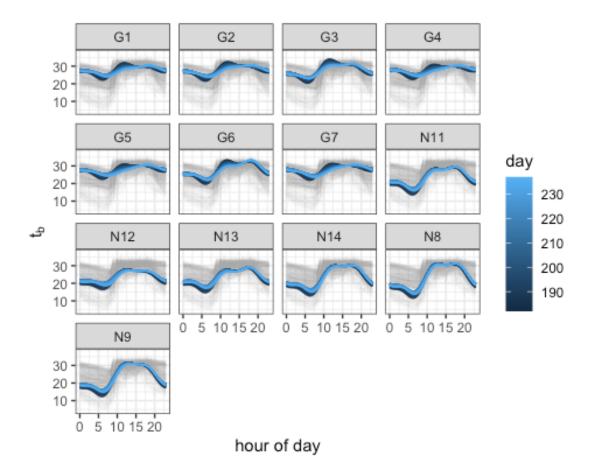
Results

The model was fit to 55 consecutive days of hourly observations of body temperatures for 7 pregnant and 6 non-pregnant snakes.

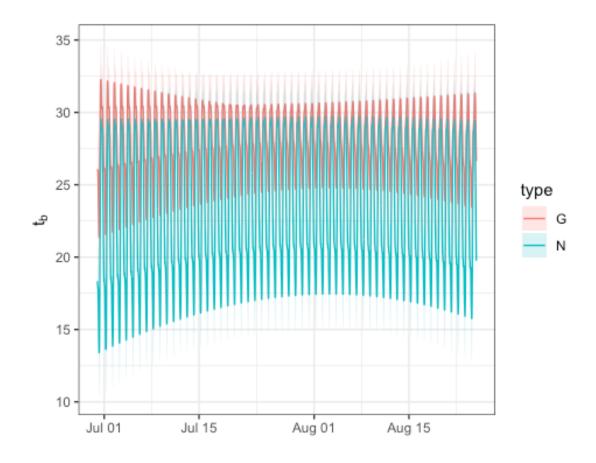
The ANOVA table below summarizes conditional F tests for fixed terms and indicates most model terms explained significant variability in mean body temperatures, except for the day-group and day-group-hour interaction terms. This suggests that estimated differences in body temperature associated with pregnancy status do not depend significantly on seasonal shifts.

	numDF	denDF	F-value	p-value
type	1	11	621.8	4.957e-11
fb.hour	6	6 17419 65.39		0
fb.day	2	17419 9.096 0.0001		0.0001126
type:fb.hour	6	17419	8.954	8.688e-10
type:fb.day	2	17419	1.713	0.1803
fb.hour:fb.day	12	17419	17.26	0
type:fb.hour:fb.day	12	17419	1.909	0.0286

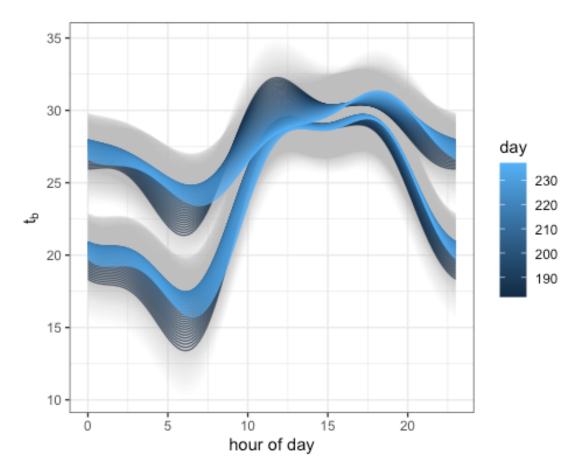
Fitted body temperatures over time by snake show substantial variability in daily fluctuations by snake. It should be noted that the model did not estimate seasonal fluctuations at the snake level, so the shifts by day shown in the figure are identical within pregnancy group.



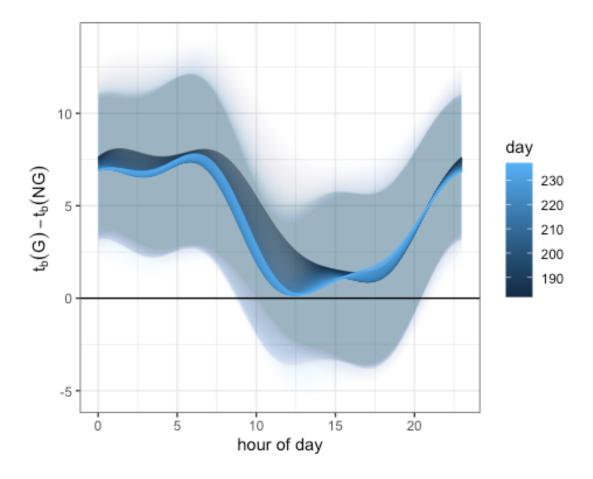
Population-level time courses with Bonferroni-adjusted 95% confidence bands are shown below by group. This displays clearly the estimated seasonal shifts by pregnancy group.



Visualizing the same information by hour of day indicates: (a) temperature differences are more pronounced at night; (b) estimated temperatures for pregnant snakes show a much more dramatic seasonal shift toward lower morning body temperatures.



Lastly, estimated differences between pregnant and non-pregnant mean body temperatures by hour and day with a Bonferroni-adjusted 95% confidence band are shown below.



Parameter estimates

Fixed parameters

	Value	Std.Error
(Intercept)	16.6	2.38
type1	5.02	2.38
fb.hoursin1	-44.5	7.2
fb.hourcos1	-17.9	6.96
fb.hoursin2	6.33	4.2
fb.hourcos2	6.98	4.15
fb.hoursin3	3.73	2.95
fb.hourcos3	-15.4	2.94
fb.daysin1	-63.4	15.5
fb.daycos1	-114	29.9
type1:fb.hoursin1	9.83	7.2
type1:fb.hourcos1	-3.12	6.96
type1:fb.hoursin2	-5.11	4.2
type1:fb.hourcos2	1.59	4.15
type1:fb.hoursin3	5.94	2.95
type1:fb.hourcos3	-1.14	2.94
type1:fb.daysin1	23.7	15.5
type1:fb.daycos1	33.5	29.9
fb.hoursin1:fb.daysin1	-207	46.1
fb.hourcos1:fb.daysin1	-155	44.8
fb.hoursin2:fb.daysin1	35.1	27.2
fb.hourcos2:fb.daysin1	83.5	26.9
fb.hoursin3:fb.daysin1	37.9	19
fb.hourcos3:fb.daysin1	-81	19
fb.hoursin1:fb.daycos1	-366	89.4
fb.hourcos1:fb.daycos1	-107	86.9
fb.hoursin2:fb.daycos1	59.9	52.7
fb.hourcos2:fb.daycos1	32.4	52.2
fb.hoursin3:fb.daycos1	7.94	36.9
fb.hourcos3:fb.daycos1	-154	36.7
type1:fb.hoursin1:fb.daysin1	40.3	46.1
type1:fb.hourcos1:fb.daysin1	-62.1	44.8
type1:fb.hoursin2:fb.daysin1	-27	27.2

	Value	Std.Error
type1:fb.hourcos2:fb.daysin1	30.7	26.9
type1:fb.hoursin3:fb.daysin1	45.8	19
type1:fb.hourcos3:fb.daysin1	-18.7	19
type1:fb.hoursin1:fb.daycos1	58	89.4
type1:fb.hourcos1:fb.daycos1	-77.9	86.9
type1:fb.hoursin2:fb.daycos1	-60.4	52.7
type1:fb.hourcos2:fb.daycos1	11.6	52.2
type1:fb.hoursin3:fb.daycos1	79.7	36.9
type1:fb.hourcos3:fb.daycos1	-27.6	36.7

Variance parameters

	Variance	StdDev
fb.hoursin1	17.5034090	4.1837076
fb.hourcos1	9.9855598	3.1599936
fb.hoursin2	1.0164998	1.0082161
fb.hourcos2	0.7006386	0.8370416
fb.hoursin3	1.0582381	1.0287070
fb.hourcos3	1.2256821	1.1071053
Residual	12.9308561	3.5959500

The autoregressive parameter was estimated to be $\hat{\phi}$ = 0.8165178

Estimated random parameters

	fb.hoursin1	fb.hourcos1	fb.hoursin2	fb.hourcos2	fb.hoursin3	fb.hourcos3
12	7.57	2.73	-0.386	-0.62	-1.31	1.37
1	1.86	1.01	0.302	0.121	-0.471	-0.00376
3	-1.76	-5.15	-0.932	1.14	1.05	-0.128
7	0.585	2.54	0.897	-0.239	-0.837	0.449
11	1.07	1.98	-0.354	0.293	0.979	0.314
13	4.22	2.26	-1.68	-0.782	1.58	0.837
4	2.36	2.75	0.415	-0.541	-0.318	0.652
2	0.52	-0.58	0.179	0.748	-0.34	0.245
5	1.85	2.09	0.306	-1.03	-0.202	0.799
6	-5.42	-2.65	-1.17	-0.196	1.11	-2.01
9	-3.15	-5.23	1.19	0.88	0.38	0.124
14	-3.21	0.381	0.761	-0.276	-1.04	-0.804
8	-6.49	-2.12	0.477	0.504	-0.588	-1.84