Basic stuff on how percentages work

What is a percentage?
A percentage is the top part of a fraction whose bottom part is 100.
So 50% means 'half of' and 25% means 'a quarter of'. 100% means the complete quantity.

$$\frac{50}{100} = \frac{1}{2} \text{ and } \frac{25}{100} = \frac{1}{4} \qquad \frac{100}{100} = 1 \text{ i.e the whole lot.}$$

Why bother with them?
Percentages are useful because they make it very easy to compare things.
For example, suppose the marks in two successive tests are 67/80 and 51/60. It is not very easy to say which of these was best. Percentages use our ordinary number system of 10's, 100's etc and, because they are out of 100 rather than 10, we avoid a lot of the decimal points which make some people twitchy.

Changing a fraction to a %
Taking the example of the test mark of 67 out of 80,

$$\text{We have } \frac{67}{80} = \frac{\text{what we want}}{100}$$

Multiplying both sides of this equation by 100 gives us

$$100 \times \frac{67}{80} = 83.75\% = \text{what we want}$$

RULE:- to change a fraction to a %, multiply it by 100.

Question:- What is the second test mark of 51/60 as a %? Try this yourself before looking.

Answer

$$\frac{51}{60} \times 100 = 85\%$$

so the mark in the second test was higher.

****Update**** (Jan 2004) I've had some emails asking for help to turn a single number into a

percentage. It can't be done! You have to know as a percentage of what.
For example, a drinks bill of £20 for a party costing £80 in total means that the drinks cost 20/80 or 1/4 or 25% of the total. The £20 on its own can't be turned into a percentage.

Changing a % to a fraction

RULE:- You simply turn it into a fraction by writing it over 100.
Then cancel down if possible.

Example:- What is 35% as a fraction?

35% is the same as $\frac{35}{100}$ or $\frac{7}{20}$

cancelling down to the simplest form by dividing the top and bottom by 5.

*** Remember that the value of a fraction remains unchanged when you multiply or divide *both the top and the bottom* by the same number. ***

Changing a decimal to a %

Dead easy, this one!
Suppose we want to write 0.27 as a %. Since a decimal is a kind of fraction, all we have to do is to multiply by 100. You just need to remember that each time you multiply by 10 the number becomes larger by a factor of 10 so the decimal point moves one place to the right. Multiplying by 100 moves it 2 places to the right. This neat rule is because decimals are fractions in our base ten number system.
So we find that 0.27 is the same as (0.27 x 100)% = 27%.

Similarly, 0.735 is the same as (0.735 x 100)% = 73.5%
and 7.46 is the same as (7.46 x 100)% = 746%.

RULE:- To change a decimal to a % we multiply by 100
which just moves the decimal point 2 places to the right.

Changing a % to a decimal

(Again, dead easy!)

RULE:- All we have to do is to divide by 100, so move the decimal point 2 places to the left.

Here are 3 examples to show you how to deal with all possible snags.
Example (1) What is 37% as a decimal?

Answer:- 37% is the same as 0.37.

Example (2) What is 25.5% as a decimal?

Answer:- 25.5% is the same as 0.255.
(Notice that the percentage had a decimal point in here too.)

Example (3) What is 50% as a decimal?

Answer:- 50% is the same as 0.50 = 0.5.
(The last zero just tells us that there is nothing in the 2nd position after the decimal point, so we can leave it out.)

Percentage increases and decreases

The easiest way to explain how to work these out is to look at some examples.

Example (1) Suppose the profits of a certain company go from �365 000 in January to �425 000 in February. What is the % increase in their profits?

RULE:- Percentage increases and decreases are always calculated with respect to the value before the change took place.

Here, the actual increase in profits is �425 000 - �365 000 = �60 000.
The % profit is �60 000 as a percentage of �365 000

$$\text{so it is } \frac{60000}{365000} \times 100 = 16{\cdot}4\% \text{ to 1 d.p.}$$

Example (2) The number of first year students at a certain university studying Law was 127 in 1996 and 114 in 1997. What was the % decrease?

The actual decrease is 127 - 114 = 13.
The % decrease is 13 as a % of 127.
13 as a fraction of 127 is 13/127.
Now, just multiply by 100 so you get

$$\frac{13}{127} \times 100 = 10{\cdot}2\% \text{ to 1 d.p.}$$

Example (3) The price of a certain model of car goes up by 8%. It used to cost �7 800. What will it cost in future?

There are two ways of finding this.
Method (1)
First find the actual increase in cost. This is 8% of �7 800 so it is

$$\frac{8}{100} \times \pounds 7\,800 = \pounds 624.$$

Therefore the new price is �7 800 + �624 = �8 424.

Method (2)
This is the all-in-one way of doing it.
The new price is 108% of the old one, so it is

$$\frac{108}{100} \times \pounds 7\,800 = \pounds 8\,424.$$

Example (4)
At the beginning of December, the price of a certain item is increased by 5% to make a bigger Christmas profit.

At the beginning of January, there is a Sale and the unsold items are labelled **5% OFF!**
Would you now be paying the same as if you had bought the item in November? If not, would you be paying more or less?

Have a go at answering this before looking.

Suppose it cost �100 in November.
Then in December the price increased by 5% to �105.
The Sale Price is now calculated as a reduction of 5% on this current price of �105.
So, using method (1), the actual reduction in price is

$$\frac{5}{100} \times £105 = £5.25$$

Therefore you would only pay �105 - �5.25 = �99.75. You would get it cheaper in January than you would have done if you had bought it in November.

I've added the next two examples (Feb 2004) because I've had emails asking for help with this kind of problem.

Example (5)
A certain computer store reckons to make 30% profit on each gizmo that it sells.
If the selling price of a particular gizmo is £1 560 what was the cost price to the dealer?

DANGER! The 30% profit is on the *cost price* of the gizmo. So we don't want to find 30% of £1 560.

We know that the selling price of £1 560 is 130% of the cost price since the dealer is making a profit of 30% on the cost price. So we can say

$$£1560 = \frac{130}{100} \times \text{cost price}.$$

Now multiply both sides of this equation by 100 and divide both sides by 130. This gives

$$\frac{100}{130} \times £1560 = \frac{100}{130} \times \frac{130}{100} \times \text{cost price} = \text{cost price}$$

since the two right-hand fractions cancel each other out.
Therefore the cost price to the dealer was £156 000/130 = £1 200.

Example (6)
In a sale, there is a rack of coats marked "All prices in this rack are reduced by 20%!".
The one I choose now has a price of £120. How much did it cost before the sale?

The working is very similar to the previous example. We know that £120 is 80% of the original selling price since 20% has been taken off this price. This time we'll save some writing in the equations by calling the original selling price P. Then we have

$$£120 = \frac{80}{100} \times P.$$

Now, multiplying both sides of this equation by 100 and dividing both sides by 80, we have

$$\frac{100}{80} \times £120 = \frac{100}{80} \times \frac{80}{100} \times P = P.$$

So the original selling price was £12 000/80 = £150.
A little bit of algebra saves a lot of writing!

I've added this next example on VAT (Jan 2005) because I've had emails asking about this.

Example (7)

People have found difficulty here when they needed to work backwards from a bill whose total includes VAT (value added tax) at 17.5% to find out what the bill would have been before the VAT was added.
The easiest way to explain how this is done is to take an actual example.
Suppose that a bill which includes VAT comes to £1602.70. We want to know what the amount was before VAT was added.

DANGER You can't work out this answer by finding 17.5% of £1602.70 and then taking it off. The reason for this is that the 17.5% is of the original amount of the bill and not the final £1602.70.

We'll save writing by calling the amount before VAT was added C.
Then, working in £, we know that

$$C + 17.5\% \text{ of } C = 1602.70$$

$$\text{so we know} \quad C + \left(\frac{17.5}{100}\right)C = 1602.70$$

$$\text{so} \left(\frac{117.5}{100}\right)C = 1602.70$$

$$\text{so } C = \frac{100 \times 1602.70}{117.5} = 1364.$$

The total of the bill before VAT was added was £1364.

We can make a general rule for this if we let P stand for the total of the bill including VAT at 17.5%.
We want to find C, the amount of the bill before VAT was added.
Using the same argument as above, we get

$$C = \frac{100 \times P}{117.5}.$$

RULE:- To find the amount of a bill before VAT at 17.5% was added, multiply the amount including VAT by 100 and divide the result by 117.5.
The working here is similar to Example (5) above but VAT seems to create special difficulties.

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