Question 1 (4 points)

Convert the following decimal numbers to their 2's complement representations (use 8 bits for each number and truncate the fractional digits if necessary):

a) 102.3

b) 6.3

c) -13

d) 0.04

Solution:

a)01100110
b)0110.0100
c)11110011
d)0.0000101

Question 2 (6 points)

Perform the following operation in 2's complement. Indicate when an overflow happens.

a) $11001010 + 11101010$

b) $01011010 + 00110101$

c) $0011.1100 + 0100.0100$

d) $11010111 - 00001011$

Solution:

a)10110100
b)10001111(overflow)
c)1000.0000(overflow)
d)11001100

Question 3 (10 points)

Write IEEE floating point representation of the following decimal numbers.

a) 19.45

b) -0.028

Solution:

a)01000001100110111001100110011001
b)10111100111001010110000001000001

Question 4 (10 points)

The following numbers are in IEEE floating point representation. Write their decimal equivalents.

a) 10111100 11010101 11100000 00000000

b) 00000000 00000100 00000000 00000000

Solution:

a)-0.0261077880859375
b)$0.03125 \times 2^{-126} \approx 3.6734198^{-40}$

Question 5 (10 points)

Assume we are considering a set of symbols consisting of the capital English letters (26 letters) plus white-space, question mark, and full stop to communicate messages. In our coding list, the letters of the alphabet precede white-space, question mark and full stop in this representation.

a) How many bits do we need to encode each symbol in this set?

b) Assume 32-bit words are used for communicating messages. Furthermore, assume the character codes cannot span over multiple words. Show how the following message is encoded, and what will be its equivalence in hexadecimal.

THIS IS A SAMPLE STRING. OR IS IT?

Solution:

a)6 bits
b)
01001100011100100001001001101000 4C721268
00100001001001101000000001101000 21268068
01001000000000110000111100101100 48030F2C
00010001101001001001001101000100 11A49344
00100000110100011001110001101000 20D19C68
00111001000101101000100001001000 39168848
01101000100001001101101100000000 6884DB00

Question 6 (10 points)

Consider the following circuit, depicted in Figure 1. What is its output for the following inputs?

a) $a_1a_2 = 10$, $b_1b_2 = 11$

b) $a_1a_2 = 01$, $b_1b_2 = 00$

Moreover, what does the circuit do?

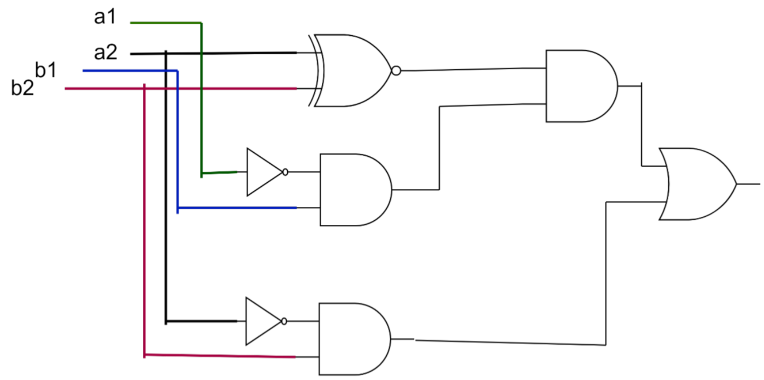


Figure 1

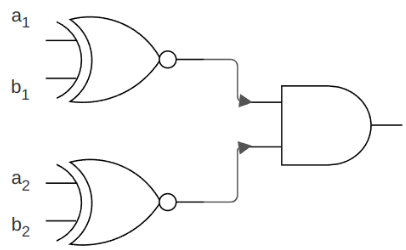
Solution:

a)1
b)0
to check ($a_1a_2 < b_1b_2$ or $a_2 < b_2$)

Question 7 (10 points)

Design a combinational logic circuit to compare two 2-bit numbers. The output should be 0 if the numbers are not equal, and 1 if they are equal.

Solution:



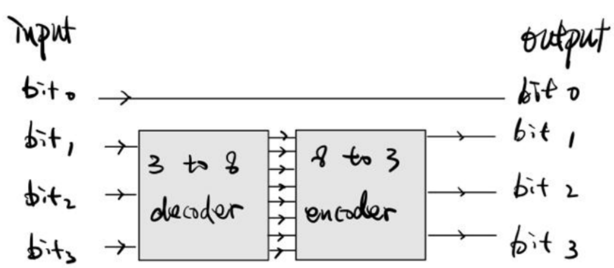
Question 8 (10 points)

Using a decoder and an encoder design a circular incrementor circuit for 4-bit positive binary numbers. The input (a number between 0 and 7) will be incremented by one if its is less than 7. If the input is 7, the output should be zero.

Solution:

ignore the first bit as 0 defined as 000 and 7 defined as 111, we connect the three bits to a 3 to 8 decoder, has output D_0 to D_7, and connect the 8 output to an 8 to 3 encoder

3	3 to 8 decoder	8 to 3 encoder
000	00000001	001
001	00000010	010
010	00000100	011
011	00001000	100
100	00010000	101
101	00100000	110
110	01000000	111
111	10000000	000



Question 9 (15 points)

We want to design a circuit to detect if an 8-bit positive binary number is divisible by 5 or not. Use a multiplexer and any necessary logic gates to design your circuit. Justify your solution.

Solution:

Firstly, we defined 5 states for the reminder of the input after division, $D_0(reminder = 0)D_1(reminder = 1)D_2(reminder = 2)D_3(reminder = 3)D_4(reminder = 4)$.
The state is encoded in 3 bits. (i.e D_0 is 000,D_1 is 001)
We take $D_0 - D_4$ as 5 input of the MUX, and take the current state as selectors $S_0S_1S_2$ also the current bit from the input as S_3
The default state will be D_0,which meanings the default reminder is 0, by multiply the reminder by 2 and add the left most digit, we can always find the next state.
we connect 8 of this MUX together, the first output will be the next MUX's selector, etc. The final output will be the reminder of the number.
Add the 3 to 1 encoder to the end, if input is 000 will set it to 1,else 0.

Question 10 (15 points)

Adding two numbers represented in scientific notation requires shifting floating points to make exponents equal before adding the fractions. Consider the following example:

$$1.34 \cdot 10^3 + 2.1 \cdot 10^{-1}$$

First, shift the decimal point of the smaller number to equalize the exponents:

$$1.34 \cdot 10^3 + 0.00021 \cdot 10^3$$

Then add the fractions, and normalize the result if necessary.

$$1.34021 \cdot 10^3$$

Follow the procedure described above to add the following numbers:

$$N = 14.25, \quad M = 0.5$$

a) Convert the numbers to IEEE floating point format.

b) Add the numbers.

c) Normalize the result to put it in IEEE format again.

Solution:

a)N=01000001011001000000000000000000
M=00111111000000000000000000000000

b)$N = 1.11001 \times 2^3$
$M = 1 \times 2^{-1} = 0.0001 \times 2^3$
$M + N = 1.11011 \times 2^3$
c)01000001011011000000000000000000