Introduction to Optimization, 2023-2024

Homework 3

1. Compute the Fenchel conjugates of the functions

   (a) $g : \mathbb{R}^N \to \mathbb{R}$, defined by $g(x) = \iota_Q(x)$, where $Q = [-\rho, \rho]^N$ with $\rho > 0$.

   (b) $h : \mathbb{R}^M \to \mathbb{R}$, given by $h(y) = \frac{1}{2}\|y\|^2 + b \cdot y$, where $b \in \mathbb{R}^M$.

2. Determine the dual (D) of the primal problem:

   $$(P) \qquad \min_{x \in \mathbb{R}^N} \left\{ g(A^T x) + h(x) \right\},$$

   where A is a real matrix of size $M \times N$. Observe (not prove, just observe!) that the proximal-gradient method can be applied to solve both (P) and (D).

3. Compare the performance of the proximal-gradient method, when applied to solve (P) and (D), in terms of the expected convergence rate and the ease of implementation.