Midterm-Rom-P

October 1, 2020

1 CP - Midterm - 2020

1.1 Instruction

- Modify this file to be Midterm-, e.g., Midterm-Chaklam-S.ipynb
- This exam accounts for 25% of the overall course assessment.
- This exam is open-booked; open-internet.
- You ARE NOT allowed to use sklearn or any libraries, unless stated.
- The completed exams shall be submitted at the Google Classroom
- All code should be complemented with comments, unless it's really obvious. I and Joe reserve the privilege to give you zero for any part of the question where the benefit of doubt is not justified

1.2 Examination Rules:

- For **offline** students, you may leave the room temporarily with the approval and supervision of the proctors. No extra time will be added to the exam in such cases.
- For **online** students, you are required to turn on your webcam during the entire period of the exam time
- Students will be allowed to leave at the earliest 45 minutes after the exam has started
- All work should belong to you. A student should NOT engage in the following activities
 which proctors reserve the right to interpret any of such act as academic dishonesty without
 questioning:
 - Chatting with any human beings physically or via online methods
 - Plagiarism of any sort, i.e., copying from internet sources or friends. Both copee and copier shall be given a minimum penalty of zero mark for that particular question or the whole exam.
- No make-up exams are allowed. Special considerations may be given upon a valid reason on unpredictable events such as accidents or serious sickness.

1.3 Question 1 (21 pts)

1). The rabbit: (5pts)

Once upon a time, there is a father rabbit lives in a far away jungle. Everyday, the father rabbit has to go out and find some carrots for his family. In his family there are mother rabbit, grampa rabbit, sister rabbit, and his son. In total there are 5 rabbits to feed. In one day, the adult rabbits

(himself, mother rabbit and sister rabbit) will eat 3 carrots while the elderly eat 2 carrots and baby rabbit eat 1 carrot.

Unfortunately, the carrots are not easy to find. The father rabbit has to travel into the scary jungle and find some carrot then bring them back to the family before the sunset at 6PM.

- Every 1 km, the rabbit will find 3 carrots.
- The rabbit will use 1 hour to travel 1 km.

In summary, in order to find the least number of carrot for each day, the rabbit will have to use (3 + 3 + 3 + 2 + 1)/3 = 4 hours. This mean that he has to leave the house at the latest 10AM (4 hours for go out and another 4 for comming back).

This daily work has to be done exactly on time, leaving to late will cause whether his life or his family life. Would you like to help the rabbit?

```
[]: # print 'yes' to help the rabbit or 'no' to refuse the challenge. (if yes →> 1⊔ →pt)

#Father Rabbit 3 carrots
#Mother Rabbit 3 carrots
#Grandpa Rabbit 2 carrots
#Sister Rabbit 3 carrots
#Son Rabbit 1 carrots

#Leaves house at the latest 10AM
#yes
```

Good to hear that young programmer!!

What I have in mind is to build a clock that when the rabbit puts the number of (adult, elderly, young) rabbit, it will calculate how many hours is required for a travel. Of cause we have to make it as a function because the number of each rabbit type will change over the time.

- Write a function carrot that takes three integers as an input in follow this format (adult, elderly, young) (1pt)
- The function will calculate number of hour required for travelling a day. (1pt)
- The function will also calculate the time to leave. Think of it as an alarm clock for leaving the house (1pt)
- The function will return a tuple (#hours, #time) (1pt)

```
return (hours,leave)
carrot(3,1,1)
```

[10]: (4.0, 10.0)

- 2). Print the shape: (16pts)
 - Write a function square that takes integer as an input. (1pt)
 - The function will return a string of * in the shape of a square with both width and height equal to the input interger. (2pts)

```
[14]: # Your code here
def square(n):
    for j in range(n):
        for i in range(n):
            print("*",end="")
            print()
```

- Write a function triangle that takes integer as an input. (1pt)
- The function will return a string of * in the shape of a triangle with level equal to the input integer. (2pts)

```
[18]: # Your code here

def triangle(n):
    for i in range(n):
        spaces = " "*(n-i-1)
        ast = "*"*(i+1)
        print(spaces,end="")
        print(ast)
```

* **

- Write a function pyramid that takes integer as an input. (1pt)
- The function will return the string of * in the shape of a pyramid with level equal to the input interger. (2pts)

```
[21]: # Your code here

def pyramid(n):
    for i in range(n):
        spaces = " "*(n-i-1)
        ast = "*"*(i*2+1)
        print(spaces,end="")
        print(ast,end="")
        print(spaces)
```

Now, let's combine the three algorithms into one single class. - Create a class named MyShape that can do the followings - Take two arguments during the class construction. The first one is an integer and the second one is a string. The names are level and shape (1pt) - Check the input arguments whether the interger is in the range of [1,10] and string is in the set of {'squ','tri','pyr'}. Raise a ValueError. (2pts) - Both attributes should be able to change via a set method only. set[attrName] (1pt) - Of cause, the set method should check the out of range too. (1pt) - To check the current setting, write a get method. get[attrName] (1pt) - Print the shape with method show. It should return the string of the current shape with the correct level (1pt)

```
[]:['''
     Example 1
     >>> ms = MyShape(2, 'tri')
     >>> ms.show()
      *
     **
     >>> ms.setLevel(3)
     >>> ms.setShape('squ')
     >>> ms.show()
     ***
     ***
     ***
     >>> ms.setShape('a')
     Traceback (most recent call last):
       File "<stdin>", line 1, in <module>
     ValueError: ......
     >>>
     111
```

```
[46]: # Your code here
      class MyShape():
          def __init__(self,level,shape):
              self.l = level
              self.s = shape
              self.checkLevel(self.1)
              self.checkShape(self.s)
          def checkLevel(self,level):
              if(level > 10 or level < 1):</pre>
                  raise ValueError("Invalid Level")
          def checkShape(self,shape):
              if(shape not in ["squ","tri","pyr"]):
                  raise ValueError("Invalid Shape")
          def setLevel(self,level):
              self.checkLevel(level)
              self.l = level
          def setShape(self,shape):
              self.checkShape(shape)
              self.s = shape
          def getLevel(self):
              return self.l
          def getShape(self):
              return self.s
          def pyramid(self,n):
              for i in range(n):
                  spaces = " "*(n-i-1)
                  ast = "*"*(i*2+1)
                  print(spaces,end="")
                  print(ast,end="")
                  print(spaces)
          def triangle(self,n):
              for i in range(n):
                  spaces = " "*(n-i-1)
                  ast = "*"*(i+1)
                  print(spaces,end="")
                  print(ast)
          def square(self,n):
              for j in range(n):
```

```
for i in range(n):
                print("*",end="")
            print()
    def show(self):
        if(self.s == 'squ'):
            self.square(self.1)
        elif(self.s == 'tri'):
            self.triangle(self.1)
        elif(self.s == 'pyr'):
            self.pyramid(self.1)
        else:
            print("HI")
ms = MyShape(3,'pyr')
print(ms.getShape())
print(ms.getLevel())
ms.show()
ms.setShape('squ')
ms.setLevel(10)
print(ms.getShape())
print(ms.getLevel())
ms.show()
```

1.4 Question 2 (10 pts)

2). ML Skill

$$y = ax + b$$

The above equation is your favorite linear equation where a,b are the constant value indicate the slope and the offset of the line in the graph.

We all know given and two points.

$$(x_1, y_1)(x_2, y_2)$$

you can find a,b very easy using Geometry

$$a = \frac{y_2 - y_1}{x_2 - x_1}$$
$$b = y_i - ax_i$$

Since we have learnt that using LinearRegression can find the value of the a,b too.

Now, do the followings.

- Write a function drawLine that takes two tuples as inputs.
- Calculate a,b using Geometry.
- Draw the first graph with scatter on the given two points and a line.
- Calculate a,b using LinearRegression with Batch Gradient Descent.
 - Generate 1000 sample data along the line.
 - Regress on the data using LinearRegression-BatchGradientDescent
- Draw the second graph with scatter on the given two points and a line.
- Does both method yeild the same outcome? Which method runs faster? (use timeit)
- What will happen if the data is normalize first? (draw another graph and timeit)
- What will happen if the data is standardize first? (draw another graph and timeit)

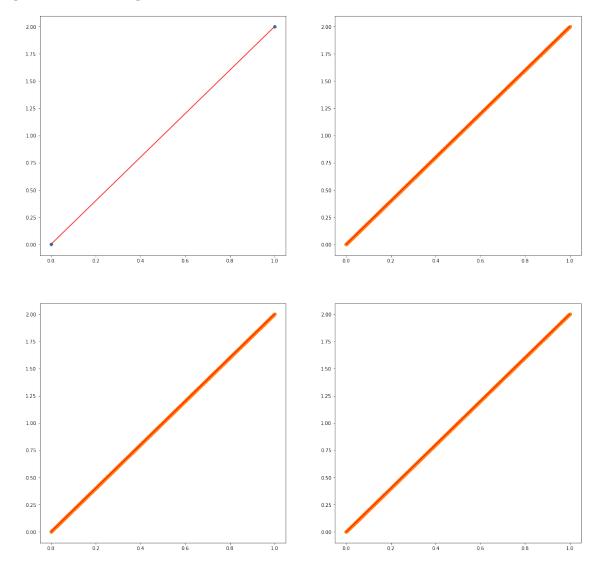
```
[160]: # Your code here
       import matplotlib.pyplot as plt
       import numpy as np
       class LinearRegressionModel:
       #1. hypothesis function
           def h(self, X, theta):
               hypothesis = X@theta
               return hypothesis
       #2. cost function
           def cost(self, X, y, theta, average = False):
               #expects X to be a design matrix, y to be a column vector and theta t_{0,1}
        ⇒be a column vector
               if(average == False):
                   J = 1/2*(self.h(X,theta)-y).T@(self.h(X,theta)-y)
                   J = 1/(2*X.shape[0])*(self.h(X,theta)-y).T@(self.h(X,theta)-y)
               return J
       #3. gradient function
           def gradient(self, X, y, theta, average = False):
               if(average == False):
```

```
dJ = X.T@(self.h(X,theta)-y)
        else:
            dJ = X.T@(self.h(X,theta)-y)/(X.shape[0])
        return dJ
#4. batch gradient descent
    def batch_gd(self, X, y, initial_theta, max_iteration, alpha, tolerance = __
\rightarrow 0, average = False):
        cost = []
        theta = initial_theta
        iteration = 0
        cost.append(self.cost(X,y,theta,average))
        for n in range(max_iteration):
            gradient = self.gradient(X,y,theta,average)
            theta = theta - alpha*gradient
            cost.append(self.cost(X,y,theta,average))
            iteration += 1
            if(self.mean_squared_error(X,y,theta) < tolerance):</pre>
                return theta, cost, iteration
        cost = np.array(cost)
        return theta, cost, iteration
#5. normal equation
    def normal_equation(self, X, y):
        theta = np.linalg.inv(X.T@X)@X.T@y
        return theta
#5. predict
    def predict(self,X,theta):
        prediction = self.h(X,theta)
        return prediction
#6. score/error calculation
    def mean_squared_error(self,X,y,theta):
        mse = self.cost(X,y,theta,average = True)*2
        return mse
#7. plotting cost
    def plot_cost(self,cost, iteration_no):
        iteration_series = np.arange(0,iteration_no+1)
        ax = plt.axes()
        ax.plot(iteration_series, cost)
def drawLine(one,two):
    x,y = zip(one,two)
    print("timeit for using the equations")
    timeit (y[1]-y[0])/(x[1]-x[0]), y[0]-a*x[0]
```

```
a = (y[1]-y[0])/(x[1]-x[0])
   b = y[0]-a*x[0]
   fig,ax = plt.subplots(2,2,figsize = (20,20))
   ax[0,0].scatter(x,y)
   ax[0,0].plot(x,y,'r')
   x_sample = np.linspace(np.min(x),np.max(x),1000)
   y_sample = x_sample*a+b
   LR = LinearRegressionModel()
   iterations = 1000
   alpha = 0.001
   initial_theta = np.zeros(2)
   x_sample_inserted = np.insert(x_sample[:,np.newaxis],0,1,axis=1)
   print("timeit for batch gradient descent")
   %timeit LR.
→batch_gd(x_sample_inserted,y_sample,initial_theta,iterations,alpha)
   theta, cost, iteration = LR.
→batch_gd(x_sample_inserted,y_sample,initial_theta,iterations,alpha,tolerance=1¢-7)
   y_pred = LR.predict(x_sample_inserted,theta)
   #LR.plot_cost(cost,iteration)
   print("iteration:",iteration)
   MSE = LR.mean_squared_error(x_sample_inserted,y_sample,theta)
   print("MSE =",MSE)
   ax[0,1].scatter(x,y)
   ax[0,1].plot(x,y,'r')
   ax[0,1].scatter(x_sample,y_pred)
   mini = np.min(x_sample)
   maxi = np.max(x_sample)
   x_norm = (x_sample-mini)/(maxi-mini)
   x_norm_inserted = np.insert(x_norm[:,np.newaxis],0,1,axis=1)
   iterations2 = 1000
   alpha2 = 0.001
   initial_theta2 = np.zeros(2)
   print("timeit for normalized data")
   %timeit LR.
→batch_gd(x_norm_inserted,y_sample,initial_theta2,iterations2,alpha2)
   theta2,cost2,iteration2 = LR.
→batch_gd(x_norm_inserted,y_sample,initial_theta2,iterations2,alpha2,tolerance=1e-7)
```

```
y_pred2 = LR.predict(x_norm_inserted,theta2)
    print("iteration2",iteration2)
    MSE2 = LR.mean_squared_error(x_norm_inserted,y_sample,theta2)
    print("MSE =",MSE2)
    ax[1,0].scatter(x,y)
    ax[1,0].plot(x,y,'r')
    ax[1,0].scatter(x_norm*(maxi-mini)+mini,y_pred2)
    mean = np.mean(x_sample)
    std = np.std(x sample)
    x_stan = (x_sample-mean)/std
    x_stan_inserted = np.insert(x_stan[:,np.newaxis],0,1,axis=1)
    iterations3= 1000
    alpha3 = 0.001
    initial_theta3 = np.zeros(2)
    print("timeit for standardized data")
    %timeit LR.
 →batch_gd(x_stan_inserted,y_sample,initial_theta3,iterations3,alpha3,average=True)
    theta3,cost3,iteration3 = LR.
 →batch_gd(x_stan_inserted,y_sample,initial_theta3,iterations3,alpha3,tolerance=1e-7)
    y_pred3 = LR.predict(x_stan_inserted,theta3)
    print("iteration3",iteration3)
    MSE3 = LR.mean_squared_error(x_stan_inserted,y_sample,theta3)
    print("MSE =",MSE3)
    ax[1,1].scatter(x,y)
    ax[1,1].plot(x,y,'r')
    ax[1,1].scatter(x_stan*std+mean,y_pred3)
    print("slope =",a,"intercept =",b)
drawLine((1,2),(-0,0))
timeit for using the equations
250 ns \pm 11.2 ns per loop (mean \pm std. dev. of 7 runs, 1000000 loops each)
timeit for batch gradient descent
30.2 ms \pm 1.63 ms per loop (mean \pm std. dev. of 7 runs, 10 loops each)
iteration: 107
MSE = 9.514767563793411e-08
timeit for normalized data
30.3 ms ± 1.94 ms per loop (mean ± std. dev. of 7 runs, 10 loops each)
```

```
iteration2 107 MSE = 9.514767563793411e-08 timeit for standardized data 31.8 ms \pm 1.63 ms per loop (mean \pm std. dev. of 7 runs, 10 loops each) iteration3 1 MSE = 6.848366140997965e-32 slope = 2.0 intercept = 0.0
```



1.5 Question 3 (69 pts)

- 1). **Exploratory Data Analysis**: Load the data "howlongwelive.csv" to pandas and print the first 5 and last 5 rows of data (1 or 0pt)
 - Print the shape, feature names, and summary (describe) of the data (1 or 0pt)

- Check whether there is missing data. (1 or 0pt)
- Fix all missing data using means or mode (1 or 0pt)
- Since Hepatatis B has a lot of nans, and highly correlate with Diptheria, simply drop column Hepatatis. Also drop column Population since there are way too many nans (1 or 0pt)
- If there are any features which are string and you want to use them as features, we need to convert them to int or float. For now, convert Status to 0 or 1 (1 or 0pt)
- Rename column thinness_1-19_years to thinness_10-19_years (1 or 0pt)
- Perform a group country and plot their life expectancy. Which country has the low-est/highest life expectancy? (1 or 0pt)
- Plot average life expectancy of developed country vs. developing country. (1 or 0pt)
- Perform a t-test of life expectancy between developed and developing countries. Is the result significant? (1 or 0pt)
- Perform a pairplot to see which features are likely to have strong predictive power for life expectancy. Identify the most 3 important features. (1 or 0pt)
- Perform a histogram of life expectancy. Is it normal? (1 or 0pt)
- 2). **Regression** Prepare your X and y into Numpy array (you have to map from Pandas to numpy). For X, prepare two versions of them. For first X_selected, you have to choose the most 3 important features from above, and for second X_all, simply use all features (you may want to omit Country since they are categorical). Set y to life expectancy. (1 or 0pt)
 - Perform standardization using Numpy way (NOT sklearn way). (1 or 0pt)
 - Perform train-test split by using Numpy way (NOT sklearn way). Use test size of 0.3. (1 or 0pt)
 - Perform assertion whether your splitting is correct accordingly (1 or 0pt)
 - Write a class Regression(X, y, grad_method, max_iter, alpha, tol, decay, decay_iter, decay_rate, stop_delay_counter, verbose, lam, poly, poly_deg) that can perform the followings:
 - Mini-batch, Stochastic, and Batch Gradient Descent (each 2pts)
 - Polynomial of degree k (2 or 0pt)
 - Decay learning rate (1 or 0pt)
 - * Decay learning rate is a learning rate that becomes smaller after certian iteration. For example, after 5 iterations, the learning rate will reduce to 95% of the current learning rate.
 - * To implement it, simply multiply current learning rate with some constant decay_rate. For now, set it to 0.9
 - Regularization with ridge (2 or 0pt)
 - Must have at least four methods for fit() (i.e., for finding weights) predict() (i.e., for predicting X_test data), score() (i.e., for returning r^2 score), and mse() (return mse) (each 1pt)
 - Accepts X, y, grad_method (default set to "batch"), alpha (learning rate), max_iter, tol, decay (whether to use decay learning rate; default set to False), decay_iter (after how

many iterations will the decay apply), stop_delay_counter (this is the maximum number of times that decay the learning rate), verbose (default is set to False, whether model will display the Cost for each iteration), lam (this is the ridge regularization parameter), poly (default is set to False), and poly_deg (default is set to 2) (each 1/13pt)

- Create the following 3 models **from your class** (For any unspecified parameters, feel free to use any :D)
 - 1. For the first model, transform your feature using polynomial degree 3, then perform linear regression with batch gradient descent with early stopping of tol 1e-3 (1 or 0pt)
 - 2. For the second model, perform linear regression with mini-batch gradient desent with early stopping of tol 1e-3 (1 or 0pt)
 - 3. For the third model, perform ridge regression with stochastic gradient desent with early stopping of tol 1e-3 and decay set to True and lam to 1e-4 (1 or 0pt)
- Create Lasso model from Sklearn with default parameters (1 or 0pt)
- For these four models, using two different versions of X, perform a cross validation of 10 folds, comparing the four models * two versions of X. Here you should implement cross validation. Report which one is the best candidate model (3pts for implement from scratch or 1pt for using sklearn)
 - Recall that in a 10 folds cross validation, you split your data into 10 even pieces. Then you run 10 iterations, where in each iteration, you pick 1 of this piece as the validation set, and the rest as training set. Once you reach the 10th iteration, you would have already exhaust all the 10 pieces as validation set.
- Using the best model, fit again with the training data. Plot the weights using bar charts along the feature names. Before you actually plot the weights, we need to multiply these weights by their feature standard deviation, so to reduce these weights to same unit of measure. Interpret these weights and what they imply. (For those who are curious why we need to multiply with std, you may read this > https://scikit-learn.org/stable/auto_examples/inspection/plot_linear_model_coefficient_interpretation.html#interpreti coefficients-scale-matters (2 or 0pt)
- Perform predictions on testing data. Print adjusted r^2 and mse. (1 or 0pt)
- Plot the predicted values against actual values (1 or 0pt)

3). Classification

- Change your y to discrete value. Here split y into three class, {0, 1, 2}, where 0 belongs to low life expectancy group, and 2 for the high life expectancy group. (1 or 0pt)
- Write a class for multinomial logistic regression with stochastic gradient descent. Must have at least six methods for fit() (i.e., for finding weights) predict() (i.e., for predicting X_test data), accuracy() (i.e., for returning accuracy score), recall(), precision(), and f1() (each 1pt)
- Using the best X_train of the two suggested by the cross validation step, fit the data with your class. (1 or 0pt)
- Perform predictions on testing data. Print accuracy, recall, precision, and f1_score from your class. (1 or 0pt)

• Plot the decision boundary with the X_test data. To plot this, you may want to choose only 2 features. (1 or 0pt)

4). Final verdict

- Attempt to do whatever ways including sklearn or scratch or change your features, or do feature engineering such that your mse is lowest possible. (0 to 5pts following class normal distributions)
- 1). **Exploratory Data Analysis**: Load the data "howlongwelive.csv" to pandas and print the first 5 and last 5 rows of data (1 or 0pt)
 - Print the shape, feature names, and summary (describe) of the data (1 or 0pt)
 - Check whether there is missing data. (1 or 0pt)
 - Fix all missing data using means or mode (1 or 0pt)
 - Since Hepatatis B has a lot of nans, and highly correlate with Diptheria, simply drop column Hepatatis. Also drop column Population since there are way too many nans (1 or 0pt)
 - If there are any features which are string and you want to use them as features, we need to convert them to int or float. For now, convert Status to 0 or 1 (1 or 0pt)
 - Rename column thinness_1-19_years to thinness_10-19_years (1 or 0pt)
 - Perform a group country and plot their life expectancy. Which country has the low-est/highest life expectancy? (1 or 0pt)
 - Plot average life expectancy of developed country vs. developing country. (1 or 0pt)
 - Perform a t-test of life expectancy between developed and developing countries. Is the result significant? (1 or 0pt)
 - Perform a pairplot to see which features are likely to have strong predictive power for life expectancy. Identify the most 3 important features. (1 or 0pt)
 - Perform a histogram of life expectancy. Is it normal? (1 or 0pt)

```
[193]: # Your code here
import pandas as pd

data = pd.read_csv("howlongwelive.csv")
```

[194]: data.head()

[194]:	Country	Year	Status	Life expectancy	Adult Mortality	\
0	Afghanistan	2015	Developing	65.0	263.0	
1	Afghanistan	2014	Developing	59.9	271.0	
2	Afghanistan	2013	Developing	59.9	268.0	
3	Afghanistan	2012	Developing	59.5	272.0	
4	Afghanistan	2011	Developing	59.2	275.0	

infant deaths Alcohol percentage expenditure Hepatitis B Measles ... \

```
2
                     66
                             0.01
                                                73.219243
                                                                   64.0
                                                                              430 ...
       3
                     69
                             0.01
                                                78.184215
                                                                   67.0
                                                                             2787
       4
                     71
                             0.01
                                                 7.097109
                                                                   68.0
                                                                             3013 ...
                 Total expenditure Diphtheria
                                                   HIV/AIDS
                                                                          Population \
          Polio
                                                                     GDP
            6.0
                               8.16
                                                                          33736494.0
       0
                                            65.0
                                                         0.1 584.259210
           58.0
                               8.18
                                            62.0
       1
                                                         0.1 612.696514
                                                                            327582.0
       2
           62.0
                               8.13
                                            64.0
                                                         0.1 631.744976
                                                                          31731688.0
           67.0
       3
                               8.52
                                            67.0
                                                        0.1 669.959000
                                                                           3696958.0
           68.0
                               7.87
                                            68.0
                                                        0.1
                                                               63.537231
                                                                           2978599.0
           thinness 1-19 years
                                   thinness 5-9 years \
       0
                           17.2
                                                 17.3
       1
                            17.5
                                                 17.5
       2
                            17.7
                                                 17.7
       3
                            17.9
                                                 18.0
       4
                            18.2
                                                 18.2
          Income composition of resources
                                            Schooling
       0
                                     0.479
                                                 10.1
       1
                                     0.476
                                                 10.0
       2
                                                  9.9
                                     0.470
       3
                                     0.463
                                                  9.8
       4
                                     0.454
                                                  9.5
       [5 rows x 22 columns]
[195]: data.tail()
                                  Status Life expectancy
[195]:
              Country Year
                                                             Adult Mortality \
       2933 Zimbabwe 2004 Developing
                                                      44.3
                                                                       723.0
                                                                       715.0
       2934 Zimbabwe 2003
                             Developing
                                                       44.5
       2935 Zimbabwe 2002
                             Developing
                                                                        73.0
                                                       44.8
       2936 Zimbabwe 2001
                             Developing
                                                       45.3
                                                                       686.0
       2937 Zimbabwe 2000 Developing
                                                       46.0
                                                                       665.0
                            Alcohol percentage expenditure Hepatitis B Measles
             infant deaths
       2933
                                4.36
                         27
                                                          0.0
                                                                      68.0
                                                                                   31
       2934
                                                          0.0
                                                                       7.0
                         26
                                4.06
                                                                                 998
       2935
                         25
                                4.43
                                                          0.0
                                                                      73.0
                                                                                 304
       2936
                         25
                                1.72
                                                          0.0
                                                                      76.0
                                                                                 529
       2937
                        24
                                1.68
                                                          0.0
                                                                      79.0
                                                                                1483
             ... Polio Total expenditure Diphtheria
                                                                                \
                                                         HIV/AIDS
                                                                           GDP
                                     7.13
       2933
                 67.0
                                                  65.0
                                                              33.6 454.366654
```

71.279624

73.523582

65.0

62.0

1154 ...

492 ...

0.01

0.01

62

64

0

1

```
2935
                 73.0
                                     6.53
                                                  71.0
                                                              39.8
                                                                     57.348340
       2936
                 76.0
                                     6.16
                                                  75.0
                                                              42.1 548.587312
                 78.0
                                     7.10
                                                              43.5 547.358879
       2937
                                                  78.0
             Population
                          thinness 1-19 years
                                                  thinness 5-9 years
       2933 12777511.0
                                            9.4
                                                                  9.4
                                            9.8
                                                                  9.9
       2934 12633897.0
                                            1.2
                                                                  1.3
       2935
               125525.0
       2936 12366165.0
                                            1.6
                                                                  1.7
       2937 12222251.0
                                           11.0
                                                                 11.2
             Income composition of resources Schooling
       2933
                                        0.407
                                                     9.2
       2934
                                        0.418
                                                     9.5
       2935
                                        0.427
                                                    10.0
       2936
                                        0.427
                                                     9.8
       2937
                                        0.434
                                                     9.8
       [5 rows x 22 columns]
[196]: print("shape:",data.shape)
       print("features:",data.columns)
       data.describe()
      shape: (2938, 22)
      features: Index(['Country', 'Year', 'Status', 'Life expectancy ', 'Adult
      Mortality',
             'infant deaths', 'Alcohol', 'percentage expenditure', 'Hepatitis B',
             'Measles ', ' BMI ', 'under-five deaths ', 'Polio', 'Total expenditure',
             'Diphtheria ', ' HIV/AIDS', 'GDP', 'Population',
             'thinness 1-19 years', 'thinness 5-9 years',
             'Income composition of resources', 'Schooling'],
            dtype='object')
[196]:
                     Year
                           Life expectancy
                                              Adult Mortality
                                                                infant deaths
                                                                  2938.000000
       count
              2938.000000
                                 2928.000000
                                                  2928.000000
              2007.518720
      mean
                                   69.224932
                                                   164.796448
                                                                    30.303948
       std
                 4.613841
                                    9.523867
                                                   124.292079
                                                                   117.926501
      min
              2000.000000
                                   36.300000
                                                     1.000000
                                                                     0.000000
       25%
              2004.000000
                                   63.100000
                                                    74.000000
                                                                     0.000000
       50%
              2008.000000
                                   72.100000
                                                   144.000000
                                                                     3.000000
       75%
              2012.000000
                                   75.700000
                                                   228.000000
                                                                    22.000000
              2015.000000
                                   89.000000
                                                   723.000000
                                                                  1800.000000
       max
                           percentage expenditure Hepatitis B
                                                                       Measles
                  Alcohol
                                                                                 \
       count
              2744,000000
                                       2938.000000 2385.000000
                                                                    2938.000000
```

6.52

68.0

36.7 453.351155

7.0

2934

mean std min 25% 50% 75%	4.602861 4.052413 0.010000 0.877500 3.755000 7.702500	738.251295 80.940461 2419.592240 1987.914858 25.070016 11467.272489 0.000000 1.000000 0.000000 4.685343 77.000000 0.000000 64.912906 92.000000 17.000000 441.534144 97.000000 360.250000					
max	17.870000	19479.911610 99.000000 212183.000000					
count mean std min 25%	BMI 2904.000000 38.321247 20.044034 1.000000 19.300000 43.500000	2938.000000 2919.000000 2712.00000 42.035739 82.550188 5.93819 160.445548 23.428046 2.49832 0.000000 3.000000 0.37000 0.000000 78.000000 4.26000 4.000000 93.000000 5.75500	\				
75% max	56.200000 87.300000	28.000000 97.000000 7.49250 2500.000000 99.000000 17.60000					
count mean std min 25% 50% 75% max	Diphtheria 2919.000000 82.324084 23.716912 2.000000 78.000000 93.000000 97.000000	HIV/AIDS GDP Population \ 2938.000000 2490.000000 2.286000e+03 1.742103 7483.158469 1.275338e+07 5.077785 14270.169342 6.101210e+07 0.100000 1.681350 3.400000e+01 0.100000 463.935626 1.957932e+05 0.100000 1766.947595 1.386542e+06 0.800000 5910.806335 7.420359e+06 50.600000 119172.741800 1.293859e+09					
	thinness 1-19 years thinness 5-9 years \						
count mean	2904.000000 2904.000000 4.839704 4.870317						
std		4.420195 4.508882					
min		0.100000 0.100000					
25%		1.600000 1.500000					
50%		3.300000 3.300000					
75%		7.200000 7.200000					
max		27.700000 28.600000					
	Income composition of resources Schooling						
count		2771.000000 2775.000000					
mean	0.627551 11.992793						
std min	0.210904 3.358920						
min 25%	0.000000 0.000000 0.493000 10.100000						
50%	0.493000 10.100000						
75%	0.779000 14.300000						
max		0.948000 20.700000					

```
[197]: print("amount of missing data for each column:")
       print(np.sum(data.isnull()))
      amount of missing data for each column:
      Country
      Year
                                             0
      Status
                                             0
      Life expectancy
                                            10
      Adult Mortality
                                            10
      infant deaths
                                             0
      Alcohol
                                           194
      percentage expenditure
                                             0
      Hepatitis B
                                           553
      Measles
                                             0
       BMI
                                            34
      under-five deaths
                                             0
      Polio
                                            19
      Total expenditure
                                           226
      Diphtheria
                                            19
       HIV/AIDS
                                             0
                                           448
      GDP
                                           652
      Population
                                            34
       thinness 1-19 years
       thinness 5-9 years
                                            34
      Income composition of resources
                                           167
      Schooling
                                           163
      dtype: int64
[198]: data.fillna(data.mean(),inplace=True)
       print("No more missing data:")
       print(np.sum(data.isnull()))
      No more missing data:
      Country
                                           0
      Year
                                           0
      Status
                                           0
      Life expectancy
                                           0
      Adult Mortality
                                           0
      infant deaths
                                           0
      Alcohol
                                           0
      percentage expenditure
                                           0
      Hepatitis B
                                           0
      Measles
                                           0
       BMI
                                           0
      under-five deaths
                                           0
      Polio
                                           0
      Total expenditure
                                           0
```

0

Diphtheria

```
GDP
                                           0
                                           0
      Population
       thinness 1-19 years
                                           0
       thinness 5-9 years
                                           0
      Income composition of resources
                                           0
      Schooling
                                           0
      dtype: int64
[205]: data.drop(columns = ["Hepatitis B", "Population"], inplace=True)
[209]: data["Status"]
[209]: 0
               Developing
               Developing
       1
       2
               Developing
       3
               Developing
               Developing
       2933
               Developing
       2934
               Developing
       2935
               Developing
       2936
               Developing
       2937
               Developing
       Name: Status, Length: 2938, dtype: object
[213]: labels, levels = pd.factorize(data["Status"])
       data["Status"] = labels
[215]: data
[215]:
                 Country
                           Year
                                 Status
                                          Life expectancy
                                                             Adult Mortality \
       0
             Afghanistan
                           2015
                                       0
                                                      65.0
                                                                        263.0
                                       0
                                                      59.9
                                                                        271.0
       1
             Afghanistan
                           2014
       2
             Afghanistan 2013
                                       0
                                                      59.9
                                                                        268.0
       3
             Afghanistan 2012
                                                      59.5
                                       0
                                                                        272.0
       4
             Afghanistan
                           2011
                                       0
                                                      59.2
                                                                        275.0
       2933
                Zimbabwe
                          2004
                                       0
                                                      44.3
                                                                       723.0
       2934
                Zimbabwe
                          2003
                                       0
                                                      44.5
                                                                        715.0
       2935
                Zimbabwe
                           2002
                                       0
                                                      44.8
                                                                         73.0
       2936
                Zimbabwe
                           2001
                                       0
                                                      45.3
                                                                        686.0
       2937
                Zimbabwe
                           2000
                                                      46.0
                                                                        665.0
                                                                            BMI
             infant deaths
                             Alcohol
                                       percentage expenditure
                                                                Measles
                                                                                  \
       0
                         62
                                0.01
                                                    71.279624
                                                                    1154
                                                                            19.1
       1
                         64
                                0.01
                                                    73.523582
                                                                     492
                                                                            18.6
```

0

HIV/AIDS

```
2
                         0.01
                                                               430
                  66
                                              73.219243
                                                                     18.1
3
                  69
                         0.01
                                              78.184215
                                                              2787
                                                                     17.6
4
                  71
                         0.01
                                               7.097109
                                                              3013
                                                                     17.2
2933
                  27
                         4.36
                                               0.000000
                                                                31
                                                                     27.1
2934
                  26
                         4.06
                                               0.000000
                                                               998
                                                                     26.7
2935
                         4.43
                                                                     26.3
                  25
                                               0.000000
                                                               304
2936
                  25
                         1.72
                                               0.000000
                                                               529
                                                                     25.9
2937
                         1.68
                                               0.000000
                  24
                                                              1483
                                                                     25.5
      under-five deaths
                            Polio Total expenditure Diphtheria
                                                                      HIV/AIDS \
0
                       83
                              6.0
                                                 8.16
                                                               65.0
                                                                            0.1
1
                       86
                            58.0
                                                 8.18
                                                               62.0
                                                                            0.1
2
                       89
                             62.0
                                                 8.13
                                                               64.0
                                                                            0.1
3
                       93
                             67.0
                                                 8.52
                                                               67.0
                                                                            0.1
                                                               68.0
4
                       97
                             68.0
                                                 7.87
                                                                            0.1
                                                                 •••
2933
                       42
                            67.0
                                                 7.13
                                                               65.0
                                                                           33.6
2934
                             7.0
                                                 6.52
                                                               68.0
                                                                           36.7
                       41
2935
                                                               71.0
                       40
                            73.0
                                                 6.53
                                                                           39.8
2936
                       39
                            76.0
                                                 6.16
                                                               75.0
                                                                           42.1
2937
                       39
                            78.0
                                                 7.10
                                                               78.0
                                                                           43.5
                    thinness 1-19 years
             GDP
                                             thinness 5-9 years \
0
      584.259210
                                     17.2
                                                            17.3
1
      612.696514
                                     17.5
                                                            17.5
                                                            17.7
2
      631.744976
                                     17.7
3
      669.959000
                                     17.9
                                                            18.0
4
                                     18.2
       63.537231
                                                            18.2
2933 454.366654
                                      9.4
                                                             9.4
                                      9.8
                                                             9.9
2934 453.351155
                                      1.2
                                                             1.3
2935
       57.348340
                                                             1.7
2936 548.587312
                                      1.6
2937
     547.358879
                                     11.0
                                                            11.2
      Income composition of resources Schooling
0
                                  0.479
                                               10.1
1
                                  0.476
                                               10.0
2
                                  0.470
                                                9.9
3
                                  0.463
                                                9.8
4
                                  0.454
                                                9.5
•••
2933
                                  0.407
                                                9.2
2934
                                  0.418
                                                9.5
2935
                                  0.427
                                               10.0
2936
                                  0.427
                                                9.8
```

2937 0.434 9.8

[237]: data.rename(columns={" thinness 1-19 years":

[2938 rows x 20 columns]

```
→"thinness_10-19_years"},inplace=True)
[239]: data
[239]:
                  Country
                                  Status
                                          Life expectancy
                                                              Adult Mortality \
                            Year
       0
              Afghanistan
                            2015
                                        0
                                                        65.0
                                                                         263.0
       1
             Afghanistan
                           2014
                                        0
                                                        59.9
                                                                         271.0
                                                        59.9
       2
             Afghanistan 2013
                                        0
                                                                         268.0
       3
             Afghanistan 2012
                                                        59.5
                                                                         272.0
                                        0
       4
             Afghanistan 2011
                                        0
                                                        59.2
                                                                         275.0
       2933
                 Zimbabwe
                           2004
                                                        44.3
                                        0
                                                                         723.0
       2934
                 Zimbabwe
                            2003
                                        0
                                                        44.5
                                                                         715.0
       2935
                            2002
                                                        44.8
                 Zimbabwe
                                        0
                                                                          73.0
                                                        45.3
       2936
                 Zimbabwe
                            2001
                                        0
                                                                         686.0
       2937
                 Zimbabwe
                           2000
                                        0
                                                        46.0
                                                                         665.0
              infant deaths
                              Alcohol
                                       percentage expenditure
                                                                 Measles
                                                                              BMI
                                                                                    \
       0
                                 0.01
                                                      71.279624
                                                                              19.1
                          62
                                                                      1154
       1
                          64
                                 0.01
                                                      73.523582
                                                                       492
                                                                              18.6
       2
                                 0.01
                          66
                                                      73.219243
                                                                       430
                                                                              18.1
                                 0.01
                                                                      2787
       3
                          69
                                                      78.184215
                                                                              17.6
       4
                          71
                                 0.01
                                                       7.097109
                                                                      3013
                                                                              17.2
                                                       0.000000
                                                                        31
                                                                              27.1
       2933
                         27
                                 4.36
       2934
                         26
                                 4.06
                                                       0.000000
                                                                       998
                                                                              26.7
       2935
                                 4.43
                                                                              26.3
                          25
                                                       0.000000
                                                                       304
                                                                              25.9
       2936
                          25
                                 1.72
                                                       0.000000
                                                                       529
       2937
                          24
                                 1.68
                                                       0.000000
                                                                      1483
                                                                              25.5
             under-five deaths
                                   Polio
                                           Total expenditure Diphtheria
                                                                               HIV/AIDS \
       0
                                     6.0
                                                         8.16
                               83
                                                                       65.0
                                                                                    0.1
       1
                               86
                                    58.0
                                                         8.18
                                                                       62.0
                                                                                    0.1
       2
                               89
                                    62.0
                                                         8.13
                                                                       64.0
                                                                                    0.1
       3
                               93
                                    67.0
                                                         8.52
                                                                       67.0
                                                                                    0.1
       4
                               97
                                    68.0
                                                         7.87
                                                                       68.0
                                                                                    0.1
                                                                         •••
                                                         7.13
       2933
                               42
                                    67.0
                                                                       65.0
                                                                                   33.6
       2934
                               41
                                     7.0
                                                         6.52
                                                                       68.0
                                                                                   36.7
       2935
                                                         6.53
                               40
                                    73.0
                                                                       71.0
                                                                                   39.8
       2936
                               39
                                    76.0
                                                         6.16
                                                                       75.0
                                                                                   42.1
       2937
                               39
                                    78.0
                                                         7.10
                                                                       78.0
                                                                                   43.5
```

```
2
             631.744976
                                          17.7
                                                               17.7
                                          17.9
       3
             669.959000
                                                               18.0
              63.537231
                                          18.2
                                                               18.2
       2933 454.366654
                                           9.4
                                                                9.4
       2934 453.351155
                                           9.8
                                                                9.9
       2935
              57.348340
                                           1.2
                                                                1.3
       2936 548.587312
                                           1.6
                                                                1.7
       2937 547.358879
                                          11.0
                                                               11.2
             Income composition of resources
                                               Schooling
       0
                                        0.479
                                                    10.1
       1
                                        0.476
                                                    10.0
       2
                                        0.470
                                                     9.9
       3
                                        0.463
                                                     9.8
       4
                                        0.454
                                                     9.5
       2933
                                                     9.2
                                        0.407
       2934
                                        0.418
                                                     9.5
       2935
                                        0.427
                                                    10.0
       2936
                                        0.427
                                                     9.8
       2937
                                        0.434
                                                     9.8
       [2938 rows x 20 columns]
[243]: data.columns
[243]: Index(['Country', 'Year', 'Status', 'Life expectancy ', 'Adult Mortality',
              'infant deaths', 'Alcohol', 'percentage expenditure', 'Measles ',
              'BMI', 'under-five deaths', 'Polio', 'Total expenditure',
              'Diphtheria ', ' HIV/AIDS', 'GDP', 'thinness_10-19_years',
              'thinness 5-9 years', 'Income composition of resources', 'Schooling'],
             dtype='object')
[255]: grouped = data.groupby('Country')
       fig = plt.figure(figsize=(30,10))
       ax = plt.axes()
       grouped.mean()['Life expectancy '].plot(ax = ax)
      /home/rom/Desktop/AIT/Programming/lib/python3.6/site-
      packages/pandas/plotting/_matplotlib/core.py:1235: UserWarning: FixedFormatter
      should only be used together with FixedLocator
```

GDP

ax.set_xticklabels(xticklabels)

584.259210

612.696514

0

1

thinness_10-19_years

17.2

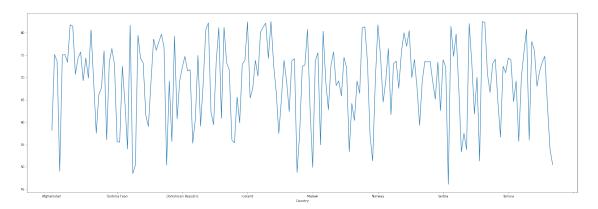
17.5

thinness 5-9 years \

17.3

17.5

[255]: <AxesSubplot:xlabel='Country'>



```
[270]: #print(grouped.mean()['Life expectancy '])
       series = grouped.mean()['Life expectancy ']
       series.sort_values()
```

[270]: Country

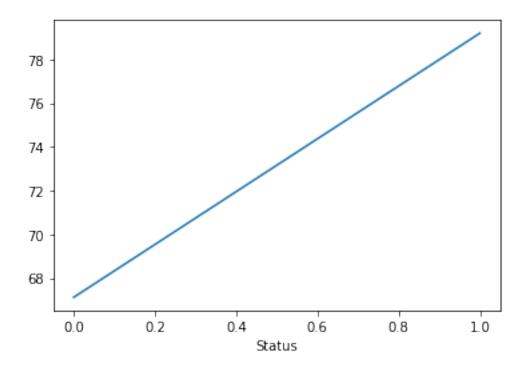
Sierra Leone 46.11250 48.51250 Central African Republic Lesotho 48.78125 49.01875 Angola Malawi 49.89375

France 82.21875 Switzerland 82.33125 Iceland 82.44375 Sweden 82.51875 Japan 82.53750

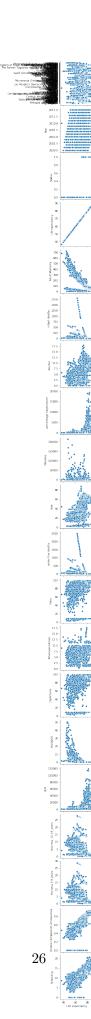
Name: Life expectancy , Length: 193, dtype: float64

[274]: grouped = data.groupby("Status") grouped.mean()['Life expectancy '].plot() print(levels)

Index(['Developing', 'Developed'], dtype='object')

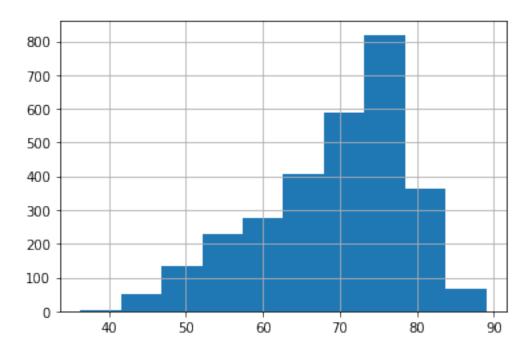


[294]: <seaborn.axisgrid.PairGrid at 0x7f119c563f28>



```
[292]: data['Life expectancy '].hist()
#its a skewed graph so it's not normal
```

[292]: <AxesSubplot:>



- 2). **Regression** Prepare your X and y into Numpy array (you have to map from Pandas to numpy). For X, prepare two versions of them. For first X_selected, you have to choose the most 3 important features from above, and for second X_all, simply use all features (you may want to omit Country since they are categorical). Set y to life expectancy. (1 or 0pt)
 - Perform standardization using Numpy way (NOT sklearn way). (1 or 0pt)
 - Perform train-test split by using Numpy way (NOT sklearn way). Use test size of 0.3. (1 or 0pt)
 - Perform assertion whether your splitting is correct accordingly (1 or 0pt)
 - Write a class Regression(X, y, grad_method, max_iter, alpha, tol, decay, decay_iter, decay_rate, stop_delay_counter, verbose, lam, poly, poly_deg) that can perform the followings:
 - Mini-batch, Stochastic, and Batch Gradient Descent (each 2pts)
 - Polynomial of degree k (2 or 0pt)
 - Decay learning rate (1 or 0pt)
 - * Decay learning rate is a learning rate that becomes smaller after certian iteration. For example, after 5 iterations, the learning rate will reduce to 95% of the current

learning rate.

- * To implement it, simply multiply current learning rate with some constant decay_rate. For now, set it to 0.9
- Regularization with ridge (2 or 0pt)
- Must have at least four methods for fit() (i.e., for finding weights) predict() (i.e., for predicting X_test data), score() (i.e., for returning r^2 score), and mse() (return mse) (each 1pt)
- Accepts X, y, grad_method (default set to "batch"), alpha (learning rate), max_iter, tol, decay (whether to use decay learning rate; default set to False), decay_iter (after how many iterations will the decay apply), stop_delay_counter (this is the maximum number of times that decay the learning rate), verbose (default is set to False, whether model will display the Cost for each iteration), lam (this is the ridge regularization parameter), poly (default is set to False), and poly_deg (default is set to 2) (each 1/13pt)
- Create the following 3 models **from your class** (For any unspecified parameters, feel free to use any :D)
 - 1. For the first model, transform your feature using polynomial degree 3, then perform linear regression with batch gradient descent with early stopping of tol 1e-3 (1 or 0pt)
 - 2. For the second model, perform linear regression with mini-batch gradient desent with early stopping of tol 1e-3 (1 or 0pt)
 - 3. For the third model, perform ridge regression with stochastic gradient desent with early stopping of tol 1e-3 and decay set to True and lam to 1e-4 (1 or 0pt)
- Create Lasso model from Sklearn with default parameters (1 or 0pt)
- For these four models, using two different versions of X, perform a cross validation of 10 folds, comparing the four models * two versions of X. Here you should implement cross validation. Report which one is the best candidate model (3pts for implement from scratch or 1pt for using sklearn)
 - Recall that in a 10 folds cross validation, you split your data into 10 even pieces. Then you run 10 iterations, where in each iteration, you pick 1 of this piece as the validation set, and the rest as training set. Once you reach the 10th iteration, you would have already exhaust all the 10 pieces as validation set.
- Using the best model, fit again with the training data. Plot the weights using bar charts along the feature names. Before you actually plot the weights, we need to multiply these weights by their feature standard deviation, so to reduce these weights to same unit of measure. Interpret these weights and what they imply. (For those who are curious why we need to multiply with std, you may read this > https://scikit-learn.org/stable/auto_examples/inspection/plot_linear_model_coefficient_interpretation.html#interpreticoefficients-scale-matters (2 or 0pt)
- Perform predictions on testing data. Print adjusted r^2 and mse. (1 or 0pt)
- Plot the predicted values against actual values (1 or 0pt)

[311]: col

```
[311]: ['Country',
        'Year',
        'Status',
        'Life expectancy ',
        'Adult Mortality',
        'infant deaths',
        'Alcohol',
        'percentage expenditure',
        'Measles ',
        ' BMI ',
        'under-five deaths ',
        'Polio',
        'Total expenditure',
        'Diphtheria',
        ' HIV/AIDS',
        'GDP',
        'thinness_10-19_years',
        ' thinness 5-9 years',
        'Income composition of resources',
        'Schooling']
[331]: X = data.loc[:,["Adult Mortality","Income composition of
       →resources", "Schooling"]].values.astype(float)
       X all = data.drop(columns=['Country', 'Life expectancy ']).values.astype(float)
       y = data['Life expectancy '].values.astype(float)
       print(X.shape)
       print(X_all.shape)
       mean = np.mean(X,axis=0)
       std = np.std(X,axis=0)
       X_norm = (X-mean)/std
       mean_all = np.mean(X_all,axis=0)
       std_all = np.std(X_all,axis=0)
       X_all_norm = (X_all-mean_all)/std_all
       ix = np.arange(X.shape[0])
       np.random.shuffle(ix)
       m = X.shape[0]
       percentage = 0.7
       ix_train = ix[:int(m*percentage)]
       ix_test = ix[int(m*percentage):]
       X_norm_train = X_norm[ix_train]
       X_all_norm_train = X_all_norm[ix_train]
       X_norm_test = X_norm[ix_test]
```

```
X_all_norm_test = X_all_norm[ix_test]
       y_train = y[ix_train]
       y_test = y[ix_test]
       print(X_norm_test.shape[0]/(X_norm_train.shape[0]+X_norm_test.shape[0]))
       assert X_norm_test.shape[0]/(X_norm_train.shape[0]+X_norm_test.shape[0]) < 0.31
        →and X_norm_test.shape[0]/(X_norm_train.shape[0]+X_norm_test.shape[0]) > 0.29
      (2938, 3)
      (2938, 18)
      0.30020422055820284
[332]: class Regression():
           def
        →__init__(self,X,y,grad_method,max_iter,alpha,tol,decay,decay_iter,decay_rate,stop_delay_cou
               self.X = X
               self.y = y
               self.grad_method = grad_method
               self.max_iter = max_iter
               self.alpha = alpha
               self.tol = tol
               self.decay = decay
               self.decay_iter = decay_iter
               self.decay_rate = decay_rate
               self.stop_delay_counter = stop_delay_counter
               self.verbose = verbose
               self.lam = lam
               self.poly = poly
               self.poly_deg = poly_deg
           #1. hypothesis function
           def h(self, theta):
               hypothesis = self.X@theta
               return hypothesis
       #2. cost function
           def cost(self, theta):
               J = 1/(2*X.shape[0])*(self.h(X,theta)-y).T@(self.h(X,theta)-y)
               return J
           def cost_reg(self, X, y, theta,lamb):
               J = 1/(2*X.shape[0])*(self.h(X,theta)-y).T@(self.h(X,theta)-y) +_{\Box}
        →lamb*np.sum(theta@theta)
               return J
```

```
#3. gradient function
   def gradient_reg(self, X, y, theta, average = False):
        dJ = X.T@(self.h(X,theta)-y)/(X.shape[0]) + theta*lamb
        return dJ
   def gradient(self, X, y, theta, average = False):
       dJ = X.T@(self.h(X,theta)-y)/(X.shape[0])
       return dJ
   def mini_batch():
       pass
   def stochastic():
       cost = []
       theta = initial_theta
        iteration = 0
        cost.append(self.cost(X,y,theta,average))
       for n in range(max_iteration):
            for i in X.shape[0]:
                if(self.grad_method == 'ridge'):
                    gradient = self.gradient_reg(X[i],y,theta,average)
                else:
                    gradient = self.gradient(X[i],y,theta,average)
                theta = theta - alpha*gradient
                cost.append(self.cost(X[i],y,theta,average))
                iteration += 1
        cost = np.array(cost)
       return theta, cost, iteration
   def batch():
       cost = []
        theta = initial_theta
        iteration = 0
        cost.append(self.cost(X,y,theta,average))
       for n in range(max_iteration):
            gradient = self.gradient(X,y,theta,average)
            theta = theta - alpha*gradient
            cost.append(self.cost(X,y,theta,average))
            iteration += 1
        cost = np.array(cost)
       return theta, cost, iteration
   def polynomial_features():
       pass
   def decay_learning_rate():
        pass
```

```
def ridge():
    pass

def fit():
    pass

def predict():
    pass

def score():
    pass

def mse():
    pass

#would've been better if the class wasn't fixed to the form Regression(X, y, u sprad_method, max_iter, alpha, tol, decay, decay_iter, decay_rate, u stop_delay_counter, verbose, lam, poly, poly_deg)
#because i have written a class but not in this way
```