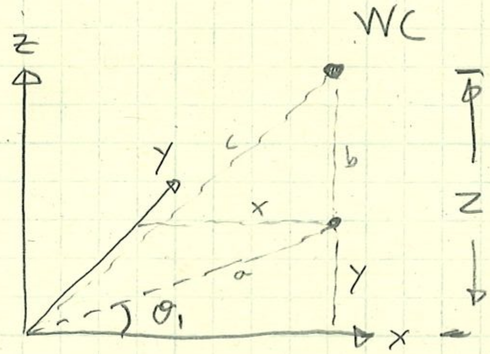
Theta 1

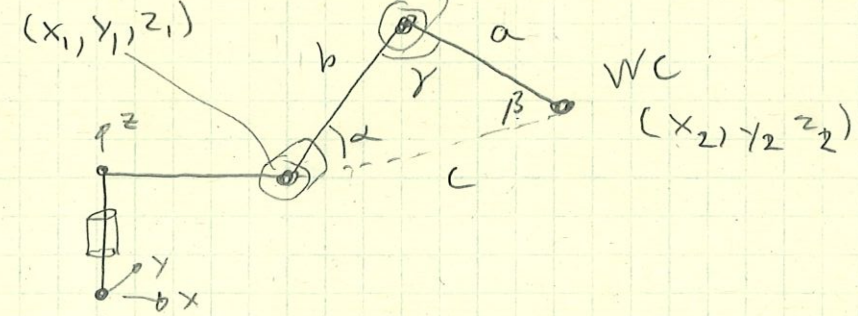


$$\tan(\theta_1) = Y/X$$

$$\theta_1 = \operatorname{atan}(Y/X)$$

$c^2 = a^2 + b^2 \quad a = \sqrt{c^2 - b^2}$

Theta 2, 3



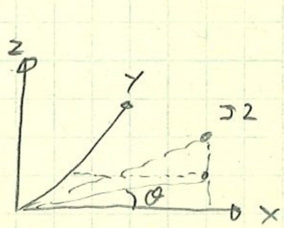
$$c^2 = a^2 + b^2 - 2ab\cos(\gamma)$$

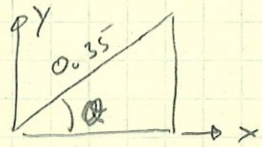
Length a + b are known
a = 1.501
b = 1.25

c is determined by calculating the distance between two points

$$c = \pm\sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$

where x_2 is the wrist center
and x_1 is the location of Jont 2





Machine dimensions and theta 1 determine the x_1, y_1, z_1 positions

$\sin\theta_1 = y/0.35$ $\qquad y_1 = 0.35\sin\theta_1$

$\cos\theta_1 = x_1/.35$ $\qquad x_1 = 0.35\cos\theta_1$

$z_1 = 0.75$

Note: Use positive solution as negative would not physically make sense.

The arm would have a drastically reduced reach.

Theta 2, 3

Rearrange Law of Cosines

$$c^2 = a^2 + b^2 - 2ab\cos(\gamma)$$

$$2ab\cos(\gamma) = a^2 + b^2 - c^2$$

$$\cos(\gamma) = \frac{a^2 + b^2 - c^2}{2ab}$$

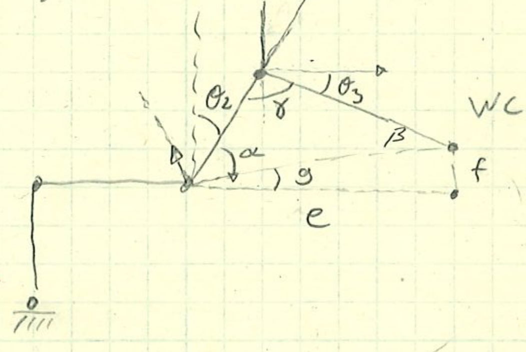
$$\gamma = a\cos\left(\frac{a^2 + b^2 - c^2}{2ab}\right) \quad (1)$$

$$a^2 = b^2 + c^2 - 2bc\cos(\alpha)$$

$$\alpha = a\cos\left(\frac{b^2 + c^2 - a^2}{2bc}\right) \quad (2)$$

$$b^2 = a^2 + c^2 - 2ac\cos(\beta)$$

$$\beta = a\cos\left(\frac{a^2 + c^2 - b^2}{2ac}\right) \quad (3)$$



Theta 2 $= \pi/2 - \alpha - g$

Theta 3 $= \pi/2 - \gamma$

Note: Kuka arm initial conditions

Theta 3 $= \pi/2 - (\gamma + 0.036)$

$f = |z_2 - z_1|$

$e = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$\tan(g) = \frac{f}{e}$

$g = a\tan(f/e)$

Theta 4, 5, 6

Derivation recreated following Mike Day

$$R_X(\theta_1) \cdot R_y(\theta_2) \cdot R_z(\theta_3) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_1 & s_1 \\ 0 & -s_1 & c_1 \end{bmatrix} \begin{bmatrix} c_2 & 0 & -s_2 \\ 0 & 1 & 0 \\ s_2 & 0 & c_2 \end{bmatrix} \begin{bmatrix} c_3 & s_3 & 0 \\ -s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Notation $s_1 = \sin\theta_1$

$c_1 = \cos\theta_1$

multiply out

$$R_X(\theta_1) R_y(\theta_2) R_z(\theta_3) = \begin{bmatrix} c_2 c_3 & c_2 s_3 & -s_2 \\ s_1 s_2 c_3 - c_1 s_3 & s_1 s_2 s_3 + c_1 c_3 & s_1 c_2 \\ c_1 s_2 c_3 + s_1 s_3 & c_1 s_2 s_3 - s_1 c_3 & c_1 c_2 \end{bmatrix}$$

$$\theta_1 = \operatorname{atan}\left(\frac{s_1 c_2}{c_1 c_2}\right) = \operatorname{atan}\left(\frac{s_1}{c_1}\right)$$

$$c_2 = \pm\sqrt{c_2^2 c_3^2 + c_2^2 s_3^2}$$

this step likely follows some annoying trig identity like $\sin^2 + \cos^2 = 1$

solution discored on next page

$$\theta_2 = \operatorname{atan}\left(\frac{-(-s_2)}{c_2}\right)$$

$$\theta_3 = \operatorname{atan}\left(\frac{c_2 s_3}{c_2 c_3}\right)$$

Alternate solution on next page

Note: arcsin not used in code due to multiple solutions (quadrant)

Theta 4, 5, 6

$$R_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_1 & -s_1 \\ 0 & s_1 & c_1 \end{bmatrix}$$

$s_1 = \sin(x)$

$s_2 = \sin(y)$

$s_3 = \sin(z)$

$$R_y = \begin{bmatrix} c_2 & 0 & s_2 \\ 0 & 1 & 0 \\ -s_2 & 0 & c_2 \end{bmatrix}$$

$$R_z = \begin{bmatrix} c_3 & -s_3 & 0 \\ s_3 & c_3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_x R_y R_z = \begin{bmatrix} c_2 c_3 & s_1 s_2 c_3 - c_1 s_3 & c_1 s_2 c_3 + s_1 s_3 \\ c_2 s_3 & c_1 c_3 + s_1 s_2 s_3 & c_1 s_2 s_3 - s_1 c_3 \\ -s_2 & s_1 c_2 & c_1 c_2 \end{bmatrix}$$

$$\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

$$g = -s_2 = -\sin(y) \Rightarrow -g = \sin(y) \Rightarrow y = \arcsin(g)$$

$$\tan(x) = \frac{h}{i} = \frac{s_1 c_2}{c_1 c_2} = \frac{s_1}{c_1} = \frac{\sin(x)}{\cos(x)} \Rightarrow x = \operatorname{atan}(h/i)$$

$$\tan(z) = \frac{d}{a} = \frac{\cancel{c_2} s_3}{\cancel{c_2} c_3} = \frac{s_3}{c_3} = \frac{\sin(z)}{\cos(z)} \Rightarrow z = \operatorname{atan}\left(\frac{d}{a}\right)$$

Alternate Solution

$$c_2 = \pm\sqrt{h^2 + i^2} = \pm\sqrt{s_1^2 c_2^2 + c_1^2 c_2^2}$$

$$\pm\sqrt{(s_1^2 + c_1^2)\, c_2^2}$$

$$= \pm\sqrt{(1) \cdot c^2}$$

Note: report discusses $\pm\sqrt{\ }$ impact

$$y = \operatorname{atan}\left(\frac{g}{\pm\sqrt{h^2 + i^2}}\right)$$