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Sum

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A sum is the result of an **addition**. For example, adding 1, 2, 3, and 4 gives the sum 10, written

$$1 + 2 + 3 + 4 = 10. \tag{1}$$

The numbers being summed are called **addends**, or sometimes summands. The summation operation can also be indicated using a capital sigma with upper and lower limits written above and below, and the index indicated below. For example, the above sum could be written

$$\sum_{k=1}^4 k = 10. \tag{2}$$

The sum of a list of numbers is implemented as `Total[list]`.

A sum

$$\sum_{i=1}^n a_i \tag{3}$$

in which each term a_i is given by some fixed rule (i.e., $\{a_i\}_{i=1}^n$ is a well-defined **sequence**) is called a (finite) **series**, and if the number of terms n is infinite, the sum is called an infinite series (or often just a "series"). A sum of the form

$$\sum_{k=1}^n r^k \tag{4}$$

is called a **geometric series**.

Conditions for convergence of a **series** can be determined in the **Wolfram Language** using `SumConvergence[a, n]`.

The general finite **power sum**

$$\sum_{k=1}^n k^p \tag{5}$$

can be given by the expression

$$\sum_{k=1}^n k^p = \frac{(B + n + 1)^{[p+1]} - B^{[p+1]}}{p + 1}, \tag{6}$$

which is equivalent to **Faulhaber's formula**, where the notation $B^{[k]}$ means the quantity in question is raised to the appropriate **power** k and all terms of the form B^m are replaced with the corresponding **Bernoulli numbers** B_m .

An amusing identity due to J. Ziegenbein (pers. comm., June 19, 2002) follows from the identity

$$n^2 - \frac{1}{2}(n-1)n = \frac{1}{2}n(n+1), \tag{7}$$

which can be written

$$n^2 - \sum_{k=1}^{n-1} k = \sum_{k=1}^n k. \tag{8}$$

Therefore, $\sum_{k=1}^{10} k = 55$, for example, can be written in the equivalent forms

$$\sum_{k=1}^{10} k = 10^2 - \left(\sum_{k=1}^9 k \right) \tag{9}$$

$$= 10^2 - \left(9^2 - \left(\sum_{k=1}^8 k \right) \right) \tag{10}$$

$$= 10^2 - \left(9^2 - \left(8^2 - \left(\sum_{k=1}^7 k \right) \right) \right) \tag{11}$$

$$= 10^2 - \left(9^2 - \left(8^2 - \left(7^2 - \left(\sum_{k=1}^6 k \right) \right) \right) \right) \tag{12}$$

and so on.

Nicomachus's theorem gives as curious expression for the **power sum** $\sum_{k=1}^n k^3$.

Special sums include

$$\sum_{j=1}^n \frac{x_j^r}{\prod_{\substack{k=1 \\ k \neq j}}^n (x_j - x_k)} = \begin{cases} 0 & \text{for } 0 \leq r < n-1 \\ 1 & \text{for } r = n-1 \\ \sum_{j=1}^n x_j & \text{for } r = n \end{cases} \tag{13}$$

and

sum

THINGS TO TRY:

- = sum
- = what is the sum of the integers 100
- = sum of the prime numbers 100000

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$$\sum_{k=1}^n \frac{\prod_{r=1}^n (x+k-r)}{r \prod_{\substack{r=1 \\ r \neq k}}^n (k-r)} = 1$$

(14)

To minimize the sum of a set of squares of numbers $\{x_i\}$ about a given number x_0

$$S \equiv \sum_i (x_i - x_0)^2$$

(15)

$$= \sum_i x_i^2 - 2 x_0 \sum_i x_i + N x_0^2,$$

(16)

take the derivative.

$$\frac{d}{d x_0} S = -2 \sum_i x_i + 2 N x_0 = 0.$$

(17)

Solving for x_0 gives

$$x_0 \equiv \bar{x} = \frac{1}{N} \sum_i x_i,$$

(18)

so S is minimized when x_0 is set to the mean.

SEE ALSO:
[Arithmetic Series](#), [Bernoulli Number](#), [Binomial Sums](#), [Clark's Triangle](#), [Convergence Improvement](#), [Cumulative Sum](#), [Dedekind Sum](#), [Double Series](#), [Einstein Summation](#), [Euler Sum](#), [Factorial Sums](#), [Faulhaber's Formula](#), [Gabriel's Staircase](#), [Gaussian Sum](#), [Geometric Series](#), [Gosper's Algorithm](#), [Hurwitz Zeta Function](#), [Infinite Product](#), [Kloosterman's Sum](#), [Lerch Transcendent](#), [Nicomachus's Theorem](#), [Odd Number Theorem](#), [Partial Sum](#), [Pascal's Triangle](#), [Power Sum](#), [Product](#), [Ramanujan's Sum](#), [Riemann Zeta Function](#), [Series](#), [Whitney Sum](#)

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