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Sum

A sum is the result of an addition. For example, adding 1, 2, 3, and 4 gives the sum 10, written

$$1+2+3+4=10. \qquad (1)$$

The numbers being summed are called addends, or sometimes summands. The summation operation can also be indicated using a capital sigma with upper and lower limits written above and below, and the index indicated below. For example, the above sum could be written

$$\sum_{k=1}^{4} k=10. \qquad (2)$$

The sum of a list of numbers is implemented as `Total`[*list*].

A sum

$$\sum_{i=1}^{n} a_i \qquad (3)$$

in which each term a_i is given by some fixed rule (i.e., $\{a_i\}_{i=1}^{n}$ is a well-defined sequence) is called a (finite) series, and if the number of terms n is infinite, the sum is called an infinite series (or often just a "series"). A sum of the form

$$\sum_{k=1}^{n} r^k \qquad (4)$$

is called a geometric series.

Conditions for convergence of a series can be determined in the Wolfram Language using `SumConvergence`[*a*, *n*].

The general finite power sum

$$\sum_{k=1}^{n} k^p \qquad (5)$$

can be given by the expression

$$\sum_{k=1}^{n} k^p=\frac{(B+n+1)^{[p+1]}-B^{[p+1]}}{p+1}, \qquad (6)$$

which is equivalent to Faulhaber's formula, where the notation $B^{[k]}$ means the quantity in question is raised to the appropriate power k and all terms of the form B^m are replaced with the corresponding Bernoulli numbers B_m.

An amusing identity due to J. Ziegenbein (pers. comm., June 19, 2002) follows from the identity

$$n^2-\tfrac{1}{2}(n-1)\,n=\tfrac{1}{2}\,n\,(n+1), \qquad (7)$$

which can be written

$$n^2-\sum_{k=1}^{n-1} k=\sum_{k=1}^{n} k. \qquad (8)$$

Therefore, $\sum_{k=1}^{10} k=55$, for example, can be written in the equivalent forms

$$\begin{aligned}
\sum_{k=1}^{10} k &= 10^2-\left(\sum_{k=1}^{9} k\right) && (9)\\
&= 10^2-\left(9^2-\left(\sum_{k=1}^{8} k\right)\right) && (10)\\
&= 10^2-\left(9^2-\left(8^2-\left(\sum_{k=1}^{7} k\right)\right)\right) && (11)\\
&= 10^2-\left(9^2-\left(8^2-\left(7^2-\left(\sum_{k=1}^{6} k\right)\right)\right)\right) && (12)
\end{aligned}$$

and so on.

Nicomachus's theorem gives as curious expression for the power sum $\sum_{k=1}^{n} k^3$.

Special sums include

$$\sum_{j=1}^{n} \frac{x_j^r}{\prod_{\substack{k=1\\ k\neq j}}^{n} (x_j-x_k)}=\begin{cases} 0 & \text{for } 0\leq r<n-1\\ 1 & \text{for } r=n-1\\ \sum_{j=1}^{n} x_j & \text{for } r=n \end{cases} \qquad (13)$$

and

$$\sum_{k=1}^{n} \frac{\prod_{r=1, r\neq k}^{n} (x+k-r)}{\prod_{r=1, r\neq k}^{n} (k-r)} = 1 \qquad (14)$$

To minimize the sum of a set of squares of numbers $\{x_i\}$ about a given number x_0

$$S \equiv \sum_i (x_i - x_0)^2 \qquad (15)$$

$$= \sum_i x_i^2 - 2 x_0 \sum x_i + N x_0^2, \qquad (16)$$

take the derivative.

$$\frac{d}{d x_0} S = -2 \sum_i x_i + 2 N x_0 = 0. \qquad (17)$$

Solving for x_0 gives

$$x_0 \equiv \bar{x} = \frac{1}{N} \sum_i x_i, \qquad (18)$$

so S is minimized when x_0 is set to the mean.

SEE ALSO:
Arithmetic Series, Bernoulli Number, Binomial Sums, Clark's Triangle, Convergence Improvement, Cumulative Sum, Dedekind Sum, Double Series, Einstein Summation, Euler Sum, Factorial Sums, Faulhaber's Formula, Gabriel's Staircase, Gaussian Sum, Geometric Series, Gosper's Algorithm, Hurwitz Zeta Function, Infinite Product, Kloosterman's Sum, Lerch Transcendent, Nicomachus's Theorem, Odd Number Theorem, Partial Sum, Pascal's Triangle, Power Sum, Product, Ramanujan's Sum, Riemann Zeta Function, Series, Whitney Sum

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