Self-driving car Nanodegree (Term 2). Project 3: Markov Localization Theory and Particle Filters

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Abstract

This report is under development

1 Introduction

In these notes I summarize the notions of Markov Localization and Particle Filters, two important tools for localization of autonomous driving vehicles.

2 Markov Localization

Localization of an object is a very important topic in many technological applications especially in self driving cars. GPS signals can be used to localize an object on every point on the earth surface, however due to the wavelenght of the GPS radio waves, weather conditions, presence of obstacles etc..., the precision of the GPS is of the order of few meters. This make this technology not useful for a more precise localization result which, for autonomous vehicles needs to be of the order of centimeters. There are essentially two kind of problems that can be investigated:

- Localization on unknown territory, i.e. without a map.
- Localization within a map.

Here we will consider the localization within a given, known map M.

A landmark based map is a sparse list of objects or landmarks, their specifications and coordinates. We can assume that the map provides precise information about the landmarks, it is updated, and for the localization purpose, it is also time independent. Other kind of maps are grid based maps.

The localization problem consists in obtaining an estimation of the location of the vehicle on the map, i.e. estimating the distribution at time t:

$$p(x_t|z_{1...t}, u_{1...t}, M) \tag{1}$$

where x_t is the position at time t, the $z_{1...t}$ is a set of observation carried out by the vehicle at previous times (the measurement history), $u_{1...t}$ is a set of actions performed by the vehicle, e.g. turning or braking, and M is the map of landmarks. Using the Bayes law and separating the contribution at current and previous times, we get:

$$p(x_t|z_{1...t}, u_{1...t}, M) = \frac{p(z_t|x_t, z_{1...t-1}, u_{1...t}, M)p(x_t|z_{1...t-1}, u_{1...t}, M)}{p(z_t|z_{1...t-1}, u_{1...t}, M)} \quad (2)$$

The first term at the numerator, i.e. the likelihood term, is also called *observation model* and it accounts for the likelihood of a specific observation given the actual system state. The second term, or the prior, is called *motion model* and it accounts for estimating the state of the system given information about the past and current system move. At the denominator we have the normalization term which we do not treat at the moment.

2.1 The motion model

The motion model can be rewritten as follows:

$$p(x_t|z_{1...t-1}, u_{1...t}, M) = \int p(x_t|x_{t-1}z_{1...t-1}, u_{1...t}, M)p(x_{t-1}|z_{1...t-1}, u_{1...t}, M)dx_{t-1} \quad (3)$$

As a first approximation for simplification we assume that the transition from x_{t-1} to x_t depends only on the actual motion, i.e.

$$p(x_t|z_{1...t-1}, u_{1...t}, M) = \int p(x_t|x_{t-1}, u_t, M)p(x_{t-1}|z_{1...t-1}, u_{1...t}, M)dx_{t-1} \quad (4)$$

For this reason the first term in the integrand is known as *transition model* and depends on the actual motion of the vehicle and possible noise terms. The second term in the integrand is instead nothing but the belief of the vehicle position at previous times, i.e. Eq. 1, showing a recursive relation between the current belief and the previous belief.

2.2 The observation model

To simplify the observation model we assume that the likelihood of a measurement does not depend on the vehicle motion nor on the previous measurements, i.e.

$$p(z_t|x_t, z_{1...t-1}, u_{1...t}, M) = p(z_t|x_t, M) \quad (5)$$

Moreover since at a given time different measurement can be performed, i.e. $z_t \equiv (z_t^{(1)}, z_t^{(2)} ....z_t^{(K)})$, we assume that all these measurement are independent from each other, and therefore

$$p(z_t|x_t, z_{1...t-1}, u_{1...t}, M) = \prod_{k=1}^{K} p(z_t^{(k)}|x_t, M) \quad (6)$$

2.3 The Markov localization

We can summarize the results of last sections as follows:

$$bel(x_t) = \eta \Big[\prod_{k=1}^{K} p(z_t^{(k)}|x_t, M) \Big] \int p(x_t|x_{t-1}, u_t, M) bel(x_{t-1}) dx_{t-1} \tag{7}$$

where η is a normalization constant. We obtained a recursive structure such that given a suitable transition model $p(x_t|x_{t-1}, u_t, M)$ and observation models $p(z_t^{(k)}|x_t, M)$ leads us to obtain the probability distribution of the vehicle position at time t as a function of that at the previous time.

In practice the algorithm needs to be initialized with some relatively correct belief, using for instance some other kind of localization method (e.g. the GPS). Once initialized the recursive formula can be directly used for estimating the belief. Convergence of the algorithm depends mostly on a suitable transition model and choice of the time step dt between the observations. In fact if the motion model introduces excessive diffusion and dt is too large, the vehicle will not be well localized in the time during two consecutive measurements.

3 Particle Filters

Among Markov localization filters, particle filters occupy an important place and are widely used in the automotive sector for precise vehicle localization. In a particle filter for vehicle localization, the system state is described by the combined state of N non interacting particles which are initialized according to a noisy localization measurement, usually via the GPS. This measurement provides coordinates X and Y plus an orientation θ with errors σ_X, σ_Y and σ_θ. The N particles have coordinates (x_i, y_i, θ_i) drawn from normal probability distributions centered on (X, Y, θ) and standard deviations $(\sigma_X, \sigma_Y, \sigma_\theta)$. This distribution describes the prior belief on the system position and can be used to update the belief at later times. The algorithm consists in a prediction and un update step:

- In the prediction step all the particles are evolved according to the system dynamics, i.e. according to the vehicle dynamical or cinematical model. This step will diffuse the particles and introduce as well additional noise.
- In the update step, the system receives measurements information about landmarks on the map. Let us assume that each measurement returns the location of a landmark within a certain radius. Also all landmarks are equivalent. Since each particle has a different position and orientation to the original vehicle, this measurement will indicate a different location on map...